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On the Electrical Resistivity of Antiferromagnetic Metals at Low Temperatures

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The electrical resistivity of antiferromagnetic metals due to electron-magnon scattering is calculated on the *s-f* model in the low temperature region by the use of the variational approach to the Boltzmann equation. The resistivity ρ varies with temperature T as $\rho \sim T^5$ for $T_N \gg T \gg D$ (T_N: Neel temperature, D: anisotropy energy of the spin system) and as $\rho \sim T \exp(-\sqrt{T_N D}/T)$ for $T_N, D \gg T$. The effects of the Umklapp scattering and the interband scattering to the total conductivity are shown to be small compared with that due to the intraband scattering. The nature of the approximation in the calculation of the resistivity which was used in a previous paper is discussed and corrected.

§ 1. Introduction

It is well known that electrical resistivity of magnetic metals is considerably higher than that of non-magnetic metals on account of electron scattering by spins which cause magnetism. Many theoretical and experimental efforts have been made to clarify various behaviour of resistivity of magnetic metals.

In the magnetic metals at low temperatures electron-magnon scattering gives an important contribution to the resistivity. Several authors[1)~4)] derived the T^2 dependence of the resistivity for the ferromagnetic metals in this low-temperature region. For antiferromagnetic metals, on the other hand, they only conjectured the T^4 dependence without making any detailed calculations: They merely replaced, in the resistivity formula for the ferromagnetic case, the magnon spectrum $\omega_q \propto q^2$ for the ferromagnet by $\omega_q \propto q$ for the antiferromagnet. However such a conjecture seems to be very doubtful for the following reasons. First, such a treatment neglects the momentum dependence of the electron-magnon coupling resulting from the Bogoliubov transformation of spins (see § 2), which will have an important effect on the temperature dependence of the resistivity. Second, in that treatment an effect of the spin order on the energy spectrum of conduction electrons is omitted. When the Fermi surface touches the magnetic zone boundaries, as realized in many cases, such an effect is thought to be serious. In this situation Elliott and Wedgwood,[5)] and Miwa[6)] calculated the effective number of electrical carriers and successfully explained the behaviour of the resistivity of rare earth metals near the transition point. Moreover for conduction electrons near the zone boundaries, i.e., on the neck part of the Fermi surface, Umklapp

processes are possible, and hence different relaxation times are expected for those electrons compared with that for the electrons on the almost spherical part of the Fermi surface.

This problem also arises in the case of electron-phonon scattering if the Umklapp process is taken into account. Lawrence and Wilkins[7] has studied the effect of the Umklapp process due to electron-phonon scattering and obtained the drastic result using the variational method for the Boltzmann equation that the effect of the Umklapp process dominates the resistivity. Unfortunately, however, their formulation derived by the variational method is inadequate in this case, and hence their results are erroneous, as will be discussed in § 3. 1.

Thus more careful treatment seems to be necessary to derive the resistivity due to the electron-magnon scattering in antiferromagnetic metals.

The main purpose of the present paper is to clarify the temperature dependence of the resistivity for the antiferromagnetic metals at low temperatures.

In calculating the resistivity we adopt the *s-f* model and use a formula appropriate for our case derived from the Boltzmann equation by a variational method, and take the first Born approximation for the electron magnon scattering. The vertex correction is shown to be higher order of J/ζ, where J and ζ are *s-f* coupling constant and the Fermi energy, respectively, and then Migdal theorem is satisfied as in the electron-phonon case.[8] Both the intra and interband scattering will be investigated. Furthermore, the difference between the relaxation times of the electrons on the neck part of the Fermi surface and that of the electrons on the belly part (almost spherical part) of the Fermi surface is taken into account.

In § 2 the model Hamiltonian is introduced, and by the use of the Holstein-Primakoff transformation the Hamiltonian is rewritten in the magnon representation. In § 3 the resistivity is calculated by the method mentioned above. Section 4 is devoted to the summary and discussion. In the Appendix it is shown briefly how to treat the problem as in the way of the variational method within the framework of the hydrodynamical approximation of the Kubo formula which was employed in a previous paper.[9]

§ 2. The Hamiltonian

The model Hamiltonian of the system is

$$\mathcal{H}=\mathcal{H}_e+\mathcal{H}_s+\mathcal{H}_{sf}\,, \tag{1}$$

$$\mathcal{H}_e=\sum_{k\sigma}\xi_k c^*_{k\sigma}c_{k\sigma}\,, \qquad \xi_k=\varepsilon_k-\zeta\,, \tag{2}$$

$$\mathcal{H}_s=-\frac{1}{2}\sum_{rr'}(V_{rr'}\boldsymbol{S}_r\cdot\boldsymbol{S}_{r'}+W_{rr'}S_r^z\cdot S_{r'}^z)-D\sum_r(S_r^z)^2, \tag{3}$$

$$\mathcal{H}_{sf}=-\frac{J}{2}\sum_r\sum_{\alpha\gamma}c^*_{r\alpha}c_{r\gamma}\boldsymbol{\sigma}_{\alpha\gamma}\cdot\boldsymbol{S}_r\,, \tag{4}$$

$$c_{r\sigma}=\frac{1}{\sqrt{N}}\sum_{k} e^{-ikr}c_{k\sigma}\,, \tag{5}$$

where D is the anisotropy constant and the other notations are the same as in Ref. 9).*) In the above Hamiltonian we assume $\mathcal{H}_{sf}$ is the small perturbation and the antiferromagnetic structure results from $\mathcal{H}_s$.

The magnetic structure is assumed to be an antiferromagnet of two interpenetrating sublattice type in a s.c. or b.c.c. crystal lattice. In this case the antiferromagnetic structure can be characterized by a wave vector $\boldsymbol{Q}$. One of some equivalent vectors of $\boldsymbol{Q}$ is $\pi/a(111)$ in a s.c. lattice and $2\pi/a(100)$ in the b.c.c. lattice, where a is the lattice parameter.

In the *s-f* model describing rare earth metals, the term $\mathcal{H}_s$ is negligible contrary to the present model, and the exchange interaction between the spins is R-K-K-Y interaction in the antiferromagnetic state. Thus when one applies the present results to the rare earth metals, he should replace the exchange interaction $V(\boldsymbol{q})$ by $J^2\chi(\boldsymbol{q})$ which is of order J^2/ζ.[9] In the model it is assumed that the system has the uniaxial anisotropy in the z direction and the Fermi surface crosses the magnetic zone at $k^z=\pm\boldsymbol{Q}/2$. In the antiferromagnetic state the original single electron band splits into two bands. In this paper we study the following case; both of the Fermi surfaces have sufficiently large belly parts (almost spherical parts) compared with the small neck parts near the zone boundaries. This case was studied by Elliot and Wedgwood[5] (the case (i) in Ref. 5)), and by Miwa[6] in the calculation of the resistivity of rare earth metals.

