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On the Electroweak Theory at High Fermion Density

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We consider several points related to the recent observation[1] that the effects due to the level crossing phenomenon in the electroweak theory lead to the instability of normal matter at high fermion density, formation of the abnormal weak matter and stability of the drops of the abnormal matter surrounded by the normal vacuum. We illustrate the basic ideas within a toy model in (1+1) dimensions and then proceed to abelian and non-abelian theories in (3+1) dimensions. We pay particular attention to the gauge invariance of the effective hamiltonian of the boson fields at high fermionic density and zero temperature, effects due to non-abelian structure of the electroweak gauge group, weak hypercharge interactions and fermion mass terms. We also present some calculational details.

§ 1. Introduction

Various aspects of spontaneously broken gauge theories at finite fermion density and both zero and non-zero temperature have been studied during the last few years.[1)~7)] An obvious motivation is that high fermion density could be present at some stages of the evolution of the early Universe (this seems to be the case in a recently proposed scenario of the late generation of the baryon asymmetry[8)]) as well as in some exotic astrophysical objects. It has been found that the effect of the Yukawa couplings of fermions to the Higgs field is that the fermion density tends to decrease the scalar condensate.[9)] One the other hand, non-zero densities of broken charges make the opposite effect[10),2)] and, in the context of the electroweak theory, lead to the W-boson condensation.[3),4)]

It has been found by Tavkhelidze and the present author[1)] that new phenomena occur in theories with chiral structure of the interactions of fermions with gauge fields. In the case of neutral (with respect to all charges) matter, the normal state becomes unstable at sufficiently high density, and the system undergoes the first order phase transition to the abnormal state, in which the number of real fermions and scalar condensate are zero, while the gauge field condensate is characterized by large vector potentials and small, long ranged field strengths. Furthermore, it has been found[1)] that sufficiently large drops of the abnormal matter are stable with respect to decay into free fermions; while interacting with ordinary matter, these drops would eat up fermions (baryons, leptons) transforming an appreciable part of the rest energy of fermions into heat. In Ref. 1) these results were applied to the standard electroweak theory; the critical fermion number at which the drop of the abnormal weak matter becomes stable was estimated to be of order of 10^{14}, the radius of the critical drop was found to be of order 10^{-12}cm.

The main purpose of this paper is to discuss some points related to the analysis of Ref. 1), such as the gauge invariance of the effective bosonic hamiltonian at high density, effects of the weak hypercharge interactions and fermion mass terms, etc. We also justify some approximations made in Ref. 1) and present some calculational details.

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Throughout this paper we consider fermionic matter with zero densities of all charges, both broken and unbroken, abelian and non-abelian. In the standard electroweak theory, the neutrality condition is[4]

$$n_{u_L}^{(\alpha)}=n_{d_L}^{(\alpha)}=n_{e^-_L}=n_\nu\,, \tag{1·1}$$

$$n_{u_R}^{(\alpha)}=n_{d_L}^{(\alpha)}=n_{e^-_R}\,, \tag{1·2}$$

where n_{f_R} and n_{f_L} denote the number densities of left-handed and right-handed fermions respectively; u, d, e^- and ν denote generically up quarks, down quarks, charged leptons and neutrinos of all generations, respectively; $\alpha=1, 2, 3$ is the color index. We neglect the radiative corrections due to bosonic loops, so the gauge and Higgs fields, A_μ and ϕ, are treated as classical ones (condensates). The neutrality of the system implies that $A_0=0$ and A and ϕ are time-independent. We comment on the case of the asymmetric matter in the last section. Throughout this paper we restrict ourselves to the zero temperature case.

This paper is organized as follows. In § 2 we present some preliminary remarks concerning the relevance[1] of the level crossing phenomenon[11)~13)] for the theories at high densities. In § 3 we consider a toy model in (1+1) dimensions which shares some properties inherent in (3+1) dimensional theories; this model is useful for discussing the gauge invariance of the effective bosonic hamiltonian. In § 4 we study an abelian ($V-A$) theory at finite density in (3+1) dimensions. In § 5 we discusss the peculiarities of the standard electroweak theory. In § 6 we consider a finite drop of the abnormal weak matter in the normal vacuum, calculate its properties (mass, radius, etc.) and establish the conditions for its stability. Section 7 is devoted to concluding remarks.

§ 2. Level crossing and gauge theories at high densities

Strictly speaking, fermionic matter in gauge theories with chiral fermions is unstable even at low density: Indeed, in these theories the fermion number is not conserved due to the θ-vacuum structure and triangle anomaly.[14)~16)] However, in spontaneously broken, weakly coupled theories this nonconservation is a tunnelling effect and has a slow rate ($\sim\exp(-16\pi^2/g^2)$). (Recall that we are considering zero-temperature case; at high temperatures the situation becomes different.[17)]) In this paper we neglect the instanton-like tunnelling effects, i.e., we assume that their rates are small compared to the time the system is observed.

Another peculiarity of the theories under study is that the fermion number can be transferred from fermions to gauge fields. This becomes important at high fermion densities.[1)] Consider, for example, an $SU(2)$ gauge model with an even number (in order to avoid the non-perturbative anomaly[18)]), f, of left-handed fermion doublets and suppose that the fermion number density of each doublet is equal to $n_F/2f$ (the total fermion number density is equal to n_F). Assume that initially the classical gauge field is absent, so that the Fermi energy (chemical potential) is related to n_F in a standard way[19)]

$$\mu_F=\left(\frac{1}{2f}6\pi^2 n_F\right)^{1/3}.$$

Switching on the classical gauge field can make the fermion energy levels move down[11)~13)] as shown in Fig. 1. Clearly, both the number of real fermions and their energy become smaller, the decrease in the energy of fermions being determined by μ_F. On the other hand, the energy of the gauge (and Higgs) field increases, but this increase is independent of μ_F. Therefore, at sufficiently large μ_F, it is energetically favourable for the system to develop the gauge field condensate.[1)]

To gain some preliminary insight into the properties of this condensate, we recall that the level crossing phenomenon takes place whenever the non-zero Chern-Simons number,

$$N_{\rm CS}=-\frac{1}{16\pi^2}{\rm Tr}\Big(F_{ij}A_k-\frac{2}{3}A_iA_jA_k\Big)\varepsilon^{ijk}d^3x\ , \tag{2·1}$$

is developed (see, e.g., Ref. 12)). Therefore, the gauge field condensate should have the non-vanishing Chern-Simons density, $N_{\rm CS}/V$, where V is the volume of the system. It is worth noting that this argument can be considered as the physical interpretation of the appearance of the term

$$\Delta E_{\rm CS}=-f\mu_F N_{\rm CS} \tag{2·2}$$

in the effective hamiltonian of the gauge field at finite fermion density. This term was obtained, within the perturbation theory, in Ref. 6) (see also Ref. 7)), where it was also mentioned that it could give rise to an instability. In fact, the precise form of the contribution (2·2) can be understood by the following simple reasoning. At small but fixed $\boldsymbol{A}$, the structure of fermionic levels with energies of order μ_F is almost the same as at $\boldsymbol{A}=0$; however, the number of real fermions in each doublet is decreased by $N_{\rm CS}$ as the gauge field changes from zero to $\boldsymbol{A}$ (the decrease in the number of real fermions is equal to the number of levels crossing zero from above minus the number of levels crossing zero from below; this difference is just equal to $N_{\rm CS}$, see, e.g., Ref. 12). Of course, the latter fact is closely related to the triangle anomaly.) This means that $N_{\rm CS}$ levels with energies equal to μ_F become unoccupied, which leads to the decrease in the energy equal to $\mu_F N_{\rm CS}$ per each doublet.

