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Johannes Bisschop and Alexander Meeraus

On the Development of a General Algebraic Modeling System in a Strategic Planning Environment

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ON THE DEVELOPMENT OF A GENERAL ALGEBRAIC MODELING SYSTEM IN A STRATEGIC PLANNING ENVIRONMENT*

Johannes BISSCHOP** and Alexander MEERAUS
Development Research Center, The World Bank, Washington, DC 20433, *U.S.A.*

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Modeling activities at the World Bank are highlighted and typified. Requirements for successful modeling applications in such a strategic planning environment are examined. The resulting development of a General Algebraic Modeling System (GAMS) is described. The data structure of this system is analyzed in some detail, and comparisons to other modeling systems are made. Selected aspects of the language are presented. The paper concludes with a case study of the Egyptian Fertilizer Sector in which GAMS has been used as a modeling tool.

Key words: Algebraic Modeling System, Modeling Language, Strategic Planning, Applications.

1. Introduction

The first portion of this paper focuses on the dynamics of modeling activities in a strategic planning environment such as the World Bank. This environment is broadly characterized by long-term, often ill-defined and poorly understood issues, which require near immediate decision making. It is the long-term impact of the decisions that make them important. Government planning agencies and corporate planning offices are other examples of a strategic planning environment. Mathematical models are a potentially powerful tool during the process of making good plans and decisions in such an environment, but their effective use has often been limited. This is not only due to the extensive resource requirements in terms of technical skills, money and time, but also because of such intangible issues as the low reliability of model generators, and the extensive communication problems that occur during the dissemination of models and their results.

The second portion of the paper focuses on our efforts to eliminate some of the current barriers to successful modeling applications, namely the development of a General Algebraic Modeling System (GAMS). The aim of this system

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** Presently at Shell Research, Amsterdam, The Netherlands.

is to provide one representation of a model which is easily understood by both humans and machines. We have chosen a rigorous algebraic representation of both data and equations, coupled with relational database-type facilities. With such a notation, the information content of the model representation is such that a machine can not only check for algebraic correctness and completeness, but also interface automatically with solution algorithms and report writers. In Section 4, we compare our choice of data structure to those underlying some of the existing modeling systems that were designed for large-scale linear programming problems. In Section 5, we provide some selected aspects of the language in GAMS in order to illustrate its use as a tool for expressing structural and partitioning information inherent in large models.

The final portion of this paper is devoted to a specific application in the industrial planning area, namely the planning of investments in the Egyptian Fertilizer industry. This section serves as an illustration of a strategic modeling exercise. In the case study, the model was used as a moderator, and was continuously modified as the planning process took place. GAMS was used as the basic modeling tool.

2. The modeling environment at the World Bank

Over the last decade, the World Bank has emerged as a prominent producer of research on development issues [3]. Both social and economic research is done with the following broad objectives: (i) to support all aspects of the Bank's operations, including the assessment of development progress in member countries, (ii) to broaden understanding of the development process, (iii) to improve the Bank's capacity to give policy advice to its members, and (iv) to help develop indigenous research capacity in member countries. The bulk of the Bank's research is organized into research projects which are usually prepared and executed within the Bank. If the expertise of the research staff is limited in a particular area, it is supplemented with that of outside consultants and other institutions. The audience for which research is done consists of policy and operating staff within the Bank, planners and policy makers in developing countries, and the international development community, including other researchers.

Mathematical models play an important role in many of the research projects of the Bank. The research on the planning of investment programs in the manufacturing sector is one example [4, 9]. This project has designed a methodology for investment planning in industrial subsectors where there are economies of scale, such as the forest industry, steel and fertilizer, and where interdependent choices must be made on scale, timing, location, product mix and technology. We will address one specific case study in the last portion of this paper. A second example is the research on income distribution where several projects have used economy-wide general equilibrium models as a framework for analyzing the effects on different income groups of policy interventions that might be undertaken to improve income distribution [1, 5]. A third example is the use of models in country economic analysis based on the construction of social accounting matrices [10]. A fourth example is the modeling framework for the projections of global growth, international trade, and capital flows underlying the Bank's World Development Reports [8]. Other examples of model use are the linear programming research studies that have been used in the formulation and evaluation of agricultural policies [6, 7].

The above examples describe a diversity of model use within the World Bank. The Bank, however, is not unique in this respect. There are many national planning agencies and corporate planning offices where a wide range of modeling activities are employed in the process of better planning. These planning environments have common characteristics and can be typified as follows. The issues under consideration are usually extremely complex, and need to be sorted through. The amount of possibly relevant information is vast. In addition, the consequences of any decision are not necessarily limited to one person or one institution. Nor are all other aspects of the decision necessarily under the jurisdiction of one person or one organization. In such an environment, mathematic models play a special role. They are used as a framework for analysis, for data collection and for discussion. They are created to improve one's conceptual understanding of the problem. If several decision makers and/or institutions are involved in a final decision or set of recommendations, models can be used as neutral moderators to guide the discussions. Different viewpoints can be tested and examined. In such an environment the actual values of model results are not so important, but the relative values resulting from testing different scenario's are of interest. The model is a learning device, and should never be expected to produce final decisions. Because of this indirect importance of a model in a strategic planning environment, there is no clear way to measure the benefits, although it is not too difficult to keep track of the (usually high) costs.

Due to the special role that mathematical models play in a strategic planning environment, there are definite requirements for the success of any modeling exercise. A model is successful if it is easy to understand, if its structure and content can be communicated effectively to others, if the results produced by the model can be explained, if changes in the model can be accomplished on short notice, and if model experiments can be easily repeated or verified by experts other than the original model builders. These high requirements have undoubtedly contributed to the limited role that mathematical models have played thus far in the planning environment of the Bank. These same requirements have also stimulated our ideas for a General Algebraic Modeling System.

The relatively limited use of models in our environment is partly due to the fact that a significant portion of total resources in a modeling exercise (measured in either time, skill or money) is spent on the generation, manipulation and reporting of models. It is evident that this must be reduced greatly if models are to become effective tools in planning and decision making. Other barriers to effective model use have come from attempts to disseminate previous and ongoing research. As modeling is a dynamic process in a planning environment, it becomes an horrendous task to document the many versions of each model, especially when they are large. In addition, experience has shown that it is difficult to communicate models to interested parties that are not part of the development team. As there are no standards in notation, it is practically impossible to judge from any write-up what exactly the model is. Without proper documentation, however, no effective dissemination of knowledge can take place. A third barrier that we have become painfully aware of, is the non-existence of a single interface with different solution routines.

The heart of it all is the fact that solution algorithms need a data structure which, for all practical purposes, is impossible to comprehend by humans, while, at the same time, meaningful problem representations for humans are not acceptable to machines. We feel that the two translation processes required (to and from the machine) can be identified as the main source of difficulties and errors. GAMS is a system that is designed to eliminate these two translation processes, thereby lifting a technical barrier to effective modeling in a strategic planning environment.

3. The development of GAMS

In the previous section, we described and typified the modeling environment in which GAMS evolved. In this section, we would like to portray our basic choice of data structure, and compare this to the apparent design choices in selected other modeling systems. The following statements should not be interpreted as absolute facts, but they do reflect our strong beliefs after several years of experience in modeling activities.

Model building in a strategic planning environment is a dynamic process, where models are used as a way to unravel the complex real-world situation of interest. This implies not only that a model builder must be able to develop and modify models continuously in a convenient manner, but, more importantly, that a model builder must be able to express all the relevant structural and partitioning information contained in the model in a convenient short-hand notation. We strongly believe that one can only accomplish this by adhering to the rigorous and scientific notation of algebra. Only by providing a capability to express partitionings, mappings, nestings and conditional information can we expect to be able to communicate the complexities inherent in large-scale real-world phenomena. With a well-specified algebraic syntax, any model representation can be understood by both humans and machines. The machine can make all the required syntactical and semantic checks to guarantee a complete and algebraically correct model. At the same time, humans with a basic knowledge of algebra can use it as the complete documentation of their model. In addition to this, the algebraic representation contains all the necessary information that is needed for an automatic interface with the various linear and nonlinear solution routines.

