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On the dynamical implications of models of B_s in the Earth's core

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SUMMARY

Recent work has shown that the zonal, equatorially symmetric, time-varying part of a model of the flow at the surface of the Earth's core can be well explained by only two standing waves, and that by making certain assumptions these waves may be inverted for rms B_s (the component of field pointing away from the rotation axis) and a quantity parametrizing friction or excitation (F) of the waves. Here, we discuss the two-wave fit, and describe the implications of models of rms B_s and friction/excitation for the dynamic state of the core, for the torque balance on axial cylinders, and for recent numerical simulations of the geodynamo. We find several possible explanations for why only two standing waves are needed to fit the data, including the possibility that it is due to the resolution of the core flow model rather than conditions within the core itself. We find that the fits of rms B_s and F suggest that the role of inertia should not be discounted in the core, and that care should be taken in constructing geodynamo simulations so that the effective friction at the core–mantle boundary does not swamp the inertial term. A ratio of the magnitude of the two appears to be $O(1)$ in the Earth's core: we believe that, ideally, a numerical model of the Earth's core should reproduce this result.

Key words: core flows, dynamo theory, geomagnetism.

1 INTRODUCTION

Inverse theory has long been used to construct maps of the geomagnetic field from observations taken by satellites, observatories and ships. This is achieved by inverting these data using Laplace's equation (Langel 1987). Similarly, models of the magnetic field and its secular variation may be inverted for maps of the fluid flow at the surface of the core through the induction equation of magnetohydrodynamics by making appropriate assumptions, although the maps are non-unique (that is, different flows may fit the data equally well). Further assumptions can be used to reduce this ambiguity in the flow (Bloxham & Jackson 1991).

In turn, it is possible to use the Navier–Stokes equation, which governs the force balance of the core fluid, to invert core flows for information related to the dynamics of the core (Zatman & Bloxham 1997, 1998) (henceforth referred to as Papers I and II), provided appropriate extra assumptions are made. One such assumption that we believe to be reasonable is that the decadal variation of the axisymmetric, equatorially symmetric zonal flow is due to torsional oscillations, which are to first order the differential rotation of cylindrical annuli coaxial with the rotation axis (see Fig. 1). Braginsky (1984) pointed out that if the component of magnetic field pointing away from the Earth's rotation axis (B_s) is around 0.2 mT—which is of the same order of magnitude as the inferred field at the core–mantle boundary (CMB)—then torsional oscillations will have a period of around 60 years. In contrast, inertial oscillations will be on diurnal timescales, and magnetostrophic oscillations are expected to have timescales of $\simeq 300$ years (Hide 1966; Gubbins & Roberts 1987).

Further evidence for torsional oscillations comes from the agreement, at least during this century, between decadal variations in core angular momentum calculated from the core flow with those inferred from geodetic observations of the rate of rotation of the mantle (Jault *et al.* 1988; Jackson *et al.* 1993; Jackson 1997). Core angular momentum is calculated from the core flow by assuming that the axisymmetric, equatorially symmetric zonal flows are invariant in the z-direction, which is the form of torsional oscillations.

Jackson *et al.* (1993) noted that the core angular momentum appears to be due to the difference between two largely opposed oscillations in the t_1^0 and t_3^0 components of the flow. Jault *et al.* (1996) then showed that the axisymmetric, equatorially symmetric, zonal part of the flow (the t_j^0 terms where j is odd) varies in space and time in a way that is suggestive of torsional oscillations. In Papers I and II, we found that this part of a time-dependent, tangentially geostrophic flow (Bloxham 1995)

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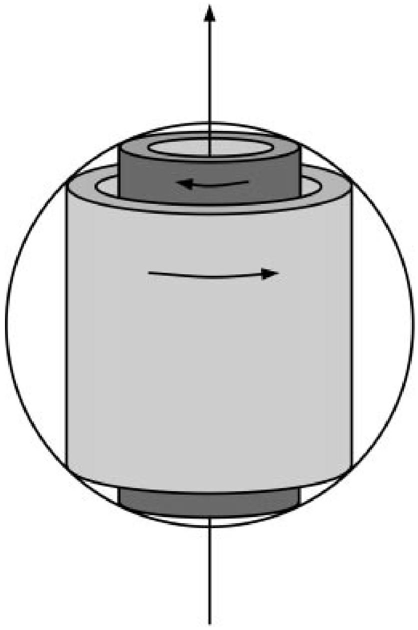

Figure 1. Torsional oscillations. The motion is the differential rotation of axial cylinders about the Earth's axis.

could be inverted for the period between 1900 and 1990 for a model of two damped harmonic waves and a steady flow, and that this model explained 98.8 per cent of the variance of the flow coefficients.

