

## ON THE EQUIVARIANT HOMOTOPY TYPE OF $G$ -ANR'S

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**ABSTRACT.** I show that every metric  $G$ -ANR has the  $G$ -homotopy type of a  $G$ -CW complex. Therefore I. James and G. Segal's results concerning equivariant homotopy type are special cases of the Whitehead theorem for  $G$ -CW complexes.

In this note  $G$  is assumed to be a compact Lie group. A metric  $G$ -space  $X$  is said to be a  $G$ -ANR if for any  $G$ -embedding  $i: X \rightarrow Y$  in a metric  $G$ -space  $Y$  such that  $iX$  is closed in  $Y$ , the image  $iX$  is a  $G$ -retract of some open invariant neighborhood of  $Y$ .

**THEOREM.** *Every metric  $G$ -ANR has the  $G$ -homotopy type of a  $G$ -CW complex.*

As a consequence of this theorem and Theorem 5.3 in [3] we obtain the following.

**COROLLARY (JAMES, SEGAL [1]).** *Let  $f: X \rightarrow Y$  be a  $G$  map between  $G$ -ANR's. Then  $f$  is a  $G$ -homotopy equivalence iff  $f^H: X^H \rightarrow Y^H$  is an ordinary homotopy equivalence for every closed subgroup  $H \subseteq G$ .*

**PROOF OF THEOREM.** Let  $X$  be a metric  $G$ -ANR. By the equivariant version of a standard argument (Lemma 4.7 of [4]), it suffices to prove that  $X$  is  $G$ -dominated by a  $G$ -CW complex. Observe that every metric  $G$ -space  $X$  may be  $G$ -embedded as a closed  $G$ -subset of a convex  $G$ -set in a Banach space of bounded real-valued functions on  $X$  with  $G$ -action given by  $g(h)(x) = h(g^{-1}(x))$  for  $g \in G$ ,  $h: X \rightarrow \mathbb{R}$ , and  $x \in X$ .

Therefore we may assume that  $X$  is a closed  $G$ -subset of a convex  $G$ -set  $C$  in a Banach  $G$ -space.

Being a  $G$ -ANR,  $X$  is a  $G$ -retract of some open neighborhood  $U$  of  $X$  in  $C$ ; in particular it is  $G$ -dominated by some  $G$ -CW complex. This is seen by an easy modification of the proof of Theorem 3.B in [2]. We obtain that  $U$  is  $G$ -dominated by a  $G$ -nerve induced by a locally finite refined slice covering and this  $G$ -nerve has the  $G$ -homotopy type of a  $G$ -CW complex (which is a direct limit of barycentric manifolds in the notation of [2]).

**REMARK.** Matumoto's proof that a barycentric manifold is a  $G$ -CW complex is incorrect because it relies on a result of Yang which relies on an incorrect result of Cairns. A correct proof that smooth manifolds are  $G$ -CW complexes is in a preprint *Triangulation of stratified fibre bundles* by Andrei Verona. The weaker

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assertion that smooth  $G$ -manifolds have the  $G$ -homotopy type of  $G$ -CW complexes (which is all that might be needed for this paper is immediate from [4]).

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#### REFERENCES

1. I. M. James and G. B. Segal, *On equivariant homotopy type*, *Topology* **17** (1978), 267–272.
2. T. Matumoto, *Equivariant  $K$ -theory and Fredholm operators*, *J. Fac. Sci. Univ. Tokyo Sect. IA Math.* **18** (1971), 109–125.
3. \_\_\_\_\_, *On  $G$ -CW complexes and theorem of J. H. C. Whitehead*, *J. Fac. Sci. Univ. Tokyo Sect. IA Math.* **18** (1971), 363–374.
4. S. Waner, *Equivariant homotopy theory and Milnor's theorem*, *Trans. Amer. Math. Soc.* **258** (1980), 351–368.

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