At first sight, the term (2·2) seems not to be gauge-invariant. We discuss this point in the next section within a simple model in (1+1) dimensions.

§ 3. A model in (1+1) dimensions

In this section we consider a toy model which is a mixture of the (1+1) dimensional Higgs model and γ^5-analogue of the Schwinger model.[20)] We shall see in the following sections that some (but not all) features of this model are inherent in more realistic theories in (3+1) dimensions. The model is defined by the following Lagrangian,

$$L=-\frac{1}{4g^2}F_{\mu\nu}^2+|D_\mu\phi|^2-\lambda(|\phi|^2-c^2)^2+\bar{\psi}i\gamma^\mu(\partial_\mu-i\gamma^5A_\mu)\psi\ ,$$

where ϕ is complex scalar field, A_μ is $U(1)$ gauge field, $D_\mu\phi=(\partial_\mu-iA_\mu)\phi$, ν, $\mu=0, 1$ in this section and γ^μ are two-dimensional γ-matrices. The model can be consistently renormalized so that the axial current (which couples to A_μ) is conserved, while the gauge-invariant fermionic current,

$$J_\mu{}^F=\bar{\psi}\gamma^\mu\psi \tag{3·1}$$

is anomalous,

$$\partial_\mu J_\mu^F = -\frac{1}{\pi}\varepsilon_{\mu\nu}F^{\mu\nu}\,. \tag{3·2}$$

We assume that the space is a large circle of length L, i.e., that the periodic boundary conditions are imposed on all fields. We also assume that

$$c \gg 1\,. \tag{3·3}$$

The neutrality condition for this model reads

$$\langle \bar{\psi}\gamma^0\gamma^5\psi\rangle = 0\,,$$

hereafter $\langle\ \rangle$ denotes the statistical average. The bosonic part of the hamiltonian reads

$$E_B = \int dx^1[|D_1\phi|^2 + \lambda(|\phi|^2 - c^2)^2]\,. \tag{3·4}$$

Here we recall that we set $A_0=0$ and take A_1 and ϕ to be time-independent.

We now calculate the number density of real fermions, $n_R[A_1]$, and fermionic energy density, $\varepsilon_F[A_1]$, assuming that the number density at $A_1=0$, $n_F \equiv n_R[A_1=0]$, is fixed. Note that the system with fixed n_F can be prepared by inserting fermions into the box which is initially empty; n_F has clear physical siginificance. In what follows we study the phase transition occurring as n_F increases.

It is straightforward to calculate the fermionic spectrum at $A_1 \neq 0$,

$$E_k = \frac{2\pi}{L}(k - N_{\text{CS}})\,,$$

$$k = 0,\ \pm 1,\ \pm 2,\ \cdots\,, \tag{3·5}$$

where

$$N_{\text{CS}} = \frac{1}{2\pi}\int A_1 dx^1 \tag{3·6}$$

is the Chern-Simons number in $(1+1)$ dimensions. The levels are two-fold degenerate (chirality ± 1). As the gauge field changes from zero to some fixed A_1, $2[N_{\text{CS}}]$ levels cross zero from above ($[N_{\text{CS}}]$ denotes an integer part of N_{CS}). This means that $2[N_{\text{CS}}]$ fermions fill the negative energy levels in the Dirac sea, see Fig. 1, and the number of remaining real fermions becomes

$$N_R[A_1] = N_F - 2[N_{\text{CS}}]\,, \tag{3·7}$$

where we assume the right-hand side to be non-negative. The number density of real fermions is

$$n_R[A_1] = n_F - 2n_{\text{CS}} + O(L^{-1})\,, \tag{3·8}$$

where $n_{\text{CS}} = N_{\text{CS}}/L$ is the average Chern-Simons density. Note that Eqs. (3·7) and

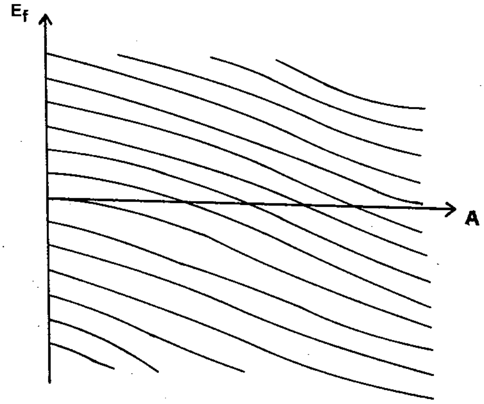

Fig. 1. Behaviour of the fermionic levels in gauge theories with chiral fermions.

(3·8) are essentially the integrated forms of the anomaly equation (3·2).

The average energy density of real fermions is

$$\varepsilon_F[A_1]=\frac{2}{L}\sum_{k=[N_{cs}]+1}^{N_R/2}\varepsilon_k \tag{3·9a}$$

$$=\frac{\pi}{2}(n_F-2n_{cs})^2+O(L^{-2})\,, \tag{3·9b}$$

where we pointed out that the correction on the right-hand side of Eq. (3·9b) is in fact $O(L^{-2})$ rather than $O(L^{-1})$. Thus, the total energy of the system is

$$E=E_B+L\varepsilon_R[A_1]\,. \tag{3·10}$$

This expression can be regarded as the effective hamiltonian of the boson fields at non-zero fermion density.

Several remarks are in order. First, we can introduce the chemical potential in a standard way,

$$\mu_F=\frac{\partial\varepsilon_F}{\partial n_F}\,.$$

Making use of Eq. (3·9b) we obtain

$$\mu_F[n_F;A_1]=\pi(n_F-2n_{cs})\,.$$

Introducing the standard Legendre transform,

$$\tilde{E}[\mu_F,A_1,\phi]=E-\mu_F N_F$$

we find

$$\tilde{E}=E_0(\mu_F)+E_B-2\mu_F N_{cs}\,, \tag{3·11}$$

where $E_0(\mu_F)\equiv-\mu^2F/2\pi$ is the term corresponding to free fermions. Note that the Chern-Simons term, $\Delta E_{cs}\equiv-2\mu_F N_{cs}$, is the only non-trivial contribution to E. This could have been expected, since the field strength vanishes in (1+1) dimensions in the case of neutral matter.