As discussed further in Paper II, we make a number of assumptions. We assume that these waves are torsional oscillations, and that the excitation and damping of the oscillations can be parametrized through a term that is proportional to the difference in zonal velocity between the torsional oscillation and the mantle at the CMB. The constant of proportionality F (with units N s m^{-3}) for which we solve is a measure of the stress per unit velocity difference at the CMB, and thus can be thought of as a coefficient of friction (or excitation). Since we are considering the motion of coaxial cylinders, we need only consider the averaged coefficient of friction on the intersection of each cylinder with the CMB (non-axisymmetric or equatorially symmetric variations in the friction are removed upon integration), so we can consider F as simply a function of distance from the rotation axis, s. With this, the formula governing torsional oscillations becomes (Paper II)

$$\frac{1}{2(r_c^2-s^2)^{1/2}}\left(\frac{-\omega\mathscr{M}(\mathscr{R}+\mathscr{R}_m)}{2\pi s}+\frac{1}{\mu_0\omega}\frac{\partial\mathscr{R}}{\partial s}\right.$$
$$\left.\times\left\{\alpha[B_sB_z]_{z_-}^{z_+}+\frac{s}{(r_c^2-s^2)^{1/2}}\left[B_{s_{z_+}}^2+B_{s_{z_-}}^2\right]\right\}\right)$$
$$+\frac{\mathscr{R}r_c}{r_c^2-s^2}F+\frac{1}{\omega\mu_0}\left(\left(\frac{\partial^2\mathscr{R}}{\partial s^2}+\frac{\partial\mathscr{R}}{\partial s}\left(\frac{3}{s}-\frac{s}{r_c^2-s^2}\right)\right)\right.$$
$$\left.+\frac{\partial\mathscr{R}}{\partial s}\frac{\partial}{\partial s}\right)\overline{B_s^2}=0\,, \qquad (1)$$

where r_c is the core radius, ω is the (complex) frequency of a torsional oscillation, $\mathscr{R}$ is the axial rotation rate of an axial cylinder with respect to the mantle, $\mathscr{R}_m$ is the perturbation rate of rotation of the mantle, $\mathscr{M}$ is the mass of the axial cylinder, z_+ and z_- mark the axial (z) coordinates of the upper and lower intersections of the axial cylinder with the CMB, μ_0 is the magnetic permeability and α is a parameter that is varied between 0 and 1 in order to test the possible effects of uncertainties in the electrical boundary condition at the CMB. We use $\{x\}$ to designate the zonal average of a quantity x, and $\overline{x}$ as the average of x on an axial cylinder. By specifying a value of α, we may invert eq. (1) for $\overline{B_s^2}$ and F as functions of s. Since there is no inclusion of inner core dynamics, this equation is applied only to the region exterior to the tangent cylinder (the axial cylinder which grazes the inner core).

The inversions are described in Papers I and II; here we plot sample results for rms B_s and F (see Figs 2 and 3). These inversions raise several interesting questions concerning the dynamical conditions of the core, which we shall discuss below. First, one might imagine that the core could sustain a whole spectrum of torsional oscillations, so we address why a model

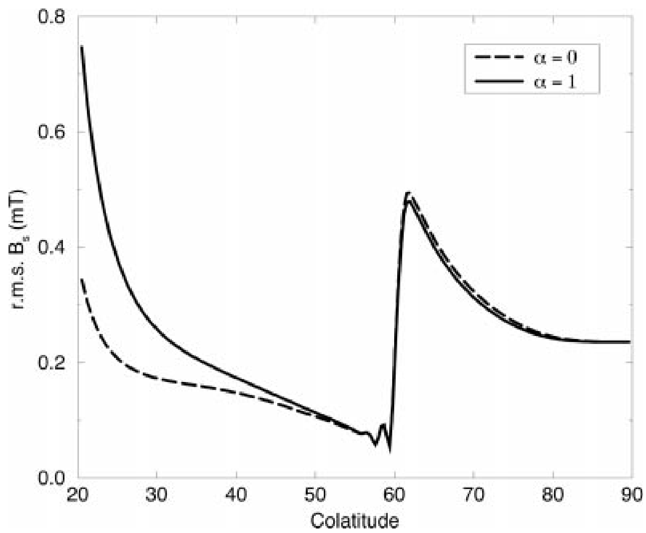

Figure 2. Rms B_s averaged over axial cylinders, showing the effects of varying α, for the inversion of wave A from Zatman & Bloxham (1998).

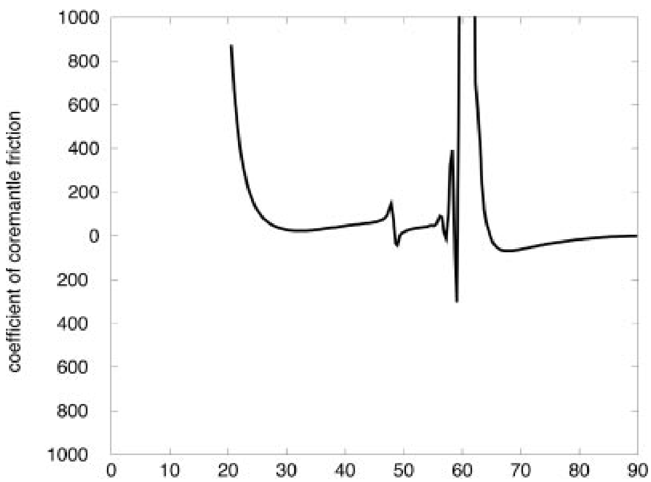

Figure 3. The coefficient of core–mantle friction (units are N m^{-3} s) for the inversion of wave A of Zatman & Bloxham (1998) with $\alpha=1$.