The data structure in GAMS resembles that of a sophisticated relational database with an added capacity to handle symbolic algebraic relationships. It does not resemble any general purpose programming language, but instead stays as close as possible to existing algebraic conventions. Some examples of the language are specified in the next section, while the last section illustrates its use in the planning of the Egyptian fertilizer industry. Although it is not possible to provide a detailed comparison of GAMS to other modeling systems in this paper, we would like to compare the underlying data structure of GAMS to the apparent choices made by others. Our selection is guided by personal experience with these systems. The intent is to make some general comments reflecting our views.

Systems such as GAMMA (developed by Bonner and Moore), MAGEN, PDS, OMNI (developed by Haverly Systems) and DATAFORM (Ketron) are some of the most successful LP data management systems in use today. They are often referred to as 'matrix generators', a rather limited description which does not acknowledge their important role in database management and report generation. The key to success for these systems has been the recognition that the major portion of most real-world LP models consists of data, and that these data must be managed efficiently. As a result, they all have an easy-to-use two-dimensional data structure allowing alpha-numeric characters as table row and column identifiers. These identifiers are then used in the generation of equation and variable names, which in turn are used for the generation of reports. The data tables used in GAMS are similar, except that all identifiers, however many there are, must be carried separately in the row and column labels for each table. This results in multi-dimensional labels whenever the data elements in a table are identified by more than two identifiers. We have chosen the more restrictive form for several reasons. In order to communicate models and their associated data, it is important that data tables are self-explanatory. In that case, any outsider familiar with the data can read the table without having to ask questions regarding the information content of each label. In addition, carrying this partitioning information along will allow the user to express subsequently all algebraic and logical relationships between the various data elements. This is a relevant factor, since many data elements occurring in the equations of a model are not collected directly, but are generated in some algebraic and/or logical fashion from more basic data.

Another principal choice for any of these systems is that they provide the user with a short-hand notation for the MPS-tape, the more than 30-year old industry standard for interfacing with an LP solver. This means that any model builder using such a language is forced into the process of concatenating strings of at most eight characters in order to generate the row and column names needed to identify the non-zero elements in the MPS-tape. Centering on the MPS-tape also implies that any structural information contained in the model can only be passed on via the labeling scheme for the rows and columns. We feel that in this case, the partitioning information is essentially lost as any understanding of the underlying model structure requires a key for decoding the information. The data structure in GAMS allows one to express all partitioning information directly via the use of algebra. The information content of this representation is much higher than that of an MPS-tape: one can program a machine to translate the algebraic representation into an MPS-tape, but not the other way around. As we already pointed out previously, the algebraic notation allows for a machine-intensive modeling technology where syntactically and semantically correct models can be interfaced automatically with a large variety of linear and nonlinear solution algorithms.

Other modeling systems such as DATAMAT, MGG and ALPS differ in one respect or another from the three systems mentioned above. DATAMAT, developed by the National Bureau of Economic Research, is a system designed around interactive LP model building. It also centers around the construction of the MPS-tape, but uses built-in macro's to allow for a quick and compact interface with the LP modeling system. This design characteristic is both the strength and the weakness of the system: although the model representation is extremely compact, it is difficult to understand and to communicate. In this respect, it is further removed from GAMS than the four LP data management systems discussed previously. A rather different system has been developed by Scicon, and is called MGG, Matrix Generator Generator. This system can be viewed as a short-hand notation for a FORTRAN program matrix generator, and is probably the first commercial system to have chosen a row-wise (equation-wise) representation of the model. In addition, being so close to a general purpose programming language, MGG has the advantage of being relatively fast during execution time. Unfortunately, it has also inherited some characteristics of FORTRAN that are less desirable for model representations. One example is that all data elements in FORTRAN are accessed by position and not by non-numeric labels, which does not permit a database-like language such as the one we have chosen in GAMS. The model notation in MGG also differs from the notation in GAMS in that the model builder is still required to specify the concatenation scheme for the eight-character equation and variable names. The first system in which the user does not have to be concerned with concatenating the eight-character equation and variable names for the MPS-tape is ALPS. ALPS stands for advanced linear programming system, and has been marketed by United Computing Systems. Like MGG, it is a FORTRAN-based and equation-oriented system, also suffering from a lack of flexible database facilities. Both the choice of database facilities and the notational restrictions in

ALPS make the system different from GAMS. One example is the restriction that data arrays in ALPS cannot be more than two dimensions, which, we feel, inhibits a natural formulation of the problem. In spirit, however, the ALPS system is closest to GAMS, as it concerns itself with a model representation, which, as far as the user is concerned, is independent of any input requirements imposed by the various solution algorithms.

The above comparisons of GAMS with other modeling systems reflect our personal views and experiences. It goes almost without saying that most ideas embodied in GAMS can be traced to one or more of the systems discussed in this section. GAMS seems to be a natural outgrowth of these systems, providing a rigorous but flexible algebraic representation (data structure) whereby a machine can take the responsibility for the correctness of the model and for the automatic interface with solution algorithms.

4. Selected aspects of the modeling language in GAMS

While the previous section concerned itself with general aspects of GAMS, this section will illustrate selected details of the language in GAMS. We should state clearly that this section is not designed to convince any reader that our choice of language is superior to any other modeling language. We merely want to illustrate that our choice of data structure does provide a framework for expressing structural information contained in large models. The examples in this section are supplemented with the application of the last section. For a more extensive description of the language, the reader is referred to [2].

A model statement in GAMS can be viewed as an integrated database. In addition to the data tables and assignment statements, there are the symbolic equations which represent data that can only be obtained via some solution algorithm. Both data and symbolic equations are used for a complete model definition within GAMS. We have restricted ourselves to a small character set which is available on most computers. We also have assumed that there is no carriage control available (i.e., no subscripts or superscripts), and that there are only capital letters. Besides the usual algebraic and logical operators found in other languages, we have introduced one new operator, namely the $-operator. This is a conditional operator which can be inserted throughout the language (see Section 4.4). There are several key words used in the language, each one identifying important components of a model. Some of the main ones are: SET, PARAMETER, TABLE, VARIABLE, EQUATION and MODEL. The following subsections each describe a selected aspect of the language.

4.1. Sets and set mappings

A simple (one-dimensional) set in GAMS is a finite collection of labels. These sets play an important role in the indexing of algebraic statements. One-dimensional sets can be related to each other in the sense that there exists a correspondence between them. The syntax for sets and set correspondences are alike. As an example, consider the following correspondence between the simple sets of regions, water zones and districts.

```
SET  RZD  REGION ZONE DISTRICT MAPPING /

          NORTH.IRRIGATED.(W-NORTH, C-NORTH, E-NORTH)
          CENTRAL.(IRRIGATED.(NW-UPPER, NE-UPPER)
                   RAINFED.(S-UPPER, W-LOWER, E-LOWER))
                   :                                  /;
```

After the key word set comes the name of the set, followed by a description of the set name (optional). The (three-dimensional) elements of the set are contained between the 'slash' separators. Note that the period is used as a separator of the dimensions embedded in each element, and that the order of the dimensions is fixed (in this case regions first, water zones second and districts third). In order to reduce unnecessary repetition of labels, parentheses have been used. The above three lines represent eight three-dimensional elements of the set RZD.

