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## **On the Empirical Implementation of Some Game Theoretic Models of Household Labor Supply**

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**Peter Kooreman**

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### **A B S T R A C T**

*This paper discusses some issues in the empirical implementation of game theoretic models of household labor supply. In particular we focus on the identification problems inherent in many of these models. As an illustration, we estimate a game theoretic model which uses data on preferred working hours as additional identifying information.*

### **I. Introduction**

During the last decade the microeconomic literature on labor supply has presented many extensions of the standard textbook model of a utility maximizing individual choosing an optimal combination of leisure and consumption. In particular, much effort has been put in taking into account the nonlinearity or nonconvexity of the budget set due to tax and social security systems and to modelling the effect of institutional constraints which restrict the choice set of the individual.

In addition, there is a tendency to extend models of individual labor supply to models of household labor supply; examples are the papers by Hausman and Ruud (1984), Kooreman and Kapteyn (1987), Lundberg (1988), and Ransom (1987). A motivation for developing household models rather than individual models is that there is some evidence that the exogeneity assumption on the variable "other household income" (which

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includes labor income of the spouse) in individual models is not always tenable; see Smith and Blundell (1986) and Lundberg (1988). More important, it is likely that there is a tight structural relation between the labor supply decisions of the male and the female partner within a household, and to understand this interrelationship, it should be taken into account explicitly in setting up the empirical model.

One of the additional problems in modelling joint decisions of household members is how to properly describe the decision-making process within the household. In empirical work researchers usually opt for a straightforward extension of the individual labor supply model by specifying a utility function (or a dual representation) with male leisure, female leisure, and total household consumption as arguments. This utility function is assumed to represent the preferences of the household as a whole. Maximization subject to the household budget constraint then yields a male and female labor supply function. However, if male and female preferences differ, this approach is generally not acceptable (cf. Samuelson 1956, and Brown and Chuang 1981).

The literature on more general models of the household's labor supply decision-making process (mainly of the bargaining type) is characterized by an abundance of theoretical refinements and very few empirical applications. One of the problems in applied work is that it is not clear which of the many proposed solution concepts should be used in an empirical model. In addition, the number of utility parameters to be estimated will in principle be about twice as large as for models based on a joint household utility function. The available data usually do not contain sufficient information to identify these parameters. A further problem is that, except for some extremely restrictive functional forms and some specific choices of the solution concept, one can generally not derive closed form expressions for the behavioral equations.

In Section II of this paper we review the existing literature on game theoretic models of household labor supply. Section III discusses the problems encountered in the empirical implementation in more detail. We also indicate what kind of additional information would be necessary to solve the identification problem inherent to many of these models. As an illustration, Section IV presents estimation results of a game theoretic model which uses data on preferred working hours as additional identifying information. Some concluding remarks follow in Section V.

## **II. Household Decision-Making as a Two Person Game: A Review**

We consider households with a male and a female partner who pool their resources and allocate them jointly. Moreover, we confine

ourselves in this section to the most common case in which only male and female hours of work, male and female wage rates (for workers only), total household consumption, and total household nonlabor income are observed.

The preferences of the  $i$ th partner ( $i = m, f$ ;  $m$  denotes male,  $f$  denotes female) can be represented by a well-behaved utility function  $U^i(l_m, l_f, y)$ , where  $l_m$  is male leisure,  $l_f$  is female leisure, and  $y$  is total household consumption. The utility function includes the possibility of egoistic agents, in which case the leisure of one spouse does not directly affect the utility of the other spouse. (In that case there will, of course, still be an indirect effect through household consumption.)