of the zonal, equatorially symmetric, axisymmetric flows at the surface of the core may be explained so well by simply a steady flow and two damped harmonic oscillations. Second, we discuss the implications of these results for the force balance in the Earth's core, particularly with reference to the relative importance of the inertial and frictional terms. Recent self-consistent numerical modelling of the geodynamo suggests that magnetic fields that are Earth-like at the CMB can be generated by dynamos that differ quite radically in internal structure, both for magnetic field and flow configuration (Kuang & Bloxham 1997). The underlying assumptions behind the Glatzmaier–Roberts dynamo (Glatzmaier & Roberts 1995, 1996) are different from those behind the Kuang–Bloxham dynamo (Kuang & Bloxham 1997) in that the boundary conditions and the relative importance of the frictional and inertial terms differ.

Here we seek observational constraints on the force balance within the core that might help guide efforts in numerical modelling. The inversions for rms B_s and F provide new information concerning the structure of the outer core outside the tangent cylinder that allows us to attempt to estimate the torque balance on axial cylinders within the core. We show that the resultant estimates are consistent with order-of-magnitude calculations that we construct without the need to consider the dynamics of torsional oscillations.

2 WHY IS THE SIGNAL EXPLAINED SO WELL BY TWO WAVES?

The flow components (in standard spherical harmonic flow notation; Bloxham & Jackson 1991) and frequencies of the two fitted oscillations are presented in Table 1.

One might imagine that the core supports a broad spectrum of oscillations on decadal timescales, as there could be a broad spectrum of torsional oscillations. However, numerical tests of confidence presented in Paper II make it clear that not only are no further waves required by the data, but even the second wave is at the edge of significance. Therefore, we wish to understand why only two waves fit the flow model so well. We consider five possible explanations.

First, there may be other waves in the data with shorter periods than the recovered waves, but with smaller amplitudes. Commonly, geophysical systems such as variations in the length of day (ΔLOD) exhibit greater power at longer periods. Fig. 4 shows that there is a trend in the decadal range of the power spectrum of ΔLOD, so that the power increases with period. The two waves which appear to be supported by the data may be the two torsional oscillations with the longest periods. Wave B, which has a period close to 2/3 that of wave A, is lower in amplitude. Thus, faster oscillations may be present but lost in the noise of the flow model.

Second, it is possible that shorter-period waves are harder to detect not because the waves are lower in amplitude but because their spatial complexity increases as period decreases. This would cause the angular momenta of more rapid oscillations to cancel out when integrated over the whole core, lowering their contribution to ΔLOD, which would explain the decadal trend in Fig. 4, where the power decreases at shorter periods. In this case, the damping of higher spherical harmonic coefficients of the flow would hinder the detection of these waves.

A third possibility is that the excitation mechanism is more efficient at long periods than at short periods, although this is difficult to assess since very little is known about the excitation process. However, excitation due to MAC waves [for which there is a balance between Lorentz, Coriolis, buoyancy and pressure forces (Moffatt 1978)] in the core may be more efficient for periods that are harmonics of those waves.

Fourth, spatial variation of the excitation or damping of the oscillations may discriminate between waves. For example, if one wave has a node where another wave has an antinode, then local damping will cause the first wave to be preferred to the second. It may happen that the waveforms of the two waves are preferred to the others.

Fifth, the flow model that we use is expanded in time by evenly spaced B-splines with knots at 5 yr intervals. Using an admittedly conservative four-to-one rule of thumb (Constable & Parker 1988), this suggests that the model only properly resolves features down to $\simeq 20$ yr, or periods down to $\simeq 40$ yr. Therefore, it is quite possible that waves with periods shorter than that of wave B will not be well resolved by this velocity model. The magnetic field model ufm1 (Bloxham & Jackson 1992) is constructed using evenly spaced B-splines with knots at 2.5 yr intervals, which allows resolution of features with periods down to $\simeq 20$ yr. Regularization of the field and flow models would further prevent the recovery of short-period variations. The resolution of such waves may require field models which better recover any rapid variation of the main field.

Table 1. Flow components (km yr^{-1}) and frequencies (yr^{-1}) of fitted oscillations. A negative imaginary part of the frequency corresponds to decay.