First we diagonalize the magnon part of $\mathcal{H}_s$. The Holstein-Primakoff transformation[10] retained up to first order of $1/S$ expansion is

$$\left.\begin{aligned} &S_l^z=S-A_l^*A_l\,, && S_m^z=-S+B_m^*B_m\,,\\ &S_l^+=\sqrt{2S}\,A_l\,, && S_m^+=\sqrt{2S}\,B_m^*,\\ &S_l^-=\sqrt{2S}\,A_l^*, && S_m^-=\sqrt{2S}\,B_m\,, \end{aligned}\right\} \tag{6}$$

where $\boldsymbol{l}$ and $\boldsymbol{m}$ are the position vectors on the A and B sublattice respectively. These vectors satisfy $e^{i\boldsymbol{Q}\cdot\boldsymbol{l}}=1$ and $e^{i\boldsymbol{Q}\cdot\boldsymbol{m}}=-1$, where $2\boldsymbol{Q}$ is one of the vectors of the crystal reciprocal lattice. By the use of the momentum representation

$$A_l=\sqrt{\frac{2}{N}}\sum_{k} e^{-ikl}A_k\,, \qquad B_m=\sqrt{\frac{2}{N}}\sum_{k} e^{ik(m+\alpha)}B_k\,, \tag{7}$$

the quadratic part of $\mathcal{H}_s$ has the form

$$\begin{aligned}\mathcal{H}_s=&-\frac{NS^2}{2}[V(\boldsymbol{Q})+W(\boldsymbol{Q})]-NS^2D\\ &+SV(\boldsymbol{Q})\sum_{k}\{[1+d-\bar{v}(\boldsymbol{k})]A_k^*A_k+[1+d-\bar{v}(\boldsymbol{k})]B_k^*B_k\\ &-\tilde{v}(\boldsymbol{k})e^{-ik\alpha}A_k^*B_k^*-\tilde{v}(\boldsymbol{k})e^{ik\alpha}A_kB_k\}\end{aligned} \tag{8}$$

*) The unit $k_B=\hbar=1$ will be used.

with

$$\left.\begin{aligned} d&=\frac{SW(\boldsymbol{Q})+D(2S-1)}{SV(\boldsymbol{Q})}, \\ \bar{v}(\boldsymbol{k})&=\frac{V(\boldsymbol{k})+V(\boldsymbol{k}+\boldsymbol{Q})}{2V(\boldsymbol{Q})}, \qquad \tilde{v}(\boldsymbol{k})=\frac{V(\boldsymbol{k})-V(\boldsymbol{k}+\boldsymbol{Q})}{2V(\boldsymbol{Q})}, \end{aligned}\right\} \tag{9}$$

where $V(\boldsymbol{k})$ and $W(\boldsymbol{k})$ are the Fourier transform of $V_{rr'}$ and $W_{rr'}$, and $\bar{v}(\boldsymbol{k})$ and $\tilde{v}(\boldsymbol{k})$ are real values due to the inversion symmetry assumed for our systems. Here $\sum_{\boldsymbol{k}}$ denotes the summation over the first magnetic Brillouin zone. The vector $\boldsymbol{a}$ is a vector which superposes the B lattice onto the A lattice and satisfies $e^{i\boldsymbol{Q}\boldsymbol{a}}=-1$. We have used the relation

$$\sum_{\boldsymbol{l}} e^{i\boldsymbol{k}\boldsymbol{l}}=\sum_{\boldsymbol{m}} e^{i\boldsymbol{k}(\boldsymbol{m}+\boldsymbol{a})}=\frac{N}{2}\sum_{\boldsymbol{P}}\delta(\boldsymbol{k}+\boldsymbol{P}). \tag{10}$$

The summation $\sum_{\boldsymbol{P}}$ is over all the vectors in the magnetic reciprocal lattice. The expression (8) is diagonalized by the canonical transformation:

$$\begin{pmatrix} A_{\boldsymbol{k}} \\ B_{\boldsymbol{k}}{}^* \end{pmatrix}=\begin{pmatrix} \cosh\phi_{\boldsymbol{k}} & -e^{-i\boldsymbol{k}\boldsymbol{a}}\sinh\phi_{\boldsymbol{k}} \\ -e^{i\boldsymbol{k}\boldsymbol{a}}\sinh\phi_{\boldsymbol{k}} & \cosh\phi_{\boldsymbol{k}} \end{pmatrix}\begin{pmatrix} \alpha_{\boldsymbol{k}} \\ \beta_{\boldsymbol{k}}{}^* \end{pmatrix} \tag{11}$$

with

$$\tanh 2\phi_{\boldsymbol{k}}=-\frac{\tilde{v}(\boldsymbol{k})}{1+d-\bar{v}(\boldsymbol{k})} \tag{12}$$

and

$$\begin{aligned} \mathcal{H}_s=&-\frac{NS^2}{2}[V(\boldsymbol{Q})+W(\boldsymbol{Q})]-NDS^2 \\ &+SV(\boldsymbol{Q})\sum_{\boldsymbol{k}}\{[\sqrt{[1+d-\bar{v}(\boldsymbol{k})]^2-\tilde{v}(\boldsymbol{k})^2}-(1+d-\bar{v}(\boldsymbol{k}))] \\ &+\epsilon_{\boldsymbol{k}}(\alpha_{\boldsymbol{k}}{}^*\alpha_{\boldsymbol{k}}+\beta_{\boldsymbol{k}}{}^*\beta_{\boldsymbol{k}})\} \end{aligned} \tag{13}$$

with

$$\epsilon_{\boldsymbol{k}}=SV(\boldsymbol{Q})\sqrt{[1+d-v(\boldsymbol{k})][1+d-v(\boldsymbol{k}+\boldsymbol{Q})]}. \tag{14}$$

From Eq. (12) we have

$$\left.\begin{aligned} (\cosh\phi_{\boldsymbol{k}}-\sinh\phi_{\boldsymbol{k}})^2&=\sqrt{\frac{1+d-v(\boldsymbol{k}+\boldsymbol{Q})}{1+d-v(\boldsymbol{k})}}, \\ (\cosh\phi_{\boldsymbol{k}}+\sinh\phi_{\boldsymbol{k}})^2&=\sqrt{\frac{1+d-v(\boldsymbol{k})}{1+d-v(\boldsymbol{k}+\boldsymbol{Q})}} \end{aligned}\right\} \tag{15}$$

with $v(\boldsymbol{k})=V(\boldsymbol{k})/V(\boldsymbol{Q})$. By the use of the transformation (6), the term $\mathcal{H}_e+\mathcal{H}_{sf}$ in Eq. (1) is written as

$$\mathcal{H}_e+\mathcal{H}_{sf}=\mathcal{H}_e{}'+\mathcal{H}_{sf}^{(+-)}+\mathcal{H}_{sf}^{(z)} \tag{16}$$

with

$$\mathcal{H}_e' = \sum_{k\sigma} \xi_{k\sigma} c_{k\sigma}^* c_{k\sigma} - \frac{JS}{2} \sum_{l\sigma} \sigma c_{l\sigma}^* c_{l\sigma} + \frac{JS}{2} \sum_{m\sigma} \sigma c_{m\sigma}^* c_{m\sigma} , \tag{17}$$

$$\mathcal{H}_{sf}^{(+-)} = -\frac{J\sqrt{2S}}{2} [\sum_l (c_{l\uparrow}^* c_{l\downarrow} A_l^* + c_{l\downarrow}^* c_{l\uparrow} A_l) + \sum_m (c_{m\uparrow}^* c_{m\downarrow} B_m + c_{m\downarrow}^* c_{m\uparrow} B_m^*)] \tag{18}$$

and

$$\mathcal{H}_{sf}^{(z)} = -\frac{J}{2} [-\sum_{l\sigma} \sigma c_{l\sigma}^* c_{l\sigma} (A_l^* A_l - \langle A_l^* A_l \rangle) + \sum_{m\sigma} \sigma c_{m\sigma}^* c_{m\sigma} (B_m^* B_m - \langle B_m^* B_m \rangle)], \tag{19}$$

where $\langle A_l^* A_l \rangle$ or $\langle B_m^* B_m \rangle$ is a certain average of the Bose particle.

