to the calorimeter were determined from experiments in which no gas was flowing through the pipe, the results obtained by him must be too low by a quantity which is uncertain to the extent of 5 per cent. of the value of the specific heat. In virtue of the form of connecting pipe used, the error probably amounts to something of the order of half this amount, which would bring the results into close agreement with the author's.

The work was carried out in Prof. Callendar's laboratory at the Imperial College of Science, and the method adopted was, in its main features, the same as that employed by Prof. Callendar for the determination of the specific heat of steam.

On the Depression of the Filament of Maximum Velocity in a Stream flowing through an Open Channel.

By A. H. Gibson, M.Sc. (Vict.).

(Communicated by Prof. J. E. Petavel, F.R.S. Received December 18, 1908,—Read January 14, 1909.)

When water flows with sinuous motion through a circular pipe the resistance introduced by the solid boundaries reduces the velocity of axial flow as the sides are approached, this velocity being greatest at the centre and least at the sides, as indicated by the transverse velocity curve of fig. 1. When flow takes place through a closed rectangular pipe, the same effect is

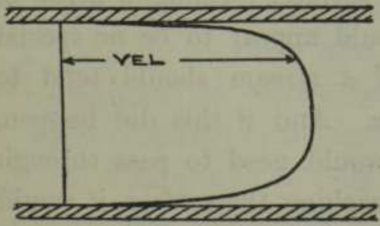

Fig. 1.

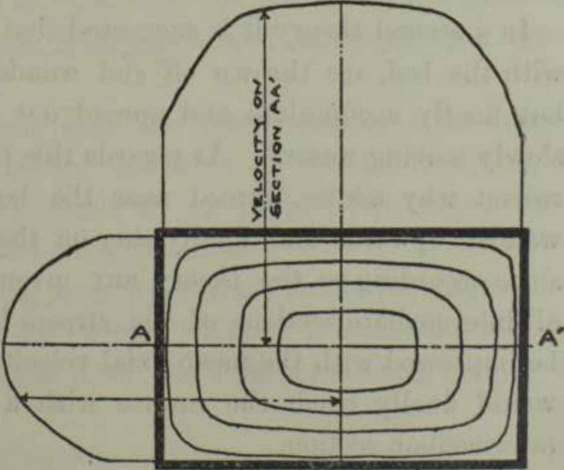

Fig. 2.

noticed, the transverse velocity curves and the curves joining points of equal velocity, or the contours of equal velocity in a cross-section, being much as shown in fig. 2. Here, again, the maximum velocity is found at the centre of the pipe.

From analogy with this latter case it might be expected that when flow takes place through an open rectangular flume such as would be obtained by taking the portion of the rectangular pipe of fig. 2 below the level AA′, or indeed through an open channel of any ordinary section, the filament of maximum velocity would be found in the water surface and in the centre of the stream.

In the majority of cases the latter assumption is fairly well justified by the results of experiment, although in some instances two points of maximum velocity have been noticed, one on each side of and at some distance from the centre. Further reference to this point will be made at a later stage of the paper.

Except, however, in the case of a broad, rapid, and shallow stream, it is found that this filament occurs at some depth below the surface. Its depth varies somewhat with the direction of the wind and with the physical characteristics of the stream, and, on a calm day, usually ranges from about one-tenth to four-tenths of the depth of the stream.

This phenomenon of the depression of the filament of maximum velocity has been much discussed, and three theories have been propounded for its explanation.

In the first of these the surface film is supposed to act in much the same way as a solid boundary in producing retardation of the surface layers. This theory is, however, discounted by the fact that, even with a downstream wind of considerably greater velocity than the stream, the filament remains below the surface.

In a second theory it is suggested that "eddies of water, stilled by contact with the bed, are thrown off and wander through all parts of the stream, but finally accumulate and spread out at the surface, forming a layer of slowly moving water." As regards this there would appear to be no special reason why eddies formed near the bottom of a stream should tend to wander upwards and finally stay on the surface. And if this did happen, since according to the theory any given eddy would need to pass through all intermediate sections of the stream before reaching the surface, it would be impressed with the mean axial velocity of each section in succession, and would finally reach the surface with a velocity greater than that of any intermediate section.