	Wave A		Wave B	
	Real part	Imaginary part	Real part	Imaginary part
t_1^0	5.6752	3.0238	−0.5110	3.3957
t_3^0	−1.5681	−2.9076	0.7500	−1.1206
t_5^0	1.9214	−0.4320	−0.6404	0.6111
t_7^0	−0.5538	−0.6259	0.3289	−0.0179
t_9^0	−0.0840	0.0782	−0.0119	−0.0516
t_{11}^0	0.0545	0.0103	0.0142	0.0363
t_{13}^0	−0.0314	0.0218	0.0039	−0.0261
Frequency	0.0131	−0.0021	0.0189	−0.0017

3 EVIDENCE OF THE TORQUE BALANCE WITHIN THE CORE

There is much discussion in geodynamo theory of the torque balance on axial cylinders. In a magnetostrophic core (balance of pressure, buoyancy, coriolis and magnetic forces), axial cylinders obey Taylor's constraint (Taylor 1963); that is, the Taylor torque γ_B satisfies

$$\gamma_B = \frac{s}{\mu_0} \int [(\nabla \wedge \mathbf{B}) \wedge \mathbf{B}]_\phi \, d\Sigma = 0 \,, \tag{2}$$

where $\int d\Sigma$ implies integration over an axial cylinder. Taylor envisaged that small deviations of γ_B from zero would be balanced by inertia (that is, they would excite motions so as to satisfy Newton's Second Law), which would lead to torsional oscillations. There are, however, many possibilities

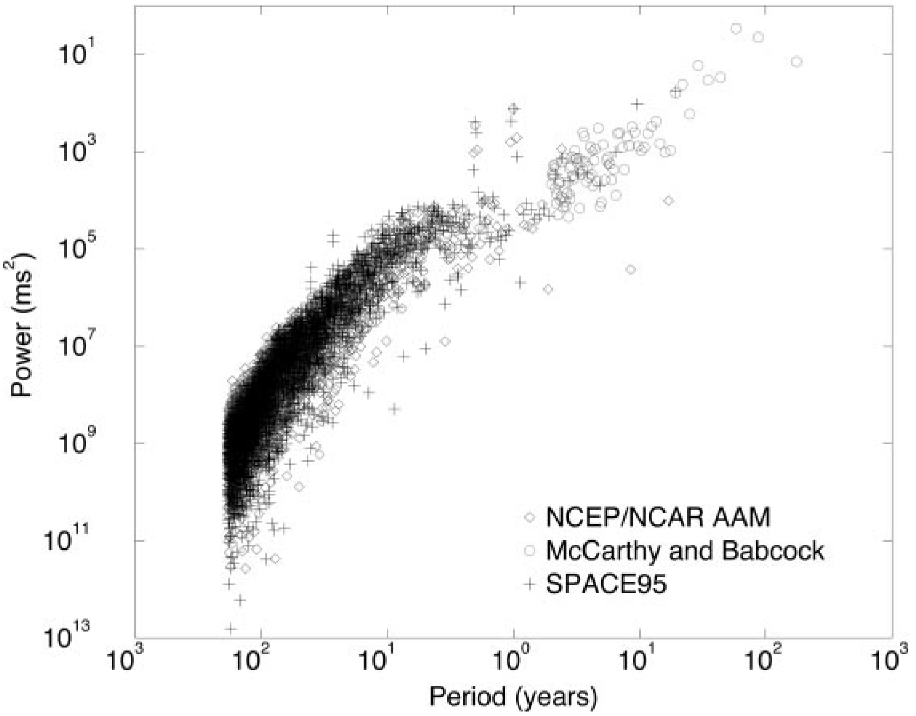

Figure 4. Power spectrum of changes in ΔLOD and AAM. SPACE95 power estimates are using daily estimates of ΔLOD from October 5, 1976 to February 9, 1996. The nine-day and fortnightly tides, as well as the annual and semi-annual signals, are clearly visible in the SPACE95 data. NCEP/NCAR AAM estimates (adjusted to ms of ΔLOD) are from January 1, 1979 to December 31, 1995. The McCarthy & Babcock power estimates are using annual estimates of ΔLOD from 1820 to 1995, with a long-term trend of 1.4 ms century^{-1} removed, from McCarthy & Babcock (1986) extended using more recent results and with the data of Morrison (1979) from 1940–1955 restored. McCarthy & Babcock apply some smoothing that may reduce the estimated power at some of the shorter periods evaluated for that data set (< 10 yr).

for balancing a Taylor torque in the real core, including the following: coherent convective transport of angular momentum; turbulent stresses between cylinders; core–mantle coupling that does not behave like friction (that is, is not proportional to the shear at the CMB); core–mantle 'friction'; and inertia. Electromagnetic coupling at the CMB can have parts that are friction-like and other parts that are not friction-like.

We shall consider a simplification by neglecting all but the last two of these possibilities; that is, assuming that when axial torques are integrated over axial cylinders, we have the balance

$$\text{moment of inertial response} = \text{frictional torque} + \text{`reduced' Taylor torque}\,, \tag{3}$$

where the 'reduced' Taylor torque ($=\gamma_{B_r}$) is the Taylor torque excluding terms which correspond to 'magnetic friction' at the CMB (Braginsky 1988). In a conducting fluid where the magnetic field is important in the force balance, we might expect there to be two asymptotic regimes: a low-friction regime, where the frictional term is small and inertia balances the reduced Taylor torque, and a high-friction regime, where friction balances the reduced Taylor torque.

