Federal Reserve Bank of New York
Staff Reports

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Suresh Sundaresan
Zhenyu Wang

Staff Report no. 448
May 2010
Revised November 2011

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JEL classification: G12, G23

Abstract

Contingent capital (CC), a regulatory debt that must convert into common equity when a bank's equity value falls below a specified threshold (a trigger), does not in general lead to a unique equilibrium in the prices of the bank's equity and CC. Multiplicity or absence of equilibrium arises because economic agents are not allowed to choose a conversion policy in their best interests. The lack of unique equilibrium introduces the potential for price manipulation, market uncertainty, inefficient capital allocation, and unreliability of conversion. Because CC may not convert to equity in a timely and reliable manner, it is not a substitute for common equity as capital buffer. The problem exists even if banks can issue new equity to avoid conversion. The problem is more pronounced when bank asset value has jumps and when bankruptcy is costly. For a unique equilibrium to exist, allowing for jumps and bankruptcy costs, we prove that, at trigger price, mandatory conversion must not transfer value between equity holders and CC investors. Besides the challenge of practically designing such a CC, absence of value transfer prevents punishment of bank managers at conversion. This is problematic because punitive conversion is desirable to generate the desired incentives for bank managers to avoid excessive risk taking.

Key words: contingent capital, banking

Sundaresan: Columbia University (e-mail: mss122@columbia.edu). Wang: Federal Reserve Bank of New York (e-mail: zhenyu.wang@ny.frb.org). This paper was previously distributed under the title "Design of Contingent Capital with a Stock Price Trigger for Mandatory Conversion." The authors are grateful for comments from participants in the New York Fed Workshop on Contingent Capital, the BIS, the ECB, Toulouse Economics Group, Columbia Business School, Columbia Engineering School, Columbia Law School, the University of Wisconsin at Madison, the Federal Reserve Banks of Richmond and New York, Moody's Credit Risk Conference, and the FIRS annual conference. They especially thank Anat Admati, Pierre Collin-Dufresne, Douglas Diamond, Mark Flannery, Kenneth Garbade, Paul Glasserman, Larry Glosten, Charles Goodheart, Christopher Hennessy, Beverly Hirtle, Ravi Jagannathan, Weiping Li, Jamie McAndrews, Bob McDonald, Stephen Morris, Stewert Myers, George Pennacchi, Ned Prescott, Adriano Rampini, Marc Saidenberg, Joao Santos, Ernst Schaumburg, Chester Spatt, Kevin Stiroh, and James Vickery for helpful suggestions, as well as Julia Dennett and Kevin Pan for excellent research assistance.

One of the lessons learned from the financial crisis of 2007–2009 is that the capital structure and financial insolvency procedures of banks and other financial institutions need a major overhaul. The bailout of Bear Stearns, the bankruptcy of Lehman Brothers, and the financial distress experienced by Citigroup, Bank of America and AIG have demonstrated the need to revisit the financial insolvency procedures that should govern banks and other financial institutions. In particular, the extensive amount of implicit guarantees, outright infusion of taxpayer money, and other direct and indirect benefits extended to large financial institutions have come under much scrutiny, and new frameworks for capital market regulation have been proposed. The United States Congress passed "The Dodd-Frank Wall Street Reform and Consumer Protection Act."[1] The Basel Committee on Banking Supervision set forth to strengthen bank regulation with Basel III capital and liquidity standards.[2] A central issue in the debate on these new regulations is the design of a prudential capital structure that ensures enough loss-absorbing capital in large financial institutions and removes the need of public bailout.[3]

In the pursuit of a prudential capital structure, there has been considerable interest in debt securities that convert into equity in periods of distress when the bank's capitalization is low. Some academic researchers and regulatory agencies believe that such a security may mitigate the "too big to fail" problem as such a debt overcomes the reluctance of raising equity in a good state and restores the level of loss-absorbing equity in a bad state.[4] They

[1]The Dodd-Frank Wall Street Reform and Consumer Protection Act (Pub.L. 111-203, H.R. 4173) is a federal statute in the United States. President Barack Obama signed the Act into law on July 21, 2010.

[2]Bank for International Settlements, Press Release, January 13, 2011, "Final elements of the reforms to raise the quality of regulatory capital issued by the Basel Committee."

[3]For example, Kashyap, Rajan, and Stein (2008) have proposed that banks buy "systemic risk insurance" and secure the payouts on insurance. Admati and Pfleiderer (2010) have argued for increasing the liability of owners (equity holders) and suggest that such a structure will mitigate the conflicts of interests between equity and debt holders and may help reduce the need for bailouts. Admati, DeMarzo, Hellwig and Pfleiderer (2010) suggest a significant increase in equity capital.

[4]See *Risk, Reward and Responsibility: the Financial Sector and Society*, HM Treasury, United Kingdom, December 2009. Also see "*Too-big-to-fail and Embedded Contingent Capital*," Remarks by Julie Dickson of the Office of the Superintendent of Financial Institutions Canada (OSFI) on May 6, 2010.

trust that the mandatory conversion can increase capital buffer in a timely manner and internalize the losses within the claim holders of the firm. It has also been argued that the potential for a "punitively dilutive" conversion of contingent debt sets the right incentives for managers to avoid excessive risk taking, and encourages them to maintain higher capital ratios (Squam Lake Working Group, 2010). Many scholars, regulators, and practitioners view punitive CC as a tool to manage both the agency problem and the capital structure.

The trigger for the conversion from debt to equity is perhaps the most important and controversial parameter in contingent capital. Since mandatory conversion happens when the stock price of the bank is low, it is unlikely to be in the interest of CC investors because they would like to convert to equity when the bank is doing well. However, conversion can be at the option of bank management, as structured in the mandatorily convertible preferred (MCP) in the 2009 U.S. Treasury's Capital Assistance Program (CAP).[5] Bolton and Samama (2011) advocate conversion at the option of bank management. Most contingent capital issued in the private sector places the mandatory conversion trigger on accounting ratios. For example, the Lloyds's Enhanced Capital Note (ECN) issued on November 5, 2009 sets the trigger at five percent of core tier 1 capital ratio, i.e., conversion must happen if core tier 1 capital falls to or below five percent of the total risk-weighted assets. Another example is the Rabobank's Senior Contingent Note (SCN) issued on March 12, 2010 that sets the trigger at seven percent of equity ratio. The Squam Lake Working Group (2009) advocates placing the conversion trigger on accounting ratios. An alternative is to let a regulator to decide when to convert the CC to equity. The Credit Suisse's Buffer Capital Note (BCN) issued on February 14, 2011 converts to equity if equity ratio hits seven percent or at the discretion of Swiss banking regulator. The Office of the Superintendent of Financial Institutions (OSFI) in Canada prefers leaving conversion to the discretion of the regulators.

[5] For the details of the MCP and CAP, see Glasserman and Wang (2011).

So far there has not been any contingent capital issuance placing the mandatory conversion trigger on market value of equity, although a group of academics strongly advocate it.[6] They recommend market trigger because other triggers may not warrant timely conversion. Leaving conversion as an option of the bank managers may not protect tax payers because bank management may be reluctant to convert and hope for the best or a bailout. This will be especially true if conversion is punitive. Setting a trigger on backward-looking accounting ratios gives the management an opportunity for manipulation, as in the cases of repo 105 in Lehman Brothers and special purpose vehicles in Enron. Another piece of evidence against placing the trigger on accounting ratios is that the accounting ratios of many troubled banks in the recent financial crisis did not provide any warning signals prior to the onset of the crisis. Leaving conversion to regulatory discretion may be problematic as well: the potential political pressure and concern about false alarms may prevent regulators from timely action. In addition, potential intervention by regulators who make decisions based on asset value can cause multiplicity or absence of equilibrium in asset value, in view of Bond, Goldstein and Prescott (2010). In contrast, setting the trigger on market value may ensure that conversion is based on criteria that are thought to be objective, timely, difficult to manipulate, and independent of regulators' intervention.

An important part of the design of a market trigger is the choice of security on which we place the trigger. The price of the security should be a timely indicator of the expected financial difficulties of the bank that issues the contingent capital. In other words, the market value of the security should fall if the bank is expected to experience large losses. For this reason, the security should be a financial claim that is *junior* to the contingent capital. If the security is a financial claim that is senior to the contingent capital, the price will not drop to signal expected losses when the contingent capital effectively protects this claim. Then,

[6]See Flannery (2002, 2009), McDonald (2010), and Calomiris and Herring (2011).

there will be no signal for the contingent capital to convert. If the price of financial claims that is senior or equal to the CC drop sharply even with the presence of the CC, the market must view that conversion of contingent capital will no longer help. In this case, conversion is likely to be too late to save the bank. Therefore, the market trigger should not be placed on the senior debt yield (or its CDS spread) or on any other claims that are not junior to contingent capital. Since contingent capital becomes common equity after conversion, the only financial claim that is junior to contingent capital is common equity. This leaves common equity as the natural choice for placing the market trigger.

Given that a market trigger on common equity is most effective in signaling future large losses, it is important to ask whether the contingent capital with equity trigger can be a valuable tool to provide stability to banks and markets consistent with the expectations of policy makers and practitioners. For economic analysis, an additional advantage of market trigger is that market value of equity is an economic variable whose dynamic stochastic process can be derived from the statistical properties of a bank's asset value. In contrast, it is unclear how we may derive the dynamic stochastic process of accounting ratios or regulatory discretion. While our paper focuses on market-equity triggers, the analysis has implications when triggers are based on accounting ratios if accounting methods are regulated to reflect market value closely.[7] Our results also have implications to supervisory triggers if regulators use market equity value in deciding when to pull the trigger.

In this paper, we show that a contingent capital with market equity trigger does *not* in general lead to a unique equilibrium in the prices of the bank's equity and CC. Multiplicity or absence of equilibrium arises because economic agents are not allowed to choose a conversion policy in their best interests. This problem exists even if banks can issue new equity to avoid conversion. The problem is more pronounced when a bank's asset value has jumps and when

[7]Mark-to-market accounting rules establish a link between market prices and accounting numbers.

bankruptcy is costly. In the case of multiple equilibria, we note that incentives of CC-holders are aligned towards the equilibrium with early conversion, whereas the incentives of equity holders are aligned towards the equilibrium that delays or avoids conversion.

The lack of unique equilibrium means contingent capital does not fit into the basic economic theory to promise a stable market price and efficient capital allocation. With multiplicity and absence of equilibrium, contingent capital may introduce the potential for price manipulation, market uncertainty, inefficient capital allocation, and unreliability of conversion. The incentives of CC and equity holders towards the different equilibria may cause them to attempt market manipulation. Davis, Prescott and Korenok's (2011) controlled experiments indicate that excessive uncertainty and inefficient allocation will reign in a market with multiplicity or absence of equilibrium. In addition, the unreliability of conversion into equity implies that CC may not become loss-absorbing equity when it is needed. Therefore, CC is not a substitute for common equity as capital buffer.

Allowing for jumps and bankruptcy cost, we prove that for a unique equilibrium to exist, at trigger price, mandatory conversion must not transfer value between equity holders and CC investors. Besides the challenge of practically designing such a CC, it also prevents punishing equity holders at conversion. This is problematic because punitive conversion is important to generate the desired incentives for bank managers to avoid excessive risk taking.

The road map for the paper is as follows. In section I, we provide intuition on the possibility of multiplicity or absence of equilibrium, and its dependence on the conversion ratio. In section II, under the assumption that bank asset value exhibits smooth diffusive shocks as well as jump risk and costly bankruptcy, we derive the condition for an equilibrium to exist and be unique and numerically characterize the range of multiple equilibria. In Section III, we discuss several additional issues. We demonstrate that a practically implementable contingent capital contract that gives unique equilibrium is possible if there is no jump risk in the asset value and verification of the conversion condition is continuous. We further show that banks' ability to issue new shares of equity does not guarantee a unique equilibrium. We also show that multiple equilibria may exist even when financial distress costs are present. In Section IV we conclude.

I The Intuition for the Pricing Problem

The main thrust of our paper is the following: When triggers for mandatory conversion are placed directly on the market value of equity, there is a need to ensure that conversion does not transfer value between equity holders and CC holders when equity value hits exactly the trigger level. The economic intuition behind this design problem is as follows: A contingent capital is essentially a junior debt that converts to equity shares when the stock price reaches a certain low threshold. This sounds like a normal and innocuous feature. However, the unusual part of the CC design is that conversion into equity is *mandatory* as soon as the stock price hits the trigger level from above. Since common equity is the residual claim of the bank's value, it must be priced together with the CC. Keeping firm value and senior bond value fixed, a dollar more for the CC value must be associated with a dollar less for the equity value. Therefore, a transfer of value between equity and CC disturbs equilibrium by moving the stock price up or down depending on the conversion ratio. To have a unique equilibrium, the design of the conversion ratio must ensure that there is no such transfer of value.

If the transfer of value never pushes the stock price across the trigger, there is no problem because, given each asset value, investors always know whether or not there will be a conversion. However, if the transfer of value pushes the stock price across the trigger from above to below, there are two possible equilibria. In the first one, all investors believe conversion will not happen, leading the equity value to stay above the trigger. In the second one, all investors believe conversion will happen, leading the equity value to hit the trigger. Since two prices are possible whenever the firm's value drops to a certain level, by combining these dual equilibria around the trigger at different times in the future, numerous expected equity values are possible well before conversion happens. These numerous values can form a range, and the whole range can be above the trigger.