The third theory suggests that, as the water is less constrained at the surface than at any other point, irregular movements of all kinds are set up here and energy is therefore utilised in giving motions, not of translation, to the water.*

This suggestion is directly opposed to the fact brought out by Osborne Reynolds in his researches on the causes of instability of flow in water, viz., that unconstrained boundaries tend to stability, not to instability, of flow.†

The unsatisfactory nature of these theories led the author to an investigation of the question, and as a result of this the following explanation of the phenomenon is offered.

In any channel, however smooth its wetted perimeter may be, the velocity of forward flow is greatest near the centre and least near the sides and bottom, and if it were possible to obtain a state of affairs in which motion might take place in stream lines parallel to the axis of the stream, we should have, with steady flow along a straight reach of the channel, the velocity greatest in the surface and at the centre of the stream, and the water surface level from side to side.

In practice this is modified by the eddy formation which always takes place at the sides of the stream, and the phenomena in question would appear to be due almost entirely to the modification thus introduced.

A consideration of the process of eddy formation as it usually occurs at the sides of a stream shows that this involves the temporary existence of a region of less than normal pressure on the downstream side of the projection causing the eddy, and, as a result of this, where eddy formation is proceeding continuously in a uniform stream with a bed which is horizontal from side to side, it is to be expected that the depth of water will in consequence be slightly less at the sides than at the centre. The cross-sectional elevation of the surface thus becomes concave to the bed of the stream.

This curvature of the surface profile has been noticed by several observers, and was commented upon by Messrs. Humphreys and Abbot in their report on the gauging of the Mississippi.‡

The superelevation of the surface near the centre creates a tendency to a general outward flow from centre to sides of the channel, this, for permanence of *régime*, being accompanied by an inward flow consisting of water projected in the form of eddies from the sides.

Now supposing an eddy, extending from the surface to the bottom, to break away from the side of a stream. Its forward velocity is somewhat less than that of the current in which it finds itself, the difference being greater the

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* Flamant, 'Hydraulique.'

† 'Roy. Inst. Proc.,' 1884; also 'Scientific Papers,' vol. 2, p. 153.

‡ Humphreys and Abbot, 'Report on the Mississippi River.'

nearer the surface. From a consideration of its direction of rotation and of the external forces acting on its mass in virtue of its rotation in a stream moving more rapidly than its mass centre, it appears that these tend to drive it towards the centre of the stream.* This effect becomes greater as the relative velocity of the mass of water forming the eddy and of the passing current becomes greater, and will therefore increase from the bottom to the surface. It follows that the drift of the "eddy current" will be greatest near the surface and least near the bottom, and, as a nett result, that a system of transverse currents will be set up consisting of an inward surface drift from each side towards the centre; an outward drift over the lower portions of the stream; and, accompanying these, a downward current near the centre and an upward current near each side.

Since the inward surface drifts consist of water which has travelled up the sides and has come from the region of minimum velocity, they will evidently have the effect of reducing the surface velocity and of depressing the filament of maximum velocity.

The sketches in figs. 3 and 4 show respectively the directions of the transverse currents, and of the resultant motion of the stream, the full lines *a′a′*, *aa*, in fig. 4 representing the direction of the surface currents, and the dotted lines *b′b′*, *bb*, those of the bottom currents.

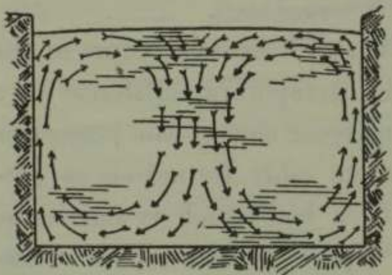

FIG. 3.