Once the male and female labor supply are determined, household consumption follows immediately from the household budget constraint

$$(1) \quad w_m l_m + w_f l_f + y = w_m T + w_f T + \mu = Y$$

where  $w_m$  and  $w_f$  are the male and female wage rate,  $T$  is total time endowment, and  $\mu$  is nonlabor income;  $Y$  is full household income.<sup>1</sup>

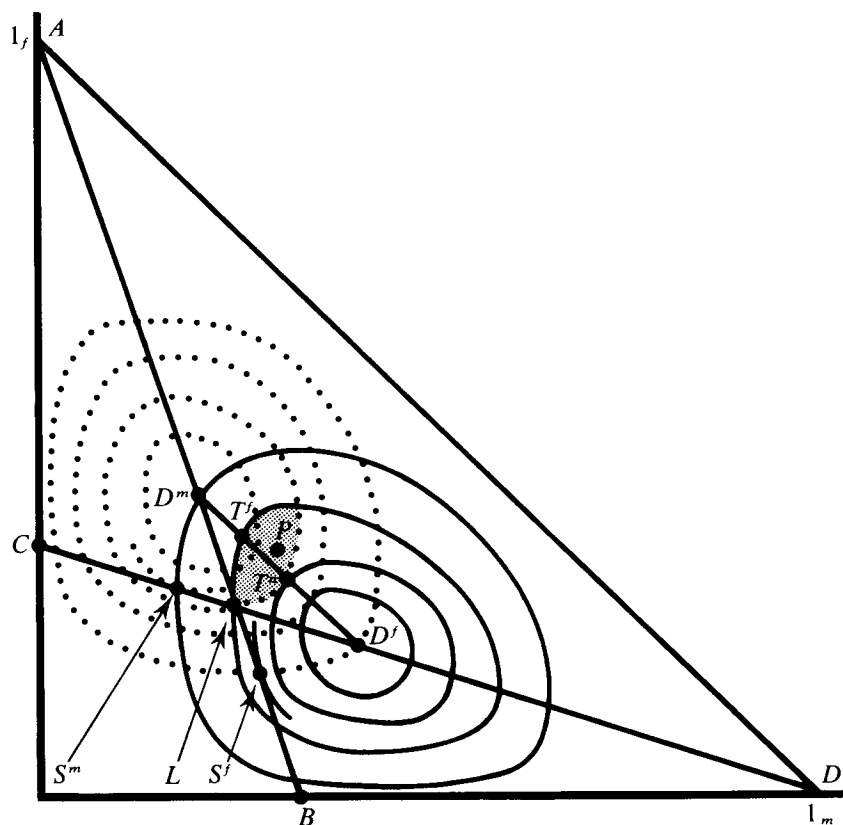
The *dictatorial point* of the  $i$ th partner is defined as the solution of maximizing  $U^i(l_m, l_f, y)$  subject to the household budget constraint (1).

After eliminating  $y$  from the utility functions using the budget constraint (1), the utility levels can be represented in the  $(l_m, l_f)$ -plane; see Figure 1.

The solid curves around the dictatorial point  $D^f$  are the indifference curves of the female partner; the dotted curves around  $D^m$  are the indifference curves of the male partner. The farther an indifference curve is removed from a dictatorial point, the lower is the utility level corresponding to this curve. The line  $AB$  in Figure 1 is the conditional demand function of the male partner, i.e., it represents the optimal male leisure demand, given the leisure demand of the female (*reaction curve*). It connects all tangency points of horizontal lines with the indifference curves around  $D^m$ . Analogously, line  $CD$  is the conditional female leisure demand function.

Using Figure 1, we can graphically represent several allocations that have been used in the literature. In an article in this Journal, Leuthold (1968) estimated a model based on the assumption that each partner maximizes his or her own utility function, given the labor supply of the spouse. Using "egoistic" Stone–Geary utility functions, she derived the two re-

1. Some papers have proposed a framework in which the decision making within households and household formation are determined simultaneously. In this paper we will confine ourselves to studying the decision-making process within the household, given its existence.