There are also economic conditions in which CC with a market trigger may not even have an equilibrium price. This happens if equity value would fall below the trigger without conversion but conversion pushes stock price above the trigger level by transferring value from CC holders to equity holders. In this case, investors cannot believe that conversion will not happen because with such a belief, equity value will fall below the trigger and the CC must convert. Investors cannot believe that conversion will happen either because with such beliefs, the equity price will stay above the trigger level and the CC must not convert. Therefore, there is no belief and stock price that are consistent with the mandatory conversion rule of the CC. Then, there is no rational expectations equilibrium in the values of equity and CC.

The only way to prevent multiplicity or absence of equilibrium is to ensure that no value is transferred when the equity value hits the trigger. If economic agents are permitted to convert in their self interest, they would select the optimal conversion strategy endogenously by comparing the value of conversion with the value of holding CC unconverted but taking into account optimal future conversion strategy. This, however, is prevented by the design of CC in which conversion is mandatory and dictated by the equity value. The zero value transfer condition requires that, at all possible conversion times, the value of shares converted at the trigger price must be exactly the same as the market value of the non-converted CC.

Although methods for pricing of subordinated debt and equity are established, the pricing of contingent capital with a market trigger poses special challenges. In this section, we illustrate these challenges in discrete time, leaving the formal analysis in dynamic continuous-time models to the next section. The analysis in discrete time demonstrate that the conversion trigger and ratio cannot be chosen arbitrarily for a unique equilibrium price to exist.

Let us first describe a bank that has a capital structure with CC. Consider a bank that has senior bondholders and common-equity holders who have claims on an asset (or a business). The asset requires an investment of A_0 dollars today (time 0). The asset is typically risky; its value at time t is a random number A_t. At time 0, the bank has also issued a security called "contingent capital." The security is in the form of a debt (or preferred equity) with face value $\bar{C}$, which is junior to the bond but converts to common equity when certain pre-specified conditions are met.[8]

The *contingent capital with market trigger* sets the conversion condition on the bank's equity value. Suppose S_t is the stock price of this bank and there are n shares outstanding. At any time t, the bank converts the junior debt under the contingent capital to m_t shares of common equity as soon as the share price S_t falls to level K_t. The quantity m_t is referred to as the *conversion ratio* and K_t as the *conversion trigger*. The conversion trigger is hit if the stock price reaches K_t/n, which is referred to as the *trigger price*. If $n = 1$, the conversion trigger and trigger price are the same. In general, the conversion ratio and trigger are either constant or pre-specified functions of observable variables. A particular contract specifies the conversion ratio $m(\cdot)$ and trigger $K(\cdot)$ as functions of observable variables over time.

The following are two examples of contingent capital contracts. The *simple form* of contingent capital can have a constant trigger K and a constant conversion ratio m. The contingent capital contracts proposed in the literature typically have a time-varying conversion trigger and ratio. The one suggested by Flannery (2002) specifies that $K_t = z \cdot \mathrm{RWA}_t/n$,

[8] To keep the analysis simple, we assume in this section that the contingent capital does not pay a coupon or dividend. We make a similar assumption for the bond. Also, we assume that the asset does not generate cash flow. We will relax these assumptions in the next section.

where RWA_t is the most recent risk-weighted asset value and z is a constant related to regulatory capital ratios. To ensure that the converted shares and the CC have the same value if conversion happens at the trigger price on maturity date, the conversion ratio should be set to $m_t = n\bar{C}/K_t$. Since the risk-weighted asset changes only at the end of each quarter, this contingent capital takes the simple form after its last change of risk-weighted asset, if it is not converted by then.

A The Pricing Restriction at Maturity

First consider the equilibrium stock price at the maturity, which is time T. The bank's asset will finish at certain value A_T, which is random. The par value of the bank's senior bond is $\bar{B}$, at maturity. The bank's contingent capital, which also matures at the same time, has a par value of $\bar{C}$. The trigger of the contingent capital is K. The bank has n shares of equity. We suppose that the contingent capital has not been converted because the asset value has been so high that the equity value is above the trigger. At maturity, if the CC is still not converted, the stock price should be

$$S_T^u = (A_T - \bar{B} - \bar{C})/n. \tag{1}$$

If the CC is converted, the stock price should be

$$S_T^c = (A_T - \bar{B})/(n + m). \tag{2}$$

The above pricing formula and the CC's trigger rule lead to the unconversion and conversion criteria in terms of asset value: (a) Since the CC stays unconverted at maturity if and only if $nS_T^u > K$, equation (1) implies that there is no conversion if and only if $A_T > \bar{B} + K + \bar{C}$; (b) Since the CC should be converted at maturity if and only if $nS_T^c \leq K$, equation (2) implies conversion happens if and only if $A_T \leq \bar{B} + K + (m/n)K$.

From the conversion criteria in terms of asset value, we can see the pricing restriction on the CC's conversion ratio. If $\bar{C} < (m/n)K$, the asset value A_T can fall into $(\bar{B}+K+\bar{C},\ \bar{B}+K+(m/n)K]$. Then, both criteria for unconversion and conversion are met. In this case, there are multiple equilibrium stock prices. One price is above the trigger price and the other is below. If $\bar{C} > (m/n)K$, the asset value A_T can fall into $(\bar{B}+K+(m/n)K,\ \bar{B}+K+\bar{C})$. Then, neither the criteria for unconversion or conversion is met. In this case, there is no equilibrium stock price. If and only if $\bar{C} = (m/n)K$, will we have either $A_T > \bar{B}+K+\bar{C}$ or $A_T \leq \bar{B}+K+(m/n)K$, but not both, for all asset value A_T. In this case, there is always a unique equilibrium. Therefore, a unique equilibrium always exists at maturity if and only if $\bar{C} = mK/n$ or $m = n\bar{C}/K$. The last equation implies that the conversion ratio is restricted by other parameters if we want to assure a unique equilibrium

As a numerical example, let $n = 1$, $\bar{B} = 90$, $\bar{C} = 10$ and $K = 5$. Notice that $n\bar{C}/K = 2$. If $m = 3$, which is higher than 2, then there can be multiple equilibria when the asset value turns out to be $A_T = 106$. One equilibrium stock price is $S_T^u = (106-90-10)/1 = 6$, which is above the trigger, and the other is $S_T^c = (106-90-10)/(1+3) = 4$, which is below the trigger. If $m = 1$, which is smaller than 2, then there is no equilibrium when the asset value turns out to be $A_T = 104$. This is because the stock price associated with unconversion is $S_T^u = (104-90-10)/1 = 4$, lower than the trigger, and stock price associated with conversion is $S_T^c = (104-90)/(1+1) = 7$, higher than the trigger. However, one can easily verify that these cases of multiple or no equilibrium do not occur if $m = 2$.

The above analysis of zero value transfer condition at maturity is simple because if the contingent capital does not convert at maturity, the value of contingent capital is simply its face value $\bar{C}$. The conversion ratio $m = n\bar{C}/K$ does not transfer value at trigger price on maturity date because if the stock price hits on the trigger, the contingent capital investors receive $mS_T = m(K/n)$ dollars, which is the same $\bar{C}$ dollars they would have received in the absence of conversion.

B The Pricing Restriction Before Maturity

The conversion ratio that guarantees zero value transfer at the trigger price on maturity date may still transfer value at the trigger price on some days before maturity, causing multiple equilibria. To ensure a unique equilibrium, the pricing restriction needs to hold at any possible conversion time t: $C_t = mK/n$. To see this intuitively, we can repeat the previous analysis by switching T to t. Then, with $\bar{C} = mK/n$, two possibilities may arise. The first possibility is $C_t = \bar{C} = mK/n$ at any time before conversion and before default.[9] The other possibility is $C_t \neq mK/n$ at some possible conversion time. This can lead to multiple or no equilibrium at this time, translating to multiplicity or absence of equilibrium in the initial price.

In a one-period discrete-time model, it is easy to show that keeping conversion ratio fixed as $m = n\bar{C}/K$ through the period allows multiple equilibrium in the initial equity and CC prices. In the discrete model, all securities are priced and traded at the initial time of the period, $t = 0$, and the terminal time, $t = T$, when both the bond and CC matures. The initial asset value is A_0, and the terminal value, A_T, is a random variable. Let $P(\cdot)$ and $p(\cdot)$ be the CDF and PDF, respectively, of the risk neutral distribution. To keep things simple, assume the risk-free rate is zero. With face value $\bar{B}$, the initial value of the bond is

$$B_0 = \bar{B}(1 - P(\bar{B})) + \int_0^{\bar{B}} A_T p(A_T) dA_T. \tag{3}$$

With face value $\bar{C}$, the initial value of the CC, if it is not converted at time 0, is

$$C_0^u = \bar{C}(1 - P(\bar{B} + \bar{C} + K)) + \frac{m}{n+m} \int_{\bar{B}}^{\bar{B}+\bar{C}+K} (A_T - \bar{B}) p(A_T) dA_T. \tag{4}$$

[9]For this to happen, the economic system needs to satisfy certain restrictive conditions later described by Theorem 3 in Section A.

Given $m = n\bar{C}/K$, the maximum payoff the CC holders may receive is $\bar{C}$. This implies $C_0^u \le \bar{C}$, which can also be verified by some trivial algebra.

The stock value at time 0 depends on whether the CC is converted. If the CC is not converted, the stock value is $S_0^u = (A_0 - B_0 - C_0)/n$. It follows from the condition for no-conversion, $nS_0^u > K$, that the CC is not converted if and only if $A_0 > B_0 + C_0 + K$. If the CC is converted, the stock value is $S_0^c = (A_0 - B_0)/(n + m)$. Because the condition for conversion is $nS_0^c \le K$, the CC is converted if and only if $A_0 \le B_0 + (m/n)K + K$. This inequality is equivalent to $A_0 \le B_0 + \bar{C} + K$, in view of $m = n\bar{C}/K$. Since $C_0 < \bar{C}$, the interval $(B_0 + C_0 + K,\ B_0 + \bar{C} + K]$ is nonempty. For every A_0 in this interval, both the conditions for no-conversion and conversion hold, and thus there are two equilibrium stock prices, S_0^u and S_0^c. The CC price associated with S_0^u is C_0^u. With conversion at time 0, the value received by the CC holders is the value of m shares: $C_0^c = mS_0^c$.

As a numerical example, let $\bar{B} = 90$, $\bar{C} = 10$, $K = 5$, $m = 2$, and $n = 1$. Notice that $m = n\bar{C}/K$ holds for these parameters. Assume that the probability distribution of A_T is discrete: $p\{A_T = 80\} = 0.25$, $p\{A_T = 100\} = 0.50$, and $p\{A_T = 120\} = 0.25$. It follows that $A_0 = E[A_T] = 100$. Straightforward calculation, using equations (3) and (4), gives $B_0 = 87.50$ and $C_0^u = 5.83$. Then, no conversion at time 0 is an equilibrium because $nS_0^u = 100 - 87.50 - 5.83 = 6.67$, which is higher than the trigger $K = 5$. Conversion is also an equilibrium because $nS_0^c = (100 - 87.50)/(1 + 2) = 4.17$, which is below the trigger. The CC values associated with the two equilibria are $C_0^u = 5.83$ and $C_0^c = 8.33$. To visualize the two equilibria intuitively, this example is displayed as a trinomial tree in Figure 1.

The above example illustrates a key problem: equity holders prefer the "no conversion" equilibrium as their value is higher in that equilibrium. On the other hand, contingent capital holders prefer the " conversion" equilibrium as their values are higher in that equilibrium. If a contingent capital of this design were to be issued by a bank, and the stock price subsequent to the issuance approaches the trigger level, equity holders would have an incentive to manipulate the stock price up and keep it above the trigger. By the same token, contingent capital holders would have the incentive to manipulate the stock price down so that it hits the trigger to force conversion. For this reason, CC holders have an incentive to sell the bank stock short. If they succeed in forcing the stock to hit the trigger, they can cover their short positions using the new shares that have been issued by the bank to fulfill the mandatory conversion. Consequently, a bank's equity price can be volatile when it approaches the trigger level.[10] Such manipulative behavior may arise due to the possibility of value transfers, which pit the equity holders against the holders of contingent capital. It may be argued that the market should be able to anticipate such value transfers ahead of time and incorporate them before the equity price approaches the trigger. This, however, is not possible when there are multiple equilibria, as there is no credible way to tell which equilibrium will result in the future.

The two equilibria leave the stock price and CC value undetermined before maturity. Multiple equilibria occurred on a node before maturity because the conversion ratio, $m = 2$, is too high at time 0 and transfers value from equity holders to contingent capital investors. To prevent such a value transfer, we need to set the conversion ratio so that, at time 0, the value of the converted shares at trigger price equals the value of the non-converted contingent capital. As shown in Figure 1, the non-converted contingent capital is valued as \$5.83 on this node. Accordingly, the conversion ratio should be set to $m = 5.83/K = 5.83/5 = 1.166$. To use this conversion ratio, we need to know the value of the non-converted contingent capital

[10] In fact, such problems have been already witnessed in the markets of securities such as barrier options, which have payoffs when a trigger is reached. During 1994–1995, "knock-in" barrier options on Venezuelan Brady bonds, which pay when the underlying bonds reach a high enough level (trigger) experienced manipulation. The fund owning the option attempted to push the price up by buying the bond, and the investment bank that sold the option attempted to keep the prices down. During the height of manipulation, about 20% of the outstanding bonds changed hands, and the prices went up by 10%. See, "Barrier grief: hedge funds," *Economist*, March 18, 1995.

on this node. To know the value of non-converted CC at every time and in every state is not practical for two reasons: first, the conversion ratio is typically specified ex-ante, and second, relying on a market price of CC for every future state to determine the conversion ratio is not judicious because the specification of a contract should not depend on the future market price of the contract itself.