A quantitative description of the balance in eq. (3), in non-dimensional form, is

$$Ro_t\, s \int \rho \frac{\partial}{\partial t}(v_\phi + v_{\phi M})\, d\Sigma = \epsilon \frac{s^2}{\cos(\theta)} \int [F_{z_+} v_{\phi_{z_+}} + F_{z_-} v_{\phi_{z_-}}]\, d\phi + \gamma_{B_r}\,, \tag{4}$$

where

$$Ro_t = \frac{B_s^*}{r_c \Omega \sqrt{\rho\mu_0}}\,, \tag{5}$$

$$\epsilon = \frac{F^*}{\rho\Omega r_c}\,. \tag{6}$$

Ro_t provides the scaling for the inertial term on the timescale of torsional oscillations, as is appropriate for considering the angular momentum balance of axial cylinders. B_s^* is a typical strength of B_s in the core, F^* a typical magnitude of F, and ρ is the density. The ratio Ro_t/ϵ is important for determining whether the system is in a low-friction or high-friction regime.

If friction is viscous, then

$$\epsilon = \left(\frac{\eta_v}{\Omega r_c^2}\right)^{1/2} = E^{1/2}\,, \tag{7}$$

where E is the Ekman number, and η_v is the kinematic viscosity (units $m^2\ s^{-1}$). Braginsky (1975, 1978, 1988) suggested that the dynamo may be in a nearly axisymmetric state where $|B_s| \ll |B_z|$ ('Model Z'). As $E \to 0$, $|B_s/B_z| \sim \epsilon^{1/3}$, and $\gamma_B \to 0$ (that is, it asymptotically satisfies Taylor's constraint). Although, of course, one may not evaluate an asymptotic relation with one data point, our results nonetheless suggest that the assumption $|B_s| \ll |B_z|$ is not true for the Earth.

It is worth considering the behaviour of the Model Z dynamo as the Ekman number is decreased. Model Z implies that $Ro_t \sim \epsilon^{1/3}|B_z|$, i.e. that $Ro_t/\epsilon \sim \epsilon^{-2/3}|B_z|$. Thus, if $|B_z|$ remains constant as $E \to 0$, the importance of the inertial term overwhelms that of the viscous term, and Model Z scaling will break down. This may be viewed as a transition from a 'high-friction' regime to a 'low-friction' regime as the friction is reduced, as can be seen in the study of Jault (1995) (see their Fig. 11).

Using the part of the zonal flow which is steady on the timescales of our study and our model of core–mantle friction, we may estimate the quasi-steady torque on cylinders due to shear at the CMB. We may then compare this with estimates of the internal Lorentz torque on the cylinders by assuming that at least the equatorially symmetric, axisymmetric, zonal steady velocities are independent of z (as we expect for the corresponding time-varying part of the flow) so as to learn something about core dynamics. This condition would be true for a Model Z-type dynamo, where the quasi-steady zonal winds in the core are primarily geostrophic (Roberts 1989). The results of this comparison may tell us whether or not it is likely that this condition is met in the core, from which we may make some inferences concerning core dynamics. In order to obtain a rough estimate of the quasi-steady internal Lorentz torque of a Model Z-type dynamo, as well as assuming that the quasi-steady zonal winds in the core are primarily geostrophic, we further assume that the field produced by shearing of B_s by the zonal wind diffuses over length scales d of order 10 times smaller than the radius of the core. The diffusion length scale may be different from the shearing length scale in the core depending on the geometry of the field within the core, but by assuming a relatively short diffusion length scale we should be erring on the side of underestimating the internal Lorentz torque.

Unfortunately, there are complications. It is likely that there are torques other than the 'frictional' torque and the internal Lorentz torque that vary on long timescales, especially if core–mantle coupling is magnetic in nature. An example might be the 'leakage' torque associated with the diffusion of toroidal magnetic field (Stix & Roberts 1984). Integrated over the whole CMB, the quasi-steady torques should balance, unless gravitational core–mantle coupling is important. In the future, if a sufficiently reliable picture of core–mantle friction can be obtained, it may be possible to estimate the magnitudes of the quasi-steady torques by this method, but this is well beyond the scope of this study. The other torques mentioned earlier that may exist between axial cylinders within the core would also need to be taken into account.