Second, both the number of real fermions, Eq.(3·7), and the effective energy functional, Eq.(3·10), seem not to be invariant under "large" gauge transformations (i.e., those with non-zero winding number of the gauge function). However, this lack of gauge invariance causes no trouble. Indeed, we defined $N_R[A_1]$ as the number of real fermions in the system *which would contain* N_F *fermions if* A_1 *was equal to zero.* In other words, this system is assumed to be obtained from that with N_F fermions and $A_1=0$ by a slow process which starts from $A_1=0$ and ends up at non-vanishing A_1. If we gauge transformed this system, the corresponding process would start from the pure gauge field, $A_1'=g\partial_1 g^{-1}$, and end up at $A_1{}^g=gA_1g^{-1}+g\partial_1 g^{-1}$. Clearly, the number of real fermions at the end of the latter process would be the same, so that $N_R[A_1]$ makes physical significance. Another way of expressing the same idea is as follows. The differences like $(N_R[A_1]-N_R[A_1'])$ are explicitly gauge invariant, as is clear from Eqs. (3·6) and (3·7). To specify the system, it is necessary and sufficient to define N_R at some fixed A_1. Suppose one knows the number of real fermions when the boson fields are in a vacuum state (this is the case if the system is prepared by adding fermions into initially empty box). However, there exist an infinite number of the topologically distinct classical vacua which differ by large gauge transformations. The choice of vacuum is arbitrary, but once this choice is made, the gauge freedom with respect to large gauge transformations is lost and $N_R[A_1]$ becomes physically significant. Our definition corresponds to the choice $A_1=0$. The θ-structure of vacuum and other physical states is straightforwardly incorporated into the above argument, provided that the instanton-like transitions are negligible (they are indeed negligible at small n_F because of Eq. (3·3)).

Let us now study the condensates of the boson fields. We use the unitary gauge, Im $\phi=0$ (in fact, the very possibility of using this gauge relies upon the neglect of the instantons). In this gauge, the energy functional, Eq. (3·10), is minimized by spatially homogeneous fields A_1 and ϕ. For these fields, the energy density reads

$$\varepsilon[A_1, \phi]=\frac{\pi}{2}\left(n_F-\frac{1}{\pi}A_1\right)^2+A_1{}^2\phi^2+\lambda(\phi^2-c^2)^2 . \qquad (3\cdot12)$$

This expression is straightforwardly analyzed. At small n_F there exists only one mimimum, for which

$$\begin{aligned} &\phi=O(c) , \\ &A_1=\frac{n_F}{2\phi^2} , \\ &n_{\rm CS}\equiv\frac{1}{2\pi}A_1\ll n_F , \end{aligned} \qquad (3\cdot13)$$

(we neglect corrections of order $O(c^{-1})$). In this state, the gauge field condensate is rather weak, and the number density of real fermions only slightly deviates from n_F.

At $n_F=\sqrt{2\lambda}\,c/\pi$ another local minimum appears at

$$\begin{aligned} &\phi=0 , \\ &A_1=\pi n_F , \\ &n_{\rm CS}=\frac{1}{2}n_F . \end{aligned} \qquad (3\cdot14)$$

This point becomes a global mimimum of Eq. (3·12) at $n_F=n_{\rm crit}^{(1)}$, where

$$n_{\rm crit}^{(1)}=\sqrt{\frac{2\lambda}{\pi}}c^2 . \qquad (3\cdot15)$$

Equation (3·15) is easy to understand. The energy density in the state (3·14) is contained in the Higgs field, so it is equal to λc^4 (the number of real fermions is zero; fermions are eaten up by the gauge field condensate). On the other hand, the energy density in the state (3·13) is contained mostly in real fermions, so it is equal to $\frac{1}{2}\pi n_F{}^2$. Comparing the two energy densities, one obtains Eq. (3·15).

At $n_F\sim n_{\rm crit}^{(1)}$, the state (3·13) is still a local mimimum of the energy functional. This local mimimum disappears at $n_F=n_{\rm crit}^{(2)}$ where

$$n_{\rm crit}^{(2)}=2\left(\frac{2}{3}\right)^{3/2}\sqrt{\lambda}\,c^3 . \qquad (3\cdot16)$$

This behaviour of the energy functional means that the phase transition is of the first order. If one neglects *all* tunnelling effects, then the transition occurs at $n_F = n^{(2)}_{\rm crit}$; the system rolls down from the state (3·13) to the state (3·14).

Once the state (3·14) is reached, the unitary gauge can no longer be used (the phase of the Higgs field becomes undefined). The state (3·14) is *not* the final state of the system; the energy decreases further when the Higgs condensate of the form $\phi(x^1) \propto \exp(iA_1 x^1)$ is developed, while A_1 remains unchanged (and thus the number of real fermions remains zero). The final state is the gauge transformed vacuum,

$$\begin{aligned} &A_1 = \pi n_F\,, \\ &\phi = c e^{iA_1 x^1}\,, \\ &n_R = 0\,. \end{aligned} \tag{3·17}$$

We conclude that although we disregarded the instanton-like *tunnelling* transitions, the system evolved into the state with no fermions and pure gauge configuration of the boson fields. This evolution proceeded classically, the disappearance of fermions still having taken place due to the anomaly.

§ 4. $U(1)$ model in (3+1) dimensions

Before considering non-abelian theories in (3+1) dimensions, it is instructive to study an abelian model with spontaneous symmetry breaking and chiral structure of the interactions of fermions with gauge field. Let $\psi_L^{(i,+)}$ and $\psi_L^{(i,-)}$, $i=1, \cdots, f$, be two sets of massless left-handed fermion doublets with opposite $U(1)$ charges and ϕ be the complex Higgs field. The model to be discussed in this section is defined by the following Lagrangian

$$\begin{aligned} L = &-\frac{1}{4g^2}F_{\mu\nu}^2 + \left|\left(\partial_\mu - i\frac{q}{\sqrt{2}}A_\mu\right)\phi\right|^2 - \lambda(|\phi|^2 - c^2)^2 \\ &+ \sum_{i=1}^{f}\left[\bar{\psi}_L{}^{(i,+)} i\sigma_\mu\left(\partial_\mu - \frac{i}{2}A_\mu\right)\psi_L{}^{(i,+)} + \bar{\psi}_L{}^{(i,-)} i\sigma_\mu\left(\partial_\mu + \frac{i}{2}A_\mu\right)\psi_L{}^{(i,-)}\right], \end{aligned} \tag{4·1}$$

where $\sigma_\mu = (1, \boldsymbol{\sigma})$, $\boldsymbol{\sigma}$ are the Pauli matrices and $\sqrt{2}q$ is the charge of the Higgs field, so that $M_W = gc$ is the vector boson mass. The gauge field is normalized in such a way that our model (modulo the Higgs sector) can be viewed as the truncated version of a non-abelian theory with an $SU(2)_L$ gauge group and f left-handed fermion doublets,

$$\psi^{(i)} = \begin{bmatrix} \psi^{(i,+)} \\ \psi^{(i,-)} \end{bmatrix}.$$

The above choice of the fermion charges makes the model renormalizable. The total fermionic current,

$$J_\mu{}^F = \sum_{i=1}^{f}(\bar{\psi}_L{}^{(i,+)}\sigma_\mu\psi_L{}^{(i,+)} + \bar{\psi}_L{}^{(i,-)}\sigma_\mu\psi_L{}^{(i,-)})$$

is anomalous

$$\partial_\mu J_\mu{}^F = -\frac{f}{32\pi^2} F_{\mu\nu}\tilde{F}_{\mu\nu} . \tag{4·2}$$

The neutrality condition in this model is

$$n_F{}^{(i,+)} = n_F{}^{(i,-)} \equiv \frac{1}{2f} n_F ,$$

where $n_F{}^{(i,\pm)}$ are the number densities at $\boldsymbol{A}=0$.

