ON THE ELASTIC BEAM-FLUID INTERACTION BASED ON STRUCTURAL MODE SHAPES

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ABSTRACT: The concept of fluid-structure interaction(FSI) has a vast range of practical engineering applications. Since the analytical solution for such a systems is not available, numerical methods are adopted. Two main approaches are monolithic and partitioned methods. Treating the coupled system of fluid and structure simultaneously as in monolithic approach will limit the flexibility, in the other hand this property is maintained by decomposing the physical system into fluid and structure. Using a partitioned approach, the solution is separately advanced in time over each partition. Interaction effects are accounted for by transmission and synchronization of coupled state variables. The strength of the coupling is traditionally maintained by correcting the interaction effects iteratively. Besides convergence's high computational cost, the physics of the interaction can also jeopardize an iterative procedure. For two-way physically coupled systems the information exchange among two partitions (fluid and structure) is relaxed to avoid numerical instabilities. In this paper, the idea of preserving coupling strength by catching the presence of the structure through an interaction law is studied and formulated. The structural modal representation is utilized to derive the interaction boundary at the fluid-structure interface. Since this approach strongly enforces the information exchange on the fluid-structure interface, the source of instabilities is abolished. As a test case, a 1D Euler-Bernoulli beam located at the bottom of a 2D tank is strongly coupled with a fluid above by means of iterative under-relaxed coupling results and also the weak coupling. The results are shown and then discussed.

Keywords: FSI, Coupled mechanical systems, Hydro-elasticity, Modal decomposition

1. INTRODUCTION

1.1 The concept of FSI

The interaction between fluids and solids is a phenomenon that can often be observed in nature, for example, the deformation of trees (elastic response) or the movement of sand dunes caused by wind (rigid structure response). In almost the same manner, wind can interact with buildings, sometimes with dramatic consequences, such as the collapse of the Tacoma-Narrows Bridge in November 1940. These processes can only be calculated using laws and equations from different physical disciplines. Examples like this, where the arising subproblems cannot be solved independently, are called multi physics applications. Two disciplines involved in these kinds of multi physics problems are fluid dynamics and structural dynamics, which can both be described by the relations of continuum mechanics. In engineering applications, in order to study the effects of FSI for a system, experimental test are performed which sometimes are extremely costly and time consuming and occasionally even infeasible. Recently the numerical techniques and computational power has improved a lot, making it possible to deliver more accurate results within a acceptable time.

2. PHYSICAL MODEL

In general there are two types of physical systems in practice: single domain systems and multi domain systems. In multi domain systems the intensity of the interaction between domains is different. The exchange of information can be either only from one to the other or in two-way. As illustrated in fig. 1 in case of fluid-structure system, two-way coupled domains are present only within a specific range of interaction.

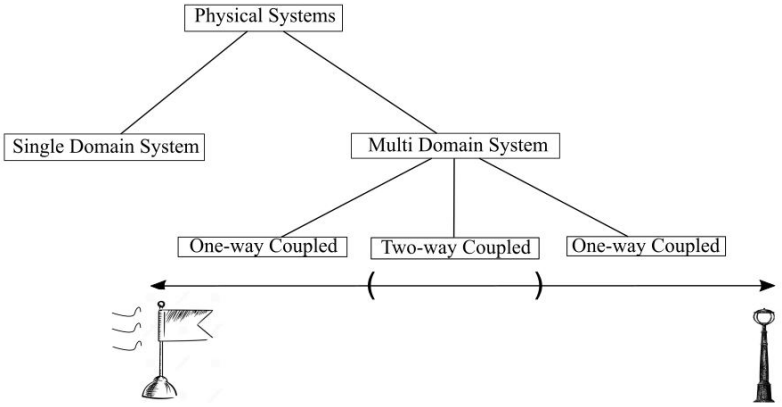

Figure 1 - Physical systems classified as single and multi domain

One-way coupled fluid-structure interaction Let's consider that the structure due to the flow field around it deforms slightly or vibrates with a small amplitude, then the flow field variation due to structure response could be negligible which allows the coupling to be in one direction. On the other hand, the structure is too flexible that follows the flow field without any distortion on the flow around it like a flag. In both cases, the coupling between these domains is only in one-way.