Nonetheless, we can still compare the quasi-steady frictional axial torques on cylinders with the quasi-steady internal Lorentz torque between cylinders due to shear in the core, since we might expect our model of F to provide a rough estimate of the magnitude of core–mantle friction (except perhaps at the possibly spurious peak), although as effects other than core–mantle friction are also represented by the fitted F, it might be best regarded as a soft upper bound on core–mantle friction (it is possible that F arises from a small difference between relatively large excitations and dampings that are locally in near balance, in which case inertia would be unimportant and some other explanation for the wave-like nature of these flows would need to be found). We do not allow for longitudinal variation in either the friction or the velocity, but this should not affect the order of magnitude of our estimates. Although the part of the velocity field that we use for this calculation (shown in Fig. 5), the steady part, may be unreliable due to diffusion of magnetic field at the top of the core (Voorhies 1993;

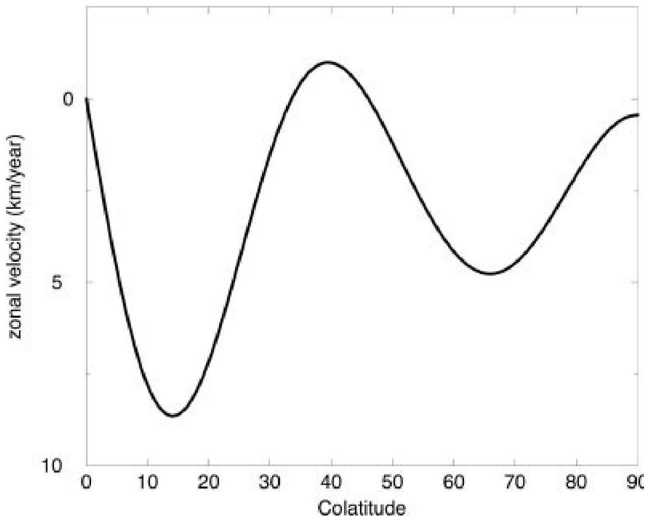

Figure 5. The steady, axisymmetric, equatorially symmetric part of the velocity. Velocity is given in km yr^{-1}.

Gubbins & Kelly 1996), we shall assume that we can still use it for calculating rough estimates in this study. The relative magnitudes of the different torques in eq. (3) are insensitive to the scale of the velocity.

The shear between geostrophically rotating cylinders ($\partial \mathscr{R}_g(s)/\partial s$) will induce some zonal field,

$$B'_\phi \simeq -\frac{s\,d^2 B_s}{\eta}\,\frac{\partial \mathscr{R}_g}{\partial s}\,, \tag{8}$$

where η is the magnetic diffusivity (units m^2 s^{-1}). Here we neglect the production of B_ϕ by other advectional effects and by boundary effects, and assume $\nabla^2 \simeq 1/d^2$. If F is the friction at the CMB, then the frictional torque on an axial cylinder will be

$$\gamma_{\mathrm{F}} = \frac{4\pi r_c s^3 \mathscr{R}_g}{\sqrt{r_c^2 - s^2}}\,F\,. \tag{9}$$

As eq. (8) excludes boundary contributions, it can be used together with eq. (2) to estimate the quasi-steady reduced Taylor torque on an axial cylinder from the basic state mainstream magnetic field:

$$\begin{aligned}
\gamma_{B_{rs}} &\simeq \frac{s}{\mu_0}\int\left(\frac{2B_sB_\phi}{s} + B_s\,\frac{\partial B_\phi}{\partial s} + B_\phi\,\frac{\partial B_s}{\partial s}\right)d\Sigma + \frac{\alpha s}{\mu_0}\int [B_zB_\phi]_{z_-}^{z_+}\,d\phi \\
&\simeq -\frac{4\pi s^2 d^2\sqrt{r_c^2-s^2}}{\mu_0\eta}\left(\left(s\,\frac{\partial^2\mathscr{R}_g}{\partial s^2} + \frac{\partial \mathscr{R}_g}{\partial s}\left(3 - \frac{s^2}{r_c^2-s^2}\right)\right)\right. \\
&\qquad \left. + s\,\frac{\partial \mathscr{R}_g}{\partial s}\,\frac{\partial}{\partial s}\right)\overline{B_s^2} - \frac{2\pi s^3 d^2}{\mu_0\eta}\,\frac{\partial \mathscr{R}_g}{\partial s} \\
&\qquad \times\left\{\alpha[B_sB_z]_{z_-}^{z_+} + \frac{s}{(r_c^2-s^2)^{1/2}}\,[B_{s_{z_+}}^2 + B_{s_{z_-}}^2]\right\}.
\end{aligned} \tag{10}$$

We shall assume $\eta \simeq 1$ and $d \simeq r_c/10$. Fig. 6 shows our estimate of the ratio of this torque to the quasi-steady frictional torque γ_{F_s}. The ratio becomes very large in places where F or $\mathscr{R}_g$ becomes small locally, but the important point to note is that the Taylor torque is always much larger than the frictional torque, with the possible exception of the region where F is strongest around a colatitude of 60°. B_s within the core appears