4.2. Data tables

Tabular arrangements are a very convenient way to describe data elements. As we discussed previously, a fundamental part of the data structure in GAMS is that all identifying information for a data element must be explicit in its description, and must be carried along in any later references. The different identifiers (set elements) can be contained in either the row or column labels. If there is more than one identifier embedded in a label, the period is again used as a separator. The following table illustrates a four-dimensional parameter, where three dimensions are captured in the row description (namely regions, crop rotations and production technologies), while the fourth dimension (time) is contained in the column label.

```
TABLE  L  LABOR COEFFICIENTS IN HOURS PER RAI

                                  JANUARY  FEBRUARY  MARCH  APRIL
NORTH-UPP.SUGARCANE.TRAD-BUFF        2         2       2      12
NORTH-UPP.SUGARCANE.MOD-TRACT        1         2       2      10
             :
+                                   MAY      JUNE     JULY  AUGUST
NORTH-UPP.SUGARCANE.TRAD-BUFF       12        35       30     45
NORTH-UPP.SUGARCANE.MOD-TRACT       12        30       30     40
```

The order of the sets used in the row and column description in the table statement must be maintained in later references to the parameter. For the above example this will be L(R,C,T,M) where R, C, T and M refer to the simple sets of regions, crop rotations, technologies and months respectively.

4.3. Assignment and equation statements

Most of the syntax used in assignment statements and equations are the same, although it is straightforward to detect if a GAMS statement is either an assignment or an equation.

An assignment statement in GAMS is an instruction to perform some data manipulation and store the result. It can be compared to a FORTRAN statement where the result of the operations performed is stored under the name that appears on the left side of the equal sign. As an example, consider the parameter DIST(I,J) indicating the distance from location I to location J, where the elements in the sets I and J are identical. Assume that initially only the lower triangular part of DIST was specified in a TABLE statement (Section 4.2), and that we are interested in specifying the entire TABLE. One should note that all values of DIST(I,J) that are not defined in the TABLE statement, are assumed to be zero. We can write the following algebraic statement.

```
DIST(I,J) = DIST(I,J) + DIST(J,I) ;
```

This statement is implicitly defined for all two-tuples of the Cartesian product of the sets I and J. The entries of DIST(I,J) on the left will be replaced in a parallel fashion with the results from the additions on the right. An equation in GAMS is a symbolic representation of one or more constraints to be used as part of a simultaneous system of equation, or an optimization model. It always begins with the equation name, possibly indexed, followed by two dots (periods). Each symbolic equation has a type associated with it. Possible types are =L= (for less than or equal to constraints), =G= (for greater than or equal to constraints), and =E= (for equality constraints). An example is given in the next section.

4.4. The $ operator

Partitioning large models by using driving indices provides an elegant shorthand notation. Complexities, however, are introduced when there are restrictions imposed on the partitionings. As these complexities arise continually in large-scale models, we have strived for an elegant and effective way to incorporate them in a model statement. Let us begin with an example. Define the sets R and D as regions and districts respectively. Assume that for each district in a region we know the level of income YD(R,D), and that we want to determine the regional income YR(R) for each of the regions. Writing the assignment statement

```
YR(R) = SUM(D, YD(R, D)) ;
```

is meaningless as not every district is contained in each region. We need to use, therefore, the relationship between the sets R and D. Let RD be the set correspondence between these two sets. Then we can write the following assignment statement

```
YR(R) = SUM(D $ RD(R, D), YD(R,D)) ;
```

Here the dollar sign is used as a conditional operator. For each specific region R it restricts the sum to be over those elements of D for which the correspondence RD(R,D) is defined.

More generally, let A be a name or an expression in GAMS, and let B be a name or a true-false expression. Then the phrase A $ B is a conditional statement in GAMS where the name A is considered or the expression A is evaluated if and only if the name B is defined or the expression B is true.

A second example illustrates the conjunctive use of the dollar operator and logical phrases contained in an assignment statement. Let the sets P, I and M denote processes, plants and machines respectively. The parameter K(M,I) denotes the number of units of available capacity of machine M in plant I, while the parameter B(M,P) describes the required number of units of capacity of machine M per unit level of process P. We want to define a zero-one parameter, PPOSS(P,I), indicating which processes P need to be considered for plant I. We can write the following set of logical relations always resulting in either a zero or one.

```
PPOSS(P,I) = SUM(M $ (K(M,I) EQ 0), B(M,P) NE 0) EQ 0;
```

Here the expression B(M,P) NE 0 will contain a value 1 if process P is dependent on machine M, and 0 otherwise. These values are summed over all machines M that are not available in plant I. If the resulting sum is zero for process P, then the process is not dependent on unavailable machines, and should therefore be considered. Note that PPOSS is one in this case. If the resulting sum is not 0, the process is dependent on at least one unavailable machine, and should therefore not be considered. The parameter PPOSS is set to zero in this case.

When the dollar operator appears in an equation statement, it is used to control the generation of equations and/or variables. As an illustration let CAP be an equation name referring to capacity constraints, and let Z be a variable name referring to levels of process operation. Using the notation of the previous paragraph, we can write the following symbolic equation.

```
CAP(M,I) $ (K(M,I) GT 0)..
  SUM(P $ PPOSS(P,I), B(M,P) * Z(P,I)) =L= K(M,I);
```

In this example, the system will generate an equation for a specific pair of machines and plants only when the capacity of that machine in that plant is strictly positive. Similarly, only those variables that refer to processes which can be operated at a positive level will be generated.

5. The Egyptian fertilizer sector; A case study using GAMS

In this section, we will report on a real-world application where GAMS was used as a model building tool. It is a case study to evaluate the present structure and development potential of the Egyptian fertilizer industry using several planning models. We will not be able to describe the entire family of models, but instead will provide some of the background, some experiments and some of the results. This section relies heavily on a recently published book called "The planning of investment programs in the fertilizer industry" [4]. We would like to point out that the GAMS representation of the one-period Egyptian fertilizer model is listed in the appendix of this book. Although the model is carefully developed in the various chapters, the GAMS version is the only document that finally presents a complete description of the model.

The problem in the Egyptian fertilizer industry is as follows. Given that fertilizer use will continue to increase from present levels, and that existing production capacity is not sufficient to meet even current demand, Egyptian planners are faced with the question as to what the best policy is for Egypt to adopt in order to meet future fertilizer demand requirements. Would it be preferable to import fertilizers, to produce them domestically, or both? If some fertilizers are to be imported, which ones should be imported? At what scale should fertilizer production take place? Which feedstocks should be used? What is the least-cost transport pattern for both imported and domestically produced fertilizers? Should intermediate products be shipped between plants? In an effort to deal with these questions from the point of view of the sectoral planner, a family of dynamic, linear, mixed-integer planning models of the Egyptian fertilizer industry was built in collaboration with Egyptian authorities.

The family of Egyptian fertilizer models serves as a typical example of strategic modeling: a reference model is continuously modified to reflect the learning process of the parties involved. The starting point of the analysis is the actual recorded use of fertilizer material by type in each of the twenty governorates of Egypt in 1975. The supply of these fertilizers originated either from domestic production facilities in 1975 or from imports. Domestic supplies are, however, subject to capacity constraints that are initially defined in terms of the actual production levels achieved in 1975. Fertilizer materials can be transported from supply sources to the various marketing centers by water, road or rail. In accordance with the actual situation, however, it is first assumed that all final products are transported by rail, and that all raw materials are transported by boat, and if necessary by train. With this specification, the model can be used to select the least-cost supply and shipment pattern for fertilizers in 1975. It can also determine whether a fertilizer should be imported or produced domestically, given the existing production facilities and their location.

The major portion of effort at this stage is the collection of data to support the simple model. GAMS was used as an organizing device for the specification of the model equations and the lay-out of the empty data tables. As such it served as a means to communicate the model and the data needs to the Egyptian planners. These tables were then updated or modified in accordance with the availability of data. By separating technical information from political and judgmental data, the simple model helped the several parties involved in focusing on relevant issues. Once the model specification was acceptable to everyone concerned, minor variations in specification were introduced.