Two-way coupled fluid-structure interaction When large structural deformations or displacements result in significant alteration of the original flow field, a feedback from structure toward fluid seems to be essential.

In the context of offshore structures different types of structures which are rather stiff or flexible are involved. As the purpose of this project is not to limit ourselves to only a specific type of structures, the ultimate aim would be solving two-way coupled systems.

3. MATHEMATICAL MODEL

In this section the governing equations for fluid-structure interaction is strong form is presented.

3.1 Fluid Model

3.1.1 Governing Equations

For an incompressible flow the continuity equation represents the conservation of mass in fluid domain $\Omega_f \in R^3$, while conservation of momentum due to transport by convection, diffusion,

pressure and the external force ($\boldsymbol{g}$) is presented in the momentum equation.

$$\boldsymbol{\nabla}.\boldsymbol{u} = 0 \qquad \text{in} \quad \Omega_f \tag{1}$$

$$\rho_f \frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u}.\boldsymbol{\nabla})\boldsymbol{u} + \boldsymbol{\nabla}.\sigma_f = \rho_f \boldsymbol{g} \qquad \text{in} \quad \Omega_f \tag{2}$$

$$\boldsymbol{\nabla}.\sigma_f = \boldsymbol{\nabla} p - \mu \Delta \boldsymbol{u} \tag{3}$$

Here $\boldsymbol{u}$ is the velocity vector $\boldsymbol{u} = [u, v, w]^T$, p is the pressure, σ_f is fluid stress tensor, μ is the fluid viscosity and ρ_f is its density.

3.2 Structural Model

According to linear elasticity theory, the deformation $\boldsymbol{d} = [d^x, d^y, d^z]^T$ follows the relation:

$$\rho_s \frac{\partial^2 \boldsymbol{d}}{\partial t^2} + \boldsymbol{\nabla}.\sigma_s = \rho_s \boldsymbol{f}_s \qquad \text{in} \quad \Omega_f \tag{4}$$

$$\sigma_s = \frac{E}{2}(\boldsymbol{\nabla}\boldsymbol{d} + (\boldsymbol{\nabla}\boldsymbol{d})^T) \qquad \text{in} \quad \Omega_f \tag{5}$$

Here $\boldsymbol{d}$ is the displacement vector, σ is the Cauchy stress tensor, ρ_s is structure density, E the module of elasticity and f_s is the external force acting on structure.

Euler-Bernoulli Beam

The one dimensional Euler-Bernoulli beam with a constant cross section (A) along its length has deformation in z-direction d due to force per length exerted on the structure in that direction f_s by the relation below:

$$\rho_s A d_{tt} + E I_y d_{xxxx} = f_s \tag{6}$$

Here ρ_s is the structure density, A the cross section area, E the Young modules of elasticity, I_y

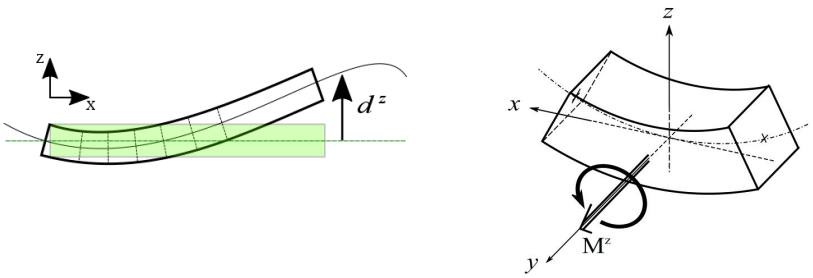

Figure 2 - Schematic of the 1D beam bending over y-axis

the second moment of area around y-axis. The beam deformation is illustrated in fig. 2.