**Figure 1**  
*Indifference Contours and Solution Concepts*

Note:

Figure 1 is based on Stone-Geary utility functions (see Section III), with  $\gamma_m = \gamma_f = \gamma_y = \delta_m = \delta_f = \delta_y = 0$ ;  $\beta_m = \alpha_f = 0.4$ ;  $\beta_f = \alpha_m = 0.2$  and  $w_m = w_f = 1$ .

actions functions. Next, estimates were obtained by estimating the reduced form of the reaction functions. This procedure is equivalent to employing a noncooperative Nash equilibrium, and is represented graphically by the intersection point  $L$  of the reaction curves  $AB$  and  $CD$ . As will be pointed out in Section III, the assumption of egoistic agents is redundant as the model estimated by Leuthold is empirically indistinguishable from one that does not make this assumption, at least in the case of Stone-Geary utility functions. A similar model, though with dif-

ferent functional forms, has been estimated by Ashworth and Ulph (1981).<sup>2</sup>

Figure 1 clearly visualizes the well-known fact that a noncooperative Nash equilibrium is generally not Pareto-optimal; both partners can improve by moving from  $L$  to, for example,  $P$ . As has been argued by Manser and Brown (1980), it seems more appropriate in a household context to employ models which only yield Pareto-optimal outcomes. The set of Pareto-allocations (i.e., the *contract curve*) can also easily be visualized in Figure 1; all tangency points of indifference curves around  $D^m$  with indifference curves around  $D^f$ , between (and including) both dictatorial points, represent Pareto-optimal allocations. So, the contract curve satisfies:

$$(2) \quad \frac{\partial V^m / \partial l_m}{\partial V^m / \partial l_f} = \frac{\partial V^f / \partial l_m}{\partial V^f / \partial l_f}$$

where  $V^i = V^i(l_m, l_f) = U^i(l_m, l_f, Y - w_m l_m - w_f l_f)$   $i = m, f$ .

Equation (2) follows immediately from the first order conditions for maximizing  $V^i(l_m, l_f)$  subject to  $V^j(l_m, l_f) = V_0 (j \neq i)$ , where  $V_0$  is some given utility level. Alternatively, (2) can be obtained by maximizing the function  $\tilde{V}$  defined by

$$(3) \quad \tilde{V} = W(V^m(l_m, l_f), V^f(l_m, l_f)).$$

Here  $W$  is a function that is increasing in both its arguments and that does not depend on  $w_m$ ,  $w_f$  or  $\mu$ . A special form of (3) is

$$(3') \quad \tilde{V} = \lambda V^m + (1 - \lambda) V^f$$

with  $0 \leq \lambda \leq 1$ . Both (3) and (3') imply (2). Note that one can always write the solution to maximization of (3) with respect to  $l_m$  and  $l_f$  as if it were the solution of maximization of (3') with a properly specified  $\lambda$ .

The shaded area in Figure 1 is the set of allocations which are better for both partners than the noncooperative Nash equilibrium  $L$ . It might be argued that only the intersection of this set and the contract curve should be considered as potential equilibria. If a point on the contract curve outside the shaded area is reached, one of the players has an incentive to behave noncooperatively. The points  $T^f$  and  $T^m$  can therefore be interpreted as threat points. Taking the existence of threat points into account explicitly, the household's behavior can be described by

2. Noncooperative equilibrium concepts have also been used by Bjorn and Vuong (1984, 1985), who estimate discrete game theoretic models of household labor force participation.

$$(4) \quad \max \bar{V} = W(V^m(l_m, l_f), V^f(l_m, l_f))$$

$$\text{s.t. } V^m(l_m, l_f) \geq \Psi_N^m(w_m, w_f, \mu)$$

$$V^f(l_m, l_f) \geq \Psi_N^f(w_m, w_f, \mu).$$

Here  $\Psi_N^m(w_m, w_f, \mu)$  and  $\Psi_N^f(w_m, w_f, \mu)$  denote the utility levels the partners attain in the case of a noncooperative Nash equilibrium. A crucial difference between problems (3) and (4) is that the solution to (4) will generally not satisfy Slutsky conditions. The same holds for (3) with  $W$  dependent on wages or nonlabor income. A special case of (4), the Nash bargaining model, has been studied by McElroy and Horney. In particular, these authors have derived a so-called Nash generalization of the Slutsky matrix. Their results are based on a general specification of the threat points, not the ones given in (4); see McElroy and Horney (1981) and McElroy (1988). In a more recently published paper (Horney and McElroy 1988) some empirical evidence in favor of their bargaining model is presented.