4.1. *Instability of the normal state*

We first study the system at small gauge fields,

$$|\boldsymbol{A}| \ll \mu_F , \tag{4·3}$$

where μ_F is the Fermi energy in the normal state, $\boldsymbol{A}=0$. We also assume that the gauge fields are long ranged,

$$k \ll \mu_F , \tag{4·4}$$

where k is the typical spatial momentum of the gauge field. Under these conditions, the fermionic contribution to the effective hamiltonian of the boson fields can be calculated perturbatively. The relevant diagram is shown in Fig. 2, where the solid line corresponds to the fermionic propagator at finite density (cf., Ref. 21))

$$G_F(p) = \frac{(p^0 + \mu_F) + \boldsymbol{\sigma p}}{(p^0 + \mu_F + i\varepsilon p^0)^2 - \boldsymbol{p}^2} .$$

The lowest (in the sense of (4·3) and (4·4)) non-vanishing contribution is the Chern-Simons term,

$$\Delta E_{\mathrm{CS}} = -\frac{f\mu_F}{32\pi^2} \int d^3x \varepsilon^{ijk} F_{ij} A_k .$$

We use the unitary gauge, $\mathrm{Im}\phi=0$, so the effective bosonic hamiltonian is (up to corrections of order $\mu_F{}^{-1}$)

$$E_B{}^{\mathrm{eff}} = \int d^3x \left[\frac{1}{4g^2} F_{ij}^2 + (\partial_i \phi)^2 + \frac{q^2}{2} \boldsymbol{A}^2 \phi^2 + \lambda(\phi^2 - c^2)^2 - \frac{f\mu_F}{32\pi^2} \varepsilon^{ijk} F_{ij} A_k \right] . \tag{4·5}$$

Unlike the effective hamiltonian of the (1+1)-dimensional example of § 3, the expression (4·5) contains no linear terms, so the normal state,

$$\boldsymbol{A} = 0 ,$$

$$\phi = c$$

is always an extremum of the effective energy. However, at large μ_F this extremum is a saddle point, rather than a minimum, of the functional (4·5). Indeed, it is straightforward to obtain the spectrum of the operator determining the energy of small fluctuations around the

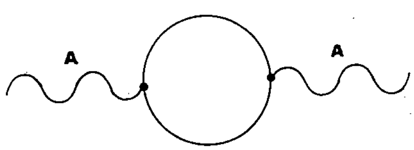

Fig. 2. Lowest order diagram contributing to the effective hamiltonian of the boson fields at finite fermion density.

normal state. Making use of Eq. (4·5) one finds that the spectrum has two branches,

$$\omega^2(k)=k^2+M_W{}^2\pm\frac{\mu_F f g^2}{16\pi^2}k\,,$$

where k is the spatial momentum of the fluctuation. At small μ_F, ω^2 is positive for any k, so the normal state is (meta)stable. A zero mode ($\omega^2=0$) appears at $\mu_F=\mu_{\rm crit}^{(2)}$, where

$$\mu_{\rm crit}^{(2)}=\frac{4\pi M_W}{f\alpha_W}\,,$$

$$\alpha_W=\frac{g^2}{4\pi}\,, \tag{4·6}$$

at slightly larger μ_F this mode becomes negative. The corresponding momentum is

$$k=M_W\,. \tag{4·7}$$

Note that Eq. (4·4) is satisfied for the zero mode in view of Eqs. (4·6) and (4·7), so our approximation is justified. Thus, at $\mu_F=\mu_{\rm crit}^{(2)}$ the normal state becomes a saddle point, rather than a minimum, of the effective energy. The system undergoes the (first order) phase transition, the point (4·6) is analogous to the second critical point of the example of § 3 (Eq. (3·16)). We shall discuss the first critical point (at which the normal state becomes a *local* minimum of energy) later on.

It is instructive to present the explicit form of the negative mode,

$$\boldsymbol{A}(\boldsymbol{x})=a(\boldsymbol{e}_1\cos\boldsymbol{kx}-\boldsymbol{e}_2\sin\boldsymbol{kx})\,, \tag{4·8}$$

where a is a small amplitude and $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are the real polarization vectors obeying

$$\boldsymbol{e}_\alpha\cdot\boldsymbol{k}=0\,,\qquad \boldsymbol{e}_\alpha\cdot\boldsymbol{e}_\beta=\delta_{\alpha\beta}\,,$$

$$\boldsymbol{e}_\alpha=-\varepsilon_{\alpha\beta}\frac{\boldsymbol{k}}{k}\times\boldsymbol{e}_\beta\,.$$

Note that the magnetic field of this perturbation is

$$\boldsymbol{H}=k\boldsymbol{A} \tag{4·9}$$

and that the Chern-Simons density,

$$n_{\rm CS}=\frac{1}{16\pi^2}\boldsymbol{HA} \tag{4·10}$$

as well as classical energy density, $(1/2g^2)H^2$, are homogeneous, in spite of the fact that $\boldsymbol{A}$ and $\boldsymbol{H}$ depend on spatial coordinates.

Let us follow the development of the instability by letting the amplitude of the unstable mode to grow. We still consider the fields obeying (4·3) and (4·4), so we use the form of the effective energy given by Eq. (4·5). Note that as long as (4·3) and (4·4) are satisfied, the Chern-Simons density is small, and the fermion number density is related to μ_F in the standard way[19)]

$$n_F=2f\frac{\mu_F{}^3}{6\pi^2}\,. \tag{4·11}$$

Therefore, at this stage fixing n_F is equivalent to fixing μ_F.

As the amplitude of the gauge field grows, the Higgs condensate decreases. Indeed, the term $\frac{1}{2}q^2A^2\phi^2$ in Eq. (4·5) acts as a positive mass term for the Higgs field. This means, in particular, that the unstable mode is not stabilized within our approximation. The Higgs condensate vanishes at

$$|\boldsymbol{A}|=2\sqrt{\lambda}\frac{M_W}{q^2g^2}.$$

Note that this field still obeys (4·3) with $\mu_F=\mu_{\rm crit}^{(2)}$.