3.3 Coupling Relation

Let Γ_{FS} denote the interface of the fluid-structure domain. The kinematic and dynamic coupling relations are :

$$\boldsymbol{u}_f = \frac{\partial \boldsymbol{d}}{\partial t} \qquad \text{on} \quad \Gamma_{FS} \tag{7}$$

$$\boldsymbol{\sigma}_f.\boldsymbol{n} = \boldsymbol{\sigma}_s.\boldsymbol{n} \qquad \text{on} \quad \Gamma_{FS} \tag{8}$$

In dynamic relation, the fluid stress tensor term has contributions of diffusion and pressure. For simplicity the effect of viscosity will be neglected, so only the pressure stresses remain from fluid part.

4. NUMERICAL MODEL

4.1 Fluid Flow Finite Volume Temporal And Spatial Discretization

The fluid equations in Eq.(1-3) are discretized with a finite volume method in a way to preserve the symmetry properties of the underlying continuous operators. The continuity equation using divergence operator ($\mathcal{M}$) in discrete form applied on discrete velocity (u) and discrete pressure (p) is :

$$\mathcal{M}\ \mathrm{u}^{n+1} = 0 \tag{9}$$

The discrete momentum equation will be:

$$\mathcal{V}\frac{\mathrm{u}^{n+1}-\mathrm{u}^{n}}{\Delta t} = -\mathcal{C}(\mathrm{u}^{n})\mathrm{u}^{n} - \mathcal{G}\ \mathrm{p}^{n+1} + \mathcal{D}\ \mathrm{u}^{n} + \mathrm{g}^{n} \tag{10}$$

In eq. 10 $\mathcal{V}$ is a diagonal matrix that contains control volume size, p is discrete pressure, g is discrete external force, $\mathcal{C}$ is discrete convective operator which is skew symmetric($\mathcal{C} = -\mathcal{C}^{T}$), $\mathcal{G}$ is discrete gradient operator and $\mathcal{D}$ is discrete diffusive operator which is symmetric and negative definite.
The velocity predictor is defined to be:

$$\tilde{\mathrm{u}}^{n} = \mathrm{u}^{n} + \Delta t\ \mathcal{V}^{-1}(-\mathcal{C}(\mathrm{u}^{n})\mathrm{u}^{n} + \mathcal{D}\mathrm{u}^{n} + \mathrm{g}^{n}) \tag{11}$$

Rearranging eq. 10 using the velocity predictor yields :

$$\mathrm{u}^{n+1} = \tilde{\mathrm{u}}^{n} - \Delta t\,\mathcal{V}^{-1}\ \mathcal{G}\ \mathrm{p}^{n+1} \tag{12}$$

By getting the divergence of eq. 12 and using continuity equation as in eq.9, the Poisson equation for pressure will be:

$$\mathcal{M}\,\mathcal{V}^{-1}\,\mathcal{G}\ \mathrm{p}^{n+1} = \frac{1}{\Delta t}\ \mathcal{M}\ \tilde{\mathrm{u}}^{n} \tag{13}$$

and since $\mathcal{M} = -\mathcal{G}^{T}$:

$$\mathcal{G}^{T}\,\mathcal{V}^{-1}\,\mathcal{G}\ \mathrm{p}^{n+1} = -\frac{1}{\Delta t}\ \mathcal{M}\ \tilde{\mathrm{u}}^{n} \tag{14}$$

The solution procedure is to predict velocities using old values as in eq.11, substituting predicted velocities in pressure Poisson and solve for new pressures (p^{n+1}) and then correcting the predicted velocities by eq.12. The computed velocities are divergence free and they satisfy the momentum equation as well.