Model (3) has *no* empirical implications over and above the Slutsky conditions implied by the standard neoclassical household labor supply model. This follows immediately from (3), since this is nothing else than a household utility function. Chiappori (1988) has considered model (3) with  $W$  dependent on  $w_m$ ,  $w_f$  and  $\mu$ .<sup>3</sup> He shows that in the general case of nonegoistic agents this model has no parametric implications for the labor supply functions.

One of the factors that have hampered the empirical implementation of models based on (3) or (4) is that in both cases it is generally impossible to obtain closed form solutions for the behavioral equations. Another crucial problem in this type of model, identification, is the subject of the next section.

### III. Identification

Obviously, the weaker the assumptions that one is prepared to make on the solution of the household bargaining problem, the weaker will be the empirical implications. In the sequel of this paper, we will primarily focus on models which are based on the mere assumption that

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3. There is one slight distinction. He distinguishes male and female consumption, so that consumption is not a public good. Since, however, the "relative price" of male and female consumption is always equal to one there is no loss of generality in combining male and female consumption from the start.

household choices are Pareto-optimal. In terms of Figure 1, this means that the household is assumed to choose a point on the contract curve between  $D^m$  and  $D^f$ . In terms of the models in the previous section, this means that the household is assumed to maximize (3'), with  $\lambda$  independent of wages and nonlabor income. Note, however, that it is perfectly possible for  $\lambda$  to vary with other characteristics, such as age and education.

As noted before, these assumptions have no other implications than the standard neoclassical properties. We will illustrate these points for the Stone–Geary specification of individual preferences:

$$(5a) \quad U^m(l_m, l_f, y) = \alpha_m \log(l_m - \gamma_m) + \alpha_f \log(l_f - \gamma_f) + \alpha_y \log(y - \gamma_y)$$

$$(5b) \quad U^f(l_m, l_f, y) = \beta_m \log(l_m - \delta_m) + \beta_f \log(l_f - \delta_f) + \beta_y \log(y - \delta_y)$$

with  $\alpha_y = 1 - \alpha_m - \alpha_f$  and  $\beta_y = 1 - \beta_m - \beta_f$ .

In the case of male dictatorship the leisure demand functions are given by

$$(6a) \quad w_i l_i = w_i \gamma_i + \alpha_i (Y - w_m \gamma_m - w_f \gamma_f - \gamma_y) \quad i = m, f$$

and in the case of female dictatorship by

$$(6b) \quad w_i l_i = w_i \delta_i + \beta_i (Y - w_m \delta_m - w_f \delta_f - \delta_y) \quad i = m, f.$$

Both cases are, of course, empirically indistinguishable from each other and from the traditional model with a joint household utility function.

The conditional demand equation for male leisure given female leisure corresponding to (5a) is given by

$$(7a) \quad w_m l_m = w_m \gamma_m + \frac{\alpha_m}{1 - \alpha_f} (Y - w_m \gamma_m - w_f l_f - \gamma_y).$$

Similarly, the conditional female leisure demand equation given male leisure corresponding to (5b) reads

$$(7b) \quad w_f l_f = w_f \delta_f + \frac{\beta_f}{1 - \beta_m} (Y - w_m l_m - w_f \delta_f - \delta_y).$$