The following refinements of the simple model were made. First, the restriction on interplant shipments of intermediate products was dropped. Then the capacity constraints were relaxed to allow 100% capacity utilization. Finally the model was altered to investigate the implications of greater flexibility in fertilizer use by specifying nutrient requirements (in terms of nitrogen, phosphorus and potassium) rather than requirements by fertilizer type. As all these model refinements were easily expressed in terms of algebra, GAMS was used as a documenting device (See Appendix).

The purpose of running these static (one period) models is to determine if there are any short-run improvements in the operation of the fertilizer sector. Given their simplified nature, however, conclusions along these lines are only tentative and indicative. In this respect, the model serves as a device to generate particular options deserving further study. In the case of introducing the interplant shipments into the model, the results showed a non-negligible 24% total cost saving. This observation led to further investigations into the possibility of interplant shipments. The second refinements, namely, allowing increases in capacity utilization in existing facilities, did not result in any substantial payoffs. The most interesting refinement of the basic model was to change the demand specification in terms of products into a demand specification in terms of nutrient requirements. Letting the model decide on the least-cost mix of fertilizer products meeting the nutrient requirements caused a drop in the objective function value of 12%. This outcome caused the Egyptian planners to focus on the likely rate of adoption by Egyptian farmers of relatively new fertilizers such as urea.

After the simple static model and its refinements were used to focus on various issues and to build confidence in the modeling exercise, a dynamic medium-term version of the model was built. This version introduces time and addresses directly the issue of economies of scale associated with production and capacity expansion over time. With a model of this type, it is standard procedure to solve the model using estimates of the parameters that the analysts consider most likely to materialize. Since many of the estimates involve projections into the future, the values eventually chosen may reflect a certain degree of compromise among the diverging opinions of the planners involved in the study. These, in turn, provide a useful demarcation of the range of values to be investigated in the subsequent sensitivity analysis. Following the basic run and some sensitivity studies of the dynamic model, specific scenarios regarding domestic production patterns, imports and exports were investigated. At this point we refer to [4] for specific details.

One qualification of the above modeling approach should be noted. The models developed in this case study are simplified representations of reality, designed to guide decision making, not to replace it. They are merely efficient tools to evaluate and quantify the implications of a certain understanding of the economic and technical relationships that typify the fertilizer industry and the environment in which the industry is supposed to function. We also should point out some of the limitations of this approach to investment planning. First and foremost, the approach requires a set of projections of demand for the final products. As the supply price for final products is not known at the outset, the demand projections need to be based on price assumptions that may turn out to be incorrect. Another limitation is that by definition the demand projection for final products excludes the possibility of substitution among products on the basis of supply price considerations. Finally, the state of art in the computational area does not permit uncertainty to be incorporated. Despite these limitations, this modeling approach has proven to be a successful aid in the planning of the Egyptian fertilizer sector, and various specific model results were judged to be meaningful at an operational level.

A complete GAMS statement of the static one-period model is provided in the Appendix as an illustration of a medium-to-large real-world model.

6. Conclusions

Mathematical models built in a strategic planning environment such as the World Bank can play a useful and even powerful role in the overall planning process. The case study of the Egyptian fertilizer industry described in Section 5 is a good example thereof. The modeling process in such an environment is a dynamic one, as models are continuously modified. This imposes special requirements on the success of any modeling exercise. Based on our experience we have concluded that the key to success is a modeling technology where only one model representation is needed to communicate with both humans and machines. The language should be a powerful notation which can express all the relevant partitioning and structural information contained in the real-world problem. In addition, the information content of the model representation should be such that a machine can take over the responsibility for verifying the algebraic correctness and completeness of the model. GAMS is a system which is designed around these principles, and is a natural outgrowth of several of the existing modeling systems. In our environment, it has grown into a standard instrument for the representation and generation of mathematical models.

Appendix. GAMS listing of Egyptian fertilizer model

Set definitions

```
SET I  PLANT LOCATIONS   /

       ASWAN
       HELWAN
       ASSIOUT
       KAFR-EL-ZT
       ABU-ZAABAL
                          /

    J  DEMAND REGIONS     /

       ALEXANDRIA      ALEXANDRIA
       BEHERA          DAMANHUR
       GHARBIA         TANTA
       KAFR-EL-SH      KARE EL-SHEIKH
       DAKAHLIA        EL MANSURA

       DAMIETTA        DAMIETTA
       SHARKIA         ZAGAZIG
       ISMAILIA        ISMAILIA
       SUEZ            SUEZ
       MENOUFIA        SHIBIN EL KOM

       KALUBIA         BENHA
       GIZA            GIZA
       BENI-SUEF       BENI-SUEF
       FAYOUM          EL-FAYOUM
       MINIA           EL-MINIA

       ASSIOUT         ASSOIUT
       NEW-VALLEY      EL KHARGA
       SOHAG           SOHAG
       QUENA           QUENA
       ASWAN           ASWAN
                                 /
    M  PRODUCTIVE UNITS          /

       SULF-A-S    SULFURIC ACID: SULFUR
       SULF-A-P    SULFURIC ACID: PYRITES
       NITR-ACID   NITRIC ACID
       AMM-ELEC    AMMONIA: WATER ELECTROLYSIS
       AMM-C-GAS   AMMONIA: COKE GAS
       C-AMM-NITR  CALCIUM AMMONIUM NITRATE
       AMM-SULF    AMMONIUM SULFATE
       SSP         SINGLE SUPERPHOSPHATE
                        /
    P  PROCESSES        /

       SULF-A-S    SULFURIC ACID: SULFUR
       SULF-A-P    SULFURIC ACID: PYRITES
       NITR-ACID   NITRIC ACID
       AMM-ELEC    AMMONIA: WATER ELECTROLYSIS
       AMM-C-GAS   AMMONIA: COKE GAS
```

Set definitions (contd.)

```
      CAN-310     CALCIUM AMMONIUM NITRATE: 31.0 PCT
      CAN-335     CALCIUM AMMONIUM NITRATE: 33.5 PCT
      AMM-SULF    AMMONIUM SULFATE
      SSP-155     SINGLE SUPERPHOSPHATE: 15.5 PCT
                                         /
   CQ NUTRIENTS        /

      N           NITROGEN
      P205        PHOSPHORUS
                                      /
   CF FINAL PRODUCTS (FERTILIZERS)    /

      UREA
      CAN-260     CALCIUM AMMONIUM NITRATE: 26.0 PCT
      CAN-310     CALCIUM AMMONIUM NITRATE: 31.0 PCT
      CAN-335     CALCIUM AMMONIUM NITRATE: 33.5 PCT
      AMM-SULF    AMMONIUM SULFATE
      DAP         DIAMMONIUM PHOSPHATE
      SSP-155     SINGLE SUPERPHOSPHATE: 15.5 PCT
      C-250-55    COMPOUND 25-5.5-0
      C-300-100   COMPOUND 30-10-0
                                 /
   CI INTERMEDIATE PRODUCTS      /

      AMMONIA
      NITR-ACID   NITRIC ACID
      SULF-ACID   SULFURIC ACID
                                                /
   CS INTERMEDIATES FOR SHIPMENT / AMMONIA, SULF-ACID /

   CR DOMESTIC RAW MATERIALS AND MISCELLANEOUS INPUTS  /

      EL-ASWAN    ELECTRICITY FROM ASWAN DAM
      COKE-GAS    COKE-OVEN GAS
      PHOS-ROCK   PHOSPHATE ROCK
      LIMESTONE
      EL-SULFUR   ELEMENTAL SULFUR
      PYRITES
      ELECTRIC    ELECTRICITY
      BF-GAS      BLAST-FURNACE GAS
      WATER       COOLING WATER
      STEAM
      BAGS
                        /
 ALIAS (I,IP);

SET C  ALL COMMODITIES ; C(CF)=YES; C(CI)=YES; C(CR)=YES;
```