4.2. *The final state*

To discuss the final state, one should go beyond the approximations (4·3)~(4·5). Also, Eq. (4·11) is no longer valid, so one should express the energy through n_F, the only fixed parameter in the model. The calculations are simplified by the observation[1] that it is sufficient to consider long ranged magnetic fields,

$$\sqrt{H}\gg k. \tag{4·12}$$

We begin with considering the system in the background field of the form (4·8). Our final purpose is to find the amplitude a and momentum k by minimizing the total energy. The reasons for choosing the ansatz (4·8) are as follows: i) The unstable mode has precisely this form; ii) Eq. (4·9) implies that for given H and k the Chern-Simons density is maximal, so the number of real fermions and their energy are minimal; iii) some physical quantities, like the Chern-Simons density, are homogeneous. In fact, our analysis is not sensitive to the precise form of the gauge field condensate; we shall discuss this point later on.

We take the momentum $\boldsymbol{k}$ to be directed along the third axis, while the polarization vectors are directed along the first and second axes respectively. The gauge field near, say, $\boldsymbol{x}=0$ can be approximated by

$$\boldsymbol{A}=a\boldsymbol{e}_1-ak\boldsymbol{e}_2x^3, \tag{4·13}$$

so that

$$\boldsymbol{H}=ak\boldsymbol{e}_1, \tag{4·14}$$

i.e., the magnetic field is considered as homogeneous (recall Eq. (4·12)). The fermionic spectrum in the background field (4·13) can be read off from Ref. 13). For charge $+\frac{1}{2}$ fermions it is labelled by two continuous variables, p_1 and p_2 (the momentum in the first direction and the position of the orbit in the (x^2, x^3) plane[22]), and one discrete variable $n=0, 1, 2, \cdots$ (the number of the orbit). The energies are

$$n=0:\qquad E=p_1-\frac{a}{2}, \tag{4·15a}$$

$$n>0:\qquad E=\sqrt{\left(p_1-\frac{a}{2}\right)^2+nH}. \tag{4·15b}$$

The spatial widths of the wave functions with $n\sim1$ are of order $\sqrt{H}$, so we can indeed use the approximation (4·13). The number of levels for fixed n and $p_1\in(p_1, p_1+dp_1)$ is[22]

$$\frac{V}{(2\pi)^2}\frac{H}{2}dp_1 . \qquad (4\cdot16)$$

Let us now evaluate the number density of real fermions, $n_R[a]$, in the system containing $N_F \equiv n_F \cdot V$ fermions at $a=0$. As the amplitude grows from a to $(a+da)$, levels with $n=0$ move down according to Eq. (4·15a), and levels with momenta $p_1 \in (a/2, (a+da)/2)$ cross zero from above. The same picture is valid for charge $(-\frac{1}{2})$ fermions. Thus, the number density of real fermions changes by (see Eq. (4·16))

$$dn_R = -2f\frac{1}{16\pi^2}Hda ,$$

where the factor $2f$ accounts for flavors. We obtain

$$n_R = n_F - \int_0^a \frac{f}{8\pi^2}Hda$$
$$= n_F - fn_{CS} , \qquad (4\cdot17)$$

where we used Eqs. (4·14) and (4·10). Thus, we explicitly recover the relation between n_R and n_{CS}. We note in passing that Eq. (4·17) can be viewed as the integrated form of the triangle anomaly.*)

It is now straightforward to calculate the energy density of real fermions. Let $\mu_F{}^{ab}$ be the Fermi energy in the abnormal state. For our purposes it is sufficient to study the case

$$\mu_F{}^{ab} < \sqrt{H} , \qquad (4\cdot18)$$

so that only $n=0$ levels are occupied. One finds from Eqs. (4·15a) and (4·16) that

$$n_R = f\frac{H\cdot\mu_F{}^{ab}}{4\pi^2} ,$$
$$\varepsilon_F = f\frac{H(\mu_F{}^{ab})^2}{8\pi^2} . \qquad (4\cdot19)$$

As observed in § 4.1, the Higgs condensate vanishes long before the final state is reached, so the total energy density for fixed a and k is

$$\varepsilon = 2\pi^2\frac{\left(n_F - \frac{f}{16\pi^2}ka^2\right)^2}{fka} + \frac{k^2a^2}{2g^2} + \lambda c^4 , \qquad (4\cdot20)$$

where we eliminated $\mu_F{}^{ab}$ by making use of Eq. (4·19) and neglected the contributions from the Dirac sea and bosonic loops (these contributions are small provided that $g^2 \log H \ll 1$).

We now have to minimize the right-hand side of Eq. (4·20) with respect to a and k. However, it is straightforward to see that the minimum does not exist: Long ranged $(k\to0)$ large $(a\to\infty)$ vector potentials make the first two terms on the right-hand side of

*) A similar discussion of the level crossing phenomenon is given in Ref. 13). However, our result, Eq. (4·17), differs from that of Ref. 13) by a factor of 1/2. The reason is that the authors of Ref. 13) consider fixed magnetic field and vary only A_1 (in our notations), while in our case $\boldsymbol{H}$ is related to $\boldsymbol{A}$ via Eq. (4·9). Equation (4·17) is the correct integrated form of the triangle anomaly for a system with periodic boundary conditions.

Eq. (4·20) arbitrarily small. So, we introduce an infrared cutoff, $k \geq k_0$, and minimize the energy density with respect to a at $k=k_0$. We find at the minimum

$$a=4\pi\sqrt{\frac{n_F}{fk_0}}+O(k_0)\,, \tag{4·21a}$$

$$H=4\pi\sqrt{\frac{n_F k_0}{f}}+O(k_0^2)\,, \tag{4·21b}$$

$$\varepsilon=\lambda c^4+8\pi^2\frac{k_0 n_F}{fg^2}+O(k_0^{5/2}n_F^{1/2})\,, \tag{4·21c}$$

$$n_R \equiv n_F - f n_{\mathrm{CS}} = O(k_0^{3/2}n_F^{1/2})\,. \tag{4·21d}$$

It is also straightforward to obtain the Fermi energy in the abnormal state,

$$\mu_F{}^{ab} \equiv \frac{\partial\varepsilon}{\partial n_F}=\frac{8\pi^2 k_0}{fg^2} \tag{4·22}$$

(alternatively, one can calculate μ_F directly by evaluating the omitted terms in Eq. (4·21) and making use of Eq. (4·19); the result coincides with Eq. (4·22)). Equations (4·21b) and (4·22) justify the approximation (4·18), while our basic assumption, Eq. (4·12), is justified by Eq. (4·21b).

Equation (4·21) show that the abnormal state is characterized by large vector potential, while the magnetic field is small compared to $n_F{}^{2/3}$, and long ranged. The fermion number is almost totally eaten up by the gauge field; the main part of the energy density is carried by the Higgs field. The latter property makes it straightforward to calculate the first critical fermion number density, $n_{\mathrm{crit}}^{(1)}$, at which the abnormal state becomes the global mimimum of the energy. One has to compare Eq. (4·21c) with the energy density in the normal state,

$$\varepsilon^{\mathrm{norm}}=\frac{3}{4}\left(\frac{3\pi^2}{f}\right)^{1/3} n_F{}^{4/3}$$

(see, e.g., Ref. 19)). One finds up to $O(k_0)$ corrections

$$n_{\mathrm{crit}}^{(1)}=\frac{1}{3}\left(\frac{8}{\pi}\right)^{1/2} f^{1/4}\lambda^{3/4}c^3\,, \tag{4·23}$$

i.e., the first critical value of the chemical potential in the normal state is

$$\mu_{\mathrm{crit}}^{(1)}=(2\pi)^{1/2}f^{-1/4}\lambda^{1/4}c\,.$$

Note that the abnormal state becomes favorable long before the normal state becomes absolutely unstable (i.e., $\mu_{\mathrm{crit}}^{(1)} \ll \mu_{\mathrm{crit}}^{(2)}$, see Eq. (4·6)); this property is characteristic to the system undergoing the strongly first order phase transition.