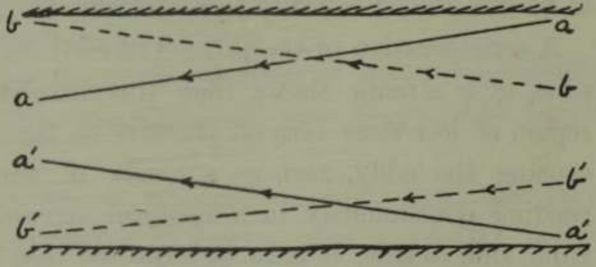

FIG. 4.

With a view of confirming these deductions, experiments were carried out on a small experimental channel in the hydraulic laboratories of the Manchester University; on a straight reach of the Mersey, a few hundred yards below the county bridge at Northenden; and on two straight reaches of the Derbyshire and Peak Forest Canal near Marple.

In the case of the model channel, which has wooden sides and bottom, the

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* Observation shows that whenever two eddies are moving down stream with different velocities of drift, and at approximately the same distance from the side, the slower tends to move centre-wards relative to the faster. Also that an eddy having greater velocity of drift than that of the current in which it finds itself always tends to move towards that side of the stream at which its direction of rotation would presume it to have been formed.

current formation was examined by means of aniline dye introduced to various parts of the stream, and also by means of threads fixed to pins in the bottom and sides.

While the latter method did not prove very satisfactory owing to the weight of the threads when wetted, it did show a very definite surface drift from side towards centre, floating threads attached to the sides making an angle of about 5° with these. The examination of the filaments of coloured water, however, showed the whole process very clearly and definitely proved the existence of the transverse currents.

The Mersey on the reach examined is practically straight, and has an almost uniform width of approximately 36 feet and a mean depth of about 6 feet. The flow throughout the reach appeared to be as nearly as possible uniform, with a mean surface velocity of about 3 feet per second. In this case the side-to-centre surface drift was very apparent. Of 10 float rods, 6 inches in length, thrown into the stream at about 2 feet from either bank, eight arrived within 2 feet of the centre of the stream before having traversed more than 100 yards of the reach. The observed behaviour of portions of water-logged leaves suspended in the stream offered substantial evidence of an upward current near the sides and of a downward current within a few feet of the centre on each side, but the dirty state of the water prevented the behaviour of such bodies being noted for a depth of more than about 6 inches.

The two reaches of the Peak Canal are each about 7 feet 6 inches wide and 5 feet deep, and were explored by means of weighted wax pellets. These gave distinct evidence of an upward current at the sides; a current commencing about 1 inch below the surface towards the centre; and a downward current commencing about 2·5 feet from the sides. Owing to the muddy character of the water it was impossible to follow the pellets to any considerable depth, and to note the depth of the return current, although their behaviour showed this to be present.

In addition to this direct experimental verification of the theory, much indirect evidence of its validity is available in that it explains several interesting phenomena which have been noted and are of importance in stream gauging, and for which the reason has not hitherto been clear.

Thus it is well known that the depth of the filament of maximum velocity—

(*a*) Depends on the physical condition of the channel, and increases as the roughness of the sides increases.

(*b*) Depends on the depth of the stream, and especially on the ratio of depth to width, its depth increasing as the latter ratio increases. In a wide shallow stream the filament is found in the surface.

(*c*) Is greater for any given vertical in a rectangular channel as that vertical approaches the side, and is at about mid depth near the side of such a channel.

(*d*) Depends on the velocity of flow, increasing as the latter diminishes.

Now it is evident that since an increase in the roughness of the sides will tend to increase the surface depression at the sides, this, by increasing the head available for producing a transverse current, will increase the magnitude and the effect of this current and will tend to depress the filament of maximum velocity. As the depth of the stream increases relatively to its width, the influence of the sides will increase, so that the effect will be the same as an increase in their roughness. On the other hand, the roughness of the bottom tends to retard the transverse current without having any compensating effect, so that an increase in the roughness of the bottom as opposed to that of the sides of the channel tends to raise the filament of maximum velocity. An increase in the width relatively to the depth of the stream will have the same effect, and with a very shallow wide stream the influence of the sides will be quite negligible.