The discussion in this section is meant to illustrate the challenge in the design of CC. To formally establish the pricing restriction, in the next section we use a dynamic continuous-time framework in which the bank asset value follows a general stochastic process, which allows for both continuous changes and discontinuous jumps. We also assume that bankruptcy is costly, deviating from Modigliani and Miller's world. In such a dynamic setting we show the pricing restriction under which we can obtain a unique equilibrium. When the restriction is breached, multiplicity or absence of equilibria may occur in the market.

II The Analysis in Dynamic Continuous-Time Model

A The Dynamic Continuous-Time Model

Valuation of contingent capital can be performed using the analytical approach developed in structural models of default pioneered by Merton (1974) and extended by Black and Cox (1976) who value default-risky senior and subordinated debt securities. These models work with the asset value of the issuing firm as the state variable and derive simultaneously the equity and debt values. The paper by Black and Cox, is particularly relevant as they explicitly model a safety covenant as a trigger for bondholders to take over the firm. The contingent claims approach has been standard for pricing corporate debt and hybrid securities in industry, as presented in detail by Garbade (2001).

We develop the ideas in the context of a structural model of default, along the lines of

Merton (1974) and Black and Cox (1976). We assume that the asset value process, denoted by A_t, is observable, but the trigger for the CC is specified in terms of the stock price S_t. Consistent with these models, we assume that pricing should exclude arbitrage profits and thus operate in a risk-neutral probability. Let the assets generate cash flows at the rate of a_t. Our analysis allows the bank asset value A_t to have time-varying drift μ_t and volatility σ_t. The analysis also allows the stochastic process of bank asset value to have jumps[11] because large downward changes in asset value are often associated with financial or economic crisis. Following Merton (1976), we assume

$$dA_t = \mu_t A_t dt + \sigma_t A_t dz_t + A_t(y_t - 1)dq_t, \tag{5}$$

where z_t is a Wiener process, q_t is a Poisson process with expected arrival rate λ_t, and y_t follows a log-normal distribution with parameters μ_y and σ_y. Let r_t be the instantaneous risk-less interest rate at time t. In risk-neutral probability measure, we should have $\mu_t = r_t - a_t - \lambda E[y_t - 1]$.

We assume that the bank has issued a senior bond with a par value $\bar{B}$ and maturity T. The coupon rate of the senior bond is b_t, which can be constant or time-varying. This allows both fixed- and floating-rate debt. Let δ be the time when the senior bond defaults. We model bankruptcy through a default barrier. Generally, the default barrier is set to limit the loss of the bond holder's investment. Upon default, bond holders take over the firm and receive its liquidation value. No value is left to securities that are subordinate to the bond. There are several ways to specify default condition, which determines the default time. In general, the time of bankruptcy is

$$\delta = \inf\{t \geq 0 : A_t \leq \Gamma_t\}. \tag{6}$$

[11]Duffie and Lando (2001) have developed a framework in which the true A_t process is continuous, but stock and bond prices exhibit discontinuities due to imperfect information. To keep our analysis simple, we work with an asset value process that exhibits jumps and is observed by the agent.

where Γ_t is the default barrier. We define $\delta = +\infty$ if A_t is above the barrier all the time. As an example, we can let the default barrier at time t be $\Gamma e^{-\gamma(T-t)}$, where Γ and γ are positive constants. This is the barrier in Black and Cox (1976). Alternately, we can model default as the choice of equity holders, who default when the value of their stake in the firm is zero. In this case, the default time is $\delta = \inf\{t \geq 0 : A_t \leq B_t\}$. The theorems derived in this section apply to both types of default conditions.

Bankruptcy is costly in practice. To bond holders, the loss after default consists of three parts: the loss of asset value relative to the par value, the liquidation discount, and legal expenses. We refer to the last two parts as bankruptcy cost. Altman (1984) examines a sample of 19 industrial firms which went bankrupt over the period of 1970-1978 and estimates the bankruptcy costs to be about 20% of the value of the firm measured just prior to bankruptcy. For banks and financial institutions, which may have much more interconnectedness, the true expected costs of bankruptcy may be significant. When a bank defaults at time δ, the loss of asset value relative to the par is $\bar{B} - A_\delta$. Let ω represent bankruptcy cost as a fraction of the asset value. The sum of the liquidation discount and legal expenses is ωA_δ. The value received by bond holders is $(1-\omega)A_\delta$.

The value function of the senior bond can be expressed in terms of the risk-free discount factor and an event indicator. Given that the instantaneous risk-free interest rate is r_t, the risk-free discount factor from time t to s is $P(t,s) = \exp\left(-\int_t^s r_u du\right)$. The event indicator 1_{event} equals either 1 or 0, depending on whether or not the event happens. The value of the bond before default ($t < \delta$) is, in rational expectation,

$$B_t = E_t\left[\bar{B}P(t,T) \cdot 1_{\delta>T} + (1-\omega)A_\delta P(t,\delta) \cdot 1_{\delta \leq T} + I_t^B\right], \tag{7}$$

where $E_t[\cdot]$ denotes the expectation, conditional on the information up to time t, and I_t^B is the discounted value of interest income:

$$I_t^B = \int_t^{\min\{\delta,T\}} b_t \bar{B} P(t,s) ds \,. \tag{8}$$

Besides senior bond, the bank capital structure consists of n shares of common equity and a contingent capital in the capital structure of the bank. The par value of contingent capital is $\bar{C}$, and it pays coupon at a rate c_t until the contingent capital converts to m_τ shares of common equity if conversion happens at time τ. After conversion, the number of outstanding shares of common equity is $n + m_\tau$. Both K_t and m_t are given functions of observable variables at time t, and they are assumed to be finite and positive.

Conversion to common equity is mandatory when the value of equity hits or runs below a trigger. The trigger condition is specified in terms of the value of common equity relative to the risk-weighted asset of the firm. The general form of conversion rules is that the contingent capital converts to m shares of common equity if the value of equity falls to or below X percent of risk-weighted asset (RWA_t). Let $K_t = \text{RWA}_t \times X/100$, which is referred to as *the conversion trigger*. Conversion happens when stock price hits or runs below K_t/n, which is referred to as *the trigger price*.

Theoretically, equity value can be compared to conversion trigger at any time, but practical CC contract must compare equity value and trigger at specified times such as daily market close. Let Λ be the set of time points when the equity value is compared to the trigger and assume $T \in \Lambda$. The first time when a stock price is found to be equal or lower than the trigger is

$$\tau = \min\left\{t \in \Lambda : nS_t \le K_t\right\} \,. \tag{9}$$

If $nS_t > K_t$ for all $t \in \Lambda$, we define $\tau = +\infty$. If condition in (9) is verified continuously, i.e., equity value is compared to the trigger at any time, then use $\Lambda_{\text{continuous}} = [0, +\infty)$ for Λ. For the contingent capital contracts that verify the conversion condition with daily or weekly closing stock prices, we use $\Lambda_{\text{daily}} = \{i/252 : i = 0, 1, 2, \cdots\}$, assuming there are 252 trading days in a year, or $\Lambda_{\text{weekly}} = \{i/52; i = 0, 1, 2, \cdots\}$, respectively. The theorem derived in this section applies to both continuous and discrete verification of the conversion condition.

After contractual coupons on the senior bond and contingent capital are paid, the cash flow generated from the assets of the bank will be paid to equity holders as dividends. Therefore, before conversion, the total dividend paid to equity holders during a short period dt is $(a_t A_t - b_t\bar{B} - c_t\bar{C})dt$. After conversion and before default of the senior bond, the total dividend paid to equity holders (including those new equity holders after conversion) during an infinitesimal period dt is $(a_t A_t - b_t\bar{B})dt$.

At any time t before the contingent capital converts ($t < \tau$), the per-share value of common equity is, in rational expectation,

$$\begin{aligned} S_t &= E_t\left[\frac{1}{n}\left\{(A_T - \bar{B} - \bar{C})P(t,T)\cdot 1_{\min\{\tau,\delta\}>T} + I_t\right\}\right] \\ &\quad + E_t\left[\frac{1}{n+m_\tau}\left\{(A_T - \bar{B})P(t,T)\cdot 1_{\tau\le T<\delta} + J_\tau P(t,\tau)\cdot 1_{\tau<\min\{\delta,T\}}\right\}\right], \end{aligned} \tag{10}$$

where I_t is the time-t value of total dividends before conversion, and J_τ is the time-τ value of the total dividends after conversion:

$$I_t = \int_t^{\min\{\tau,\delta,T\}} (a_s A_s - b_s\bar{B} - c_s\bar{C})P(t,s)ds \tag{11}$$

$$J_\tau = \int_\tau^{\min\{\delta,T\}} (a_s A_s - b_s\bar{B})P(\tau,s)ds. \tag{12}$$

The value of the contingent capital before conversion is

$$\begin{aligned} C_t &= E_t\left[\bar{C}P(t,T)\cdot 1_{\min\{\tau,\delta\}>T} + H_t\right] \\ &\quad + E_t\left[\frac{m_\tau}{n+m_\tau}\left\{(A_T - \bar{B})P(t,T)\cdot 1_{\tau\le T<\delta} + J_\tau P(t,\tau)\cdot 1_{\tau<\min\{\delta,T\}}\right\}\right], \end{aligned} \tag{13}$$

where H_t is the present value of coupon interests that the CC holders receive before conversion:

$$H_t = \int_t^{\min\{\tau,\delta,T\}} c_s \bar{C} P(t,s) ds \,. \tag{14}$$

After the contingent capital converts to m_τ shares and before the senior bond matures or defaults ($\tau \le t < \min\{\delta, T\}$), the per-share value of common equity becomes

$$S_t \;=\; \frac{1}{n+m_\tau} E_t \left[(A_T - \bar{B}) P(t,T) \cdot 1_{\delta > T} + J_t \right] . \tag{15}$$

B The Pricing Restriction

Since the value function B_t defined in equation (7) exists and is continuous in t and A_t, we focus on the value function of equity share S_t and the value of the contingent capital C_t before conversion. Given conversion trigger K_t and conversion ratio m_t, a pair of value functions, (S_t, C_t), that satisfy equations (9), (10), (13) and (15) is called a *dynamic rational expectations equilibrium* or, simply, an *equilibrium*. The equilibrium is *unique* if each of S_t and C_t has a unique value for every realization of A_t at any time t. In fact, such an equilibrium does not always exist for arbitrary specification of m_t. The next theorem presents the pricing restriction of a unique equilibrium.

Theorem 1 *For any given trigger K_t and conversion ratio m_t, a necessary condition for the existence of a unique equilibrium (S_t, C_t) is $nC_t = m_t K_t$ for every $t \in \Lambda$.*

This necessary condition is also sufficient in the following sense:

Theorem 2 *For any given trigger K_t, there exists a conversion ratio m_t and a unique equilibrium (S_t, C_t) satisfying $nC_t = m_t K_t$ for every $t \in \Lambda$.*

The pricing restriction of unique equilibrium has important implications to the design of contingent capital. These theorems say that if conversion is at the trigger price, there should be no transfer of value from CC holders to equity holders, or vice versa. To see this, we can rewrite the pricing restriction as $m_t(K_t/n) = C_t$ for every $t \in \Lambda$. If equity value hits right on the trigger at the conversion time t, we should have $S_t = K_t/n$. Then, the value of m_t shares of stock at conversion time is $m_t S_t$, which equals C_t. More importantly, the theorems imply that conversion can not be punitive to equity holders. At conversion time t, we have have $nS_t \leq K_t$. Then, $m_t S_t \leq m_t K_t/n = C_t$, which means that the value of the converted shares, $m_t S_t$, will never exceed the CC value, C_t. Therefore, in a contingent capital that entertains a unique equilibrium, conversion may punish the CC holders but never punish the equity holders.

The pricing restriction becomes $n\bar{C} = m_T K_T$ at maturity. If the conversion trigger and ratio are both constant and denoted by m and K respectively, the restriction becomes $m = n\bar{C}/K$, as seen in section A. When this restriction is violated, multiplicity or absence of equilibrium may occur at maturity, as shown by the examples in the previous section. We have also seen that the restriction at maturity does not guarantee that it will be met before maturity. The above two theorems require that the pricing restriction be satisfied at every possible conversion time. As long as the restriction can be violated at some conversion time, unique equilibrium is not assured.

It is important to point out that even without bankruptcy costs or jumps, multiplicity or absence of equilibrium may arise when the pricing restriction in Theorems 1 and 2 is violated. Thus, even in a Modigliani-Miller's world, violation of the pricing restriction can lead to multiple or no equilibrium. Since Theorems 1 and 2 still hold if the assets value follows a geometric Brownian motion, a contingent capital that violates the pricing restriction may cause stock prices to jump in a capital market even when the underlying asset prices have no jumps. Consequently, a contingent capital that violates the pricing restriction can disturb the continuity of the stochastic process of the stock price. From this point of view, contingent capital can potentially be a factor of instability, rather than an instrument to maintain stability.

