On the error bound in a combinatorial Central Limit Theorem

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Abstract. Let $\{X_{ij}: i,j = 1,\dots,n\}$ be an $n \times n$ array of independent random variables with finite third moments and let π be a random permutation of $\{1,\dots,n\}$ independent of the X_{ij}. Let $U = \sum_{i=1}^{n} X_{i\pi(i)}$ and $W = (U - \mathbb{E}U)/(\mathrm{Var}(U))^{1/2}$. A third-moment error bound on the Kolmogorov distance with an explicit constant is obtained for the central limit theorem for W by using Stein's method of exchangeable pairs and a concentration inequality. This result is more general than that of Bolthausen (1984), which is on an $n \times n$ array of real numbers and does not have an explicit constant in the error bound. This result also yields a result for sampling without replacement from a finite set of random variables whose means are not necessarily zero. This is more general than the case considered by Wolff (2012), who assumed zero means and obtained a bound on the Wasserstein distance. It is also more general than the case of sampling without replacement from a finite set of real numbers, considered by Goldstein (2007), who also obtained a bound on the Wasserstein distance.

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