Also since the effect of the current will diminish as its distance from the sides increases, this explains the greater relative depression of the filament near the sides.

In a given channel with a given depth of water, an increase in the mean velocity of flow might reasonably be expected, as is found to be the case, to diminish the relative importance of the transverse current and hence to elevate the filament of maximum velocity.

These points are well brought out in the published records of the gaugings of a very large number of rivers and channels by members of the U.S. Geological Survey.* From these the results of a number of gaugings of the experimental canal of the Cornell University have been chosen in illustration. These may be divided into two groups, denoted by A and B The experiments in series A were carried out with high velocities of flow and small depths of water, the mean velocity ranging from 2·06 to 3·16 feet per second, and the depth from 0·46 to 1·88 feet.

In series B the depths were greater and the velocities less, the depth ranging from 6·0 to 9·5 feet and the velocity from 0·23 to 2 feet per second.

The canal is of rectangular section, with concrete sides and bottom having a slope of 1 in 500, and has a width of 16 feet. Velocity measurements were made in eight verticals in a cross-section by means of carefully

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* U.S. Geological Survey, 'Water Supply and Irrigation Paper,' No. 95, p. 111.

calibrated current meters. The meter station in series A was 234 feet from the head of the canal and in series B was 280 feet from the same point.

The curves in figs. 5 and 6 show the variations of velocity in a vertical plane in typical of these experiments, each plotted point giving the mean of all eight observations at that depth in the cross-section. Here the curves of fig. 5 refer to series A and those of fig. 6 to series B.

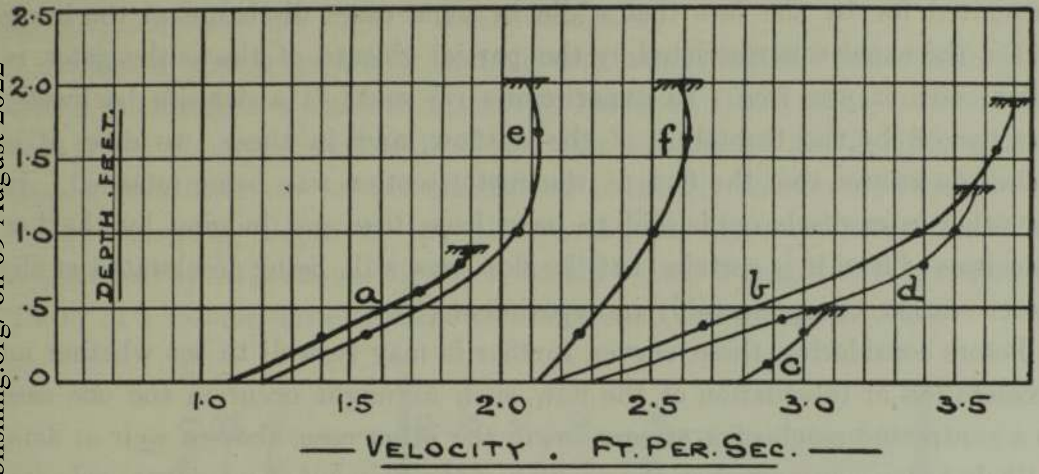

Fig. 5.

The effect of a large ratio of width to depth in raising the filament of maximum velocity is evident from a comparison of the curves of fig. 5 and of fig. 6, while the effect of an increased velocity of flow in raising the filament is evident from a comparison of the several curves of fig 6. The

