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ON THE DISCUSSIVE CONJUNCTION IN THE PROPOSITIONAL CALCULUS FOR INCONSISTENT DEDUCTIVE SYSTEMS*

Two-valued discussive systems (cf. [1]) of the propositional calculus $\mathbf{D_2}$ can be enlarged by means of the discussive conjunction $\wedge_{\mathrm{d}}$. To this end instead of the definition $\mathbf{M_2}$ def. 1 from [1] we need to posit the following definition

$\mathbf{M_2}$ def 1.1 $$p \wedge_{\mathrm{d}} q \;:=\; p \wedge \Diamond q\,.$$

After this emendation we can simplify the definition of the discussive equivalence by replacing $\mathbf{M_2}$ def. 2 by the following:

$\mathbf{M_2}$ def 2.1 $$p \leftrightarrow_{\mathrm{d}} q \;:=\; (p \rightarrow_{\mathrm{d}} q) \wedge_{\mathrm{d}} (q \rightarrow_{\mathrm{d}} p)$$

The metalogical theorem 1 (cf. [1], p. 68) remains valid in the following generalized form: *Each thesis A of the two-valued classical calculus $\mathbf{L_2}$ containing no other symbols than $\rightarrow$, $\leftrightarrow$, $\vee$ or $\wedge$ is transformed into thesis of the discussive calculus $\mathbf{D_2}$ by replacing in A functors $\rightarrow$ by $\rightarrow_{\mathrm{d}}$, $\leftrightarrow$ by $\leftrightarrow_{\mathrm{d}}$, and $\wedge$ by $\wedge_{\mathrm{d}}$, respectively.*

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The proof of the theorem contains no essential change in comparison with the proof of the metalogical theorem 1 from my original paper [1]. We must only use theorems 5–7 of $\mathbf{M_2}$ (cf. [1], p. 68) plus a new thesis of $\mathbf{M_2}$:

$\mathbf{M_2}$ 7.1 $$\Diamond(p \wedge_{\mathrm{d}} q) \leftrightarrow (\Diamond p \wedge \Diamond q).$$

The law of the inconsistency for the discussive conjunction is the following thesis of $\mathbf{D_2}$:

$\mathbf{D_2}$ 4.1 $$\neg(p \wedge_{\mathrm{d}} \neg p),$$

whereas the refuted conjunctive form [i.e., Duns Scotus Law – J.P.] is

(non $\mathbf{D_2}$) 3.1 $$(p \wedge_{\mathrm{d}} \neg p) \rightarrow_{\mathrm{d}} q$$

despite the fact that previously we had an analogous theorem for the usual [classical – J.P.] conjunction, which in my previous paper [1] is denoted by $\mathbf{D_2}$ 5 (cf. [1], p. 69).

References

[1] Stanisław Jaśkowski "Rachunek zdań dla systemów dedukcyjnych sprzecznych", *Studia Societatis Scientiarum Torunensis*, Sectio A, Vol. I, No. 5, Toruń, 1948, pp. 57–77. The first English translation "Propositional calculus for contradictory deductive systems", by O. Wojtasiewicz, appeared in *Studia Logica*, Vol. XXIV (1969), pp. 143–157. The second version, with a few modifications, including changing of notation, "A propositional calculus for inconsistent deductive systems", is published in this volume, pp. 35–56.

(translated by Jerzy Perzanowski)

Comments of the translator

1. The main result of this very short, but quite important, note is its main metatheorem that $\mathbf{D_2}$ in fact contains the full positive part of the classical logic plus observation ($\mathbf{M_2}$ 7.1) that with the new notion of *discussive* conjunction Jaśkowski's basic transformation is remarkably simplified, becoming a common homomorphism.

2. Moreover, on the ground of a modified $\mathbf{D_2}$ we have quite a lot of nice new theorems, such as the law of inconsistency ($\mathbf{D_2}$ 4.1). Indeed, on the basis of $\mathbf{M_2}$ (i.e., **S5**) we have that:

$$\begin{aligned} \neg(p \wedge_{\mathrm{d}} \neg p) &\dashv\vdash \Diamond\big(\neg(p \wedge \Diamond \neg p)\big) \\ &\dashv\vdash \Diamond(p \to \Box p) \\ &\dashv\vdash (\Box p \to \Diamond\Box p). \end{aligned}$$

3. It is clear that on the ground quite close to the modified $\mathbf{D_2}$ we can define quite a lot of new discussive connectives, including discusive negation:

$$(\neg_{\mathrm{d}}) \qquad \neg_{\mathrm{d}}\, p := \Diamond \neg p\,.$$

Indeed, in **S5** it is easy to verify that

$$\begin{aligned} \neg_{\mathrm{d}}\, p &\leftrightarrow \Diamond \neg p \\ &\leftrightarrow \big((p \to p) \wedge \Diamond \neg p\big) \\ &\leftrightarrow \big((p \to p) \wedge_{\mathrm{d}} \neg p\big). \end{aligned}$$

Also reversely,

$$\begin{aligned} (p \wedge_{\mathrm{d}} q) &\leftrightarrow (p \wedge \Diamond q) \\ &\leftrightarrow (p \wedge \Diamond \neg\neg q) \\ &\leftrightarrow (p \wedge \neg_{\mathrm{d}} \neg q). \end{aligned}$$

Discussive conjunction and discussive negation are thereby interdefinable on the ground **S5**, hence they are closely interconnected in the modified version of $\mathbf{D_2}$.

4. Of course, we have

$$\begin{aligned} \neg\neg_{\mathrm{d}}\, p &\to p \\ \neg p &\to \neg_{\mathrm{d}}\, p\,, \\ p &\to \neg_{\mathrm{d}} \neg p\,, \\ \neg_{\mathrm{d}} \neg_{\mathrm{d}}\, p &\to p\,. \end{aligned}$$

But not reversely. For in **S5** we easily obtain

$$\Diamond p \leftrightarrow \neg_{\mathrm{d}} \neg p\,,$$

whereas

$$\begin{aligned} \Box p &\leftrightarrow \neg\neg_{\mathrm{d}}\, p\,, \\ &\leftrightarrow \neg_{\mathrm{d}} \neg_{\mathrm{d}}\, p\,. \end{aligned}$$

J.P.