To diagonalize $\mathcal{H}_e'$ we first make the transformation

$$c_{l\sigma} = \sqrt{\frac{2}{N}} \sum_k e^{-ikl} a_{k\sigma} , \qquad c_{m\sigma} = \sqrt{\frac{2}{N}} \sum_k e^{-ik(m+\alpha)} b_{k\sigma} \tag{20}$$

and we have

$$\mathcal{H}_e' = \sum_{k\sigma} \Big[\Big(\frac{\xi_k + \xi_{k+Q}}{2} - \frac{\sigma JM}{2} \Big) a_{k\sigma}^* a_{k\sigma} + \Big(\frac{\xi_k + \xi_{k+Q}}{2} + \frac{\sigma JM}{2} \Big) b_{k\sigma}^* b_{k\sigma} + \frac{\xi_k - \xi_{k+Q}}{2} e^{-ik\alpha} a_{k\sigma}^* b_{k\sigma} + \frac{\xi_k - \xi_{k+Q}}{2} e^{ik\alpha} b_{k\sigma}^* a_{k\sigma} \Big] . \tag{21}$$

This expression is diagonalized by the following canonical transformation:

$$\begin{pmatrix} a_{k\sigma} \\ b_{k\sigma} \end{pmatrix} = U \begin{pmatrix} D_{k\sigma}^{(+)} \\ D_{k\sigma}^{(-)} \end{pmatrix}, \qquad U = \begin{pmatrix} u_{k\sigma}^{(+)} & u_{k\sigma}^{(-)} \\ v_{k\sigma}^{(+)} & v_{k\sigma}^{(-)} \end{pmatrix} \tag{22}$$

with

$$\left.\begin{aligned} u_{k\uparrow}^{(+)} &= u_{k\downarrow}^{(-)} = \sin\theta_k e^{in\pi} , \\ v_{k\uparrow}^{(+)} &= -v_{k\downarrow}^{(-)} = -e^{in\pi} e^{ik\alpha} \cos\theta_k , \\ u_{k\downarrow}^{(+)} &= u_{k\uparrow}^{(-)} = \cos\theta_k , \\ v_{k\downarrow}^{(+)} &= -v_{k\uparrow}^{(-)} = -e^{ik\alpha} \sin\theta_k , \end{aligned}\right\} \tag{23}$$

where

$$\left.\begin{aligned} \sin\theta_k &= e^{in\pi} \sqrt{\frac{1}{2} \Big(1 - \frac{|J|M}{2E_k} \Big)} , \\ \cos\theta_k &= \sqrt{\frac{1}{2} \Big(1 + \frac{|J|M}{2E_k} \Big)} \end{aligned}\right\} \tag{24}$$

and

$$E_k \equiv \sqrt{\Big(\frac{\xi_k - \xi_{k+Q}}{2} \Big)^2 + \Big(\frac{JM}{2} \Big)^2} . \tag{25}$$

In these equations the vector $\boldsymbol{k}$ is not limited in the first zone and the integer $n \equiv n(\boldsymbol{k})$ is defined by $k^z = k'^z + nQ$ where k'^z belongs to the first magnetic Brillouin zone, from which we have $\sin\theta_{\boldsymbol{k}+\boldsymbol{Q}} = -\sin\theta_{\boldsymbol{k}}$. The final form of $\mathcal{H}_e'$ is

$$\mathcal{H}_e' = \sum_{\boldsymbol{k}\sigma}\sum_{\nu=\pm} E_{\boldsymbol{k}\sigma}^{(\nu)} D_{\boldsymbol{k}\sigma}^{(\nu)*} D_{\boldsymbol{k}\sigma}^{(\nu)} \tag{26}$$

with

$$E_{\boldsymbol{k}\sigma}^{(\nu)} = \tfrac{1}{2}(\xi_{\boldsymbol{k}} + \xi_{\boldsymbol{k}+\boldsymbol{Q}}) + \nu E_{\boldsymbol{k}} \,. \tag{27}$$

Among the interaction part of Eq. (16), the effect of the longitudinal part $\mathcal{H}_{sf}^{(z)}$ is small in the magnon region and only the transverse part $\mathcal{H}_{sf}^{(+-)}$ is considered.

By the use of the canonical transformations for magnons and electrons obtained above, $\mathcal{H}_{sf}^{(+-)}$ has the following form in the new representation:

$$\begin{aligned}
\mathcal{H}_{sf}^{(+-)} = -\frac{J}{2}\sqrt{2S}\sqrt{\frac{2}{N}} \sum_{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q}} \sum_{\boldsymbol{P}} \delta(\boldsymbol{k}'-\boldsymbol{k}+\boldsymbol{q}+\boldsymbol{P}) \\
\times [\, f(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) e^{in'\pi} (D_{\boldsymbol{k}'\uparrow}^{(+)*} D_{\boldsymbol{k}\downarrow}^{(+)} \alpha_{\boldsymbol{q}}^* + D_{\boldsymbol{k}\downarrow}^{(+)*} D_{\boldsymbol{k}'\uparrow}^{(+)} \alpha_{\boldsymbol{q}}) \\
+ f(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) e^{-in'\pi + i\boldsymbol{P}a} (e^{i\boldsymbol{q}a} D_{\boldsymbol{k}'\uparrow}^{(-)*} D_{\boldsymbol{k}\downarrow}^{(-)} \beta_{\boldsymbol{q}} + e^{-i\boldsymbol{q}a} D_{\boldsymbol{k}\downarrow}^{(-)*} D_{\boldsymbol{k}'\uparrow}^{(-)} \beta_{\boldsymbol{q}}^*) \\
- g(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) e^{in'\pi} (e^{i\boldsymbol{q}a} D_{\boldsymbol{k}'\uparrow}^{(+)*} D_{\boldsymbol{k}\downarrow}^{(+)} \beta_{\boldsymbol{q}} + e^{-i\boldsymbol{q}a} D_{\boldsymbol{k}\downarrow}^{(+)*} D_{\boldsymbol{k}'\uparrow}^{(+)} \beta_{\boldsymbol{q}}^*) \\
- g(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) e^{-in\pi + i\boldsymbol{P}a} (D_{\boldsymbol{k}'\uparrow}^{(-)*} D_{\boldsymbol{k}\downarrow}^{(-)} \alpha_{\boldsymbol{q}}^* + D_{\boldsymbol{k}\downarrow}^{(-)*} D_{\boldsymbol{k}'\uparrow}^{(-)} \alpha_{\boldsymbol{q}}) \\
+ h(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) e^{i(n'-n)\pi} (D_{\boldsymbol{k}'\uparrow}^{(+)*} D_{\boldsymbol{k}\downarrow}^{(-)} \alpha_{\boldsymbol{q}}^* + D_{\boldsymbol{k}\downarrow}^{(-)*} D_{\boldsymbol{k}'\uparrow}^{(+)} \alpha_{\boldsymbol{q}}) \\
- h(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) e^{i\boldsymbol{P}a} (e^{i\boldsymbol{q}a} D_{\boldsymbol{k}'\uparrow}^{(-)*} D_{\boldsymbol{k}\downarrow}^{(+)} \beta_{\boldsymbol{q}} + e^{-i\boldsymbol{q}a} D_{\boldsymbol{k}\downarrow}^{(-)*} D_{\boldsymbol{k}'\uparrow}^{(+)} \beta_{\boldsymbol{q}}^*) \\
- l(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) e^{i(n'-n)\pi} (e^{i\boldsymbol{q}a} D_{\boldsymbol{k}'\uparrow}^{(+)*} D_{\boldsymbol{k}\downarrow}^{(-)} \beta_{\boldsymbol{q}} + e^{-i\boldsymbol{q}a} D_{\boldsymbol{k}\downarrow}^{(-)*} D_{\boldsymbol{k}'\uparrow}^{(+)} \beta_{\boldsymbol{q}}^*) \\
+ l(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) e^{i\boldsymbol{P}a} (D_{\boldsymbol{k}'\uparrow}^{(-)*} D_{\boldsymbol{k}\downarrow}^{(+)} \alpha_{\boldsymbol{q}}^* + D_{\boldsymbol{k}\downarrow}^{(+)*} D_{\boldsymbol{k}'\uparrow}^{(-)} \alpha_{\boldsymbol{q}})\,]
\end{aligned} \tag{28}$$