Consumption and demand data

```
106              TABLE CF75  CONSUMPTION OF FERTILIZER 1974-75 (1000 TPY)
107
108                       CAN-260    CAN-310    CAN-335   AMM-SULF       UREA
109
110     ALEXANDRIA                                 5.0        3.0        1.0
111     BEHERA              1.0                   25.0       90.0       35.0
112     GHARBIA                                   17.0       60.0       28.0
113     KAFR-EL-SH          1.0                   10.0       45.0       22.0
114     DAKAHLIA            1.0                   26.0       60.0       20.0
115
116     DAMIETTA                                   2.0       15.0        8.0
117     SHARKIA             1.0                   31.0       50.0       28.0
118     ISMAILIA                                   4.0        6.0        2.0
119     SUEZ                                       1.0
120     MENOUFIA            1.0                   24.0       21.0       30.0
121
122     KALUBIA                                   25.0       16.0        7.0
123     GIZA                                      40.0        6.0        2.0
124     BENI-SUEF           1.0                   15.0        1.0       20.0
125     FAYOUM              1.0                   20.0        6.0       20.0
126     MINIA               2.0       15.0        35.0        1.0       41.0
127
128     ASSIOUT             1.0       20.0        26.0        1.0       27.0
129     NEW-VALLEY                                                       1.0
130     SOHAG                         65.0         3.0                   7.0
131     QUENA                         95.0         2.0                   3.0
132     ASWAN                         40.0
133
134        +              SSP-155   C-250-55  C-300-100        DAP
135
136     ALEXANDRIA          8.0
137     BEHERA             64.0        1.0         .1         .1
138     GHARBIA            57.0        1.0         .2         .1
139     KAFR-EL-SH         25.0        2.0         .1
140     DAKAHLIA           52.0        1.0
141
142     DAMIETTA            5.0
143     SHARKIA            43.0        1.0         .1
144     ISMAILIA            4.0
145     SUEZ                1.0
146     MENOUFIA           33.0        2.0         .1         .1
147
148     KALUBIA            22.0        1.0                    .1
149     GIZA               14.0        1.0         .1
150     BENI-SUEF          13.0        3.0
151     FAYOUM             17.0        1.0
152     MINIA              50.0        3.0         .2         .1
153
154     ASSIOUT            35.0        5.0         .1
155     NEW-VALLEY          1.0
156     SOHAG              20.0        1.0
157     QUENA               8.0
158     ASWAN               8.0
```

Consumption and demand data (contd.)

```


          TABLE  ALPHA  NUTRIENT CONTENT

                    N        P205

   UREA           .46
   CAN-260        .26
   CAN-310        .31
   CAN-335        .335
   AMM-SULF       .206
   DAP            .18        .46
   SSP-155                   .15
   C-250-55       .25        .055
   C-300-100      .30        .10

    PARAMETERS CN75    CONSUMPTION OF NUTRIENTS 1974-75 (1000 TPY) ;

    CN75(J,CQ)        = SUM(CF, ALPHA(CF,CQ)*CF75(J,CF));
    CN75("TOTAL",CQ) = SUM(J, CN75(J,CQ));

    DISPLAY CN75;
```

Transportation data

```
             TABLE ROAD  ROAD DISTANCES (KMS)

                          ABU-KIR KAFR-EL-ZT     TALKHA ABU-ZAABAL      HELWAN

      ALEXANDRIA               16        119        187        210         244
      BEHERA                   76         42        120         50         184
      GHARBIA                 150         20         55         65         122
      KAFR-EL-SH              145         20         35        105         162
      DAKAHLIA                208         58          3        138         152

      DAMIETTA                267        131         66        216         233
      SHARKIA                 240         78         58         60         110
      ISMAILIA                365        241        146        142         173
      SUEZ                    370        246        298        224         178
      MENOUFIA                157         33         90        154         109

      KALUBIA                 190         66         81         97          76
      GIZA                    287        133        146         48           9
      BENI-SUEF               359        248        261        163         105
      FAYOUM                  341        230        243        145          88
      MINIA                   384        372        386        288         230

      ASSIOUT                 616        504        518        420         362
      NEW-VALLEY              815        703        717        619         561
      SOHAG                   715        603        617        519         461
      QUENA                   858        746        760        662         604
      ASWAN                  1134       1022       1036        938         880

          +                  SUEZ    ASSIOUT      ASWAN

      ALEXANDRIA              362        607       1135
      BEHERA                  288        547       1065
      GHARBIA                 226        485       1003
      KAFR-EL-SH              266        525       1043
      DAKAHLIA                219        515       1033

      DAMIETTA                286        596       1114
      SHARKIA                 214        473        991
      ISMAILIA                 89        536       1054
      SUEZ                               541       1059
      MENOUFIA                213        472        990

      KALUBIA                 180        439        957
      GIZA                    169        372        890
      BENI-SUEF               270        257        775
      FAYOUM                  252        308        826
      MINIA                   394        132        650

      ASSIOUT                 527                   518
      NEW-VALLEY              726        199        519
      SOHAG                   626         99        419
      QUENA                   769        242        276
      ASWAN                  1045        518
```

Transportation data (contd.)

```
        TABLE RAIL   INTERPLANT RAIL DISTANCES (KMS)

            KAFR-EL-ZT  ABU-ZAABAL  HELWAN  ASSIOUT  ASWAN
ABU-ZAABAL     85
HELWAN        142            57
ASSIOUT       504           420      362
ASWAN        1022           938     880       518


        TABLE IMPD IMPORT DISTANCES (KMS)

              BARGE      ROAD

KAFR-EL-ZT      104         6
ABU-ZAABAL      210        .1
HELWAN          183
ASSIOUT         583
ASWAN          1087        10


PARAMETERS MUF   TRANSPORT COST (LE PER TON): FINAL PRODUCTS
           MUFV  TRANSPORT COST (LE PER TON): IMPORTED FINAL PRODUCTS
           MUI   TRANSPORT COST (LE PER TON): INTERPLANT SHIPMENT
           MUR   TRANSPORT COST (LE PER TON): IMPORTED RAW MATERIALS ;

  RAIL(I,IP) = RAIL(I,IP) + RAIL(IP,I);
  ROAD(J,"IMPORT-PTS") = ROAD (J,"ABU-KIR");

  MUF(I,J)     = ( .5 + .0144*ROAD(J,I)             )$ROAD(J,I);
  MUFV(J)      = ( .5 + .0144*ROAD(J,"IMPORT-PTS") )$ROAD(J,"IMPORT-PTS");
  MUI(I,IP)    = (3.5 + .0300*RAIL(I,IP)            )$RAIL(I,IP);
  MUR(I)       = (1.0 + .0030*IMPD(I,"BARGE")       )$IMPD(I,"BARGE")
               + ( .5 + .0144*IMPD(I,"ROAD" )       )$IMPD(I,"ROAD ");

 DISPLAY MUF,MUFV,MUI,MUR;
```

Technology data

```
        TABLE A  INPUT-OUTPUT COEFFICIENT

            SULF-A-S  SULF-A-P  NITR-ACID  AMM-ELEC  AMM-C-GAS

EL-ASWAN                                     -12.0
COKE-GAS                                                -2.0
PYRITES                  -.826
EL-SULFUR    -.334
SULF-ACID    1.0         1.0
AMMONIA                            -.292      1.0       1.0
NITR-ACID                          1.0

ELECTRIC     -50         -75       -231                 -1960
BF-GAS                                                   -609
WATER         -20        -60       -.6                   -700
STEAM                                                      -4

   +       SSP-155  CAN-310  CAN-335  AMM-SULF

PHOS-ROCK    -.62
SULF-ACID    -.41                       -.76
AMMONIA               -.20     -.21     -.26
NITR-ACID             -.71     -.76
LIMESTONE             -.12     -.04
SSP-155      1.0
CAN-310               1.0
CAN-335                        1.0
AMM-SULF                                 1.0

BAGS       -22.      -23.    -23.      -22.
WATER       -6.      -49.    -49.      -17.
ELECTRIC   -14.                        -19.
STEAM                  -.4    -.4


        TABLE B  CAPACITY UTILIZATION COEFFICIENT

            SULF-A-S  SULF-A-P  NITR-ACID  AMM-ELEC  AMM-C-GAS

SULF-A-S        1
SULF-A-P                  1
NITR-ACID                           1
AMM-ELEC                                       1
AMM-C-GAS                                                 1

   +        SSP-155  CAN-310  CAN-335  AMM-SULF

SSP            1
C-AMM-NITR              1        1
AMM-SULF                                  1
```