If the pricing restriction is violated at many potential conversion points, the possible equilibrium prices may span a wide range on the initial day. We demonstrate this with a numerical example, in which the asset follows a simple geometric Brownian motion: $dA_t = rA_t + \sigma A_t dz_t$ and conversion condition is verified using daily closing prices: $\Lambda = \Lambda_{\text{daily}}$. In Table I, the column with the heading "GBM" presents the parameters used in this example and the equilibrium values.[12] We assume that the bank's initial asset level is 100. Its volatility is 4%. To keep things simple, we assume a flat term structure, anchored at 3%. The par value of the senior bond is 87 percent of the current asset, and its coupon rate is 4%. We chose the default barrier to be the par value of the bond plus the accrued coupon. There is a unique equilibrium value for the senior bond, which is 88.03, showing that the 3.34% coupon rate prices the bond over par. The par value of the CC is 5% of the bank's current asset value, and the CC converts to equity if equity value based on the daily closing price is less than or equal to 1 percent of the initial asset value. To avoid running into the case where no equilibrium exists, we set the coupon of the CC to zero. The maturities of both the senior bond and CC are five years. The range of prices generated by multiple equilibria seems substantial. Multiple equilibria produce equity values ranging from 5.86% to 6.46% of the initial asset value. They are associated with CC values ranging from 3.86% to 4.46% of initial asset value. The range of the multiple equilibrium prices is 0.6% of the initial asset value.

The range of multiple prices depends on the asset volatility, the senior bond, and the contract parameters of the CC. In Figure 2, we let one parameter vary to see how the range of multiple prices changes. Panel A shows that the price range is an increasing function of

[12] We calculate the prices with a binomial tree that approximates the diffusion process, following Cox, Ross and Rubinstein (1979).

the bank's asset volatility. Panel B shows how the range is related to the bank's leverage with the senior bond. In the first part the range is wider for a bank that has higher leverage, and in the later part the range decreases. These panels demonstrate that the bank-specific information such as asset volatility and leverage play important roles in determining the severity of multiple equilibria. In Panel C, the range widens as the par value of the CC increases. Therefore, the larger CC a bank issues, the wider its range of equity prices. Panel D shows how the range is related to the conversion trigger. The range is wider for a lower trigger than for a higher trigger, because the time to reach the trigger is longer in expectation, incorporating more conversion points in generating the multiple equilibria. The last two panels demonstrate that the severity of multiple equilibria depends on the amount and characteristics of the CC.

With jumps in the asset value, there can still be a wide range of multiple equilibrium prices. We demonstrate this by letting the asset follow the jump diffusion process in equation (5) and using conversion time Λ_{daily}. In Table I, the column with the heading "JD" presents the parameters used in this numerical example. We assume that the arrival rate of jumps is 4 times per year, reflecting the quarterly regulatory and accounting filings. The mean of logarithm jump size is -2 percent and its volatility is 3 percent. With the presence of jumps, we assume that the volatility of the continuous process is 4%, lower than the volatility assumed for the GBM. To calculate the equilibrium prices, we follow Hilliad and Schwartz (2005) to build a bi-variate tree that approximates the jump diffusion process. With jumps, the yield 3.34% prices the bond at par, which is 87% of the initial asset value. The contingent capital and equity have a bigger range of equilibrium prices. An equilibrium equity value can be as low as 3.84% or as high as 5.44% of the initial asset value, and the CC value ranges from 2.30% to 3.90% of initial asset value. The pricing range is 1.6% of the initial asset value. Since JD is different from GBM only in the parameters about jumps, these examples show that jumps enlarge the pricing range of the multiple equilibria.

It is useful to provide some perspective on why the multiple equilibria arise in the above example but do *not* generally arise in the pricing of convertible bonds or options.[13] With a convertible bond, the investor has the "option" to convert and get a pre-specified number of shares of common stock. In each state, the investor can compare values associated with different conversion decision and select the maximum. Likewise, the holder of the option can also make the optimal decision in each state. These optimal decisions can be modeled by the "smooth pasting" or the "high contact" condition pioneered by Merton (1973) and further elucidated by Dixit and Pindyck (1994). In such models, the exercise boundary itself is endogenous and not mandated, and the economic agent, acting in his self interest will select the conversion decision optimally so that there is no value transfer at the trigger price.

With mandatory conversion, no agent is allowed to optimally act at the trigger. This absence of a "smooth pasting" condition then leads to the problems we have articulated above. The smoothness breaks down if the mandatory conversion transfers value between the equity holders and CC holders at the trigger price.[14] The state-contingent conversion ratio presented in the theorems prevents the value transfer and, in effect, keeps the prices "smooth" at conversion.[15] However, conversions that are mandatory only at maturity do not pose any essential difficulty as the bond trades at par at maturity and hence the zero

[13]It should be noted that there are some studies of the multiple equilibrium in convertible bonds and options. Constantinides (1984) shows the possibility of multiple competitive equilibria, and Spatt and Sterbenz (1988) have examined sequential exercise strategies and gains to hoarding warrants. In these papers, however, multiplicity of equilibrium is caused by the distribution of ownership of warrants and reinvestment policies. Furthermore, in the context of bank runs, the possibility of multiple equilibriua has been identified by Diamond and Dybvig (1983).

[14]The effect of value transfer at mandatory conversion is similar to an exogenous value transfer caused by tax distortion. Albul et al (2010) have shown that differential tax treatment of CC's coupon interest and equity's dividend can cause multiple equilibria for a mandatory convertible debt with the trigger on asset value.

[15]In the valuation of barrier options, the exercise boundary is exogenous and their structure shares some of the features of CC. But the exercise of such options does not influence the underlying stock price itself, as there are no dilution effects to consider. These options are also in zero net supply.

value transfer restriction at maturity can be satisfied with fixed conversion trigger and ratio. The mandatory convertible preferred security in the Treasury's Capital Assistance Program in 2009 has such features at maturity.[16]

Although all our numerical examples demonstrate the case of multiple equilibrium, we should emphasize that the absence of equilibrium is equally important. While the range of multiple equilibrium offers a sense of the severity of the problem, there is no simple way to characterize the severity of no equilibrium. This does not imply that the absence of equilibrium is not a serious concern. The problem with the absence of equilibrium is demonstrated in the laboratory experiments conducted by Davis, Prescott and Korenok (2011). They let groups of heterogeneous agents trade an asset in a market where there is no equilibrium due to intervention by a market regulator and compare the results obtained in a market where there is a unique equilibrium without intervention. They observe large uncertainty in trading prices and inefficient allocation of the asset, with efficiency in their analysis measured by how much assets are allocated to the traders who value them the most. In the case of the multiple equilibria caused by regulator intervention, their experiments also show price uncertainty and allocation inefficiency.

III Additional Issues

A The Issue with Implementation

The pricing restriction presents a challenge to the implementation of the CC design: the restricted conversion ratio is tied to the market value of the contingent capital if we want a unique equilibrium. Since we cannot tell what the future market value will be, the value C_t of the nonconverted CC can be different from a pre-specified $m_t K_t/n$ at any time $t \in \Lambda$. If we set the conversion ratio to $m_t = nC_t/K_t$, it depends on the future market value of the

[16] See Glasserman and Wang (2011), who describe and value the capital assistance program.

nonconverted CC. However, if the unconverted CC is always priced at the par value, the problem will be solved by setting $m_t = n\bar{C}/K_t$.

To make a CC priced at par all the time before conversion, we need to focus on a structure that makes the market value of the CC immune to changes in interest rates and default risk. For example, if the CC had no default risk until conversion, by selecting the coupon rate at each instance to be the instantaneously risk-free rate we can ensure that the CC will trade at par. See Cox, Ingersoll, and Ross (1980) for a proof of this assertion.[17] In this case, CC will work well, because we can determine the conversion ratio ex-ante as $m_t = nC_t/K_t = n\bar{C}/K_t$. Since $\bar{C}$ and K_t are known ahead of time, we can specify the conversion ratio ahead as well. Without jumps in asset value, CC can be designed to be default-free during its life before conversion, even though the bank may have a positive probability of default on its debt claims subsequent to the expiration of CC. This idea is formalized in Theorem 3 below.

Theorem 3 *Suppose a bank's asset value follows a geometric Brownian motion, $dA_t = (r_t - \alpha_t)A_t dt + \sigma A_t dz_t$, where α_t is the rate of cash flow from the asset, r_t is the instantaneous risk-free interest rate, and z_t is a Wiener process. Given any conversion trigger K_t that is a continuous function in time, the contingent capital with coupon rate $c_t = r_t$, continuous verification $\Lambda = [0, +\infty]$, and conversion ratio $m_t = n\bar{C}/K_t$ has a unique equilibrium value, which equals the par value.*

This theorem generalizes the immunization results of Cox, Ingersoll, and Ross (1980) to a setting where there is mandatory conversion and a positive probability of default after the expiration date of CC. Since the coupons float with the risk-free rate and the principal is guaranteed at conversion, the CC is fully immunized and therefore sells at par. The economic rationale is also intuitive. Since the CC sells at par, we can design the CC with an ex-ante

[17] In the context of a CC that is exposed to default risk, the appropriate indexed coupon may also require a compensation for the mandatory conversion in addition to the risk-free rate.

conversion ratio that guarantees that, upon conversion, the CC holders will get par. This theorem demonstrates the existence of simple CC design that gives a unique equilibrium. In this CC, conversion trigger K and ratio m can be constant. To assure a unique equilibrium, we only need to set K and m so that $m = n\bar{C}/K$.

Theorem 3 appears to make a CC with market trigger implementable, but it is impracticable. Theorem 3 needs the asset price process to be continuous and the verification to be continuous. In reality, the underlying process of an asset value can have discontinuous jumps. Consequently, the bond is not free of default risk before conversion, and the theorem does not hold. Also, continuous verification is impractical; practical contract specifications are always based on closing or settlement prices, sampled over daily or other regular intervals. In addition, bank-issued CC will be less liquid than risk-free assets such as Treasury securities, and hence the coupon will have to include a component for the liquidity premium in order to make the CC value equal to par. Theorem 3 indicates the restrictive assumptions that are needed to design a CC with market trigger that produces a unique equilibrium.

B Equity Issuance and Conversion Policies

In the analysis so far, we assume that the bank does not issue new equity shares during the life of the contingent capital, particularly when the bank's equity level is low. It is a reasonable assumption because the reason for regulators to require contingent capital stems from their belief that it is too expensive or difficult for a bank to raise equity capital when the bank is under stress and highly leveraged. However, it is still interesting to ask whether the absence of a unique equilibrium may occur if banks can issue new shares to avoid conversion.

Calomiris and Herring (2011) argue that the option to issue new shares and avoid conversion eliminate the equilibriums that are disadvantageous to equity holders and assures a unique equilibrium. They also assert that the need to avoid a disadvantageous equilibrium forces banks to issue common equity. They suggest that regulators should require banks to hold contingent capital mainly to force banks to issue equity in bad times or states.

In this subsection, we will show that equity issuance may eliminate some of the equilibrium in which the CC converts and ensure unique equilibrium at maturity but *not* before maturity. This result at maturity is of limited interest as the bank becomes unlevered at maturity. In demonstrating this, we assume a frictionless world where issuing new shares does not incur additional cost beyond the shares' fair value. Finally, we show that the additional cost of issuing new shares will make multiplicity of equilibrium even more likely.

We first show how equity issuance eliminates a conversion equilibrium at the maturity of the senior bond. Let us use the same notations in Section I. To have multiple equilibria at maturity, we need to set $m > n\bar{C}/K$ so that conversion is punitive. As we have discussed before, for every $A_T \in (\bar{B} + \bar{C} + K,\ \bar{B} + (m/n)K + K]$, there are two equilibria. In one equilibrium the CC does not convert, and the stock price is $S_T^u = (A_T - \bar{B} - \bar{C})/n > K/n$. In the other equilibrium the CC converts, and the stock price is $S_T^c = (A_T - \bar{B})/(n+m) \leq K/n$. If the bank issues enough (say l) new shares to avoid conversion, the second equilibrium cannot sustain. Since conversion can be avoided, the share price with new issuance must be S_T^u. Since $S_T^c < S_T^u$, the bank will choose to issue new shares as otherwise everyone may believe conversion could happen. Consequently, the only possible equilibrium stock price is S_T^u for every $A_T \in (\bar{B} + \bar{C} + K,\ \bar{B} + (m/n)K + K]$.[18]

Equity issuance ensures a unique equilibrium in the above example because the bank is not leveraged after stock issuance. If the firm continues to be leveraged at time T, issuing new shares of equity will increase the safety of the outstanding bond and CC and raise their

[18] It is worth pointing out that no matter how large the conversion ratio m is, equity issuance does not eliminate the chance of conversion. Conversion must happen when $A_T \leq \bar{B} + \bar{C} + K$ because $S_T^u \leq K/n$ in this case. Issuing new shares will not increase S_T^u. Therefore, the probability of conversion is at least as large as $P\{\bar{B} + \bar{C} + K\}$, which is independent of m.

values. This transfers some value from equity holders to bond holders. Suppose, at time T, the bond value without issuing equity turns out to be equal to B and the bond gains $\Delta B > 0$ with the issuance of l shares of new equity. Then, the bond value with equity issuance at T is $B_T^i = B + \Delta B$. The equity price with the issuance of new equity, denoted by S_T^i, should satisfy $(A_T + l \cdot S_T^i - B_T^i - \bar{C})/(n + l) = S_T^i$. Solving for S_T^i, we obtain that the stock price should be $S_T^i = (A_T - B_T - \bar{C})/n = (A_T - B - \Delta B - \bar{C})/n$, which is smaller than S_T^u.