with

$$\left.\begin{aligned}
f(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) &= \sin\theta_{\boldsymbol{k}'}\cos\theta_{\boldsymbol{k}}\cosh\phi_{\boldsymbol{q}} - e^{i\boldsymbol{P}a}\cos\theta_{\boldsymbol{k}'}\sin\theta_{\boldsymbol{k}}\sinh\phi_{\boldsymbol{q}}\,, \\
g(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) &= \sin\theta_{\boldsymbol{k}'}\cos\theta_{\boldsymbol{k}}\sinh\phi_{\boldsymbol{q}} - e^{i\boldsymbol{P}a}\cos\theta_{\boldsymbol{k}'}\sin\theta_{\boldsymbol{k}}\cosh\phi_{\boldsymbol{q}}\,, \\
h(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) &= \sin\theta_{\boldsymbol{k}'}\sin\theta_{\boldsymbol{k}}\cosh\phi_{\boldsymbol{q}} + e^{i\boldsymbol{P}a}\cos\theta_{\boldsymbol{k}'}\cos\theta_{\boldsymbol{k}}\sinh\phi_{\boldsymbol{q}}\,, \\
l(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P}) &= \sin\theta_{\boldsymbol{k}'}\sin\theta_{\boldsymbol{k}}\sinh\phi_{\boldsymbol{q}} + e^{i\boldsymbol{P}a}\cos\theta_{\boldsymbol{k}'}\cos\theta_{\boldsymbol{k}}\cosh\phi_{\boldsymbol{q}}\,,
\end{aligned}\right\} \tag{29}$$

where $n' \equiv n(\boldsymbol{k}')$ and $n \equiv n(\boldsymbol{k})$. Except for $\theta_{\boldsymbol{k}}, \phi_{\boldsymbol{k}}$ or $n(\boldsymbol{k})$, all quantities in the new representation are periodic functions of wave vector with period $\boldsymbol{Q}$. The factor $e^{in\pi}$ or $e^{in'\pi}$ is retained in the above expression because the extended zone scheme is employed in the next section but, as is seen from the hermitness of the above expression, they have no effect on the physical quantities.

§ 3. Calculation of resistivity

3.1. *Variational method*

Let us briefly discuss a formula derived by the variational method for elec-

trical resistivity appropriate for the present case.[7),11),12)] The Boltzmann equation for the electron distribution $f(\boldsymbol{k})$ in the presence of a unit electric field $\hat{\boldsymbol{E}}$ is

$$-\nabla_{\boldsymbol{k}} f_0^i(\boldsymbol{k})\cdot\hat{\boldsymbol{E}}e=\sum_{\boldsymbol{k}'ij} P^{ij}(\boldsymbol{k},\boldsymbol{k}')\,[\psi^i(\boldsymbol{k})-\psi^j(\boldsymbol{k}')]\equiv H\psi^i(\boldsymbol{k}), \tag{30}$$

where $f_0^i(\boldsymbol{k})$ is the equilibrium distribution function for a conduction electron with a wave vector $\boldsymbol{k}$, and the deviation function $\psi^i(\boldsymbol{k})$ can be thought of as the shift of single particle energies $E^{(i)}(\boldsymbol{k})$ due to the applied field:

$$f^i(\boldsymbol{k})=f_0^j(\boldsymbol{k})-\psi^i(\boldsymbol{k})\frac{\mathrm{d}f_0^i(\boldsymbol{k})}{\mathrm{d}E^i(\boldsymbol{k})}. \tag{31}$$

In the above the conduction electrons of both the bands in the first magnetic zone is divided into several parts specified by i corresponding to each case considered below. (See Fig. 1.) This classification is important, as will be shown below, for the variational method with the use of a usual trial function when different relaxation times are expected for these parts.

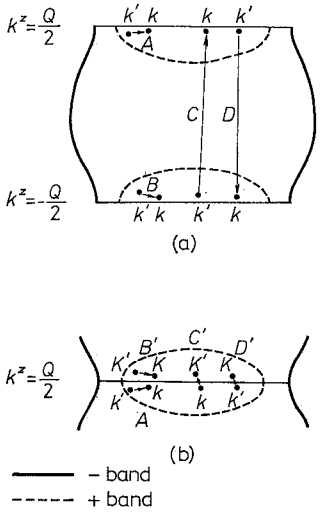

Fig. 1.

The conductivity σ is written as

$$\sigma=\sum_{\boldsymbol{k}i}\psi^i(\boldsymbol{k})H\psi^i(\boldsymbol{k})\equiv\sum_i\langle\psi^iH\psi^i\rangle. \tag{32}$$

We consider a case of two parts $i=\pm$ and take trial functions ϕ_t^i for ψ^i. Then inserting $\phi^i=\psi^i+\lambda_i\phi_t^i$ into the right-hand side of the above equation we obtain

$$\sum_i\langle\phi^iH\phi^i\rangle=\sum_i\langle\psi^iH\psi^i\rangle+2\sum_i\lambda_i\langle\phi_t^iH\psi^{-i}\rangle+\sum_i\lambda_i^2\langle\phi_t^iH\phi_t^i\rangle$$
$$+\tfrac{1}{2}\sum_{ij}(\lambda_i-\lambda_j)^2\langle\phi_t^iP\phi_t^{-i}\rangle. \tag{33}$$

The positive definite character of $\sum\langle\phi^iH\phi^i\rangle$ leads to an inequality similar to the Schwartz inequality in the form[12)]

$$\sigma\geq\frac{A_+\eta_-^2+A_-\eta_+^2-2B\eta_+\eta_-}{A_+A_--B^2}, \tag{34}$$

where

$$\left.\begin{aligned}&A_i=\langle\phi_t^iH\phi_t^i\rangle+B,\\&B=\langle\phi_t^iP\phi_t^{-i}\rangle\equiv\sum_{\boldsymbol{k}\boldsymbol{k}'}\phi_t^iP^{i-i}(\boldsymbol{k},\boldsymbol{k}')\phi_t^{-i}(\boldsymbol{k}'),\end{aligned}\right\} \tag{35}$$

$$\eta_i=\langle\phi_t^iH\psi^i\rangle. \tag{36}$$

Now taking the usual trial function $\phi_t^i(\boldsymbol{k})=V^i(\boldsymbol{k})\cdot\hat{\boldsymbol{E}}$, where $V^i(\boldsymbol{k})$ is the ve-

locity of the conduction electron, and the direction of the electric field as z direction, we obtain

$$\eta_i = e\sum_{k\sigma}[V^{zi}(\boldsymbol{k})]^2\left[-\frac{\mathrm{d}f_0^i(\boldsymbol{k})}{\mathrm{d}E_0^i(\boldsymbol{k})}\right]\equiv\frac{e}{m}n_i^z, \tag{37}$$

where m is the effective mass of the conduction electrons in the absence of the magnetic order and n_i^z defined by Eq. (37) can be regarded as the effective carrier number for the i-th part of electrons. On the other hand, roughly speaking, A_i and B is proportional to inverse of the relaxation time due to the electron-magnon scattering.

When the inter part scattering is neglected, i.e., $B=0$, then Eq. (34) reduces to

$$\sigma\geq\frac{\eta_+^2}{A_+}+\frac{\eta_-^2}{A_-}\equiv\sigma_1\,. \tag{38}$$

This equation tells us that this circuit is parallel, as it should be so. On the other hand, if we do not divide the conduction electrons into the two parts, i.e., set $\lambda_i=\lambda$, we have another (Schwartz) inequality from Eq. (33) as

$$\sigma\geq\frac{(\eta_++\eta_-)^2}{A_++A_-}\equiv\sigma_2\,. \tag{39}$$

It follows from this equation that the scattering effects average over both the parts and the circuit becomes series like. As can be easily shown, $\sigma_1>\sigma_2$, and the value of σ_1 is more accurate than σ_2. It should be noted, however, that σ_2 leads to the unphysical results in some circumstances: If $A_+\ll A_-$ and η_+ is of the same order η_-, then $\sigma_1\simeq\eta_+^2/A_+$ while $\sigma_2\simeq\eta_-^2/A_-$.