Prices

```
  PARAMETER PV  IMPORT PRICE (CIF US$ PER TON 1975)     /

 PYRITES       17.5
 EL-SULFUR     55

 UREA         150
 CAN-260       75
 CAN-310       90
 CAN-335      100
 AMM-SULF      75
 DAP          175
 SSP-155       80
 C-250-55     100
 C-300-100    130


         TABLE PD  DOMESTIC RAW MATERIAL PRICES

            LIMESTONE   COKE-GAS  EL-ASWAN  PHOS-ROCK
*              LE/TON   LE/MNCF     LE/MWH     LE/TON

 KAFR-EL-ZT                                       5.0
 ABU-ZAABAL                                       4.0
 HELWAN           1.2        16
 ASSIOUT                                          3.5
 ASWAN            1.2                  1


  PARAMETER PMISC  MISC. MATERIAL COST     /

 ELECTRIC  .007   LE/KWH
 BF-GAS    .007   LE/CM
 WATER     .031   LE/CM
 STEAM    1.25    LE/TON
 BAGS      .28    LE/UNIT
                                               /;


  PD(I,CR)$PMISC(CR) = PMISC(CR);

  DISPLAY PD;
```

Capacity data

```
        TABLE DCAP  DESIGN CAPACITY OF PLANTS (T/DAY)

            SULF-A-S  SULF-A-P  NITR-ACID  AMM-ELEC  AMM-C-GAS

ASWAN                             800         450
HELWAN                            282                   172
KAFR-EL-ZT    200        50
ASSIOUT       250
ABU-ZAABAL    242       227

    +       SSP  C-AMM-NITR  AMM-SULF

ASWAN              1100
HELWAN              364         24
KAFR-EL-ZT  600
ASSIOUT     600
ABU-ZAABAL  600


PARAMETER K INITIAL CAPACITY (1000 TPY); K(M,I) = .33*DCAP(I,M);

SCALARS EK   EXCHANGE RATE    /.4/
        UTIL UTILAZATION      /.85/ ;

SETS  MPOS    PRODUCTIVE UNITS POSSIBILITIES
      PPOS    PROCESS POSSIBILITIES
      CPOS    COMMODITY POSSIBILITIES
      CPOSP   COMMODITY PRODUCTION POSSIBILITIES
      CPOSN   COMMODITY CONSUMPTION POSSIBILITIES ;

   MPOS (M,I) = K(M,1);
   PPOS (P,I) = SUM(M$(NOT MPOS(M,I)), B(M,P) NE 0) EQ 0 ;
   PPOS ("CAN-310","HELWAN")=NO;
   PPOS ("CAN-335","ASWAN")=NO;
   CPOSP(C,I) = SUM(P$PPOS(P,I), A(C,P) GT 0) ;
   CPOSN(C,I) = SUM(P$PPOS(P,I), A(C,P) LT 0) ;
   CPOS(C,I)  = CPOSP(C,1) + CPOSN(C,I) ;

DISPLAY MPOS, PPOS, CPOSP, CPOSN, CPOS ;
```

Equations

```
   VARIABLES  Z     PROCESS LEVEL                              (1000 TPY)
              XF    DOMESTIC SHIPMENT ACTIVITY: FINAL PRODUCTS (1000 TPY)
              XI    DOMESTIC SHIPMENT ACTIVITY: INTERMEDIATES  (1000 TPY)
              VF    IMPORTS: FINAL PRODUCTS                    (1000 TPY)
              VR    IMPORTS: RAW MATERIALS                     (1000 TPY)
              U     DOMESTIC RAW MATERIAL PURCHASES            (UNITS)
              PSI   TOTAL COST (DISCOUNTED)     (1000 LE)
              PSIP  DOMESTIC RECURRENT COST     (1000 LE PER YEAR)
              PSIL  TRANSPORT COST              (1000 LE PER YEAR)
              PSII  IMPORT COST                 (1000 LE PER YEAR)

   EQUATIONS  OBJ   OBJECTIVE FUNCTION  (1000 LE DISCOUNTED)
              MBD   MATERIAL BALANCE ON DEMAND: NUTRIENT  (1000 TPY)
              MBDB  MATERIAL BALANCE ON DEMAND: MATERIAL  (1000 TPY)
              MB    MATERIAL BALANCE                 (1000 TPY)
              CC    CAPACITY CONSTRAINT
              XIFIX INTERPLANT SHIPMENT FIX

              AP    ACCOUNTING: DOMESTIC RECURRENT COST(1000 LE PER YEAR)
              AL    ACCOUNTING: TRANSPORT COST         (1000 LE PER YEAR)
              AI    ACCOUNTING: IMPORT COST            (1000 LE PER YEAR)
                             ;

MBD(CQ,J)..   SUM(CF, ALPHA(CF,CQ)*( SUM(I$CPOSP(CF,I), XF(CF,I,J))
                                    + VF(CF,J))) =G= CN75(J,CQ);


MBDB(CF,J)$CF75(J,CF)..
  SUM(I$CPOSP(CF,I), XF(CF,I,J)) + VF(CF,J) =G= CF75(J,CF);


MB(C,I)..  SUM(P$PPOS(P,I), A(C,P)*Z(P,I))
         + SUM(IP, XI(C,IP,I)$(CPOSP(C,IP)*CPOSN(C,I))
                 - XI(C,I,IP)$(CPOSN(C,IP)*CPOSP(C,I)))$CS(C)
         + (VR(C,I)$PV(C) + U(C,I)$(PD(I,C)))$(CR(C)*CPOSN(C,I))
         - SUM(J$CPOSP(C,I), XF(C,I,J))$CF(C) =G= 0 ;


XIFIX(CS,I,IP)$(CPOSP(CS,I)*CPOSN(CS,IP)).. XI(CS,I,IP) =E= 0 ;


CC(M,I)$MPOS(M,I)..  SUM(P$PPOS(P,I), B(M,P)*Z(P,I)) =L= UTIL*K(M,I) ;