Then, for every $A_T \in (B+\bar{C}+K,\ B+(m/n)K+K]$, there are two equilibria. In the first equilibrium, all investors believe that CC will not convert. In this case, bank management does not issue new shares because new issuance will lead to lower stock price. Without conversion and issuance, the stock price is S_T^u. In the second equilibrium, all investors believe that the CC converts if no new shares are issued to avoid the conversion. In this case, the result of the equilibrium depends on the magnitude of ΔB. If ΔB is so small that $S_T^i > S_T^c$, then the bank management prefers issuing new shares to conversion. Consequently, CC does not convert, new shares are issued, and the stock price is S_T^i, which is larger than S_T^c but smaller than S_T^u. However, if $\Delta B \geq n(A_T - B)/(n+m) - \bar{C}$, it is easy to verify that $S_T^i \leq S_T^c$. In this case, the bank management will be better off by letting the CC convert. Therefore, the converted share price S_T^c should be the equilibrium price, and no new shares are issued.

The above analysis suggests that equity issuance does not ensure a unique equilibrium in a dynamic setting. Take the example of one-period discrete model in Section B but set conversion ratio to $m = 3$. Recall that $\bar{B} = 90$, $\bar{C} = 10$, $K = 5$, and $n = 1$. The probability distribution of A_T is: $p\{A_T = 120\} = .25$, $p\{A_T = 100\} = 0.50$, and $p\{A_T = 80\} = 0.25$. With the assumption of zero risk-free rate, the initial asset value is $A_0 = 100$, and bond value is $B_0 = 87.50$, as shown in panel A of Figure 3. Similarly as in Section B, without issuance of new shares, we obtain two equilibria: $(C_0^u, S_0^u) = (6.25, 6.25)$ and $(C_0^c, S_0^c) = (9.38, 3.13)$, as shown in panel B of Figure 3.

Now, assume that the bank plans to issue new shares today so that the total number of shares enlarges by fifty percent. Suppose the issuance price is \$5.26, which will be shown in the next paragraph to be the equilibrium stock price with the issuance. For simplicity, assume that the proceeds of the new shares can be reinvested to enlarge the bank asset and earn the same return.[19] The asset value with reinvestment of the proceeds is $A_0^i = 100 + 0.5 \times 5.26 = 102.63$. If the original asset value A_T is 120, 100, or 80, the value of the enlarged asset, A_T^i, is 123.16, 102.63, or 82.10, respectively. These asset values are shown in panel A of Figure 3. The enlarged asset makes the bond safer and increases the bond value from 87.50 to 88.03, as shown in the same panel.

With new equity issuance being allowed, no conversion today is still an equilibrium as shown in panel B of Figure 3 because the stock price, $S_0^u = 6.25$, is above the trigger. However, if all investors believe that conversion would happen if the bank does not avoid it, issuing new shares today is another equilibrium as shown in panel C of Figure 3. Issuing new shares is a strategy that dominates conversion because $S_0^i = 5.26$ is higher than $S_0^c = 3.13$. Also notice that $S_0^i = 5.26$ is the same as the issuance price we assumed at the beginning of the previous paragraph. This confirms that $S_0^i = 5.26$ is an equilibrium price with issuance. Therefore, using equity issuance to avoid conversion, we still have two equilibria: $(C_0^u, S_0^u) = (6.25, 6.25)$ and $(C_0^i, S_0^i) = (6.71, 5.26)$.

It is difficult to analyze a dynamic continuous time model with optimal equity issuance, but the above analysis and example in discrete models are sufficient to demonstrate that optimal equity issuance does not guarantee a unique equilibrium, even if we assume equity issuance is possible and costless. Given the complicated pricing dynamics of contingent cap-

[19]That is, we assume that the bank asset has constant returns to scale. If we assume that the asset has a decreasing return to scale, it strengthens the case for multiple equilibria.

ital, the incentives of contingent capital to bank managers are largely uncertain. Therefore, using a contingent capital requirement appears to be an indirect way to force banks to issue common equity. It may be more direct and simple to set a regulatory policy that requires banks to issue common equity when the market equity ratio is low.

C Financial Distress

When deriving the condition of unique equilibrium in Section II, we allow for bankruptcy costs but not for financial distress. We assume that the cash outflows for paying coupons of the bonds and contingent capital come from operating cash flows and, when needed, from equity holders. This assumption allows us to derive the necessary and sufficient condition for the unique equilibrium in a general setting.

An argument frequently cited in favor of contingent capital is, however, that it can be converted into equity if the bank is under financial distress. The conversion thereby conserves capital as the bank is relieved of paying the coupons associated with the CC. In periods of financial distress when banks may not be able to raise capital, contractual coupon obligations may be a burden and carry significant costs. In particular, meeting such obligations may result in asset depletion, which may further exacerbate the financial distress. One reason for introducing contingent capital is to reduce the likelihood for a financial institution to experience default because the chance of costly bankruptcy destroys the value of the firm that is under financial distress.

Given that financial distress is assumed away in Section II, it is natural to ask whether there are multiple equilibria under financial stress if the contingent capital has a constant conversion ratio. This question is difficult to analyze in a setting that is as general as in Section II. Nevertheless, using a two-period discrete model, we are able to demonstrate that under financial distress when debt service depletes assets, there can still be multiple equilibria of CC and equity values. In the case of financial distress, there can even be multiple equilibria of firm values and senior bond values. In other words, contingent capital can lead the equity, bond and firm values to be all different in different equilibria. More generally, the analysis in this two-period model suggests that with financial distress a contingent capital does not always have unique equilibrium even if we place the conversion trigger on any combination of the claims of the firm.[20]

Our two-period model has three dates: dates 0, 1 and 2. For simplicity, we assume that the risk-free rate is zero. During each period, the risky asset value either has positive return R or a negative return $-R$ with equal probability. Thus, on date 1, the asset value will be $A_1 = A_0(1 \pm R)$ with equal probability for each value. On date 2, however, the asset value will depend on what happens to the bond and CC on dates 0 and 1. Assume there is a bond with face value $\bar{B}$ and coupon rate b per period and a CC with face value $\bar{C}$ and coupon rate c per period. Both the bond and CC start from date 0 and mature on date 2. Unlike in the previous section, we assume that the bank has to sell assets to serve debt obligations. Then, if neither the bond defaults nor the CC converts on date 0 or 1, the asset value on date 2 is $A_2 = (A_1 - b\bar{B} - c\bar{C})(1 \pm R)$, which has four possible values with equal probability. If the bond does not default on date 0 or 1 but the CC converts on date 0 or 1 when the asset value is A_1, the asset value on date 2 is $A_2 = (A_1 - b\bar{B})(1 \pm R)$, which has two possible values with equal probability conditioning on A_1.

The firm and bond values also depend on whether the bond is defaulted. If the bond is not defaulted up to date i, the firm and bond values on date i are

$$F_i = \begin{cases} A_2 & \text{if } i = 2 \\ E[F_{i+1}] & \text{if } i = 0, 1 \end{cases} \quad \text{and} \quad B_i = \begin{cases} (1+b)\bar{B} & \text{if } i = 2 \\ E[B_{i+1}] + b\bar{B} & \text{if } i = 1 \\ E[B_{i+1}] & \text{if } i = 0, \end{cases} \tag{16}$$

where $E[F_{i+1}]$ and $E[B_{i+1}]$ are the expected firm and bond values on date $i+1$, respectively.

[20]For example, Pennacchi (2011) suggests that placing the trigger on the sum of the equity and CC values may ensure a unique equilibrium.

Notice that the bond value is "cum-dividend." In this model, we assume that default happens if and only if the equity value is zero. Thus, the condition for default on date i is that the firm value F_i is smaller than or equal to the bond value conditioning on the bond not defaulting. This default strategy maximizes the shareholders value. Bankruptcy is costly, and the cost is a fraction (ω) of the assets. Then, if the bond has defaulted by date i, the firm and bond values on date i are

$$F_i = B_i = (1 - \omega)A_i. \tag{17}$$

The CC and equity values depend on whether the CC is converted, besides depending on the status of the bond. Let K be the trigger level for contingent capital and m the conversion ratio. Assume there is one share outstanding on date 0. If the CC has not been converted on date i, the CC and equity values on date i are

$$C_i = \begin{cases} (1+c)\bar{C} & \text{if } i = 2 \\ E[C_{i+1}] + c\bar{C} & \text{if } i = 1 \\ E[C_{i+1}] & \text{if } i = 0 \end{cases} \quad \text{and} \quad S_i = F_i - B_i - C_i. \tag{18}$$

If the CC has been converted on date i but the bond has not been defaulted, the CC and equity values on the date are

$$C_i = mS_i \quad \text{and} \quad S_i = \frac{1}{1+m}(F_i - B_i). \tag{19}$$

If the bond has been defaulted on date i, the CC and equity value on date i are

$$C_i = 0 \quad \text{and} \quad S_i = 0. \tag{20}$$

The set of the firm, bond, CC, and equity values, $\{(F_i, B_i, C_i, S_i)\}_{i=0,1,2}$, is a dynamic rational expectations equilibrium in the model if the values satisfy equations (16)–(20). The equilibrium in this model is not always unique. This can be shown by a numerical example. Let $A_0 = 100$, $R = 0.06$, $\omega = 0.1$, $\bar{B} = 85$, $b = 0.02$, $\bar{C} = 6$, $c = 0.04$ and $K = 1$. We have two equilibria, which are displayed in Figure 4. Panel A is an equilibrium in which conversion does not occur, and the bank has to pay coupons to CC holders on date 1. This reduces the assets and as a consequence increases the likelihood of default on date 2. Panel B is an equilibrium in which conversion occurs on date 1, and hence the bank is able to conserve its capital and avoids bankruptcy on date 2.

The following are worth noting. First, the trees in Figure 4 are not recombining as the assets are reduced to meet contractual coupon payments under financial distress. Studies of a multi-period model with non-recombining trees are often difficult, and this is the reason we limit ourselves to a two-period model. Second, the asset value at each node is potentially different from the bank's firm value since the latter will be the asset value *minus* the expected costs of default, which are the financial distress costs.

Of the two equilibria, the conversion equilibrium in panel B is welfare improving in the sense that it results in lower dead-weight losses. Notice that the date 0 firm value (100) in the equilibrium with conversion is higher than the value (97.84) in the equilibrium without conversion. This shows that the conversion equilibrium results in lesser dead-weight losses as the bank avoids paying coupons when "bad states" are reached. On the other hand, the no-conversion equilibrium results in higher dead-weight losses. This example suggests that contingent capital can be potentially welfare improving in the sense of reducing the expected dead-weight losses, but there is no credible way to select this equilibrium ex-ante. In fact, equity holders would prefer the no-conversion equilibrium because the equity value in this equilibrium is 6.00, which is larger than the equity value (5.36) in the other equilibrium.

IV Conclusion

Contingent capital with a mandatory conversion feature can be designed by placing the trigger for conversion on the value of securities issued by the bank. Our paper shows that depending on the design of the CC and the underlying asset dynamics, one can obtain unique, multiple, or no equilibrium. Contingent capital and other securities are claims on the same assets and that their prices (which reflect conversion policies) often need to be determined simultaneously. Since no agent is allowed to act in his interest with mandatory conversion, conversion rules must ensure that, at the trigger, conversion does not change the value of the security on which the trigger is placed.

Our paper shows, for equity triggers, that the conversion ratio that gives a unique equilibrium must produce no value transfer. Hence, the design of "dilutive" ratios in order to penalize bank managers or to promote coercive equity issuance will lead to multiple equilibria. Multiple equilibria imply that CC holders and equity holders have precisely the opposite motives, which can lead to potential manipulation of market prices when the equity price approaches the trigger level. Under some conditions, we show that equity triggers can also result is no equilibrium, which does not promote stability. Although our paper mainly focuses on the equity price trigger, our analysis has implications for all triggers that depend on market value of equity either directly or indirectly.

The pricing restriction developed in this paper offers better understanding of the literature developed recently on pricing contingent capitals. Albul, Dwight and Tchistyi (2010) obtain a closed-form solution of a unique equilibrium by assuming that the bond and contingent capital are both perpetual. The most striking distinction between the work of Albul et al and our paper is that they assume that the conversion trigger is on the level of asset value, rather than the price of a financial claim on the asset. The assumption of an asset value trigger ensures the existence of a unique equilibrium, although the assumption makes their contingent capital very different from all the proposed CC. The work of Albul et al and the pricing restriction developed by us together show that placing the trigger on asset value is not equivalent to placing the trigger on the market equity ratio.

Pennacchi (2010) takes a different approach to avoid the problem of multiple or no equilibrium. He focuses on a bank that, besides having short-term deposits and common equity, issues contingent capital but no long-term bond. The short-term deposits are always priced at par, and its total value is assumed to have an exogenous stochastic process. He suggests placing the trigger on the ratio of the asset value to the combined value of the CC and equity. Since the asset value is the sum of the deposits, equity and CC values, his approach is equivalent to placing the trigger on the asset-to-deposits ratio. This equivalence arises due to the assumption that financial distress/bankruptcy are costless. This ensures a unique equilibrium in stock and CC prices because the ratio is independent of the conversion of CC. However, this mechanism of obtaining a unique equilibrium will not work in a world with bankruptcy and distress costs. Pennacchi's work confirms the necessity of placing the trigger on variables that are unaffected by the conversion of contingent capital.