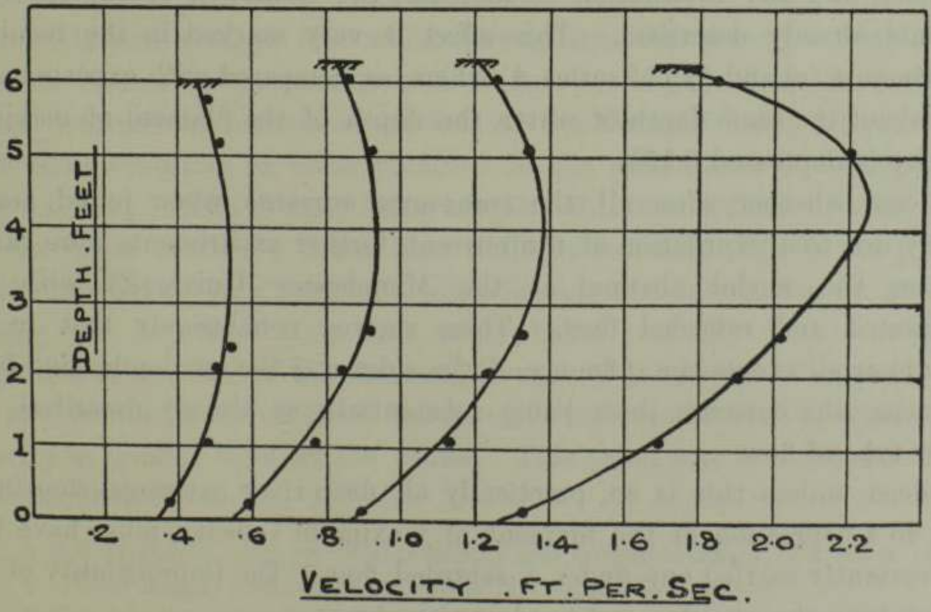

Fig. 6.

relationship between the velocity of flow and depth of filament, as obtained from the whole of the experiments of series B, is as follows:—

Mean velocity of flow, feet per second......	0·45	0·80	1·4	1·9
Depth of filament of maximum velocity...	0·44h	0·42h	0·34h	0·29h

where h is the depth of the stream.

The great difference between the several curves of fig. 5 is probably accounted for by the fact that while in some cases discharge at the lower end of the canal was restricted by the partial closure of the outlet gates, in other cases it was free. In experiments (e) and (f) a definite backwater was caused by the throttling of the outflow, and in these two cases it is definitely known that the flow at the meter section was being retarded. In the other cases discharge is said to have been free, and judging by the low velocities of flow it is certain that the flow was still being accelerated at the meter section, except possibly in experiment (C).

Before considering these curves further it may be well to see whether an acceleration or retardation of the flow, such as might occur in the one case in a contracted reach of a stream, or, in the other case, above a weir or dam, is likely to have any effect on the position of the filament of maximum velocity.

This is evident if it be considered that since any acceleration in the mean flow will be most strongly marked in the central portion of the stream, the surface level at any section which undergoes acceleration will fall to a greater extent at the centre than at the sides. The converse holds if the flow is being retarded. Thus any acceleration of the stream will tend to diminish, and any retardation to increase, the formation of the transverse currents already described. This effect is very marked in the results of experiments (e) and (f) of series A, where, as compared with experiment (b), with about the same depth of water, the depth of the filament of maximum velocity is depressed 0·15h.

To test whether, after all, the transverse currents, when found, are not mainly due to a retardation of the current, further experiments were carried out on the model channel at the Manchester University with both accelerated and retarded flow. These showed conclusively that in this channel at all events the influence of the sides was the predominating factor producing the currents, these being substantially as already described, with either type of flow.

Indeed, unless this is so, practically all deep-river gaugings, showing as they do a depression of the filament of maximum velocity, must have been inadvertently carried out under a retarded flow. The improbability of this having been the case does not need any emphasis.

A further verification of this conclusion is afforded by the gauging of the Farad flume of the Truckee River General Electric Company.* This flume has timber sides and bottom, and was gauged at two cross-sections 200 feet apart. The width was 10·09 feet throughout. The depth at the upper section was 5·98 feet, and at the lower section was 5·96 feet, while the filament of maximum velocity, this being the mean of the maxima in six verticals, had a depth of 0·50h at the upper and 0·42h at the lower section, in spite of the slightly accelerated flow.