Note that  $\gamma_f$  and  $\delta_m$  do not appear in these equations. Solving  $l_m$  and  $l_f$  from (7a) and (7b), we obtain the leisure demand equations corresponding to the *noncooperative Nash equilibrium*:

$$(8a) \quad w_m l_m = w_m \left[ \frac{\gamma_m(1 - \alpha)}{1 - \alpha\beta} \right] - w_f \left[ \frac{\delta_f \alpha(1 - \beta)}{1 - \alpha\beta} \right] \\ + Y \left[ \frac{\alpha(1 - \beta)}{1 - \alpha\beta} \right] + \left[ \frac{\alpha(\beta\delta_y - \gamma_y)}{1 - \alpha\beta} \right]$$



$$(8b) \quad w_f l_f = w_f \left[ \frac{\delta_f(1 - \beta)}{1 - \alpha\beta} \right] - w_m \left[ \frac{\gamma_m \beta(1 - \alpha)}{1 - \alpha\beta} \right] \\ + Y \left[ \frac{\beta(1 - \alpha)}{1 - \alpha\beta} \right] + \left[ \frac{\beta(\alpha\gamma_y - \delta_y)}{1 - \alpha\beta} \right]$$

where  $\alpha = \alpha_m/(1 - \alpha_f)$  and  $\beta = \beta_f/(1 - \beta_m)$ . Inspection of (8) shows that one can only identify  $\alpha_m/(1 - \alpha_f)$ ,  $\beta_f/(1 - \beta_m)$ ,  $\gamma_m$ ,  $\gamma_y$ ,  $\delta_f$  and  $\delta_y$ . First, this result implies that the model using the additional assumption that spouses are egoistic (i.e.,  $\alpha_f = \beta_m = 0$ ), as in Leuthold (1968), is empirically indistinguishable from the model without this assumption. Second, a test of the noncooperative Nash model against the traditional model would be tantamount to testing only whether  $\gamma_y = \delta_y$ . Such a test is not likely to be very powerful.

Applying (2) to derive the contract curve, we obtain

$$(9) \quad \frac{l_f - \gamma_f}{l_m - \gamma_m} \cdot \frac{\alpha_m(y - \gamma_y) - \alpha_y w_m(l_m - \gamma_m)}{\alpha_f(y - \gamma_y) - \alpha_y w_f(l_f - \gamma_f)} \\ = \frac{l_f - \delta_f}{l_m - \delta_m} \cdot \frac{\beta_m(y - \delta_y) - \beta_y w_m(l_m - \delta_m)}{\beta_f(y - \delta_y) - \beta_y w_f(l_f - \delta_f)}$$

Equation (9) is an implicit expression in  $l_m$  and  $l_f$ , defining a nonlinear contract curve. If we make the additional assumption  $\gamma_i = \delta_i (= \eta_i)$ ;  $i = m, f, y$ , (9) can be simplified to

$$(10) \quad (\alpha_f - \beta_f)w_m(l_m - \eta_m) + (\beta_m - \alpha_m)w_f(l_f - \eta_f) \\ + (\alpha_m\beta_f - \alpha_f\beta_m)(Y - w_m\eta_m - w_f\eta_f - \eta_y) = 0.$$

Thus in that case the contract curve reduces to a linear segment in the  $(l_m, l_f)$ -plane. Alternatively, this segment can be represented as:

$$(11a) \quad w_m l_m = w_m \eta_m + \{\lambda \alpha_m + (1 - \lambda) \beta_m\} (Y - w_m \eta_m - w_f \eta_f - \eta_y)$$

$$(11b) \quad w_f l_f = w_f \eta_f + \{\lambda \alpha_f + (1 - \lambda) \beta_f\} (Y - w_m \eta_m - w_f \eta_f - \eta_y)$$

for  $0 \leq \lambda \leq 1$ . Now the marginal budget shares are weighted averages of the marginal budget shares in the corresponding male and female dictatorial equations. Without further restrictions,  $\lambda$  can easily take on values which imply an equilibrium outside the shaded area in Figure 1, i.e., such that the inequality restrictions in (4) are violated. Rather than imposing restrictions which would prevent this from happening, we will report the number of cases in our empirical illustration where the allocation is outside the shaded area (Section IV).

In this special case with  $\gamma_i = \delta_i$ , the functional form of the demand functions does not differ from the traditional LES demand equations. If

$\gamma_i \neq \delta_i$ , the demand curves will differ. This is not at variance with the earlier observation that, under the mere assumption of Pareto-optimality, a game theoretic and a neoclassical model are empirically indistinguishable. One can interpret the differences that arise as merely a difference in assumptions about the functional form of the household utility function.