4.2 Structural finite element method solver

By substituting the dynamic relation as in eqn. 8 in the governing equilibrium equation of the one dimensional beam as eq. 6, the equation of motion of the beam will become:

$$\rho_{s} A\ \mathrm{d}_{tt} + EI\ \mathrm{d}_{xxxx} = -\mathrm{p}\ b \tag{15}$$

Here d is the discrete displacements in the z-direction, p is discrete pressures and b is the beam width in y-direction. The hermitian shape functions (N_A) which are C1 continuous in space describe the displacement vector as:

$$\mathrm{d} = N_A \mathrm{d}_A \qquad\qquad A = 1, \cdots, 4 \tag{16}$$

In which there is an implied sum over all the shape modes multiplied by their displacement participation factor. The weak form can be written using eq.15 in discrete form and applying the divergence theorem as :

$$\begin{aligned} &\int_{x_a}^{x_b} (N_b \rho_s A N_A \mathrm{d}_{A,tt} + N_{b,xx} E I_y N_{A,xx} \mathrm{d}_A + N_b \mathrm{p} b) \mathrm{d}x = \\ &+ N(x_a) V(x_a) - (-\frac{dN}{dx}\Big|_{x_a}) M(x_a) - N(x_b) V(x_b) - (-\frac{dN}{dx}\Big|_{x_b}) M(x_b) \end{aligned} \tag{17}$$

Taking the integrals of the shape functions for each term, will give us the discrete mass operator (M_A) and discrete stiffness operator (K_A). Eq.15 then is decomposed into three discrete equations as:

$$\mathrm{M}_A \mathrm{d}_{tt,A} + \mathrm{K}_A \mathrm{d}_A = \mathrm{p}_A \tag{18}$$

$$\mathrm{d} = N_A \mathrm{d}_A \tag{19}$$

$$\mathrm{p} = \mathrm{N}_A \mathrm{p}_A \tag{20}$$

4.3 Coupling

Solution strategies for FSI coupling are mainly divided into monolithic and partitioned approaches. The required strength of coupling is a function of the physics of the problem.

4.3.1 Monolithic Approach

In a monolithic approach all equations of motion and coupling relations are gathered and solved simultaneously. Monolithic treatment implies a specific and global handling of the all physical systems which are involved. For this reason its application to multi domain systems where individual models are naturally simulated using different formulations or discretization (e.g. Eulerian vs Lagrangian) is seriously restricted[1].

4.3.2 Partitioned Approach

The field models are computationally treated as isolated entities that are separately stepped in time. Interaction effects are viewed as forcing effects that are communicated between the individual components using prediction, substitution and synchronization techniques[2].

Weak coupling: This method typically describes the pure mapping of the physical properties resulting from the analysis of a fluid-model to another solid-model. The results from fluid model are transfered to the solid model only once per time step as an external load. As illustrated in fig.3(a) no feedback is to the fluid solver[3]. In other words, if the intention is to reduce computational cost as much as possible there is no other way but to apply one-way coupling then at each step the physical information (coupling relation: for these systems forces) at the interface is transferred from fluid to the structure and then structural response is computed. This method seriously lacks accuracy since the effect of structure response on the fluid is not considered that can give unreliable results .

Strong coupling: In case of a strong coupling the load mapping is performed per time step in an iterative loop i.e. the results (e.g. forces) of fluid model are transferred to solid model and the results (e.g. displacements) of solid model are in turn transferred back to the fluid model so as to influence the behavior of fluid part. This iterative process will continue until convergence is found or the process is stopped manually. This method requires solving fluid flow as the number of of iterations required for FSI convergence. This means it can be a computationally expensive method. In case of high added mass ratios or physically two-way coupled systems, it could even break down. In order to avoid that under-relaxation for information exchange on the interface is applied as illustrated in fig.3(b), which will increase simulation time even more.
The question is how to make a partitioned coupling more efficient?
The response is hidden in the source of instability. By extracting the most important part of structural response and adding it to fluid solver, the physics of interaction is ruled by the time step and no independent source of instability will exist. The schematic of the quasi-simultaneous approach is presented in fig.13(c)