It would appear probable that in the comparatively few instances in which the filament of maximum velocity has been found to be in the surface of a river of any considerable depth this is due to the gauging station having been fixed at a section undergoing a strongly accelerated flow.

The increasing depth of the filament as the sides of the channel are approached is well shown in fig. 7, which is an example of a gauging by Darcy of a rectangular channel 0·25 metre deep and 0·8 metre wide.

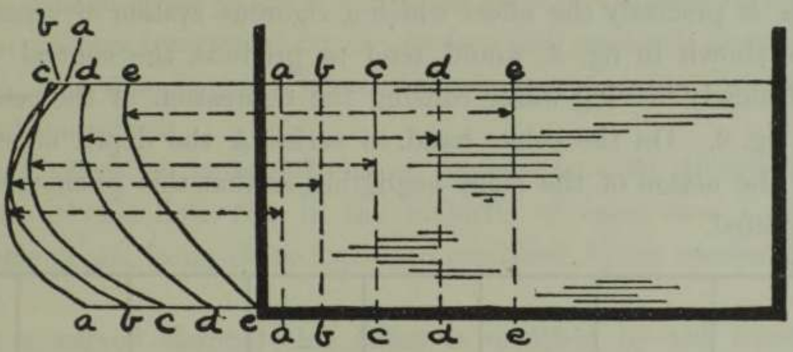

FIG. 7.

There is still another fact which strengthens the theory, and which will be made clearer by an examination of figs. 8 and 9. These show velocity curves obtained in horizontal sections of the Cornell Canal, the curves in fig. 8 being examples of those obtained in the accelerated flow experiments of series A, and those of fig. 9 being samples of those obtained in series B.†

From these it will be noted that while in series A (fig. 8) the point of maximum velocity in a horizontal plane is approximately in the centre of the stream, the departure from this point being probably due to a greater roughness of one side of the channel,‡ in each of the curves of series B there are two points in each section, one on each side of the centre, at which the velocity is greater than at the centre. This point was brought out in each experiment of the series.

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* U.S. Geological Survey, 'Water Supply and Irrigation Paper,' No. 95, p. 111.

† 'Water Supply and Irrigation Paper,' No. 95, pp. 73 and 74.

‡ This fact is mentioned in the report.

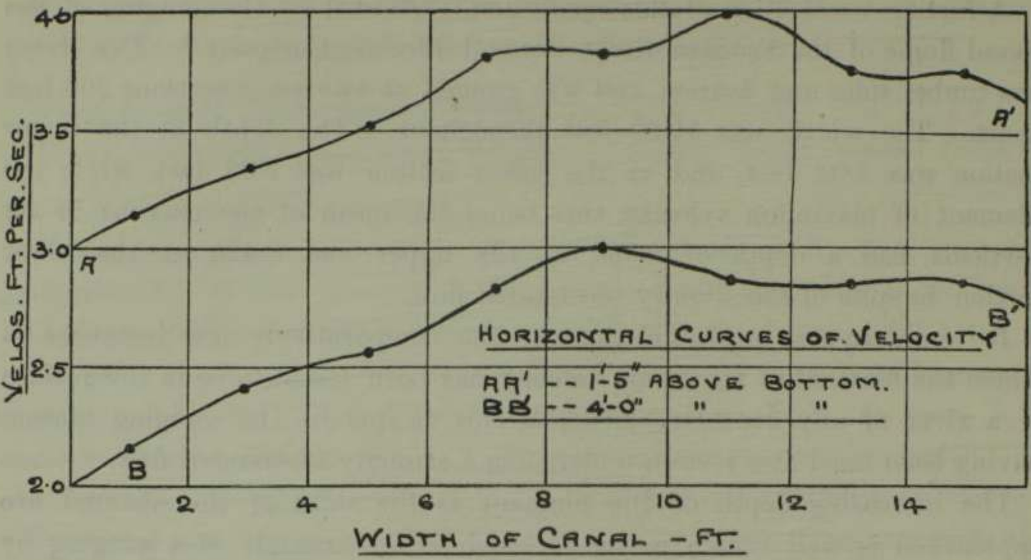

FIG. 8.