Note that, having information on actual working hours only, model (11) is underidentified. The marginal budget shares contain five structural parameters ( $\alpha_m$ ,  $\alpha_f$ ,  $\beta_m$ ,  $\beta_f$ , and  $\lambda$ ), whereas we would only be able to obtain estimates for  $\tau_m = \lambda\alpha_m + (1 - \lambda)\beta_m$  and  $\tau_f = \lambda\alpha_f + (1 - \lambda)\beta_f$ .

In view of the underidentification that seems to be inherent in this type of models a question that arises is what kind of additional information would be necessary to identify all the parameters. McElroy (1988) suggests that information on singles or divorced individuals might be used to identify the individual preferences. This is an interesting suggestion, although its empirical implementation is not without difficulties. One of the problems is that it requires modelling household formation and dissolution to take into account the selective nature of the subsamples of singles and divorced people.

Another possibility is to use data that, in some way or another, contain additional information on respondents' preferences. For example, in the 1982 Dutch Labor Mobility Survey, respondents were asked how many hours they would like to work at their current wage rate. In the following section we illustrate how these data can be used as additional identifying information.

#### IV. An Empirical Illustration

We start out from household utility function (3'), with  $\lambda$  independent of wages and nonlabor income, and the Stone–Geary specification for individual preferences. In addition, we assume that the partners agree on the level of the subsistence quantities in such a way that the bargaining process is based on (5), with  $\gamma_i$  and  $\delta_i$  replaced by  $\eta_i = \max(\gamma_i, \delta_i)$ ,  $i = m, f, y$ . As shown in the previous section, this assumption implies that the contract curve is linear, with allows us to derive closed form expressions for the Pareto-optimal leisure demand equations. The observed actual working hours in the data are assumed to represent a particular Pareto-optimal outcome chosen by the household. Tacking on additive error terms, we have the following actual hours equations:

$$(12a) \quad w_m l_m^a = w_m \eta_m + \{\lambda\alpha_m + (1 - \lambda)\beta_m\} \\ \cdot (Y - w_m \eta_m - w_f \eta_f - \eta_y) + \varepsilon_m^a$$

$$(12b) \quad w_f l_f^a = w_f \eta_f + \{\lambda \alpha_f + (1 - \lambda) \beta_f\} \\ \cdot (Y - w_m \eta_m - w_f \eta_f - \eta_y) + \varepsilon_f^a.$$

The parameter  $\lambda$  indicates which Pareto-optimal outcome (i.e., which point on the contract curve) is actually chosen by the household. In the extreme cases we have male dictatorship ( $\lambda = 1$ ) or female dictatorship ( $\lambda = 0$ ).

The identification problem can (partly) be solved by using additional information on preferred hours. In the 1982 Dutch Labor Mobility Survey respondents were also asked how many hours per week they would like to work at their going wage rate. We assume that this variable is determined exclusively on the basis of the respondent's own preferences. This interpretation is supported by the fact that in the phrasing of the question there is no reference at all to the partner's behavior or preferences. Moreover, partners answered these questions separately.<sup>4</sup> It should be noted, however, that nevertheless the individual might take his/her partner's preferences into account to some extent when answering the question. Also, the number of preferred hours may be affected by the respondent's perception of what number of hours would be acceptable from an employer's point of view. In that case, the answer may not only reflect preferences, but possibly also factors related to the demand side of the labor market.