We now show that our result for the final state, Eq. (4·21), is not sensitive to the ansatz for the gauge field condensate, Eq. (4·8). The condensate developed in the system should lead to the energy density of the same order of magnitude as Eq.(4·21c) or less. This means, in particular, that the fermion number should be totally eaten up by the gauge field, i.e., the gauge field should obey

$$N_{\rm CS} \equiv \frac{1}{16\pi^2}\int \boldsymbol{A}\boldsymbol{H} d^3x = \frac{N_F}{f} \tag{4·24}$$

up to $O(k_0)$ corrections. The gauge field condensate should minimize the classical energy, $\int \boldsymbol{H}^2 d^3x/2g^2$, under the condition (4·24). Keeping the infrared cutoff, we find the general form of the field obeying these requirements,

$$\boldsymbol{A}(\boldsymbol{x}) = \frac{1}{\sqrt{2}}\sum_{|\boldsymbol{k}|=k_0} a_{\boldsymbol{k}} e^{i\boldsymbol{k}\boldsymbol{x}}(\boldsymbol{e}_1(\boldsymbol{k}) + i\boldsymbol{e}_2(\boldsymbol{k}))\ ,$$

where $a_{-\boldsymbol{k}} = -a_{\boldsymbol{k}}^*$, and the amplitudes obey

$$\sum_{|\boldsymbol{k}|=k_0} |a_{\boldsymbol{k}}|^2 = 16\pi^2 \frac{n_F}{k_0 f}\ .$$

For this gauge field, Eqs. (4·21a~d) are still valid, if a and H are understood as follows,

$$a = \sqrt{\frac{1}{V}\int \boldsymbol{A}^2 d^3x}\ ,$$

$$H = \sqrt{\frac{1}{V}\int \boldsymbol{H}^2 d^3x}\ ,$$

while ε and n_R are spatial averages of the energy density and number density of real fermions. This is the desired result.

§ 5. Peculiarities of the standard electroweak theory

We now turn to the discussion of the dense neutral fermionic matter in the standard electroweak theory. We follow the line of § 4 and first consider the instability of the normal state. To see this instability, we need only those terms in the effective bosonic hamiltonian which are quadratic in the gauge field; the non-abelian nature of the electroweak $SU(2)_L \times U(1)_Y$ group plays no role at this point. However, there are two peculiarities to be discussed, namely, the effects of the fermionic mass terms and mixing between the third component of the $SU(2)_L$ gauge field, A_μ^3, and the $U(1)_Y$ gauge field B_μ. We shall see that the former is irrelevant, while the latter leads to slight changes in the formulas of § 4. We then consider the abnormal electroweak matter and pay special attention to the effects related to the non-abelian structure of the theory.

5.1. *Critical fermion density*

In order to see the effect of the fermionic mass, one has to insert the massive fermionic propagator, instead of massless one, into the internal lines of the diagram of Fig. 2. The calculation is straightforwardly performed with the help of the technique developed in Ref. 21). The result is that the expression for the effective energy, Eq. (4·5), should be modified by substituting p_F for μ_F, where $p_F = \sqrt{\mu_F^2 - m_F^2}$ is the Fermi momentum. This expression is valid as long as $k \ll p_F$ (instead of (4·4)). Since $m_F \ll \mu_{\rm crit}^{(2)}$ at least for the known fermions, the above modification is not relevant for the problem of the stability of the normal state.

We now turn to the discussion of A^3-B mixing. It is straightforward to generalize Eq. (4·5) to the case of the standard electroweak theory (one only has to count the group factors in the diagram of Fig. 2; neglecting the fermionic masses, one obtains

$$E_B{}^{\rm eff}=E_B-\mu_F f N_{\rm CS}^{(2)}[\boldsymbol{A}]-\mu_F\Big[\sum_{\rm doublets} Y_L{}^2-\frac{1}{2}\sum_{\rm singlets} Y_R{}^2\Big]N_{\rm CS}^{(2)}[\boldsymbol{B}]\,, \tag{5·1}$$

where E_B is the classical energy functional of the bosonic sector of the Weinberg-Salam model, f is the total number of the left-handed fermion doublets, Y_L and Y_R are weak hypercharges of the left-handed and right-handed fermions respectively, $\boldsymbol{A}$ is the $SU(2)_L$ gauge field and $N_{\rm CS}^{(2)}$ is the bilinear part of the Chern-Simons number, Eq. (2·1) (only this part is relevant for the problem of stability of the normal state). In Eq. (5·1) it is assumed that the neutrality conditions, Eqs. (1·1) and (1·2), are valid, and moreover, the densities of the left-handed and right-handed fermions are equal to each other. Inserting the actual values of Y_L and Y_R into Eq. (5·1), one finds

$$E_B{}^{\rm eff}=E_B-4f_g N_{\rm CS}^{(2)}[\boldsymbol{A}]+4f_g N_{\rm CS}^{(2)}[\boldsymbol{B}]\,, \tag{5·2}$$

where f_g is the number of the fermion generations. Note that after the standard rotation to the physical Z-boson and photon fields, the sum of the Chern-Simons terms in Eq. (5·2) does not contain terms depending only on the electromagnetic vector potential: The electromagnetic field cannot produce any change in the number of real fermions because of the vector structure of the electromagnetic interactions.

It is now straightforward to study the stability of the normal state under small perturbations of the gauge fields. One finds that the negative mode of the $W^{\pm}$ fields appears at $\mu_F=\mu_{\rm crit}^{(2)}$, where $\mu_{\rm crit}^{(2)}$ is given by Eq. (4·6). The negative mode of A^3 and B appears at $\mu_F=\mu_{\rm crit}^{(2)\prime}$, where

$$\mu_{\rm crit}^{(2)\prime}=\frac{4\pi M_Z}{f\alpha_W}\cos^2\theta_W=\mu_{\rm crit}^{(2)}\cdot\cos\theta_W\,.$$

This negative mode contains both Z-boson field and electromagnetic field. Note that $\mu_{\rm crit}^{(2)\prime}<\mu_{\rm crit}^{(2)}$, so the mixing between A^3 and B makes the normal state unstable at slightly smaller density.

As discussed in § 4, the development of the instability leads to the disappearance of the scalar condensate. The quark-antiquark condensates formed due to the strong interactions disappear as well: The $\bar{q}q$ condensates produce masses to the weak vector bosons,[23] so they vanish for the same reason as the Higgs field does.