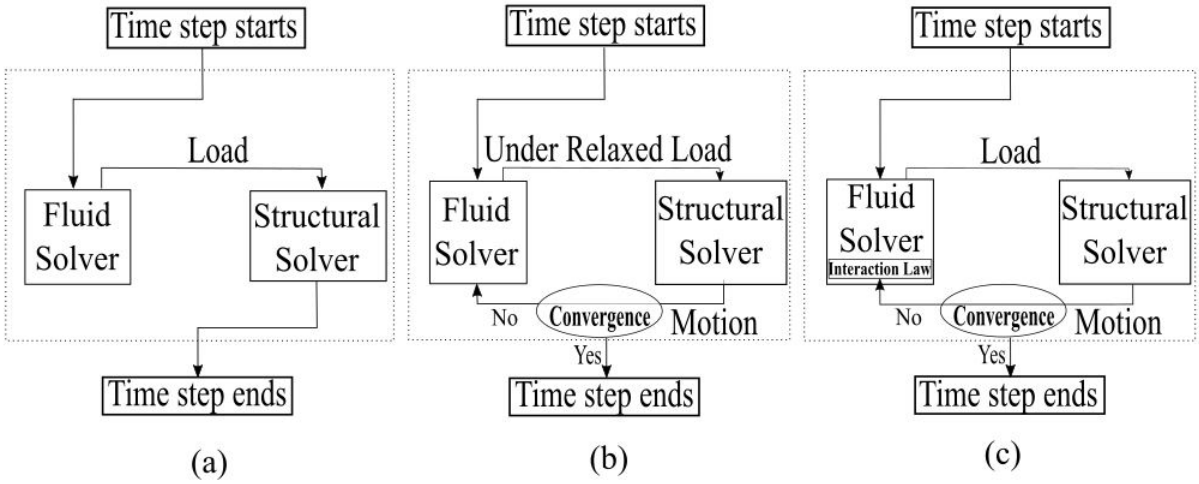

Figure 3 - Partitioned fluid-structure system solved weakly(a), strong but with under relaxation of load(b) and with the interaction law or quasi-simultaneous(c).

4.4 Modal representation of the structure

In this section, it is intended to derive the discrete modal representation of a structure which is the primary ingredient of the interaction law. The nodal discrete equation of motion of a three dimensional structure is in form of:

$$\mathrm{M}\mathrm{d}_{tt} + \mathrm{K}\mathrm{d} = \mathrm{B}\mathrm{p} \tag{21}$$

where M and K denote mass, stiffness matrix and on the right hand side the nodal pressures are projected onto the normal surface vector.
In order to derive a modal representation of the structure,Initially the free motion of the structure is considered:

$$\mathrm{M}\mathrm{d}_{tt} + \mathrm{K}\mathrm{d} = 0 \tag{22}$$

The solution to this equation is in the form of :

$$\mathrm{d} = \varphi(x)e^{i\omega t} \tag{23}$$

By substituting eq. 23 in eq. 22:

$$(\mathrm{K}-\mathrm{M}\lambda)\varphi e^{i\omega t}=0 \tag{24}$$

The trivial solution to this equation is $\varphi=0$. For a non-trivial solution, the determinant of the matrix $(\mathrm{K}-\mathrm{M}\omega^2)$ is set to be zero:

$$|\mathrm{K}-\mathrm{M}\lambda|=0 \tag{25}$$

Solving eq. 25 gives us structural eigen frequencies (λ^m) with their own corresponding eigen mode ($\varphi^m=[\varphi_1^m,\varphi_2^m,...,\varphi_n^m]^T$ in which n is number of nodal points), m representing the mode number related to each discrete degree of freedom. The solution to eq. 22 is:

$$\mathrm{d}=\sum_{m=1}^{N}\varphi^m(x)e^{i\omega^m t} \tag{26}$$

Here N denotes the number of the degrees of freedom of the structure. This sum can be truncated to approximate the displacement. The problem reduces to computing the time dependent term i.e. participation factor ($z^m=e^{i\omega^m t}$) for each mode (m) as:

$$\mathrm{M}\sum_{m=1}^{N}(\varphi^m z_{tt}^m)+\mathrm{K}\sum_{m=1}^{N}(\varphi^m z^m)=\ \mathrm{p.n} \tag{27}$$