Neglecting these possibilities, we have the following preferred hours equations

$$(13a) \quad w_m l_m^p = w_m \gamma_m + \alpha_m (Y - w_m \gamma_m - w_f \gamma_f - \gamma_y) + \varepsilon_m^p$$

and

$$(13b) \quad w_f l_f^p = w_f \delta_f + \beta_f (Y - w_m \delta_m - w_f \delta_f - \delta_y) + \varepsilon_f^p.$$

Equations (13) identify  $\alpha_m$ ,  $\beta_f$ ,  $\gamma_m$ ,  $\gamma_f$ ,  $\gamma_y$ ,  $\delta_m$ ,  $\delta_f$ , and  $\delta_y$ . Combined with the information provided by the estimation of (12), we have four "reduced form" estimates ( $\alpha_m$ ,  $\beta_f$ ,  $\tau_m = \lambda \alpha_m + (1 - \lambda) \beta_m$ , and  $\tau_f = \lambda \alpha_f + (1 - \lambda) \beta_f$ ) and five "structural" parameters ( $\alpha_m$ ,  $\alpha_f$ ,  $\beta_m$ ,  $\beta_f$ , and  $\lambda$ ) for the marginal budget share parameters. So, for complete identification, we need one additional restriction. We choose  $\alpha_y = \beta_y$ . If we would also have information on how many hours a respondent would like his/her partner to work, for example, no additional restriction would be necessary. In that case, the preferred hours responses of the male partner would completely identify the male utility function, whereas the female's preferred hours responses would completely identify her utility function. The actual hours would then identify  $\lambda$ .

4. The phrasing of the question was as follows: "Suppose your income per hour would remain the same. How many hours per week would you like to work in your present job?"

Assuming the error terms to be normally distributed with zero mean and unrestricted covariance matrix  $\Sigma$ , we estimate Equations (12) and (13) jointly by means of maximum likelihood, taking into account all cross equations restrictions, including  $\eta_i = \max(\gamma_i, \delta_i)$ .

In the estimation only households are used where both the male and the female partner work in a paid job for at least 15 hours per week. The 15 hours cutoff point is dictated by the survey design by which certain items of information such as the number of preferred hours are not collected for people who work less than 15 hours per week. As a result, we analyze a subsample of 139 households for whom sufficient information has been collected to be able to estimate the model. Some sample statistics are presented in Table 1.

In the estimation, the 15 hours sample selection rule has been taken into account by maximizing the likelihood function

$$(14) \quad L = \prod_n \frac{f_1^n(l_m^a, l_f^a, l_m^p, l_f^p)}{\int_0^{T-15} \int_0^{T-15} f_2^n(l_m^a, l_f^a) dl_m^a dl_f^a}$$

Here  $f_1^n(l_m^a, l_f^a, l_m^p, l_f^p)$  is the joint density function of  $l_m^a, l_f^a, l_m^p$  and  $l_f^p$  implied by (12) and (13) for the  $n$ th household and  $f_2^n(l_m^a, l_f^a)$  is the joint marginal density function of  $l_m^a$  and  $l_f^a$  for the  $n$ th household.

The likelihood function is maximized using a quasi-Newton algorithm which requires no (analytical) derivatives, as provided by the NAG-Library (ED04JBF). The (asymptotic) covariance matrix of the maximum likelihood estimators is estimated by the inverse of the (numerically calculated) Hessian of the min-loglikelihood function.

The estimation results are given in Table 2.

In the first place, we note that the estimated  $\alpha$ 's,  $\beta$ 's, and  $\lambda$  fall between zero and one, as they should. As has been noted before, the utility functions (5) are only well-defined if the observed quantities exceed the subsistence quantities. We have checked per observation point whether these conditions are satisfied; see Table 3.

In view of the restrictive functional forms of the labor supply equations implied by the additive utility functions (5) the numbers in Table 3 look reasonable.

For households for which regularity for actual hours is satisfied, we have checked whether the Pareto-optimal solution falls within the shaded area, i.e., we have checked whether the inequalities in (4) are satisfied. This turns out to be the case for all households but two.