```

Equations (contd.)

```
OBJ.. PSI =E= PSIP + PSIL + PSII ;

AP.. PSIP =E= SUM((CR,I)$CPOSN(CR,I), PD(I,CR)*U(CR,I));

AL.. PSIL =E= SUM(CF, SUM((I,J)$CPOSP(CF,I), MUF(I,J)*XF(CF,I,J))
                    + SUM(J, MUFV(J)*VF(CF,J)))
            + SUM((CS,I,IP)$(CPOSP(CS,I)*CPOSN(CS,IP)), MUI(I,IP)*XI(CS,I,IP
            + SUM((CR,I)$(CPOSN(CR,I)$PV(CR)), MUR(I)*VR(CR,I));

AI.. PSII/ER  =E= SUM((CF,J), PV(CF)*VF(CF,J))
                + SUM((CR,I)$CPOSN(CR,I), PV(CR)*VR(CR,I));

 MODEL STAT1  / MBDB, MB, XIFIX, CC, OBJ, AP, AL, AI /
       STAT2  / MBDB, MB, CC, OBJ, AP, AL, AI /
       STAT4  / MBD, MB, CC, OBJ, AP, AL, AI / ;

SOLVE STAT1 MINIMIZING PSI USING APEX1 ;

PARAMETERS RXF   DOMESTIC SHIPMENT ACTIVITY : FINAL PRODUCTS (1000 TPY)
           TDS   TOTAL DOMESTIC SUPPLY : FINAL PRODUCTS (1000 TPY)
           TIF   TOTAL IMPORT : FINAL PRODUCT (1000 TPY)
           TL    TRANSPORTATION LOAD (1000 TON_KM) ;

  RXF(I,J,CF) = XF.L(CF,I,J) ;
          TDS = SUM((CF,I,J), XF.L(CF,I,J));
          TIF = SUM((CF,J), VF.L(CF,J));
  TL("RAIL")  = SUM((CS,I,IP), RAIL(I,IP)*XI.L(CS,I,IP)) ;
  TL("ROAD")  = SUM((CR,I), VR.L(CR,I)*IMPD(I,"ROAD"))
              + SUM((CF,I,J), XF.L(CF,I,J)*ROAD(J,I))
              + SUM((CF,J), VF.L(CF,J)*ROAD(J,"IMPORT-PTS"));
  TL("BARGE") = SUM((CR,I), VR.L(CR,I)*IMPD(I,"BARGE")) ;

DISPLAY MBDB.LO, MBDB.M, MB.M, CC.UP, CC.M, RXF, TDS, TIF, VF.L, Z.L, VR.L,
        U.L, TL ;
```

Reference maps

```
VARIABLES   TYPE     REFERENCES

A           PARAM          REF       408       409       444   DEFINED       276       DCL       276
AI          EQU            REF       467       468       469   DEFINED       463       DCL       435
AL          EQU            REF       467       468       469   DEFINED       458       DCL       434
ALPHA       PARAM          REF       178       438   DEFINED       162       DCL       162
AP          EQU            REF       467       468       469   DEFINED       456       DCL       433
B           PARAM          REF       405       452   DEFINED       311       DCL       311
C           SET            REF       408       409     2*410       444     3*445     4*446     6*447     3*448   DEFINED
                         3*103   CONTROL       408       409       410       444       DCL       103
CC          EQU            REF       467       468       469     2*487   DEFINED       452       DCL       430
CF          SET            REF     2*178     3*438       439       441     4*442       448     2*458       459     2*463
                           478       479       480       483       484   DEFINED        67   CONTROL       103       178
                           438       441       458       463       478       479       480       483       484       DCL
                            67
CF75        PARAM          REF       178       441       442   DEFINED       106       DCL       106
CI          SET        DEFINED        79   CONTROL       103       DCL        79
CN75        PARAM          REF       179       181       439   DEFINED       178       179       DCL       176
CPOS        SET            REF       412   DEFINED       410       DCL       400
CPOSN       SET            REF       410       412       445       446       447       450       456       460       461
                           464   DEFINED       409       DCL       402
CPOSP       SET            REF       410       412       438       442       445       446       448       450       458
                           460   DEFINED       408       DCL       401
CQ          SET            REF       178       179       438       439   DEFINED        62   CONTROL       178       179
                           438       DCL        62
CR          SET            REF     2*368       447     3*456     3*461     3*464       482       485   DEFINED        87
                       CONTROL       103       368       456       461       464       482       485       DCL        87
CS          SET            REF       446     3*450     3*460       481   DEFINED        85   CONTROL       450       460
                           481       DCL        85
DCAP        PARAM          REF       393   DEFINED       374       DCL       374
ER          PARAM          REF       463   DEFINED       395       DCL       395
I           SET            REF       101     2*263     2*266     2*268     2*269     2*270       393       404       405
                           408       409     2*410     2*438     2*442     2*444     2*445     2*446     4*447     2*448
                         2*450     4*452     3*456     3*458     3*460     3*461     2*464       478       479     2*481
                         2*482     2*483     2*485   DEFINED         4   CONTROL       263       266       268       269
                           368       393       404       405       408       409       410       438       442       444
                           450       452       456       458       460       461       464       478       479       481
                           482       483       485       DCL         4
IMPD        PARAM          REF     2*269     2*270       482       485   DEFINED       247       DCL       247
IP          SET            REF     2*263     2*268     2*445     2*446     2*450     3*460     2*481   CONTROL       263
                           268       445       450       460       481       DCL       101
```