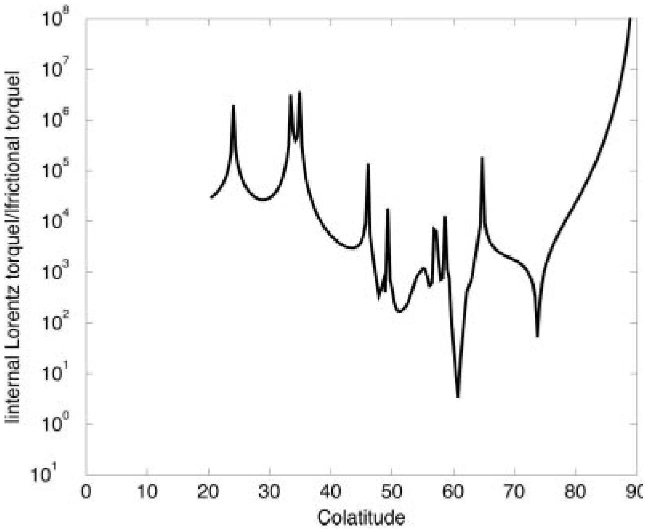

Figure 6. Estimated amplitude of the ratio of quasi-steady 'reduced' Taylor torque to quasi-steady frictional torque at the CMB, using rms B_s and F from the inversion shown as the solid line in Fig. 2 (i.e. wave A only, $\alpha=1$). The ratio becomes very large locally where F or $\mathscr{R}_g$ is small. The important point is not the detail in the graph (whose resolution could be suspect) but that except near 60°, the ratio is large.

to be too large to allow much geostrophic motion: the motions have to vary spatially so that the Taylor torque very nearly cancels itself out when integrated over the axial cylinders. This is in contrast to a Model Z-like dynamo, which is characterized by lines of magnetic field which are elongated in the axial direction (Braginsky 1988), leading to a core with relatively little B_s.

Naturally, this conclusion depends on the evidence at hand, and there is some uncertainty in the reliability of F as an estimate of the core–mantle coefficient of friction. However, a simple scaling argument leads to a similar result—the quasi-steady Lorentz torque on an axial cylinder from the basic state mainstream magnetic field should scale as

$$\gamma_{B_{rs}} \sim \frac{4\pi r_c^2 B_s B_\phi}{\mu_0} \simeq \frac{4\pi r_c^2 d^2 \mathscr{R}_g B_s^2}{\mu_0 \eta}\,, \tag{11}$$

and hence the ratio of this torque to the frictional torque will scale as

$$\left|\frac{\gamma_{B_{rs}}}{\gamma_{F_s}}\right| \sim \frac{d^2 B_s^2}{r_c F \eta \mu_0}\,. \tag{12}$$

From our inversion, $F \sim 10^{1-2}$ N m^{-3} s except close to 60°. The geodetically derived estimate of the amplitude of the decadal oscillation of $\Gamma_z \sim 10^{18}$ N m, which (from the observation that the amplitude of the oscillations in $u_\phi \sim 10$ km yr^{-1}) suggests $F \sim 10$ N m^{-3} s. Both of these estimates of F suggest that the ratio $|\gamma_{B_s}/\gamma_{F_s}| \sim 10^{1-2}$. This is probably too low, as we assume that $\partial/\partial s \sim 1/r_c$ except in the diffusion equation, which underestimates $\partial/\partial s$ for the inverted B_s^2 and $\mathscr{R}$. This supports the previous conclusion that either there are other torques involved, or the quasi-steady magnetic field and flow are arranged so as to reduce the actual $\gamma_{B_{rs}}$ to a value much smaller than the 'naïve' estimate. This is in fact exactly the role of torsional oscillations envisaged by Taylor (1963)—if there is a large Taylor torque, then torsional oscillations are excited which, on dying away, would leave the axial cylinders within the core reoriented so as to set the Taylor torque to zero.

It is possible to carry out a more direct exploration of the relative importance of friction and inertia for these motions. Defining an 'equivalent torque' from the inertial term of the force balance equation:

$$\gamma_I = \omega \mathscr{M} s^2 (\mathscr{R}_g + \mathscr{R}_m) \simeq \frac{\omega \mathscr{M} s^2 \mathscr{R}_g}{1 + \dfrac{I_c}{I_m}}\,. \tag{13}$$

The mantle rotation rate $\mathscr{R}_m$ enters into this equation because the core flow is modelled in the mantle frame, and hence there is a contribution to the inertial term from changes in the rotation rate of this frame compared with the inertial frame. The timescale, τ_e, on which $\gamma_I = \gamma_F$ is given by

$$\tau_e \simeq \frac{\mathscr{M}\sqrt{r_c^2 - s^2}}{2 r_c s \left(1 + \dfrac{I_c}{I_m}\right) F}\,. \tag{14}$$

This is plotted in Fig. 7. It is interesting to note that the timescales are of the same order of magnitude as those of the torsional oscillations: this suggests that the core may be quite efficient at relaxing to a Taylor state via torsional oscillations, but also that $Ro_t \sim \epsilon$. If $\gamma_F \gg \gamma_I$ on the timescales of torsional oscillations, then they would be very difficult to excite. However, in places (especially around 60° and near the equator) γ_F is relatively strong: perhaps torsional oscillations would be difficult to excite in these regions. Alternatively, if torsional oscillations are being excited in these regions, then $|F|$ might predominantly represent excitation, and may overestimate core–mantle friction. This is more likely where F is positive (and therefore corresponds with excitation rather than damping). We should mention that towards the equator, we would expect $\gamma_F \to \infty$ whilst $\gamma_I \to 0$ (due to geometrical effects): however, the consequent damping of the core velocity near the equator may not be recovered in the flow model due to its finite resolution, in which case τ_e may be overestimated next to the