Although it is tempting to interpret the estimated  $\lambda$  as an indication of the relative bargaining power of the spouses in the decision regarding joint labor supply, it should be borne in mind that this result is likely

**Table 1**  
*Sample Statistics<sup>a</sup>*

	Mean	Standard Deviation	Minimum	Maximum
<b>Male</b>				
preferred hours	36.0	8.2	0	70
actual hours	41.9	6.2	20	70
wage rate	13.2	3.8	7.6	28.9
<b>Female</b>				
preferred hours	25.8	8.2	12	50
actual hours	30.0	9.1	15	50
wage rate	9.5	1.5	6.3	14.4
Family nonlabor income	17.2	30.1	0	140.6

a. Hours are per week; wage rates are in Dfl. per hour, nonlabor income is in Dfl. per week.

to be sensitive to the identifying assumptions that have been imposed. Moreover, the estimated standard error of  $\lambda$  is relatively large. Therefore we shall abstain from an interpretation of  $\lambda$  in terms of relative power.

Finally, we have tested whether the utility function of both spouses are significantly different. Since we have already assumed  $\alpha_y = \beta_y$ , the null hypothesis of equal utility functions becomes  $\alpha_m = \beta_m$ ,  $\gamma_m = \delta_m$ ,  $\gamma_f = \delta_f$  and  $\gamma_y = \delta_y$ . On the basis of a Wald test, the hypothesis is rejected.<sup>5</sup> Brown and Manser (1978) and Horney and McElroy (1988) come to similar conclusions.

## V. Conclusions

The empirical implications of Pareto-optimal behavior by household members are limited. The reason is simply that Pareto-optimal behavior can generally be rationalized by the maximization of some household social welfare function. In many instances this social welfare function is indistinguishable from a standard neoclassical utility function and hence restrictions on observable behavior are equivalent to Slutsky

5. The Wald statistic follows a  $\chi^2$ -distribution with 4 degrees of freedom. The critical levels for 5 percent and 2.5 percent are 9.49 and 11.1, respectively. The statistic is computed at 47.7.

**Table 2**  
*Estimation Results*

Parameter	Estimate	Standard Error		
$\alpha_m$	0.29	0.10		
$\alpha_f$	0.02	0.13		
$\beta_m$	0.17	0.07		
$\beta_f$	0.14	0.05		
$\gamma_m$	119.2 <sup>a</sup>	8.1		
$\gamma_f$	140.8 <sup>a</sup>	22.4		
$\gamma_y$	348.1	249.7		
$\delta_m$	123.4 <sup>a</sup>	3.3		
$\delta_f$	141.3 <sup>a</sup>	2.6		
$\delta_y$	644.6	91.9		
$\lambda$	0.36	0.74		
$\Sigma^b =$	5,436	·		
	0.08	10,170		
	-0.05	-0.08	14,170	
	-0.13	0.04	0.66	3,961

a.  $T$  is set equal to 168 hours per week. The estimates of  $(T - \gamma_i)$  and  $(T - \delta_j)$  ( $i = m, f$ ), however, are independent of the choice of  $T$ .

b. Diagonal elements are variances; off-diagonal elements are correlation coefficients.

**Table 3**  
*Percentage Observations Satisfying Regularity Conditions*

	$\gamma_m$	$\gamma_f$	$\gamma_y$	$\delta_m$	$\delta_f$	$\delta_y$
Preferred hours	98	55	98	96	55	70
Actual hours	89	41	100	82	41	89

conditions. Only if additional assumptions are made, for instance that the social welfare function is dependent on prices, like in the approach by McElroy and Horney (1981), then implications are different from the standard neoclassical model. Even then the interpretation of the implications is ambiguous. For example, the McElroy and Horney approach could be viewed as neoclassical with price dependent preferences (cf. Pollak 1977).

Where it is now generally recognized that identification of competing paradigms solely on the basis of functional forms is too shaky a basis for inference, the only alternative is to invoke more information. In this paper we have used such additional information in the form of each spouse's preferred hours. Even then complete identification of the model turned out not to be quite possible.

The empirical part of the paper has been purely illustrative. Clearly, if one takes bargaining approaches to the explanation of household behavior seriously, then more solid information on each spouse's preferences is required. For the empirical worker this means that the usual data sources are no longer sufficient. The content of surveys of labor supply will have to be increased.

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