One important property of the eigen modes is that they are orthogonal with respect to M and K matrices.

$$(\varphi^{m_1})^T\mathrm{M}(\varphi^{m_2})=0 \quad \text{if} \quad m_1\neq m_2 \tag{28}$$

$$(\varphi^{m_1})^T\mathrm{K}(\varphi^{m_2})=0 \quad \text{if} \quad m_1\neq m_2 \tag{29}$$

By multiplying eq. 27 from left by $(\varphi^m)^T$ and exploiting orthogonality of eigen modes with respect to M and K, we will have :

$$(\varphi^m)^T\mathrm{M}\varphi^m z_{tt}^m+(\varphi^m)^T\mathrm{K}\varphi^m z^m=\ (\varphi^m)^T\mathrm{Bp} \tag{30}$$

This equation shows that eq. 22 is decoupled and decomposed into N(degrees of freedom of the discrete system) scaler equations (m is mode number). The eigen modes can be scaled in a way that $(\varphi^m)^T\mathrm{M}\varphi^m=1$ which makes the second term to be $(\varphi^m)^T\mathrm{K}\varphi^m=\lambda^m$. Finally the equation of motion will become:

$$z_{tt}^m+\lambda^m z^m=\ (\varphi^m)^T\mathrm{Bp} \qquad\qquad m=1,\cdots,N \tag{31}$$

Having the eigen frequencies (λ^m) and eigen modes (φ^m) for each mode (m) these equations are solved for their corresponding participation factor (z^m) independently.

4.5 Derivation of the interaction boundary condition

In this case all the terms are evaluated at the new time level $(n+1)$, as:

$$z_{tt}^{m,n+1}+\lambda^m z^{m,n+1}=\ (\varphi^m)^T\mathrm{Bp}^{n+1} \tag{32}$$

In which m represents the mode number and superscript $n+1$ is for the time level. Using backward Euler time integration relations as following:

$$z_{tt}^{n+1}=\frac{z_t^{n+1}-z_t^n}{\Delta t} \tag{33}$$

$$z^{n+1}=z^n+\Delta t {z_t}^n \tag{34}$$

Substituting eq. 33 and eq. 34 in eq. 32 will lead to :

$$z_t^{m,n+1} = (\frac{1}{\Delta t} + \lambda^m \Delta t)^{-1} (\varphi^m)^T \mathrm{p}^{n+1}.\mathrm{n} + r^{m,n} \tag{35}$$

In which $r^{m,n}$ are all other terms at the old time level.

$$r^{m,n} = (\frac{1}{\Delta t} + \lambda^m \Delta t)^{-1} (\frac{1}{\Delta t} z_t^{m,n} - \lambda^m z^{m,n}) \tag{36}$$

If we assume the mode shapes dependency on time to be negligible then the exact velocity at the interface is:

$$\mathrm{u} = \mathrm{d}_t = \sum_{m=1}^{N} \varphi^m z_t^m \tag{37}$$

The velocity contribution due to mode m is :

$$\mathrm{u}^m = \mathrm{d}_t^m = \varphi^m z_t^m \tag{38}$$

By multiplying eq. 35 by φ^m from left, then the velocity due to mode m will be:

$$\mathrm{u}^{m,n+1} = (\frac{1}{\Delta t} + \lambda^m \Delta t)^{-1} \varphi^m (\varphi^m)^T \mathrm{B}\mathrm{p}^{n+1} + \mathrm{u}^{m,n} \tag{39}$$