The simplest form of the hydrodynamical approximation of the Kubo formula which was employed in the previous paper corresponds to Eq. (39) and gives incorrect results concerning the power of temperature in the case of anisotropic scattering such as interband or Umklapp scattering, although the qualitative features of the magnetoresistance remains unchanged as they reflect the magnetic field dependence of the magnon spectrum.

Recently Lawrence and Wilkins have studied the effect of the Umklapp process due to electron-phonon scattering on the low temperature resistivity of polyvalent metals using the variational method. Unfortunately, however, they used Eq. (39) and hence their drastic result that the contribution due to the Umklapp process is larger than that due to the normal process is erroneous.

3.2. *Effects of the intraband scattering*

The approximate Hamiltonian of our system is

$$\mathcal{H}=\mathcal{H}_e'+\mathcal{H}_s+\mathcal{H}_{sf}^{(+-)} \tag{40}$$

and $\mathcal{H}_e'$, $\mathcal{H}_s$ and $\mathcal{H}_{sf}^{(+-)}$ is given by Eqs. (26), (13) and (28) respectively.

Taking $\mathcal{H}_e'+\mathcal{H}_s$ as the unperturbed state, we obtain the electrical resistivity in the first Born approximation with respect to $\mathcal{H}_{sf}^{(+-)}$. As was mentioned in the introduction, since the vertex correction is higher order J/ζ, the first Born approximation gives the leading term with respect to J/ζ.

In this section we obtain the resistivity at sufficiently low temperatures, so that the effects of the interband scattering are negligibly small and only intraband scattering dominates, and also such that the transfered momentum in scattering is so small that the electrons near the neck part (region N) of the Fermi surface does not strongly mix with the electrons on the belly part (region B). These conditions are satisfied if

$$q_c \gg q_T: \; q_c=\frac{SJm}{Q}, \qquad q_T=\frac{T}{c}, \quad c: \text{ magnon velocity}, \tag{41}$$

where q_T is the wave vector of the thermally excited magnon. By the use of $c=z|V|S\sqrt{2\alpha}$ (see below Eq. (60)), the temperature region is $(JS/\zeta)T_N\equiv T_1 \gg T$.

In the calculation of resistivity, when this condition is satisfied, the conduction electrons in each band are roughly grouped into two portions according to their relaxation rates, i.e., the electrons in the region N and those in the region B. In this case the variational method discussed in § 3. 1 or the hydrodynamical method lead to

$$\sigma=\sum_{\nu=\pm}\sum_{i=N,B}\sigma_i^{(\nu)}, \qquad \sigma_i^{(\nu)}\equiv\frac{((e/m)n_i^{\nu})^2}{\sum_{\boldsymbol{k}\boldsymbol{k}'}^{(\nu,i)}P(\boldsymbol{k}\boldsymbol{k}')\frac{1}{6}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{k}'}|^2} \tag{42}$$

with

$$n_i^{\nu}=\frac{m}{3}\sum_{\boldsymbol{k}\sigma}^{(\nu,i)}\boldsymbol{V}_{\boldsymbol{k}}^2\left(-\frac{\mathrm{d}f_0}{\mathrm{d}E_{\boldsymbol{k}\sigma}^{(\nu)}}\right), \qquad \boldsymbol{V}_{\boldsymbol{k}}=\mathrm{grad}_{\boldsymbol{k}}E_{\boldsymbol{k}\sigma}^{(\nu)}, \tag{43}$$

where the summation $\sum^{(\nu,i)}$ denotes the summation over the region i in the ν band, and $f_0\equiv f_0(E_{\boldsymbol{k}\sigma}^{(\nu)})$ is the Fermi distribution function. The directional average is performed roughly, i.e., separately for the denominator and the numerator in Eq. (42). This procedure gives the lower limit of the directional average of the conductivity, since $\sum_i(A_i/B_i)\geq\sum_i A_i/\sum_i B_i$ is satisfied for the positive values of A_i and B_i. For the belly parts, the anisotropic effect of scattering is easily shown to be negligibly small, and hence the present averaging procedure gives the correct results. On the other hand, for the neck part where the anisotropic effect is very important, this procedure should lead to the poor results. Fortunately, the contribution of the neck part to σ is negligibly small, and the present treatment gives the correct leading term of the conductivity. The denominator (the scattering term) of Eq. (42) is explicitly given as

$$\sum_{\boldsymbol{k}\boldsymbol{k}'}^{(\nu,i)}P(\boldsymbol{k}\boldsymbol{k}')\frac{1}{6}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{k}'}|^2$$
$$=\frac{\pi}{3}V^2\left(\frac{1}{4\pi^3}\right)^2\frac{SJ^2}{N}\int\frac{\mathrm{d}S_{\boldsymbol{k}}}{|\boldsymbol{V}_{\boldsymbol{k}}|}\frac{\mathrm{d}S_{\boldsymbol{k}'}}{|\boldsymbol{V}_{\boldsymbol{k}'}|}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{k}'}|^2\sum_{\boldsymbol{P}}\delta(\boldsymbol{k}'-\boldsymbol{k}+\boldsymbol{q}+\boldsymbol{P})$$

$$\times[|f(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P})|^2+|g(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{P})|^2]F(\beta\epsilon_{\boldsymbol{q}}), \tag{44}$$

here $F(x)\equiv x/[(e^x-1)(1-e^{-x})]$, and the surface integration is performed over the Fermi surface in the region i of ν band. We evaluate Eq. (42) to first order in J/ζ. The range of integration over k^z in each region is

$$\frac{Q}{2}\geq\left|k^z-\frac{Q}{2}\right|\geq q_c \qquad \text{for } (\nu,i)=(-,B), \tag{45}$$

$$\sqrt{2m\zeta}-\frac{Q}{2}>\left|k_z-\frac{Q}{2}\right|>q_c \qquad \text{for } (\nu,i)=(+,B) \tag{46}$$

and

$$q_c>\left|k^z-\frac{Q}{2}\right|\geq 0 \qquad \text{for } (\nu,i)=(\pm,N). \tag{47}$$

The effective carrier number $n_i^{(\nu)}$ can be integrated directly over k^z and we have the following results:

$$\left.\begin{aligned} n_B^{(-)}&=n_0\left(\frac{Q}{2P_F}-\frac{q_c}{P_F}\right),\\ n_N^{(-)}&=n_0\left[\frac{2m\zeta}{P_F^2}-\frac{\pi}{4}\left(\frac{Q}{2P_F}\right)^2\right]\frac{q_c}{P_F},\\ n_B^{(+)}&=n_0\left(\sqrt{\frac{2m\zeta}{P_F^2}}-\frac{Q}{2P_F}-\frac{q_c}{P_F}\right),\\ n_N^{(+)}&=n_0\left[\frac{2m\zeta}{P_F^2}-\frac{\pi}{4}\left(\frac{Q}{2P_F}\right)^2\right]\frac{q_c}{P_F} \end{aligned}\right\} \tag{48}$$

with

$$n_0\equiv\frac{V}{3\pi^2}P_F^3,$$

where n_0 is the total number of conduction electrons. Summing up all the above terms, one has

$$n_{\text{eff}}\equiv\sum_{\nu=\pm}(n_B^{(\nu)}+n_N^{(\nu)})=n_0\left(1-\frac{\pi}{16}\frac{Q}{P_F}\frac{SJ}{\zeta}\right). \tag{49}$$

This result is consistent with that of Elliot and Wedgwood,[5] and Miwa.[6]