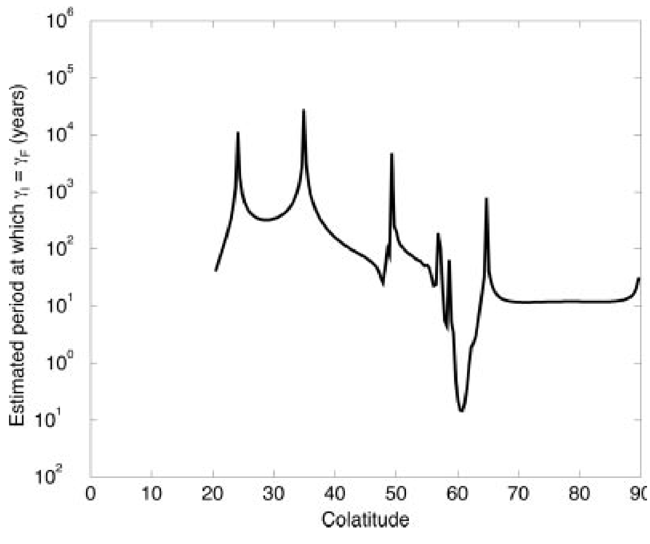

Figure 7. Period at which $\gamma_I = \gamma_F$ for axial cylinders in the core. Longer periods indicate relatively weak friction.

equator. It is conceivable that reduced Taylor torques might tend to be balanced by inertia in some parts of the core and by core–mantle friction or other interactions in other parts, leading to a core in a mixed dynamic state.

These results imply that inertia is at least as important as friction in determining the torque balance of the core, and the high-friction regime is inapplicable.

4 DISCUSSION AND CONCLUSIONS

Although the dynamical state of the Earth's core is difficult to infer from surface observations, any model that purports to describe a 'geodynamo' should be tested against what little we can infer. In this paper, we have described some evidence that suggests that inertia is at least as important as core–mantle friction for determining the torque balance of axial cylinders in the core. This does not mean that there are not other, unmodelled torques that are significant in the real core. The nature of core–mantle 'friction' will vary between models and cannot simply be estimated from the Ekman number when there is hyperviscosity (Zhang & Jones 1997), enhanced boundary layer viscosity, or a finitely conducting mantle. Therefore, it is not a trivial matter to estimate this parameter—it may be preferable instead to determine ϵ through numerical experiment.

From eqs 5 and 6, if $F \simeq 10$ N m^{-3} s and $B_s^* \simeq 2 \times 10^{-4}$ mT then $Ro_t/\epsilon \sim O(1)$, also suggested by the fact that friction becomes similar in magnitude to the inertial response on the timescales of the oscillations themselves (shown in Fig. 7). This somewhat surprising result could be mere coincidence, or it could perhaps reflect some deeper underlying physics that relates the magnitude of B_s (or perhaps the poloidal field B_p) to the strength of core–mantle coupling. The simple fact that torsional oscillations appear to exist suggests that the inertial term is important in comparison with the friction term on decadal timescales (as this study finds) because otherwise damping in the system would probably be too large to allow oscillatory behaviour. The approximate equivalence of the magnitudes of Ro_t and ϵ is less expected, although perhaps suggested by the fact that the damping timescale of the torsional oscillations is of the same order of magnitude as their periods.

One might suppose that omitting the inertial term in a simulation would simply filter out the torsional oscillations, without affecting the rest of the model. This is incorrect: without the inertial term the model would not relax towards a Taylor state, and the geostrophic winds that are excited in order to let core–mantle friction satisfy the force balance might significantly alter the character of the solution.

The Ekman number of the fluid outer core is poorly constrained. The viscosity found from a numerical model of molten iron suggests $E \simeq 4 \times 10^{-15}$ (de Wijs *et al.* 1998), but if a turbulent viscosity is appropriate then perhaps $E \sim O(10^{-6})$ (Gubbins & Roberts 1987). If core–mantle friction is mainly viscous, this would suggest $10^{-7.5} < \epsilon < 10^{-3}$, which includes the estimate $Ro_t \sim O(10^{-5})$ obtained by assuming $B_s^* \simeq 2 \times 10^{-4}$ T and $\rho^* \simeq 10^4$ kg m^{-3}.

The fact that the axisymmetric, equatorially symmetric part of our zonal flow model could be well fitted by only two waves does not mean that there are only two torsional oscillations in the core: it could also be an artefact of the lack of time resolution of the flow model. Future flow models with higher resolution may be very productive in elucidating the nature of core dynamics on short timescales, and should be aided by the upcoming series of geomagnetic satellites.

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