But this is precisely the effect which a vigorous system of cross currents, such as is shown in fig. 3, would tend to produce, the central downward current of slowly moving water causing the depression of the centre of the curves of fig. 9. On the other hand, in series A the depth of stream was small and the action of the sides negligible, so that this phenomenon would not be expected.

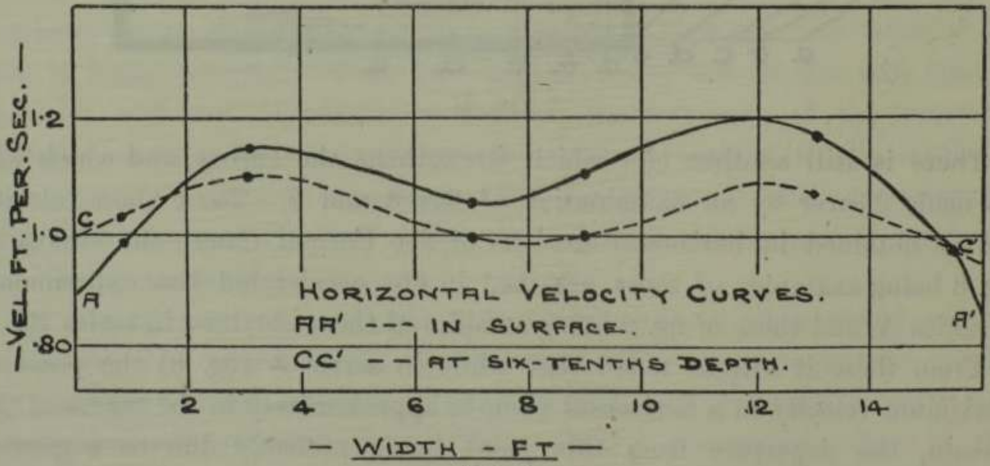

FIG. 9.

So far the channel has been assumed to be straight. If it is curved, the state of affairs is further complicated by a current which, as shown by James Thomson,[*] sweeps up the inner bank, and then, as a surface current, extends from the inner to the outer bank of the bend, thus spreading a layer of slowly moving water over the surface of the stream.

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[*] 'Roy. Soc. Proc.,' 1877, p. 356.

This will have the effect of depressing the filament of maximum velocity by an amount which will be greatest at the inner bank, and will probably only be slightly felt at the outer bank. In fact, it is probable that, owing to the sweeping of high-velocity water from the centre to the outer side of the stream, the filament of maximum velocity at all points between the centre and outer bank will be in the surface. This conclusion is confirmed by the published results of gaugings of the Oswego River, at Battle Island, New York,* these gaugings being taken in close proximity to a bend.

In the majority of rivers this action will sensibly modify the action of the sides and the distribution of the transverse currents.

Conclusions.

As a result of the investigation, the following points would appear to be well established :—

(1) The depression of the filament of maximum velocity in a straight reach of a river or canal is due to the action of the sides of the channel in producing transverse currents inwards along the surface and outwards along the bed of the stream, thus distributing a layer of slowly moving water over the central portions of the stream.

(2) This effect is increased by a retardation and diminished by an acceleration of the flow, but, in the majority of cases in a stream of any considerable depth, is never so greatly diminished by an acceleration as not to be felt.

(3) In a curved channel this effect is modified by the formation of a transverse current, due to the centrifugal action of the water and extending over the surface from the inner to the outer bank, this current tending, on the whole, to depress the filament of maximum velocity at all points between the inner bank and the centre of the stream, and to elevate it at points between the centre and the outer bank.

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* 'Water Supply and Irrigation Paper,' No. 95, p. 129.