To avoid the problem with the lack of unique equilibrium, McDonald (2010) directly assumes that the firm's equity value follows a geometric Brownian motion exogenously. Then, he places conversion trigger directly on the equity value as well as a broad market index, which is unaffected by the firm that issues CC. In contrast, Glasserman and Nouri (2010) assume that the asset value follows an exogenous geometric Brownian motion and place the conversion trigger on the ratio of "book value" of equity to the firm value, where the "book value" is obtained by subtracting the par values and obligated coupons of the bond and CC from the asset value. The lesson we can learn from these two papers is that as long as the variables for the conversion trigger are exogenous, we can steer clear of the multiplicity and absence of equilibrium and calculate a price of the contingent capital.

The pricing problem demonstrated in this paper and reflected in the literature shows the challenge regulators typically face when they interact with markets. The challenge can come in two ways: (1) regulation with a good intention may interfere with the markets and cause instability with unintended consequences; and (2) regulation's function may be constrained by the markets, causing it to become ineffective. In the example of contingent capital with market trigger, a conversion that is punitive to equity holders may introduce instability because it creates multiple equilibria. The unique equilibrium restriction strips off the incentive function of the CC. In view of the problems with bank manager option, accounting trigger, and regulator's discretion that we have discussed earlier, and the challenges of designing a CC with market trigger that we have shown, it may not be practical to design the security so that it converts to common equity in a timely and reliable manner when a bank is under stress. As a result, contingent capital may not be a close substitute of common equity as a capital buffer.

Some researchers assert that contingent capital is a cheaper substitute of equity because the CC's coupon payments qualify for tax deduction. This assertion is inconsistent with the current U.S. tax code. According to Revenue Rule 85-119, the feature of paying back the par value at maturity, if a CC is not converted, appears to make the CC a debt for tax purpose. However, the coupon payments are not tax deductible according to Section 163(l) of the Internal Revenue Codes. A security is a disqualified debt instrument in IRC Sec. 163(l) if (a) a substantial amount of the principal and interest of the security is required to be paid in or converted into the equity of the issuer, (b) a substantial amount of the principal or interest is required to be determined by reference to the value of such equity, or (c) the indebtedness is part of an arrangement which is reasonably expected to result in a transaction described in (a) or (b). Clearly, contingent capital is not tax deductible in the U.S. unless the U.S. government changes its tax codes to exempt CC particularly. However, we should not forget that banks' saving from tax deduction is taxpayers' cost.

Regulators around the world have taken positions on the credibility and impracticality of contingent capital as a loss-absorbing capital for banks and financial institutions. We summarize below the recent development of policies related to contingent capital. This provides a context for our study and helps us to put a perspective on the policy debate.

Daniel Tarullo, a Federal Reserve Governor, has argued that it is unclear that contingent capital can be designed in a way so as to be cheaper than equity but still structured so as to convert in a timely, reliable fashion. The concern is around the trigger for conversion, which has been the focus of our paper: the possibility of manipulation of prices around trigger and the likelihood of "death spiral" being two of the main concerns. Our paper shows that there is a potential for manipulation around the trigger level because the forces of supply and demand do not cause the market to converge to a unique equilibrium.

The Office of the Superintendent of Financial Institutions (OSFI) in Canada has expressed the view that the conversion trigger should be activated relatively late in the deterioration of a bank's health, when the supervisor has determined that the bank is no longer viable as currently structured. They have argued that an identifiable conversion trigger event could be when the regulator is ready to seize control of the institution because problems are so deep that no private buyer would be willing to acquire shares in the bank, or when a government injects capital into (or otherwise provides guarantees to) a bank. Upon occurrence of a trigger event, each contingent security would convert into common equity. Thus a regulatory trigger appears to be the preferred policy initiative in Canada.

Basel III, in its consultative document has concluded that contingent capital should not be used by global systemically important banks (G-SIB) for loss-absorbency, on the ground that absorbing losses at the point of non-viability violates the spirit of the "going-concern" objectives. The Basel committee noted several similarities between equity capital and CC (such as the possibility that both can be issued in good states so that they can offer protection in bad states, and both are pre-funded, which increases the liquidity of the banks in good states). After enumerating the pros and cons of CC, the Basel committee concluded that the G-SIB be required to meet the loss absorbency requirement with common equity Tier-1 capital only. Their recommendations exclude CC from the core 7% capital requirements and the 2.5% surcharge on G-SIB.

Although Basel III leaves to regional regulators to decide on the use of CC in any additional capital requirements, it issued guidelines on the design of contingent capital. The guidelines contain three important requirements. First, a contingent capital that qualifies for "additional tier-1" should trigger conversion at equity ratio not lower than 7% of the risk-weighted assets, whereas those qualify for tier-2 should trigger conversion at equity ratio not lower than 3%. Second, a contingent capital should include a regulatory trigger that forces conversion when the bank is non-viable without public assistance. Third, the contingent capital must impose a cap on share issuance. It is unclear how to set triggers on claims other than equity to meet the first requirement. The second requirement appears to reflect Basel committee's doubts about the reliability of equity triggers. The third requirement excludes any design that involves unlimited issuance of common equity shares.

Several regional regulators issued rules that are consistent with Basel III and echoes concerns expressed by the Basel committee. Switzerland has imposed additional capital requirements on its big banks. It requires that big banks maintain at least 19% capital ratio, comprised of 10% equity, 3% tier-1 CC and 6% tier-2 CC. European Commission (EC) in its Capital Requirements Directive mandates that banks maintain 8% of their risk-weighted assets in capital, of which 4.5% should be in equity, 1.5% in tier-1 CC and 2% in tier-2, along with surcharges of 7.5%, which may only be in common equity. In U.K., the Independent Commission on Banking (ICB) in its final report concluded: Equity is the only form of loss-absorbing capacity that works both pre- and post-resolution. In particular, it is the only form of loss-absorbing capacity that certainly absorbs losses before a bank fails. Contingent capital may be of some value in this regard, but this is not yet proven.

We interpret the actions of regulators as cautionary in the use of CC for the purposes of loss-absorbency of banks. The potential problems such as the uncertainty about whether the CC will convert to equity in a timely fashion, the propensity for manipulation around the trigger and the market instability associated with the multiplicity and the absence of equilibrium suggest that the role of CC as a capital instrument to internalize banks losses must be monitored carefully.

Appendix Proofs of the Theorems

Before proving the theorems, it is useful to make the following observation. If there were no CC, at any time t before maturity and default ($t \leq \min\{\delta, T\}$), the equity value would have been

$$U_t \;\; = \;\; E_t\left[(A_T - \bar{B})P(t,T) \cdot 1_{\delta > T} + J_t\right] , \tag{A1}$$

where J_t is defined in equation (12) by replacing τ with t. Since it is known from Merton (1974) and Black and Cox (1976) that U_t is a measurable function of t and A_t, we can define another hitting time based on U_t and the given K_t and m_t:

$$\upsilon = \inf\left\{t \in \Lambda : \frac{n}{n + m_t} U_t \leq K_t\right\} . \tag{A2}$$

The following lemma will be useful for all the proofs.

Lemma 1 *If (S_t, C_t) is the stock and CC prices in an equilibrium, then $nS_t + C_t = U_t$ when $t < \min\{\tau, \delta\}$, and $S_t = U_t/(n + m_\tau)$ when $\tau \leq t < \delta$.*

A Proof of Lemma 1

Suppose $t < \min\{\tau, \delta\}$. It follows from equations (10) and (13) that

$$nS_t + C_t \;\; = \;\; E_t\left[(A_T - \bar{B})P(t,T) \cdot 1_{\min\{\tau,\delta\} > T} + I_t + H_t\right]$$

$$+ E_t\left[(A_T - \bar{B})P(t,T)\cdot 1_{\tau\le T<\delta} + J_\tau P(t,\tau)\cdot 1_{\tau<\min\{\delta,T\}}\right]. \qquad \text{(A3)}$$

Substituting equations (11), (12) and (14) for I_t, J_τ and H_t, respectively, in equation (A3), we obtain

$$\begin{aligned} nS_t + C_t &= E_t\left[(A_T - \bar{B})P(t,T)\cdot 1_{\min\{\tau,\delta\}>T}\right] \\ &+ E_t\left[\int_t^{\min\{\tau,\delta,T\}}(a_s A_s - b_s\bar{B} - c_s\,\bar{C})P(t,s)ds\right] \\ &+ E_t\left[\int_t^{\min\{\tau,\delta,T\}} c_s\bar{C}P(t,s)ds\right] \\ &+ E_t\left[(A_T - \bar{B})P(t,T)\cdot 1_{\tau\le T<\delta}\right] \\ &+ E_t\left[\int_\tau^{\min\{\delta,\,T\}}(a_s A_s - b_s\bar{B})P(\tau,s)dsP(t,\tau)\cdot 1_{\tau<\min\{\delta,T\}}\right]. \end{aligned} \qquad \text{(A4)}$$

Combining the terms in equation (A4), we obtain

$$\begin{aligned} nS_t + C_t &= E_t\left[(A_T - \bar{B})P(t,T)\cdot 1_{\delta>T}\right] \\ &+ E_t\left[\int_t^{\min\{\tau,\delta,T\}}(a_s A_s - b_s\bar{B})P(t,s)ds\right] \\ &+ E_t\left[\int_\tau^{\min\{\delta,\,T\}}(a_s A_s - b_s\bar{B})P(t,s)ds\cdot 1_{\tau<\min\{\delta,T\}}\right]. \end{aligned} \qquad \text{(A5)}$$

In equation (A5), we can rewrite the integral in the second term into

$$\int_t^{\min\{\delta,T\}}(a_s A_s - b_s\bar{B})P(t,s)ds\cdot 1_{\tau\ge\min\{\delta,T\}} + \int_t^\tau (a_s A_s - b_s\bar{B})P(t,s)ds\cdot 1_{\tau<\min\{\delta,T\}}. \qquad \text{(A6)}$$

Combining the integrals of the last terms in equations (A5) and (A6), we obtain

$$\begin{aligned} nS_t + C_t &= E_t\left[(A_T - \bar{B})P(t,T)\cdot 1_{\delta>T}\right] \\ &+ E_t\left[\int_t^{\min\{\delta,T\}}(a_s A_s - b_s\bar{B})P(t,s)ds\cdot 1_{\tau\ge\min\{\delta,T\}}\right] \\ &+ E_t\left[\int_t^{\min\{\delta,T\}}(a_s A_s - b_s\bar{B})P(t,s)ds\cdot 1_{\tau<\min\{\delta,T\}}\right]. \end{aligned} \qquad \text{(A7)}$$

The last two terms in equation (A7) can be combined to give

$$\begin{aligned} nS_t + C_t &= E_t\left[(A_T - \bar{B})P(t,T)\cdot 1_{\delta>T} + \int_t^{\min\{\delta,T\}}(a_s A_s - b_s\bar{B})P(t,s)ds\right] \\ &= E_t\left[(A_T - \bar{B})P(t,T)\cdot 1_{\delta>T} + J_t\right] = U_t\ . \end{aligned}$$

This proves $nS_t + C_t = U_t$.

For $\tau \le t < \delta$, equations (15) and (A1) imply $S_t = U_t/(n + m_\tau)$. *Q.E.D.*

B Proof of Theorem 1

For $\tau < \min\{\delta, T\}$, it follows from Lemma 1 that $S_\tau = U_\tau/(n + m_\tau)$. Since $nS_\tau \le K_\tau$ by equation (9), we have $nU_\tau/(n + m_\tau) \le K_\tau$, which implies

$$\upsilon \le \tau \tag{A8}$$

in view of equation (A2). On the other hand, equation (A2) implies $nU_\upsilon/(n + m_\upsilon) \le K_\upsilon$, and thus conversion is an equilibrium at time υ. If $nS_\upsilon > K_\upsilon$, then no conversion is also an equilibrium at time υ, contradicting to the assumption of unique equilibrium. Thus, the uniqueness of the equilibrium implies $nS_\upsilon \le K_\upsilon$. It follows that

$$\tau \le \upsilon\,, \tag{A9}$$

in view of equation (9). Combining equations (A8) and (A9), we have $\tau = \upsilon$. It then follows from equations (9), (A2) and Lemma 1 that

$$\inf\{t \in \Lambda : U_t \le K_t + C_t\} = \inf\{t \in \Lambda : U_t \le K_t(n + m_t)/n\}\,. \tag{A10}$$

The above equation holds for all possible paths of U_t if and only if $K_t + C_t = K_t(n + m_t)/n$ for all $t \in \Lambda$, which implies $m_t = nC_t/K_t$ for all $t \in \Lambda$. Therefore, in order to have a unique equilibrium, the conversion ratio must satisfy $m_t = nC_t/K_t$ for $t \in \Lambda$. *Q.E.D.*