4.2. *Abnormal weak matter*

Let us now discuss the abnormal state. Since the $U(1)_Y$ coupling constant is smaller than $SU(2)_L$ one, we can disregard the $U(1)_Y$ interactions: Indeed, Eq. (4·21c) implies that only the gauge field with the largest coupling constant plays a role. So, we concentrate here on the peculiarities due to the non-abelian structure of the gauge group.

At first sight it seems that the state with $\boldsymbol{A}^3=\boldsymbol{A}$, $\boldsymbol{A}^{1,2}=0$ where $\boldsymbol{A}$ is given by Eqs. (4·8), (4·21a) is a good candidate for the ground state in non-abelian theory as well, since the energy density, Eq. (4·21c), is close to the lower bound, λc^4, of the energy density of any state with the vanishing Higgs condensate. However, in the non-abelian theories this state is still unstable with respect to small perturbations. To see this, we first recall that the gauge fields are massless in this state. Now, the fluctuations of the massless gauge fields in the homogeneous background magnetic field were studied in Ref. 24) where it has been found that there exist negative modes. Let us show that this observation is relevant in our case as well.

We begin with abelian state, $\boldsymbol{A}^3=\boldsymbol{A}$, $\boldsymbol{A}^{1,2}=0$ and use the approximate form of the gauge field, Eq. (4·13). This form differs from that studied in Ref. 24) only by the x-independent term (and the spatial direction of the magnetic field $\boldsymbol{H}^3$). Therefore, the modes expected to be unstable are the same as in Refs. 24) ~ 27) up to a gauge transformation,

$$A_3{}^1+iA_3{}^2=i\left(A_2{}^1+iA_2{}^2\right)=u\,,$$

$$u(\boldsymbol{x})=e^{-2iax_1}e^{-Hx_3{}^2/2}F(z)\,, \tag{5·3}$$

where $z=-x_3+ix_2$ and $F(z)$ is an arbitrary analytic function. The typical spatial scale of these modes is $\sqrt{H}$; since $\sqrt{H}\gg k_0$, we can indeed use the approximation, Eq. (4·13), for the field (4·8). The formation of the fields (5·3) decreases the magnetic field,

$$H^3=H-2|u|^2\,,$$

$$H^{1,2}=0\,. \tag{5·4}$$

On the other hand, a straightforward calculation shows that the Chern-Simons density remains unchanged. Therefore, the formation of the field (5·3) leads to no change in the number of real fermions, although the fermionic spectrum gets modified. We now recall that the number of real fermions in the state (4·21) is so small that their energy is negligible compared to the energy of the gauge field. Therefore, the leading effect of modes (5·3) is that they lower the energy of the gauge field. We conclude that these modes are unstable in our case as well.

It has been argued in Refs. 26) and 27) that the development of the latter instability leads to the formation of the inhomogeneous state consisting of domains, namely, magnetic flux tubes directed along the initial magnetic field. Their size in the orthogonal plane is of order $\sqrt{H}$ which, in our case, is small compared to the characteristic scale, $k_0{}^{-1}$, of the spatial variation of the initial field. The net effect of these flux tubes on the averaged energy density is that the "effective" coupling constant $g^2_{\text{eff}}=g^2/C$ should be substituted for g^2 in Eq. (4·21c). Here C is a numerical constant; the estimate of Ref. 27) is

$$C\approx 0.14\,. \tag{5·5}$$

So, we argue that the abnormal weak matter has two spatial scales. The larger scale is provided by the infrared cutoff k_0; this scale determines the variation of the direction of the magnetic field $\boldsymbol{H}^3$ and is relevant for the formation of the Chern-Simons density, just as in the abelian model of § 3. The smaller scale, $H^{-1/2}\sim(n_F k_0)^{-1/4}$, characterizes the size of the magnetic flux tubes and the separation between them.

It is worth noting that the arguments leading to Eq. (4·23) are valid also in the non-abelian case, so the abnormal state is energetically more favourable than normal one at $n_F>n_{\text{crit}}^{(1)}$.

§ 6. Stable drops of abnormal weak matter

In this section we consider drops of the abnormal weak matter surrounded by the normal vacuum. As discussed in § 5, the net effect of the non-abelian modes on the energy density can be absorbed into the modification of the coupling constant, $g^2 \to g^2/C$, so we concentrate here on the abelian part of the weak gauge field, i.e., we use the model of § 4.

Let N_F be the fermion number of a drop, i.e., the number of real fermions which would contain the system if $\boldsymbol{A}$ was equal to zero everywhere in the space. Since we neglect the instanton-like transitions, we can regard N_F as a fixed number which characterizes the drop. If the drop would be able to decay into free baryons and leptons, the decay products would contain N_F fermions (quarks and leptons). Equations (1·1) and (1·2) imply that the number of baryons in the decay products would be larger than (or equal to)$\frac{1}{4}N_F$, i.e., the drop is stable with respect to decay into free fermions provided that

$$M_{\text{drop}} \le \frac{1}{4} N_F m_p \,, \tag{6·1}$$

where M_{drop} is the mass of the drop and m_p is the proton mass.

Let R be the radius of the drop. To find its actual value and M_{drop}, we have to evaluate the energy of the drop, $E(R)$, at arbitrary R and minimize $E(R)$. Clearly, the infrared cutoff k_0 is of order R^{-1}, so we can neglect corrections to Eq. (4·21c,d) as long as $n_F R^3 \gg 1$, i.e., as long as N_F is large enough. We shall see that the drop is stable if N_F is indeed large, so we consider this case. Equation (4·21d) means that (almost) all fermions are eaten up by the gauge field, i.e., the gauge field condensate should obey Eq. (4·24). The energy of the drop consists of three parts: i) the energy of the Higgs field, $\lambda c^4 V$, where V is the volume of the drop; (ii) the energy of the gauge field,

$$E_{\text{gauge}} = \int_V \frac{1}{2g^2} \boldsymbol{H}^2 d^3x \tag{6·2}$$

iii) the surface energy. Let us consider spherically symmetric drop. Then the energy of the Higgs field is fixed provided that R is fixed. The surface energy is of order $R^2 c^3$, up to some function of the coupling constants; the surface energy is much less than the volume energy provided that $R \ggg c^{-1}$, which we shall shortly find to be the case for stable drop. Thus, the only unknown contribution is that of the gauge field itself, Eq. (6·2); to find it, we have to minimize the integral (6·2) under the condition (4·24), the vector potential should vanish at the boundary,

$$\boldsymbol{A}(R) = 0 \,. \tag{6·3}$$

To solve the latter problem, we use the standard Lagrange multiplier technique, i.e., we find an extremum of the functional

$$\tilde{E}(\nu) = E_{\text{gauge}}[\boldsymbol{A}] - 32\pi^2 \nu N_{\text{CS}}[\boldsymbol{A}] \tag{6·4}$$