To calculate the temperature dependence and the order of magnitude of the scattering term of Eq. (42), the shape of the Fermi surface of the B and N parts are simplified into completely spherical and cylindrical form respectively. By the use of the relation $dS_{\boldsymbol{k}}dS_{\boldsymbol{k}'}=dS_{\boldsymbol{k}}qdq+O(q^2)$,[7] the following results are obtained for the belly part:

$$\sum_{\boldsymbol{k}\boldsymbol{k}'}^{(-,B)}P(\boldsymbol{k},\boldsymbol{k}')\frac{1}{6}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{k}'}|^2=\frac{\pi}{3}\left(\frac{1}{4\pi^3}\right)^2\frac{V^2}{N}SJ^2\left(\frac{Q/2-q_c}{P_F}\right)X(T) \tag{50}$$

and

$$\sum_{\boldsymbol{k}'\boldsymbol{k}}^{(+,B)} P(\boldsymbol{k}\boldsymbol{k}')\frac{1}{6}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{k}'}|^2=\frac{\pi^2}{3}\left(\frac{1}{4\pi^3}\right)^2\frac{V^2}{N}SJ^2\left(\frac{\sqrt{2m\zeta}-Q/2-q_c}{P_F}\right)X(T) \tag{51}$$

with

$$X(T)=2\int_0^{q_D}\mathrm{d}qq^3\sqrt{\frac{1+d-v(\boldsymbol{q}+\boldsymbol{Q})}{1+d-v(\boldsymbol{q})}}F(\beta\epsilon_{\boldsymbol{q}}), \tag{52}$$

where we have used the fact that, in this almost spherical region, only the scattering process with $P=0$, i.e., the normal process is possible in Eq. (44).

When the conduction electrons are on the neck region of the Fermi surface, scattering with both $P=0$ (Fig. 1(a): A and B) and $P=\pm Q$ (Fig. 1(a): C and D) are possible. The scattering term of the process A in Fig. 1(a) has the form (band index is dropped for simplicity):

$$\begin{aligned}&\sum_{(\text{process A})} P(\boldsymbol{k}\boldsymbol{k}')\frac{1}{6}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{k}'}|^2\\&=\frac{\pi}{3}\left(\frac{1}{4\pi^3}\right)^2\frac{V^2}{N}SJ^2\int\frac{\mathrm{d}S_{\boldsymbol{k}}}{|\boldsymbol{V}_{\boldsymbol{k}}|}\frac{\mathrm{d}S_{\boldsymbol{k}'}}{|\boldsymbol{V}_{\boldsymbol{k}'}|}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{k}'}|^2\\&\quad\times[|f(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:0)|^2+|g(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:0)|^2]F(\beta\epsilon_{\boldsymbol{q}}).\\&\qquad\left(q_c\geq\frac{Q}{2}-k^z\geq 0,\quad q_c\geq\frac{Q}{2}-k'^z\geq 0\right)\end{aligned} \tag{53}$$

The scattering term of the process C has the form

$$\begin{aligned}&\sum_{(\text{process C})} P(\boldsymbol{k}\boldsymbol{k}')\frac{1}{6}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{k}'}|^2\\&=\frac{\pi}{3}\left(\frac{1}{4\pi^3}\right)^2\frac{V^2}{N}SJ^2\int\frac{\mathrm{d}S_{\boldsymbol{k}}}{|\boldsymbol{V}_{\boldsymbol{k}}|}\frac{\mathrm{d}S_{\boldsymbol{k}'}}{|\boldsymbol{V}_{\boldsymbol{k}'}|}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{k}'}|^2\\&\quad\times[|f(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{Q})|^2+|g(\boldsymbol{k}'\boldsymbol{k}\boldsymbol{q}:\boldsymbol{Q})|^2]F(\beta\epsilon_{\boldsymbol{q}}).\\&\qquad\left(q_c\geq\frac{Q}{2}-k^z\geq 0,\quad q_c\geq k'^z-\frac{Q}{2}\geq 0\right)\end{aligned} \tag{54}$$

If the variable $\boldsymbol{k}'$ in Eq. (54) is changed by the variable $\boldsymbol{k}'$ defined by $\boldsymbol{k}'+\boldsymbol{Q}=\boldsymbol{K}'$ and if we use Eq. (29) for f and g and the periodicity of other quantities, then this process C corresponding to the process C′ in Fig. 1(b) has the form

$$\begin{aligned}&\sum_{(\text{process C})} P(\boldsymbol{k}\boldsymbol{k}')\frac{1}{6}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{k}'}|^2\\&=\frac{\pi}{3}\left(\frac{1}{4\pi^3}\right)^2\frac{V^2}{N}SJ^2\int\frac{\mathrm{d}S_{\boldsymbol{k}}}{|\boldsymbol{V}_{\boldsymbol{k}}|}\frac{\mathrm{d}S_{\boldsymbol{K}'}}{|\boldsymbol{V}_{\boldsymbol{K}'}|}|\boldsymbol{V}_{\boldsymbol{k}}-\boldsymbol{V}_{\boldsymbol{K}'}|^2\\&\quad\times[|f(\boldsymbol{K}'\boldsymbol{k}\boldsymbol{q}:0)|^2+|g(\boldsymbol{K}'\boldsymbol{k}\boldsymbol{q}:0)|^2]F(\beta\epsilon_{\boldsymbol{q}})\\&\qquad\left(q_c\geq\frac{Q}{2}-k_z\geq 0,\quad q_c\geq K'^z-\frac{Q}{2}\geq 0\right)\end{aligned} \tag{55}$$

and the integrand becomes identical with Eq. (53) except for the region of the integration.

In the same way, the expressions of the integrand of scattering terms of the process B and D in Fig. 1(a) become identical with the process A by the similar transformation of the variables and correspond to the process B′ and D′ in Fig. 1(b). The scattering term for the neck region including all the possible scattering processes ($\boldsymbol{P}=0, \pm\boldsymbol{Q}$) thus reduces to the single expression in the form of the normal process:*)

$$\sum^{(N)} P(\boldsymbol{kk'})\frac{1}{6}|\boldsymbol{V_k}-\boldsymbol{V_{k'}}|^2$$
$$=\frac{\pi}{3}\left(\frac{1}{4\pi^3}\right)^2\frac{V^2}{N}SJ^2\int\frac{dS_{\boldsymbol{k}}}{|\boldsymbol{V_k}|}\frac{dS_{\boldsymbol{k'}}}{|\boldsymbol{V_{k'}}|}|\boldsymbol{V_k}-\boldsymbol{V_{k'}}|^2$$
$$\times[|f(\boldsymbol{k'kq}:0)|^2+|g(\boldsymbol{k'kq}:0)|^2]F(\beta\epsilon_{\boldsymbol{q}}).$$
$$\left(q_c\geq\left|\frac{Q}{2}-k^z\right|\geq 0,\quad q_c\geq\left|\frac{Q}{2}-k'^z\right|\geq 0\right) \tag{56}$$

As in the case of n_i^{ν}, the above expression is estimated with the use of the simplified Fermi surface of the cylindrical form with the width $2q_c$. Putting the various quantities by the values at $k^z=Q/2$, we have

$$\sum_{\boldsymbol{kk'}}^{(\nu,N)} P(\boldsymbol{kk'})\frac{1}{6}|\boldsymbol{V_k}-\boldsymbol{V_{k'}}|^2=\frac{\pi^2}{3}\left(\frac{1}{4\pi^3}\right)^2\frac{V^2}{N}SJ^2\frac{4(1+\nu(Q/2q_c))Q^2q_c}{k_r^{(\nu)}(mJS)^2}Y(T) \tag{57}$$

with

$$Y(T)=2\int_0^{2\pi}d\psi(\cos\psi)^4\int_0^{q_D}dqq^5(\cosh\phi_{\boldsymbol{q}})^2F(\beta\epsilon_{\boldsymbol{q}}), \tag{58}$$

where $k_r^{(\nu)}$ is the radius of the Fermi surface at the neck $k^z=Q/2$ in the ν band.