Reference maps (contd.)

```
J           SET                REF       178       179       264     2*266     2*267       438     2*439       441     3*442
                               448     2*458     2*459       463       478       479       480     2*483     2*484   DEFINED
                                13   CONTROL       178       179       264       266       267       438       441       448
                               458       459       463       478       479       480       483       484       DCL        13
K           PARAM              REF       404       452   DEFINED       393       DCL       393
M           SET                REF       393       404     2*405     3*452   DEFINED        39   CONTROL       393       404
                               405       452       DCL        39
MB          EQU                REF       467       468       469       487   DEFINED       444       DCL       429
MBD         EQU                REF       469   DEFINED       438       DCL       427
MBDB        EQU                REF       467       468     2*487   DEFINED       441       DCL       428
MPOS        SET                REF       405       412       452   DEFINED       404       DCL       398
MUF         PARAM              REF       272       458   DEFINED       266       DCL       258
MUFV        PARAM              REF       272       459   DEFINED       267       DCL       259
MUI         PARAM              REF       272       460   DEFINED       268       DCL       260
MUR         PARAM              REF       272       461   DEFINED       269       DCL       261
OBJ         EQU                REF       467       468       469   DEFINED       454       DCL       426
P           SET                REF       405     2*408     2*409     3*444     3*452   DEFINED        50   CONTROL       405
                               408       409       444       452       DCL        50
PD          PARAM              REF       370       447       456   DEFINED       344       368       DCL       344
PMISC       PARAM              REF     2*368   DEFINED       356       DCL       356
PPOS        SET                REF       408       409       412       444       452   DEFINED       405       406       407
                               DCL       399
PSI         VAR                REF       454       471       DCL       421
PSII        VAR                REF       454       463       DCL       424
PSIL        VAR                REF       454       458       DCL       423
PSIP        VAR                REF       454       456       DCL       422
PV          PARAM              REF       447       461       463       464   DEFINED       328       DCL       328
RAIL        PARAM              REF     2*263     2*268       481   DEFINED       238       263       DCL       238
ROAD        PARAM              REF       264     2*266     2*267       483       484   DEFINED       184       264       DCL
                               184
RXF         PARAM              REF       487   DEFINED       478       DCL       473
STAT1       MODEL              REF       471   DEFINED       467       DCL       467
STAT2       MODEL          DEFINED       468       DCL       468
STAT4       MODEL          DEFINED       469       DCL       469
TDS         PARAM              REF       487   DEFINED       479       DCL       474
TIF         PARAM              REF       487   DEFINED       480       DCL       475
TL          PARAM              REF       488   DEFINED       481       482       485       DCL       476
U           VAR                REF       447       456       488       DCL       420
UTIL        PARAM              REF       452   DEFINED       396       DCL       396
VF          VAR                REF       439       442       459       463       480       484       487       DCL       418
VR          VAR                REF       447       461       464       482       485       487       DCL       419
XF          VAR                REF       438       442       448       458       478       479       483       DCL       416
XI          VAR                REF       445       446       450       460       481       DCL       417
XIFIX       EQU                REF       467   DEFINED       450       DCL       431
Z           VAR                REF       444       452       487       DCL       415
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Reference maps (contd.)

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SETS
C           ALL COMMODITIES
CF          FINAL PRODUCTS (FERTILIZERS)
CI          INTERMEDIATE PRODUCTS
CPOS        COMMODITY POSSIBILITIES
CPOSN       COMMODITY CONSUMPTION POSSIBILITIES
CPOSP       COMMODITY PRODUCTION POSSIBILITIES
CQ          NUTRIENTS
CR          DOMESTIC RAW MATERIALS AND MISCELLANEOUS INPUTS
CS          INTERMEDIATES FOR SHIPMENT
I           PLANT LOCATIONS
IP          ALIAS FOR I
J           DEMAND REGIONS
M           PRODUCTIVE UNITS
MPOS        PRODUCTIVE UNITS POSSIBILITIES
P           PROCESSES
PPOS        PROCESS POSSIBILITIES

PARAMETERS
  A          INPUT-OUTPUT COEFFICIENT
  ALPHA      NUTRIENT CONTENT
  B          CAPACITY UTILIZATION COEFFICIENT
  CF75       CONSUMPTION OF FERTILIZER 1974-75 (1000 TPY)
  CN75       CONSUMPTION OF NUTRIENTS 1974-75 (1000 TPY)
  DCAP       DESIGN CAPACITY OF PLANTS (T/DAY)
  ER         EXCHANGE RATE
  IMPD       IMPORT DISTANCES (KMS)
  K          INITIAL CAPACITY (1000 TPY)
  MUF        TRANSPORT COST (LE PER TON): FINAL PRODUCTS
  MUFV       TRANSPORT COST (LE PER TON): IMPORTED FINAL PRODUCTS
  MUI        TRANSPORT COST (LE PER TON): INTERPLANT SHIPMENT
  MUR        TRANSPORT COST (LE PER TON): IMPORTED RAW MATERIALS
  PD         DOMESTIC RAW MATERIAL PRICES
  PMISC      MISC. MATERIAL COST
  PV         IMPORT PRICE (CIF US$ PER TON 1975)
  RAIL       INTERPLANT RAIL DISTANCES (KMS)
  ROAD       ROAD DISTANCES (KMS)
  RXF        DOMESTIC SHIPMENT ACTIVITY : FINAL PRODUCTS (1000 TPY)
  TDS        TOTAL DOMESTIC SUPPLY : FINAL PRODUCTS (1000 TPY)
  TIF        TOTAL IMPORT : FINAL PRODUCT (1000 TPY)
  TL         TRANSPORTATION LOAD (1000 TON_KM)
  UTIL       UTILAZATION

VARIABLES
  PSI        TOTAL COST (DISCOUNTED)     (1000 LE)
  PSII       IMPORT COST                 (1000 LE PER YEAR)
  PSIL       TRANSPORT COST              (1000 LE PER YEAR)
  PSIP       DOMESTIC RECURRENT COST     (1000 LE PER YEAR)
  U          DOMESTIC RAW MATERIAL PURCHASES             (UNITS)
  VF         IMPORTS: FINAL PRODUCTS                     (1000 TPY)
  VR         IMPORTS: RAW MATERIALS                      (1000 TPY)
  XF         DOMESTIC SHIPMENT ACTIVITY: FINAL PRODUCTS  (1000 TPY)
  XI         DOMESTIC SHIPMENT ACTIVITY: INTERMEDIATES   (1000 TPY)
  Z          PROCESS LEVEL                               (1000 TPY)

EQUATIONS
  AI         ACCOUNTING: IMPORT COST           (1000 LE PER YEAR)
  AL         ACCOUNTING: TRANSPORT COST        (1000 LE PER YEAR)
  AP         ACCOUNTING: DOMESTIC RECURRENT COST(1000 LE PER YEAR)
  CC         CAPACITY CONSTRAINT
  MB         MATERIAL BALANCE                (1000 TPY)
  MBD        MATERIAL BALANCE ON DEMAND: NUTRIENT  (1000 TPY)
  MBDB       MATERIAL BALANCE ON DEMAND: MATERIAL  (1000 TPY)
  OBJ        OBJECTIVE FUNCTION  (1000 LE DISCOUNTED)
  XIFIX      INTERPLANT SHIPMENT FIX
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Reference maps (contd.)

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MODELS

  STAT1
  STAT2
  STAT4


    UNIQUE ELEMENTS IN ENTRY ORDER

     1  ASWAN        HELWAN       ASSIOUT      KAFR-EL-ZT   ABU-ZAABAL   ALEXANDRIA   BEHERA       GHARBIA      KAFR-EL-SH   DAKAHLIA
    11  DAMIETTA     SHARKIA      ISMAILIA     SUEZ         MENOUFIA     KALUBIA      GIZA         BENI-SUEF    FAYOUM       MINIA
    21  NEW-VALLEY   SOHAG        QUENA        SULF-A-S     SULF-A-P     NITR-ACID    AMM-ELEC     AMM-C-GAS    C-AMM-NITR   AMM-SULF
    31  SSP          CAN-310      CAN-335      SSP-155      N            P205         UREA         CAN-260      DAP          C-250-55
    41  C-300-100    AMMONIA      SULF-ACID    EL-ASWAN     COKE-GAS     PHOS-ROCK    LIMESTONE    EL-SULFUR    PYRITES      ELECTRIC
    51  BF-GAS       WATER        STEAM        BAGS         TOTAL        ABU-KIR      TALKHA       BARGE        ROAD         IMPORT-PTS
    61  RAIL

    UNIQUE ELEMENTS IN SORTED ORDER

     1  ABU-KIR      ABU-ZAABAL   ALEXANDRIA   AMMONIA      AMM-C-GAS    AMM-ELEC     AMM-SULF     ASSIOUT      ASWAN        BAGS
    11  BARGE        BEHERA       BENI-SUEF    BF-GAS       CAN-260      CAN-310      CAN-335      COKE-GAS     C-AMM-NITR   C-250-55
    21  C-300-100    DAKAHLIA     DAMIETTA     DAP          ELECTRIC     EL-ASWAN     EL-SULFUR    FAYOUM       GHARBIA      GIZA
    31  HELWAN       IMPORT-PTS   ISMAILIA     KAFR-EL-SH   KAFR-EL-ZT   KALUBIA      LIMESTONE    MENOUFIA     MINIA        N
    41  NEW-VALLEY   NITR-ACID    PHOS-ROCK    PYRITES      P205         QUENA        RAIL         ROAD         SHARKIA      SOHAG
    51  SSP          SSP-155      STEAM        SUEZ         SULF-ACID    SULF-A-P     SULF-A-S     TALKHA       TOTAL        UREA
    61  WATER
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References

[1] I. Adelman and S. Robinson, *Income distribution policy in the developing countries: A case study of Korea* (Oxford University Press, Oxford, 1978).

[2] J. Bisschop and A. Meeraus, "Selected aspects of a general algebraic modeling language", in: A.V. Balakrishnan and M. Thoma, eds., *Lecture Notes in Control and Information Sciences* 23 (*Springer-Verlag, Berlin*, 1980).

[3] *H.B. Chenery and R. Weaving, "An overview of research at the World Bank", World Bank Research News* 1 (1) (1980) 1–9.

[4] A.M. Choksi, A. Meeraus and A.J. Stoutjesdijk, *The planning of investment programs in the fertilizer industry* (Johns Hopkins University Press, London, 1980).

[5] K. Dervis and S. Robinson, "A general equilibrium analysis of the causes of a foreign exchange crisis", *Journal of Political Economy*, to appear.

[6] J.H. Duloy, G. O'Mara, S. Ting and A. Brooke, "Programming and designing investment: The Indus Basin—A preliminary report on validation of and some water allocation experiments with the Indus Basin Model", Development Research Center, World Bank, mimeograph (1979).

[7] L.M. Goreux and A.S. Manne, *Multi-level planning case studies in Mexico* (North-Holland, Amsterdam, 1973).

[8] S. Gupta, A. Schwartz and R. Padula, "The World Bank model for global interdependence: A quantitative framework for the World Development Report", *Journal of Policy Modeling* (1979) 179–200.

[9] D. Kendrick and A. Stoutjesdijk, *The planning of industrial investment programs: A methodology* (Johns Hopkins University Press, London, 1979).

[10] F.G. Pyatt and J.I. Round, "Social accounting matrices for development planning", *The Review of Income and Wealth* 23 (1977) 339–364.

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