C Proof of Theorem 2

To prove Theorem 2, we use the hitting time υ defined in (A2) as the conversion time of a CC. For any conversion ratio m_t, in rational expectations, the stock price and the CC value before conversion, default, and maturity ($t < \min\{\upsilon,\, \delta,\, T\}$) are

$$
\begin{aligned}
S_t^* &= E_t\left[(A_T - \bar{B} - \bar{C})P(t,T)\cdot 1_{\min\{\upsilon,\,\delta\}>T} + I_t^*\right] \\
&\quad + E_t\left[\frac{1}{n+m_\upsilon}\left\{(A_T - \bar{B})P(t,T)\cdot 1_{\upsilon\le T<\delta} + J_\upsilon^* P(t,\upsilon)\cdot 1_{\upsilon<\min\{\delta,\,T\}}\right\}\right], \quad \text{(A11)} \\
C_t^* &= E_t\left[\bar{C}P(t,T)\cdot 1_{\min\{\upsilon,\,\delta\}>T} + H_t^*\right] \\
&\quad + E_t\left[\frac{m_\upsilon}{n+m_\upsilon}\left\{(A_T - \bar{B})P(t,T)1_{\upsilon\le T<\delta} + J_\upsilon^* P(t,\upsilon)\cdot 1_{\upsilon<\min\{\delta,\,T\}}\right\}\right], \quad \text{(A12)}
\end{aligned}
$$

$$H_t^* = \int_t^{\min\{\upsilon,\,\delta,\,T\}} c_s \bar{C} P(t,s)ds \quad \text{(A13)}$$

$$I_t^* = \int_t^{\min\{\upsilon,\,\delta,\,T\}} (a_s A_s - b_s \bar{B} - c_s \bar{C})P(t,s)ds \quad \text{(A14)}$$

$$J_\upsilon^* = \int_\upsilon^{\min\{\delta,T\}} (a_s A_s - b_s \bar{B})P(\upsilon,s)ds. \quad \text{(A15)}$$

The stock price after conversion ($\upsilon \le t < \min\{\delta, T\}$) is

$$S_t^* = \frac{1}{n+m_\upsilon} E_t\left[(A_T - \bar{B})P(t,T)\cdot 1_{\delta>T} + J_t^*\right]. \quad \text{(A16)}$$

Following the proof of Lemma 1, we can use equations (A11)–(A16) to derive similarly $nS_t^* + C_t^* = U_t$, which implies

$$nS_t^* = U_t - C_t^*\,. \quad \text{(A17)}$$

Now, we use S_t^* to define another hitting time: $\tau^* = \inf\{t \in \Lambda : nS_t^* \le K_t\}$. In view of equation (A17), we have $\tau^* = \inf\{t \in \Lambda : U_t \le K_t + C_t^*\}$. Setting $m_t = nC_t^*/K_t$ for all $t \in \Lambda$, we have

$$
\begin{aligned}
\tau^* &= \inf\{t \in \Lambda : U_t \le K_t + C_t^*\} \\
&= \inf\{t \in \Lambda : U_t \le K_t(n+m_t)/n\} = \upsilon\,. \quad \text{(A18)}
\end{aligned}
$$

Therefore, (S_t^*, C_t^*) satisfies equations (9), (10), (13) and (15) and thus is an equilibrium.

If (S_t, C_t) is another equilibrium with the conversion ratio $m_t = nC_t/K_t$, following similar reasoning in the derivation of equation (A18), we can show that the conversion time $\tau =$

$\inf\{t \in \Lambda : nS_t \leq K_t\}$ equals υ, which gives $\tau = \tau^*$. Therefore, the values of the common stock and CC calculated in equations (10), (13), (A11) and (A12) imply $S_t = S_t^*$ and $C_t = C_t^*$. This proves the uniqueness of the equilibrium. *Q.E.D.*

D Proof of Theorem 3

In this theorem, $c_t = r_t$ for all t. With the conversion ratio $m_t = n\bar{C}/K_t$, we first show that, in any equilibrium, the value of unconverted contingent capital equals its par value, i.e., $C_t = \bar{C}$ for all $t \leq \tau$.

Let (C_t, S_t) be an equilibrium. The value of the unconverted contingent capital satisfies equation (13). Using $c_t = r_t$ and the definition of the discount factor, we can write equation (14) as

$$H_t = \int_t^{\min\{\tau,\delta,T\}} r_s \bar{C} e^{-\int_t^s r_u du} ds = -\bar{C} \int_t^{\min\{\tau,\delta,T\}} e^{-\int_t^s r_u du}\, d\left(-\int_t^s r_u du\right). \qquad \text{(A19)}$$

It follows from the fundamental law of calculus that

$$H_t = -\bar{C}\, e^{-\int_t^s r_u du}\Big|_t^{\min\{\tau,\delta,T\}} = \bar{C}\left[1 - P(t,\ \min\{\tau,\ \delta,\ T\})\right]. \qquad \text{(A20)}$$

The last term can be split into two terms to give

$$H_t = \bar{C}\left[1 - P(t,T) \cdot 1_{\min\{\tau,\delta\}>T} - P(t,\ \min\{\tau,\delta\}) \cdot 1_{\min\{\tau,\delta\}\leq T}\right]. \qquad \text{(A21)}$$

Substituting the above expression for H_t back into the valuation function of C_t in equation (13) and using the properties of iterated expectations, $P(t,T) = P(t,\tau)P(\tau,T)$ and $1_{\tau\leq T<\delta} = 1_{T<\delta}1_{\tau\leq\min\{\delta,T\}}$, we obtain

$$\begin{aligned} C_t \;&=\; \bar{C} - E_t\left[\bar{C}P(t,\min\{\tau,\ \delta\}) \cdot 1_{\min\{\tau,\delta\}\leq T}\right] \\ &\quad + E_t\left[\frac{m_\tau}{n+m_\tau}\left\{(A_T - \bar{B})P(\tau,T) \cdot 1_{T<\delta} + J_\tau\right\} P(t,\tau) \cdot 1_{\tau\leq\min\{\delta,T\}}\right], \end{aligned} \qquad \text{(A22)}$$

Since equation (15) implies

$$\frac{1}{n+m_\tau} E_\tau\left[(A_T - \bar{B})P(\tau,T) \cdot 1_{T<\delta} + J_\tau\right] = S_\tau, \qquad \text{(A23)}$$

the value of contingent capital with the floating coupon rate r_t equals

$$C_t = \bar{C} - E_t\left[\bar{C}P(t, \min\{\tau,\, \delta\}) \cdot 1_{\min\{\tau,\,\delta\}\leq T}\right] + E_t\left[m_\tau S_\tau P(t,\tau) \cdot 1_{\tau\leq\min\{\delta,\,T\}}\right]. \tag{A24}$$

Since the asset follows a continuous process and the verification is continuous, default cannot occur before conversion, i.e., $\tau \leq \delta$. Hence, the above equation can be written as

$$C_t = \bar{C} - E_t\left[\bar{C}P(t,\tau) \cdot 1_{\tau\leq\min\{\delta,\,T\}}\right] + E_t\left[m_\tau S_\tau P(t,\tau) \cdot 1_{\tau\leq\min\{\delta,\,T\}}\right]. \tag{A25}$$

The continuous process and verification also imply $nS_\tau = K_\tau$ at the conversion time τ. Substituting $S_\tau = K_\tau/n$ and $m_\tau = n\bar{C}/K_\tau$, we obtain

$$C_t = \bar{C} - E_t\left[\bar{C}P(t,\tau) \cdot 1_{\tau\leq\min\{\delta,\,T\}}\right] + E_t\left[\frac{n\bar{C}}{K_\tau}\frac{K_\tau}{n}P(t,\tau) \cdot 1_{\tau\leq\min\{\delta,\,T\}}\right] = \bar{C}, \tag{A26}$$

which shows that the CC is priced at par.

Since the contingent capital is always priced at par, the equilibrium price of CC is unique. Then, Lemma 1 implies $S_t = (U_t - \bar{C})/n$ before conversion and $S_t = U_t/(n + m_\tau)$ after conversion. Consequently, the equilibrium price of the common stock is also unique. *Q.E.D.*

References

Acharya, Viral, Thomas Cooley, Matthew Richardson, and Ingo Walter (2009), *Real Time Solutions for Financial Reform, An NYU Stern Working Group on Financial Reform*.

Admati, Anat, Peter DeMarzo, Martin Hellwig, and Paul Pfleiderer (2010), "Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Expensive," Stanford Graduate School of Business Research Paper No. 2065.

Admati, Anat, and Paul Pfleiderer (2010), "Increased-Liability Equity: A Proposal to Improve Capital Regulation of Large Financial Institutions," Stanford University.

Albul, Boris, Jaffee Dwight M., and Tchistyi, Alexei (2010), "Contingent Capital Debt and Capital Structure Decisions," Working paper, Hass School of Business, UC Berkeley.

Altman, Edward (1984), "A Further Empirical Investigation of the Bankruptcy Cost Question," *Journal of Finance* 39, 1067–1089.

Black, Fischer, and John Cox (1976), "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions", *The Journal of Finance* 31, 351–367.

Bolton, Patrick, and Frederic Samama (2010), "Contingent capital and long term investors: a natural match?" Working paper, Columbia University.

Bond, Philip, Itay Goldstein, and Edward Prescott (2010), "Market-Based Corrective Actions," *Review of Financial Studies* 23, 781–820.

Calomiris, Charles, and Richard Herring (2011), "Why and how to design a contingent convertible debt requirement," Working paper, University of Pennsylvania.

Constantinides, George (1984), "Strategic Analysis of the Competitive Exercise of Certain Financial Options." *Journal of Economic Theory* 32 (February 1984), 128-38.

Cox, John, Jonathan Ingersoll, and Stephen Ross (1980), "An Analysis of Variable Rate Loan Contracts", *The Journal of Finance* 35, 389–403.

Cox, John, Stephen Ross, and Mark Rubinstein (1979), "Option pricing: A simplified approach," *Journal of Financial Economics* 3, 229–263.

Davis, Douglas, Edward Prescott, and Oleg Korenok (2011), "Market-based corrective actions: an experimental investigation," Working paper 11–01, Federal Reserve Bank of Richmond.

Diamond, Douglas, and Philip Dybvig (1983), "Bank runs, deposit insurance and liquidity," *Journal of Political Economy* 91, 401-419.

Dixit, Avinash, and Robert Pindyck (1994), *Investment Under Uncertainty*, Princeton University Press.

Duffie, Darrell and David Lando (2001), "Term Structures of Credit Spreads with Incomplete Accounting Information," *Econometrica* 69, 633–664.

Flannery, Mark (2002), "No Pain, No Gain? Effecting Market Discipline via Reverse Convertible Debentures," University of Florida, working paper.

Flannery, Mark (2009): "Stabilizing Large Financial Institutions with Contingent Capital Certificates," University of Florida, working paper.

Garbade, Kenneth (2001): *Pricing Corporate Securities as Contingent Claims*, The MIT Press, Cambridge, Massachusetts.

Glasserman, Paul, and Behzad Nouri (2010), "Contingent Capital with a Capital-Ratio Trigger," Working paper, Columbia University.

Glasserman, Paul, and Zhenyu Wang (2011), "Valuing the Treasury's Capital Assistance Program," *Management Science* 57, 1195–1211.

Hilliard, Jimmy, and Adam Schwartz (2005), "Pricing European and American derivatives under a jump-diffusion process: a bivariate tree approach," *Journal of Financial and Quantitative Analysis* 40, 671–691.

Kashyap, Anil, Raghuram Rajan, and Jeremy Stein (2008), "Rethinking Capital Regulation," Federal Reserve Bank of Kansas City Symposium.

McDonald, Robert (2010), "Risks Contingent Capital with a Dual Price Trigger," Working paper, Kellogg School of Management, Northwestern University.

Merton, Robert (1973), "Theory of Rational Option Pricing," *Bell Journal of Economics* 4, 141–183, Spring.

Merton, Robert (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance* 29, 449–470.

Merton, Robert (1976), "Option pricing when underlying stock returns are discontinuous," *Journal of Financial Economics*, 3, 125–144.

Pennacchi, George (2010), "A Structural Model of Contingent Bank Capital," University of Illinois, College of Business.

Spatt, Chester and Frederic Sterbenz (1988), "Warrant Exercise, Dividends, and Reinvestment Policy," *Journal of Finance* 43, 493–506.

Squam Lake Working Group (2009), "An Expedited Resolution Mechanism for Distressed Financial Firms: Regulatory Hybrid Securities," working paper, Council on Foreign Relations, Center for Geoeconomic Studies.