The form of the temperature dependence is determined by $X(T)$ (Eq. 52)) or $Y(T)$ (Eq. (58)). For simplicity we assume the coupling constant of the exchange interaction satisfies $V(\boldsymbol{q}+\boldsymbol{Q})=-V(\boldsymbol{q})=z|V|\gamma_{\boldsymbol{q}}$, as in the case of the nearest neighbour interaction, then $X(T)$ and $Y(T)$ have the following forms in two limiting cases:

$$X(T)=\begin{cases} T^5\dfrac{\sqrt{2\alpha}}{c^5}5\Gamma(5)\zeta(5) & \text{for } T_N\gg T\gg D, \\ T\dfrac{4}{c^4\sqrt{\dfrac{d}{2+d}}}[z|V|S\sqrt{d(2+d)}]^3\exp\left[-\dfrac{1}{T}z|V|S\sqrt{d(2+d)}\right] & \\ \qquad\qquad \text{for } T_N, D\gg T, & \end{cases} \tag{59}$$

*) The origin of this simplification is related to the assumption that the s-f coupling J is independent of momentum. If it is dependent on momentum, one must use $J(\boldsymbol{q}\pm\boldsymbol{Q})$ for the process C' and D', and $J(\boldsymbol{q})$ for the process A' and B' instead of J, and the expression including the Umklapp-process becomes complex as in the electron-phonon case.

$$Y(T)=\begin{cases} T^5\dfrac{3\pi\sqrt{2+d}}{8\sqrt{\alpha}\,c^5}5\Gamma(5)\zeta(5) & \text{for } T_N\gg T\gg D, \\ T^2\dfrac{3\pi}{c^6}\left(\sqrt{\dfrac{d}{2+d}}+\sqrt{\dfrac{2+d}{d}}+1\right)[z|V|S\sqrt{d(2+d)}]^4 & \\ \quad\times\exp\left[-\dfrac{1}{T}z|V|S\sqrt{d(2+d)}\right] & \text{for } T_N, D\gg T, \end{cases} \tag{60}$$

where c and α are given by $c=z|V|S\sqrt{2\alpha}$ and $\gamma_q=1-\alpha q^2+0(q^2)$. We have also

$$\frac{Y(T)}{X(T)}=\frac{1}{\alpha}\left(\frac{T}{T_N}\right)^{\varepsilon},\qquad \varepsilon=\begin{cases}0 & \text{for } T_N\gg T\gg D,\\ 1 & \text{for } T_N, D\gg T.\end{cases} \tag{61}$$

By the use of above results, the ratio of the conductivity due to electrons on the spherical and neck parts is estimated as

$$\frac{\sum_\nu \sigma_B^{(\nu)}}{\sum_\nu \sigma_N^{(\nu)}}\approx\left(\frac{P_F}{q_c}\right)^3\left(\frac{1}{q_c\sqrt{\alpha}}\right)^2\left(\frac{T}{T_N}\right)^{\varepsilon}\approx\left(\frac{\zeta}{SJ}\right)^5\left(\frac{T}{T_N}\right)^{\varepsilon}, \tag{62}$$

where we have used $(P_F/q_c)\sim(\zeta/JS)$ and $\alpha\sim P_F^{-2}$. Combining with the condition (41), the contribution of the $\sigma_N^{(\nu)}$ to the total conductivity is negligibly small in the temperature region

$$T_1\equiv\left(\frac{JS}{\zeta}\right)T_N\gg T\gg\left(\frac{JS}{\zeta}\right)^5 T_N \tag{63}$$

and using Eqs. (42), (48), (50) and (51) for $\sigma_B^{(\nu)}$ we have

$$\sigma=\sum_{\nu=\pm}\sigma_B^{(\nu)}=\frac{e^2}{m}n'_{\text{eff}}\tau_B \tag{64}$$

with

$$n'_{\text{eff}}=n_0\left(1-\frac{P_F}{Q}\frac{JS}{\zeta}\right) \tag{65}$$

and

$$\tau_B=\left\{\frac{V}{m}\frac{P_F^2}{6\pi^2}\Big/\frac{\pi^2}{6}\left(\frac{1}{4\pi^3}\right)^2\frac{V^2}{N}\frac{SJ^2}{P_F}X(T)\right\}. \tag{66}$$

If we take the value $SJ/\xi=1/10\backsim1/100$, $T_N\sim100$°K, the upper limit of Eq. (63) is the order of a few degrees and the lower limit is $10^{-3}\backsim10^{-8}$°K which is exceedingly small in the available temperature region. The temperatrue dependence of the resistivity due to the intraband scattering is determined by $X(T)$ as

$$\frac{1}{\sigma}=\begin{cases}\sim T^5 & \text{for } T_1\gg T\gg D,\\ \sim T\exp\left(-\dfrac{\sqrt{T_N D}}{T}\right) & \text{for } T_1, D\gg T.\end{cases} \tag{67}$$

3.3. *Effects of the interband scattering*

When the temperature exceeds T_1, the interband scattering begins to take place near the neck region. The expression for the conductivity is

$$\left.\begin{aligned}&\sigma=\sigma_B+\sigma_N\,,\\&\sigma_B=\sum_{\nu=\pm}\frac{\langle\psi_t^{\nu}H\psi^{\nu}\rangle^2}{\langle\psi_t^{\nu}H\psi_t^{\nu}\rangle}\,,\\&\sigma_N=\frac{A_{+N}\eta_{-N}^2+A_{-N}\eta_{+N}^2-2B\eta_{+N}\eta_{-N}}{A_{+N}A_{-N}-B^2}\,,\end{aligned}\right\}\tag{68}$$

where A_{+N} etc. are obtained from Eq. (36) by replacing i by $(+N)$ etc. The symbol $+(-)$ denotes the region of the Fermi surface of $+(-)$ band where only intraband scattering is possible, and $+N(-N)$ denotes the region of the $+(-)$ band where both the intra and interband scattering are possible. Above equation is derived by neglecting the scattering term $P^{(\pm,\pm N)}$ which represents the scattering between $(\pm)$ and $(\pm N)$ region, inclusion of which will change only numerical value of the coefficient of T^5 as it is intraband scattering in nature.

The term σ_N is evaluated first. Putting $\psi_t^{\alpha}=V_{\boldsymbol{k}}^{z}$ (k belongs to region α),*) the effective carrier number of the $(\pm N)$ region is estimated roughly from the area of the cylinder of the radius $P_F\sqrt{1-(Q/2P_F)^2}$ and width $2q_T(cq_T\approx T)$ as

$$\eta_{+N}=\eta_{-N}=\frac{e}{m}n_N\,,\qquad n_N=n_0\frac{T}{T_N}\sqrt{1-\left(\frac{Q}{2P_F}\right)^2}\,.\tag{69}$$

The largest contribution in the scattering term A_{+N}, A_{-N} and B comes from the interband scattering $P^{(+N)(-N)}$ and, in the free electron approximation, we have

$$\left.\begin{aligned}&A_{+N}=A_{-N}=\gamma T^2+C\,,\\&B=\gamma T^2-C\end{aligned}\right.\tag{70}$$

with

$$\gamma=\left(\frac{Q}{P_F}\right)^2\pi V^2\left(\frac{1}{4\pi^3}\right)^2\frac{SJ^2}{N}\,\frac{8\pi^2P_F^2}{Qc^2}\,\frac{8\sqrt{2}}{\sqrt{\alpha}}\Gamma(2)\zeta(2)\,.\tag{71}$$

The small additive term C comes from the k dependence of $\psi_t^{\alpha}(=V_{\boldsymbol{k}}^{z})$. Then we have

$$\sigma_N=\left(\frac{en_0}{m}\right)^2\left[1-\left(\frac{Q}{2P_F}\right)^2\right]\frac{1}{\gamma T_N}\,.\tag{72}$$

It is expected that in $(\pm)$ region the conduction electrons will have the relaxation time τ_B [Eq. (66)] because only the intraband scattering dominates even when $T>T_1$ in this region, but the carrier number will be reduced by an amount

*) Among the three equivalent trial functions $V_{\boldsymbol{k}}^{x}$, $V_{\boldsymbol{k}}^{y}$, $V_{\boldsymbol{k}}^{z}$, $V_{\boldsymbol{k}}^{z}$ gives the largest scattering in σ_N through the q independence of the velocity transfer term.

n_N. Then

$$\sigma_B = \frac{e^2}{m}(n_0 - n_N)\tau_B \tag{73}$$

and σ_B is proportional to $1/T^5$. The ratio is $\sigma_N/\sigma_B \approx \{(T/T_N)^5/(1-T/T_N)\}$ which is small in the magnon region. The less accurate expression for σ_N is

$$\sigma_N = \frac{\eta^2}{A}, \tag{74}$$

where η and A is given by Eq. (36) in which the index i denotes the region both $(+N)$ and $(-N)$. It is easily verified that Eq. (74) gives also the expression Eq. (72) in the lowest approximation.