4.6 Applying The New Boundary Condition

For partitioned fluid-structure system, conventionally the structure motion i.e. its velocity is applied as a boundary on fluid per time step. In eq.12 the new velocities is derived in terms of both pressure and velocity. Rewriting eq.12 by adding the modal boundary condition in eq.40 will lead to:

$$\mathrm{u}^{n+1} = [\tilde{\mathrm{u}}^n + \mathrm{u}^{m,n}|_{\text{interface}}] + [-\Delta t \, \mathcal{V}^{-1} \, \mathcal{G} \, \mathrm{p}^{n+1} + (\frac{1}{\Delta t} + \lambda^m \Delta t)^{-1} \varphi^m (\varphi^m)^T \mathrm{B}\mathrm{p}^{n+1}|_{\text{interface}}] \tag{40}$$

The first term on right hand side is the velocity boundary condition at the F-S interface and the second term represents the new boundary conditions for pressure Poisson.

5. RESULTS AND DISCUSSION

The test case that is chosen to verify the mathematical theory, is a 1D Euler-Bernoulli beam with properties shown in table.1 which is located at the bottom of a 2D fluid domain as illustrated in fig.4. This study is performed on fluid solver software package ComFlow which is getting extended to deal with the interaction of elastic-structure and the fluid. An approximation for added mass is the mass of column of water above the beam elements. In this case this value is computed to be 40. This value will alter due to the variation of beam deformation along its length. In order to trigger interaction, the beam is set to have an initial deflection.

5.1 Weak vs Strong Coupling

The 1st mode deflection is initialized and the displacement of the node in the middle of the beam in two cases of weakly and strongly coupled partitions is shown in fig.4. As mentioned, for problems with high added mass ratio the weak coupling can have completely different behavior than the real system. The first natural frequency of this beam form analytical relation eq.41 is

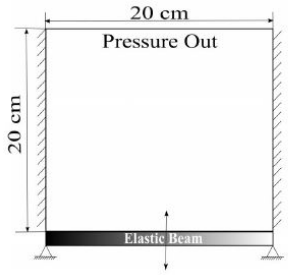

Figure 4 - Euler Bernoulli Beam interaction with fluid setup

Module of Elasticity	3.4	(GPa)
Density	1000	(kg/m^3)
Thickness	0.5	(cm)
Width	1.0	(cm)

Table 1 - Beam properties

computed to be 104.5 (Hz). This value is roughly represented by the oscillation frequency of weak coupling results that is 90.82 (Hz). On the other hand, the oscillation frequency captured by strong coupling (17.96 Hz) representing the wet natural frequency that is 17.83 (Hz). It can be concluded that using weak coupling for high added mass ratio problems like this, will be greatly mimic dry structural response. Beside the natural frequency, the damping ratio is noticeably more in case of weakly coupled partitions.

$$f_1 = \frac{1}{2\pi}[\frac{\pi}{L}]^2[\frac{EI}{\rho I}]^{\frac{1}{2}} \tag{41}$$