Table and Figures

Table I
Numerical Examples for the Range of Multiple Equilibria

	Parameter	Notation	GBM	JD
Asset				
	Initial asset value	A_0	100.00	100.00
	Risk-free interest rate	r	3.00%	3.00%
	Asset volatility	σ	4.00%	4.00%
	Arrival rate of jumps	λ		4.00
	Mean of log(jump size)	μ_y		-1.00%
	Volatility of jump size	σ_y		3.00%
Bond				
	Par value of bond	$\bar{B}$	87.00	87.00
	Coupon rate of bond	b	3.34%	3.34%
	Years to Maturity	T	5.00	5.00
	Bankruptcy cost	ω	10.00%	10.00%
CC				
	Par value of CC	$\bar{C}$	5.00	5.00
	Coupon rate of CC	c	0.00	0.00
	Years to Maturity	T	5.00	5.00
	Trigger on equity value	K	1.00	1.00
Value				
	Firm value	F_0	98.35	94.74
	Bond value	B_0	88.03	87.00
	Equity value	S_0	[5.86 , 6.46]	[3.84 , 5.44]
	CC value	C_0	[3.86 , 4.46]	[2.30 , 3.90]
	Price range		0.60	1.60

Figure 1 Multiple Equilibria in a One-Step Trinomial Tree Model

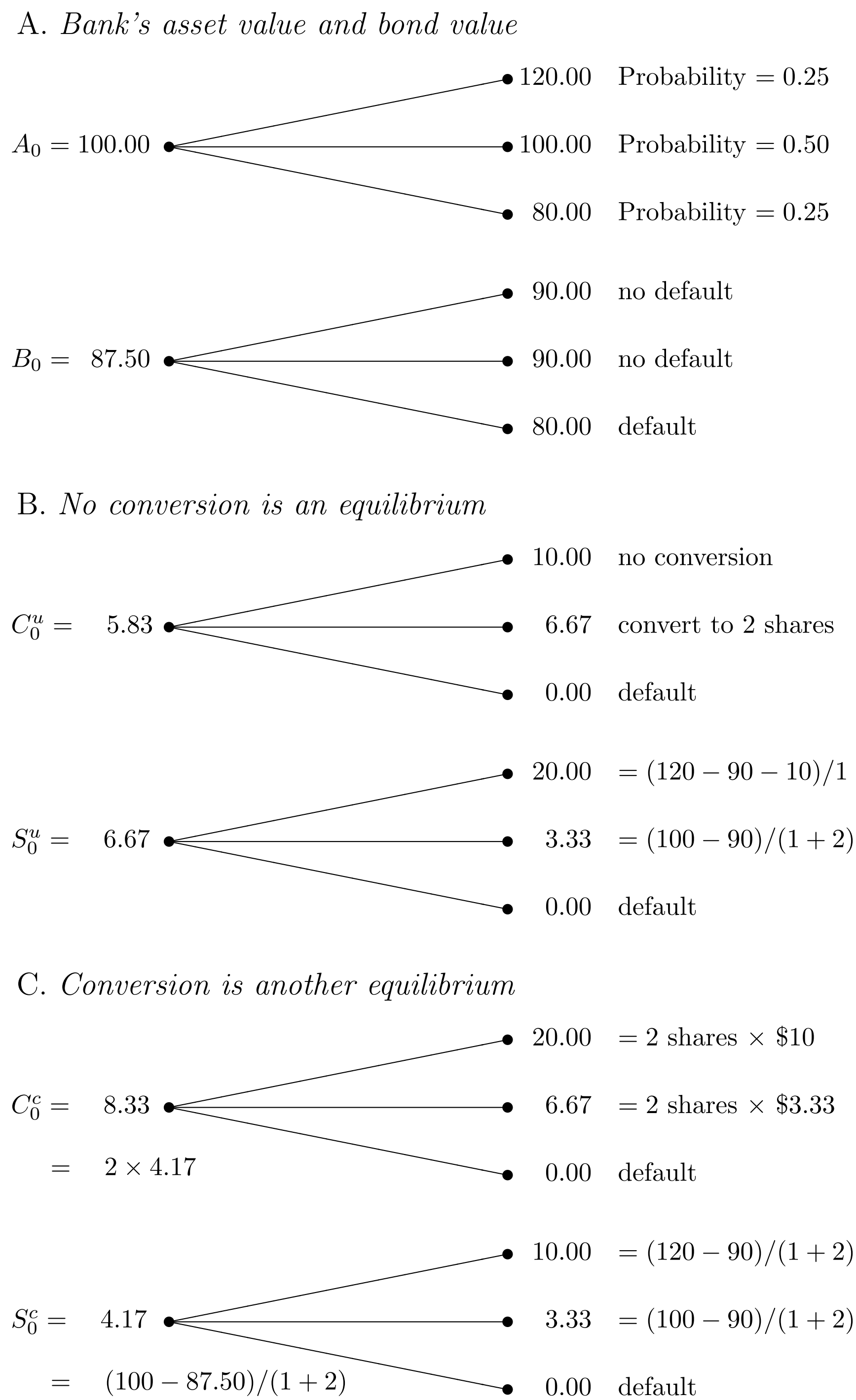


Figure 2 The Range of Multiple Equilibria

The range of multiple equity and CC prices depend on the asset volatility, the leverage in terms of bond and CC, as well as the trigger level. The solid lines represent the upper and lower bounds of the multiple equity prices, and the dot lines represent the bounds of CC values. The parameters used for the figure are the same as those in the second-last column of Table I, except the one that varies in a range indicated by the horizontal axis. For the varying parameter, the value in Table I is indicated by the vertical dash line.

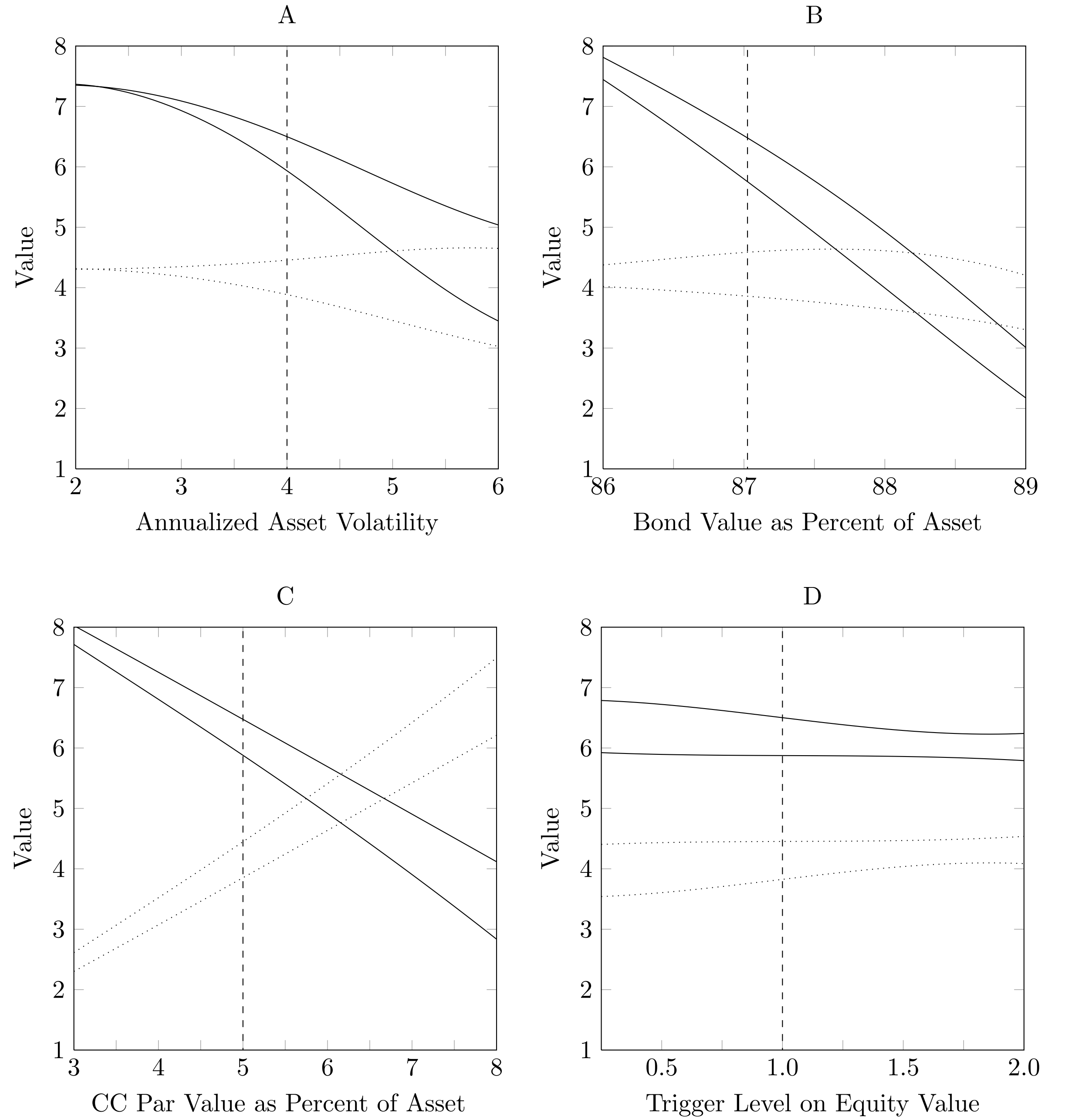


Figure 3 Multiple Equilibria When Equity Issuance Is Allowed

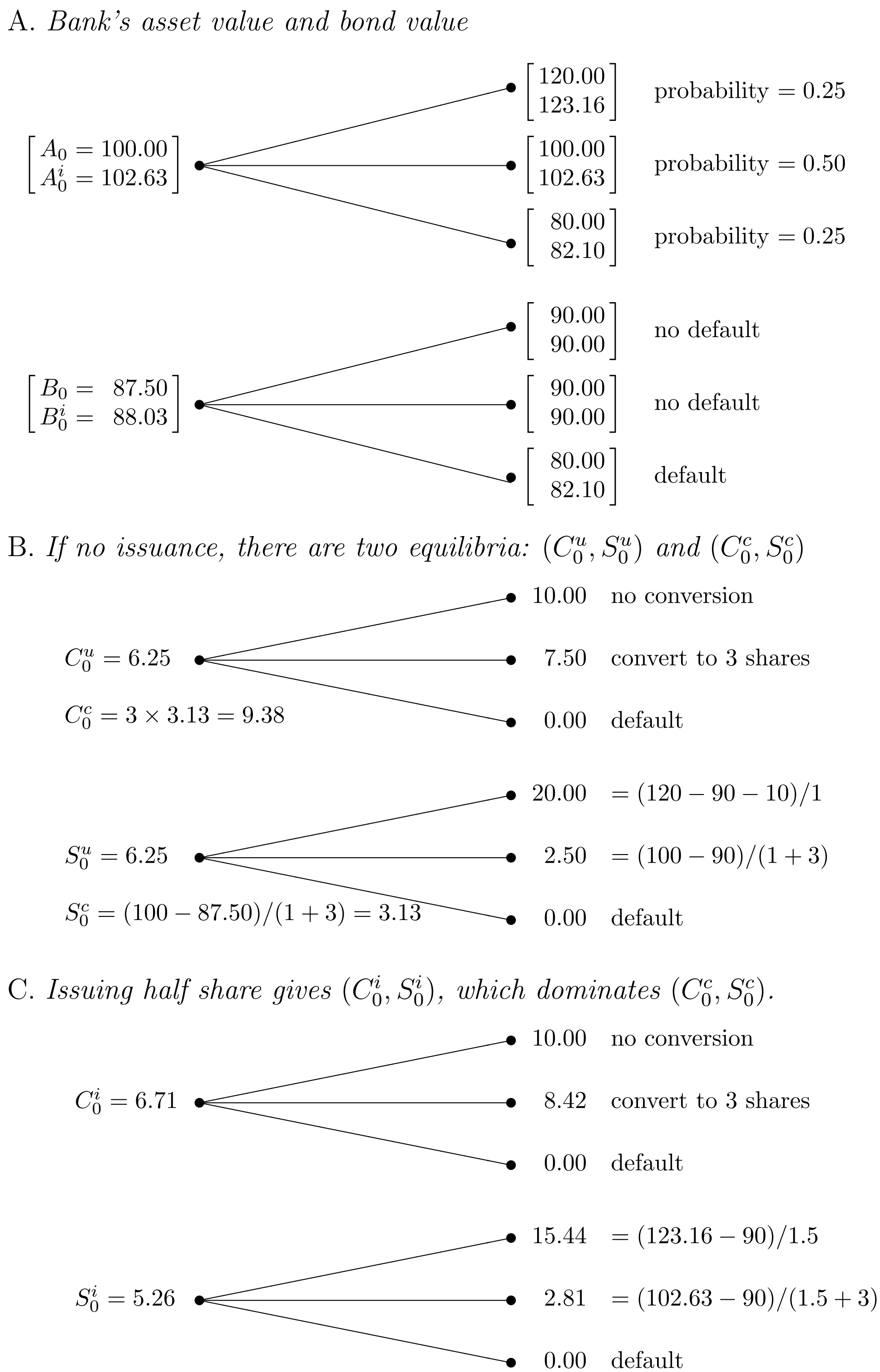
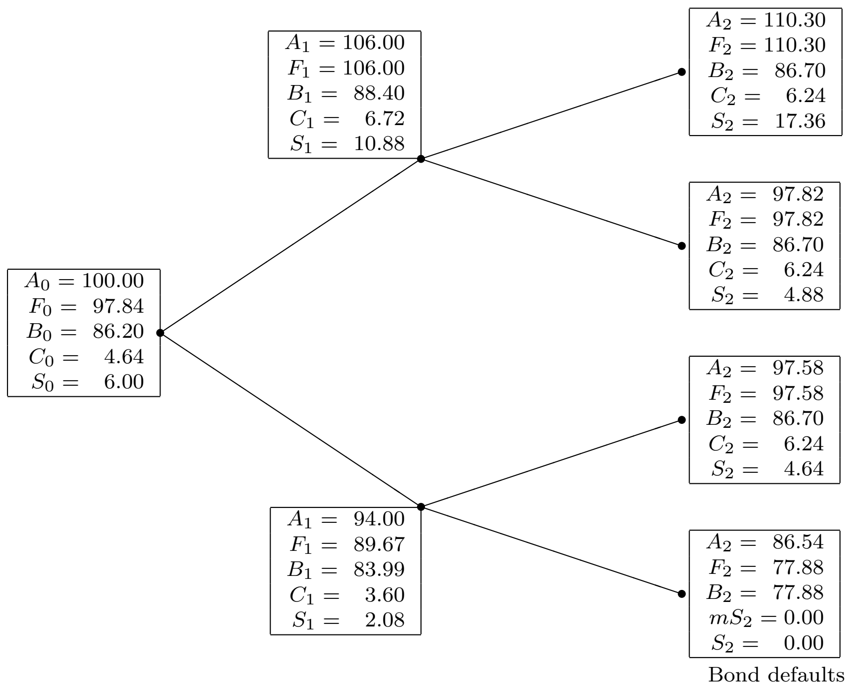


Figure 4 Multiple Equilibria under Financial Distress

A. *An equilibrium without conversion*



B. *An equilibrium with conversion*