§ 4. Summary

Contrary to the T^4 dependence conjectured by some authors, the T^5 dependence was derived for the resistivity of an antiferromagnetic metal at low temperatures. The origin of this extra power is the q dependence of the new coupling constant f or g in Eq. (29). In the case of normal scattering on the spherical Fermi surface and vanishing anisotropy energy, they are proportional to q:

$$|f(\boldsymbol{k'kq}:0)|^2 = |g(\boldsymbol{k'kq}:0)|^2 = (\cosh\phi_{\boldsymbol{q}} - \sinh\phi_{\boldsymbol{q}})^2 \sim \frac{c}{2z|V|S}q \tag{75}$$

and thus it is insufficient only to replace the ferromagnetic magnon spectrum by the antiferromagnetic one.

When the temperature is smaller than the anisotropy energy D, the resistivity varies as $\rho \sim T\exp(-\sqrt{T_N D}/T)$.

When the temperature is low enongh such as $T \ll T_1$, neither the interband nor the Umklapp scattering of the electrons on the neck part is possible and only the intraband scattering dominates. However, the contribution to the conductivity of these electrons is small compared to that of the electrons on the belly part as long as the inequality $\zeta \gg J$ holds as is seen from Eq. (62), and also under the inequality $p_F^2 - (Q/2)^2 \gg mJ$ which indicates that the area of the belly part is sufficiently larger than that of the neck part (the case (i) in Ref. 5) and the present case).

When the temperature raised and interband scattering begins $(T \gg T_1)$, the contribution to the conductivity of the electrons on the neck part remains still small, since these electrons suffer additional scattering such as interband or Umklapp scattering and have smaller relaxation times than that of the belly electrons.

It follows from these that the contribution of the neck part of the Fermi surface to the conductivity is always small throughout the magnon region, and

the main part of the conductivity is determined by the intraband scattering of the electrons on the belly part of the Fermi surface. These situations do not change when the Fermi surface has several magnetic zone boundaries rather than two (present case) as long as the inequalities $\zeta \gg J$ and $P_F^2-(Q/2)^2 \gg mJ$ are satisfied.

However, the contribution to the conductivity of such electrons near the neck part is underestimated in our variational treatment, and must increase if the better trial function is employed in this region.

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Appendix

We derive the expression of the conductivity which is of the form of Eq. (42) by the hydrodynamical approximation of the Kubo formula. That the simplest form of this expression (Nakano's expression) may lead to incorrect result in the case of anisotropic scattering has been correctly pointed out by Vonsowsky.[4]

By the use of the notations in the Appendix of Ref. 9), the low frequency conductivity is given by

$$\sigma(\omega)=\sum_{\alpha\gamma}\sqrt{\langle \mathcal{J}_\alpha, \mathcal{J}_\alpha\rangle\langle \mathcal{J}_\gamma, \mathcal{J}_\gamma\rangle}\,\Phi_{\alpha\gamma}(\omega) \tag{A1}$$

and $\Phi_{\alpha\gamma}$ obeys the hydrodynamical equations

$$\sum_\beta(\omega\delta_{\alpha\beta}+i\Gamma_{\alpha\beta})\Phi_{\beta\gamma}(\omega)=i\delta_{\alpha\gamma} \tag{A2}$$

with

$$\Gamma_{\alpha\beta}=\frac{\langle \dot{\mathcal{J}}_\alpha, \dot{\mathcal{J}}_\beta\rangle_{\omega\to 0+i\delta}}{\sqrt{\langle \mathcal{J}_\alpha, \mathcal{J}_\alpha\rangle\langle \mathcal{J}_\beta, \mathcal{J}_\beta\rangle}}\,. \tag{A3}$$

When dividing the current operator $\mathcal{J}\equiv\sum_k \mathcal{J}_k=\sum_\alpha \mathcal{J}_\alpha$, one must group the region α in k space such that the electrons in this region suffers the same degree of scattering and so have the life time of the same order. In the case of $T>T_1$ as in § 3. 3, the symbol represents the region $(\pm)$ and $(\pm N)$, and along the treatment of that section, we set $\Gamma(\pm N,\pm)=\Gamma(\pm,\pm N)=0$. By solving Eq. (A2) we obtain from Eq. (A1) the static conductivity corresponding to Eq. (68)

$$\sigma=\sum_{\alpha=\pm}\frac{\langle \mathcal{J}_\alpha, \mathcal{J}_\alpha\rangle^2}{\mathcal{R}_e\langle \dot{\mathcal{J}}_\alpha, \dot{\mathcal{J}}_\alpha\rangle_{\omega\to 0+i\delta}}+\frac{A'_{+N}\eta'^2_{-N}+A'_{-N}\eta'^2_{+N}-2B'\eta'_{+N}\eta'_{-N}}{A'_{+N}A'_{-N}-B'^2}\,, \tag{A4}$$

$$\left.\begin{aligned} &A'_{+N}=\mathcal{R}_e\langle \dot{\mathcal{J}}_{+N}, \dot{\mathcal{J}}_{+N}\rangle_{\omega\to 0+i\delta}\,, \qquad A'_{-N}=\mathcal{R}_e\langle \dot{\mathcal{J}}_{-N}, \dot{\mathcal{J}}_{-N}\rangle_{\omega\to 0+i\delta}\,,\\ &B'=\mathcal{R}_e\langle \dot{\mathcal{J}}_{+N}, \dot{\mathcal{J}}_{-N}\rangle_{\omega\to 0+i\delta}=\mathcal{R}_e\langle \dot{\mathcal{J}}_{-N}, \dot{\mathcal{J}}_{+N}\rangle_{\omega\to 0+i\delta}\,,\\ &\eta'_{+N}=\langle \mathcal{J}_{+N}, \mathcal{J}_{+N}\rangle\,, \qquad \eta'_{-N}=\langle \mathcal{J}_{-N}, \mathcal{J}_{-N}\rangle\,. \end{aligned}\right\} \tag{A5}$$

If we divide the Fermi surface into three parts; (+), (−) and (N), grouping the ($\pm N$) region into one region (N), and discard interscattering terms, then we have the less accurate expression:

$$\sigma=\sum_{\alpha=\pm}\frac{\langle \mathcal{J}_\alpha, \mathcal{J}_\alpha\rangle^2}{\mathcal{R}_e\langle \dot{\mathcal{J}}_\alpha, \dot{\mathcal{J}}_\alpha\rangle_{\omega\to 0+i\delta}}+\frac{\langle \mathcal{J}_N, \mathcal{J}_N\rangle^2}{\mathcal{R}_e\langle \dot{\mathcal{J}}_N, \dot{\mathcal{J}}_N\rangle_{\omega\to 0+i\delta}} \tag{A6}$$

which corresponds to Eq. (74).

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