with the boundary condition (6·3). Here ν is the Lagrange multiplier, the factor $32\pi^2$ is introduced for convenience. The Euler-Lagrange equations corresponding to Eq. (6·4) are

$$-\partial_i F_{ij} - 2\nu\varepsilon_{ikj}F_{ik} = 0\,. \tag{6·5}$$

One can show that the solution minimizing E_{gauge} for fixed N_{CS} is the p-wave,

$$A_i = n_i n_a f_1(r) + \frac{\delta_{ai} - n_i n_a}{r} f_2(r) + \varepsilon_{jai}\frac{n_j}{r} f_3(r)\,, \tag{6·6}$$

where a is fixed (say, $a=3$). Inserting Eq. (6·6) into Eq. (6·5) we find the following solution,

$$f_3 = A\sqrt{r}\,J_{3/2}(\nu r)\,,$$

$$\partial_r f_2 - f_1 = \nu f_3\,,$$

where A is yet undetermined amplitude and $J_{3/2}$ is the Bessel function. Equation (6·3) gives

$$\nu = \frac{\xi_0}{R}\,,$$

$$f_2 = 0\,,$$

where ξ_0 is the first root of $J_{3/2}$. The amplitude A is to be found from Eq. (4·24). After straightforward manipulations we obtain the desired expression for E_{gauge},

$$E_{\text{gauge}} = \frac{8\pi^2}{g^2}\frac{\xi_0}{R} N_{\text{CS}}\,.$$

The total energy of the drop at fixed R reads

$$E(R) = \frac{4\pi}{3} R^3 \lambda c^4 + \frac{8\pi^2}{g^2} C\xi_0 \frac{N_F}{fR}\,.$$

Minimizing this expression with respect to R, we obtain the actual radius and energy of the drop,

$$R = \left(\frac{C\xi_0}{2\lambda f\alpha_W}\right)^{1/4} \frac{1}{c} N_F^{\,1/4}\,, \tag{6·7}$$

$$E = \frac{4}{3}(4\pi\lambda)^{1/4}\left(\frac{2\pi C\xi_0}{f\alpha_W}\right)^{3/4} c N_F^{\,3/4} \tag{6·8}$$

An interesting property of Eq. (6·8) is that the energy grows slowly with N_F. This means that at sufficiently large N_F the relation (6·1) is satisfied, i.e., the drop is stable. The critical value of N_F is

$$N_F^{\text{crit}} = \left(\frac{16}{3}\right)^4 4\pi\lambda \left(\frac{2\pi C\varepsilon_0}{f\alpha_w}\right)^3 \left(\frac{c}{m_p}\right)^4. \tag{6·9}$$

The only unknown parameter entering Eqs. (6·7) ~ (6·9) is the Higgs coupling constant, λ. To estimate the properties of the critical drop we take $\lambda \sim g^2$; recalling that $f=12$ for three generations, $\alpha_W \simeq 1/30$, $c=180$ GeV, $\xi_0=4.49$ and using the estimate (5·5), we obtain

$$N_F^{\text{crit}} \approx 5\cdot 10^{15}\,,$$

while the mass and radius of the critical drop are

$$M_{\text{crit}\cdot\text{drop}} \approx 1\cdot 10^{15}\ \text{GeV}$$

$$R_{\text{crit}\cdot\text{drop}} \approx 6\ \text{Fm}$$

Making use of Eq. (6·8) we find that the Fermi energy inside the drop, $\mu_F{}^{\text{drop}} \equiv \partial E/\partial N_F$ is less than $\frac{1}{3}m_p$, provided that $N_F > N_F{}^{\text{crit}}$, i.e., provided that the drop is stable with respect to decay into free fermions. This means, that the drop interacting with ordinary matter will eat up nucleons, the considerable part of the rest energy of the latter being transformed into heat. Indeed, the extra quarks will sit down on the empty levels inside the drop; this will increase the energy of the drop only by $3\mu_F{}^{\text{drop}}$, and the energy release will be equal to $(m_p - 3\mu^{\text{drop}})$. We note that $\mu_F{}^{\text{drop}}$ is very small at $N_F \gg N_F{}^{\text{crit}}$ (this is also clear from Eqs. (4·22) and (6·7); note that $k_0 \sim R^{-1}$), so that the energy release is equal to m_p in this case.

Up to now we neglected the gravitational effects on the properties of a drop. However, these effects are essential at very large N_F. Indeed, the radius of the drop evaluated according to Eq. (6·7) grows like $N_F{}^{1/4}$, while the gravitational radius, $R_g \sim M_{\text{drop}}/M_{\text{pl}}^2$, grows like $N_F{}^{3/4}$. To estimate N_F at which the gravitational effects become important, we take $R \sim R_g$ and get

$$N_F \sim \frac{f\alpha_W}{8\pi^2\lambda}\left(\frac{M_{\text{pl}}}{c}\right)^4 \sim 10^{65}\,.$$

For this N_F, the mass and radius of a drop are

$$M_{\text{drop}} \sim 10^{52}\ \text{GeV}\,,$$

$$R \sim 1\ \text{cm}\,.$$

At larger N_F, the gravitational effects presumably make a drop to be a black hole.

§ 7. Discussion

The results obtained in Ref. 1) and in this paper show that the electroweak interactions play a critical role in dense fermionic matter. However, there are some points which should be understood better. First, the picture drawn in Refs. 24) ~ 27) and utilized in this paper has not yet been worked out in detail. In fact, this picture has been recently critisized;[28),29)] although this criticism does not apply directly to the problem studied here, we think that further investigations are required to confirm this picture. Second, we neglected the instanton-like transitions throughout this paper. The rates of these transitions are indeed very small is vacuum;[14)] however, these rates might turn out to be relatively large in the abnormal weak matter. In that case the fate of the fermionic matter would be even more drastic: The fermion number would (partially?) disappear, the energy density would be transformed into heat, in analogy to the two-dimensional model of § 2.

Throughout this paper the fermionic matter was assumed to be neutral with respect to all (broken and unbroken) charges. However, the general case of matter with the non-vanishing densities of broken charges might be of interest for applications. We expect that in the latter case the effects related to the level crossing will play a role even at lower densities as compared to the neutral matter. Indeed, it has been found in Refs.

3) and 4) that if the neutrality conditions, Eqs. (1·1) and (1·2), are violated, then at $\mu_F \gtrsim M_W$, the state with the spatially homogeneous W-boson condensate has less energy density than the normal state (without the W-boson condensate). This means that the effective hamiltonian of the gauge field contains a term like $\int U(A)d^3x$, where $U(A)$ has a minimum at $A\neq 0$. The long range spatial variation of this condensate would further decrease the energy density, because of the existence of the Chern-Simons term. So, the presence of this term would be essential at $\mu_F \gtrsim M_W$; what are its other effects remains an open question.

It is of course of interest to investigate whether the drops of the abnormal weak matter could be formed at some early stage of the evolution of the Universe, say, from the remnants of the symmetric vacuum. This possibility can be relevant for the problem of the hidden mass of the Universe and galaxies. The results of this paper might also have some implications in astrophysics.

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