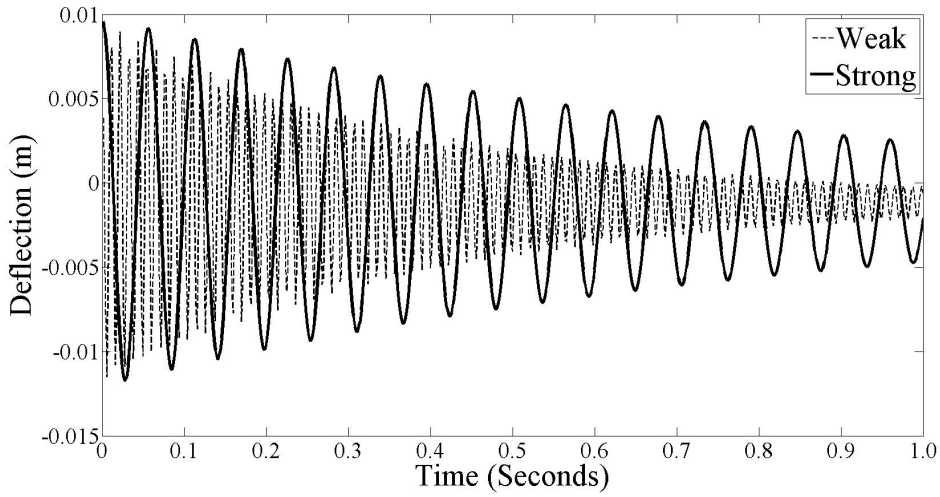

Figure 4 - Euler Bernoulli Beam interaction with fluid setup

5.2 Strong Coupling

It is needed to be mentioned that in order to avoid the break down of fluid-structure iterations the under relaxation is applied. According to numerical simulations, the iterative procedure instability occurs for relaxation factors more than 3%. This value is consistent with added mass ratio and the fact that this iteration will converge if the added mass ratio is kept below one. In this study the relaxation factor is set t0 2% in order to preserve a margin for added mass variation.

The beam local added mass is a function of time and it is directly dependent on initial deflection of the beam. To study this, two different initial conditions are applied and the fluid-structure

iteration number of each cycle is shown in fig.5. The total simulation time is one second.

- According to fig.5 the number of commencing cycle's iterations when second mode is initially excited is higher than the 1st mode. This is due to the fact that beam added mass is a function of the local added mass. The local added mass is variating in a more violating way for the second mode, as the natural frequency of this mode is higher.
- After 300 cycles or 0.6 seconds, the number of iterations are very close to each other for both cases. These is due to the forcing on the beam. The bulk of water above the beam is more likely to excite the first mode because its loading shape. This means the second mode deflection will damp by time and only the first one remains.
- This simulation is also done for the case of 3rd mode of the the beam. The results show the same behavior. The number of iterations are much higher and the time step required also need to get smaller.
- It can be concluded while setting the relaxation factor to be 2%, this iterative procedure will not break down but the number of required iterations will increase. This is an indication that the source of instability is suppressed for this problem, or in other words the added mass ratio of this problem will not exceed 50 for these three cases that has been simulated.
- To make the coupling added mass ratio independent, the interaction law need to be implemented.

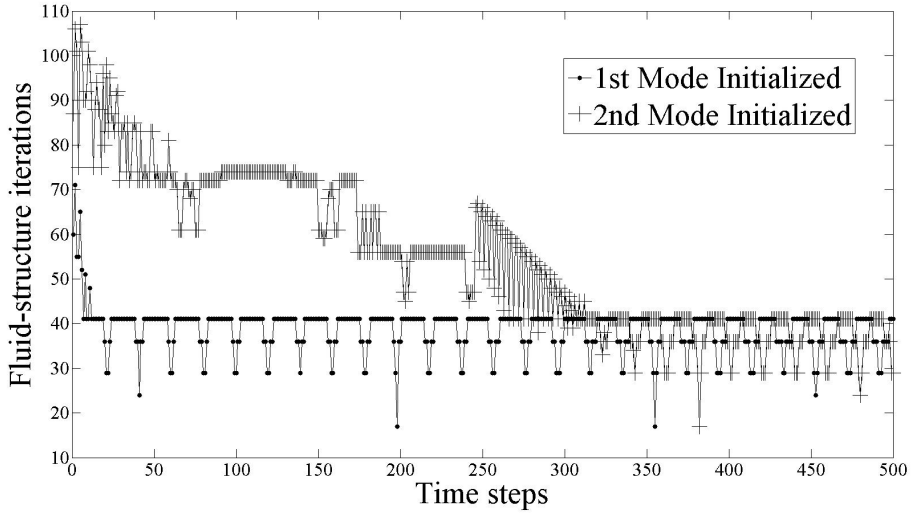

Figure 5 - Euler Bernoulli Beam interaction with fluid setup

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