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On the evolution of cosmological adiabatic perturbations in the weakly non-linear regime

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Summary. The purpose of this paper is to investigate the weakly non-linear stage in the evolution of adiabatic density fluctuations. It is shown that tidal processes may lead to the disruption of large scale inhomogeneities into smaller units. This phenomenon may change significantly the Doroshkevich, Sunyaev & Zeldovich scenario for development of structure in the expanding Universe. Accurate *N*-body experiments are necessary to decide whether the approach used in this paper is reasonable.

1 Introduction

In the standard gravitational instability picture, galaxies and clusters condense from small but finite amplitude density fluctuations, existing at the epoch of recombination. The fluctuations may be either adiabatic, isothermal, or some combination of both (for a review see Rees 1978; Zeldovich & Novikov 1975, hereafter ZN). It is generally assumed that linear stability analysis of the Friedmann models provides an adequate description of the growth of the fluctuations up to the redshift Z_f, when the relative density contrast $\delta\rho/\rho$ reaches unity. This is a reasonable assumption at early epochs, when the perturbations are small. However, it is not so at $Z \gtrsim Z_f$, when $\delta\rho/\rho$ is only slightly smaller than 1; the validity of the linear approach must be proved in this regime. As yet such a proof exists only for the model in which the primordial fluctuations are purely isothermal (Zeldovich 1965; Peebles 1974; Doroshkevich & Zeldovich 1975).

The aim of this work is to investigate the effect of non-linear phenomena, occurring at $Z \gtrsim Z_f$, on the behaviour of purely adiabatic fluctuations. In Section 2 the main assumptions and approximations used in the calculation are given and the equations of motion derived in Section 3. In Section 4 it is shown that non-linear interaction between the long wave adiabatic perturbations acts as a source of short wave perturbations and in Section 5 possible consequences of this effect for the theory of galaxy formation are described.

2 Assumptions

A number of assumptions and approximations are used in the calculation. In an effort to minimize confusion I list the main points here.

(i) I assume that the cosmological model is the standard Friedmann model with the present Hubble constant $H_0 = 50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and density parameter $\Omega = 8\pi G\rho_0/3H_0^2 = 0.1$, where ρ_0 is the present mean density of matter in the Universe. These values of H_0 and Ω are suggested by a number of different lines of evidence (Gott *et al.* 1974). For $Z_{\mathrm{rec}} \gg Z \gg \Omega^{-1}$, where $Z_{\mathrm{rec}} = 10^3$ is the redshift of recombination, the expansion of the Universe may be approximated by the Einstein–de Sitter model, so that

$$R^{-1}dR/dt = (8\pi G\rho/3)^{1/2} = 2/3t, \tag{1}$$

where t is the proper cosmic time, ρ is the mean density and R is the expansion parameter.

(ii) The large scale matter distribution can be treated as a perfect pressureless fluid. The typical length scale for matter irregularities is assumed to be small compared with the particle horizon so the non-relativistic (Newtonian) approximation may be used to describe the irregularities.

(iii) I assume that the density fluctuations can be approximated as a homogeneous and isotropic Gaussian field. Let $\rho^*(\mathbf{x}, t)$ be the density of matter at the comoving position $\mathbf{x}$ in a frame expanding with the unperturbed model. The density contrast $\mu \equiv (\rho^* - \rho)/\rho$ can be represented by a stochastic Fourier integral (Yaglom 1962)

$$\mu(\mathbf{x},t) = \int \exp(i\mathbf{k}\cdot\mathbf{x})\,a(\mathbf{k},t)\,d^3\mathbf{k}, \tag{2}$$

where different Fourier components are not statistically correlated with each other. This assumption plays a crucial role for the interpretation of the results obtained in this paper and will be discussed in more detail in Section 5. The power spectrum of the fluctuations is defined by

$$\langle a(\mathbf{k},t)\,a(\mathbf{l},t)\rangle = S(\mathbf{k},t)\,\delta^3(\mathbf{k}+\mathbf{l}), \tag{3}$$

where the angle brackets denote the average over the ensemble of all possible density fluctuations.

Let $M(k)$ be the mass within a sphere of diameter $2\pi R/k$. Then we may write the variance of the density fluctuations as

$$\langle \mu^2 \rangle = 4\pi \int_0^\infty Sk^2\,dk = \int_{-\infty}^{+\infty} (\delta\rho/\rho)^2\,d(\ln M),$$

where

$$\delta\rho/\rho \equiv (4\pi k^3 S/3)^{1/2} \tag{4}$$

is the contribution to the variance for unit increment of the logarithm of mass, or the 'rms value of the density contrast on mass scale $M(k)$'. Following the generally accepted convention I will use this quantity as a description of the degree of inhomogeneity of the matter distribution on different scales.

(iv) I will assume that primordial density fluctuations are purely adiabatic. Moreover, at sufficiently early times, corresponding to $Z \gg Z_{\mathrm{rec}}$ their spectrum is of power form $S \propto k^{n+2}$, or $\delta\rho/\rho \propto M^{-(5+n)/6}$, where n is a parameter. Prior to and during the recombination epoch, adiabatic perturbations on all scales below the Silk mass $M_{\mathrm{D}} = 3\times 10^{12}(\Omega h^2)^{-5/4} M_\odot$, where $h = H_0/50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, are damped by the photon viscosity (Silk 1974). The resulting spectrum at $Z = Z_{\mathrm{rec}}$ is (Doroshkevich, Sunyaev & Zeldovich 1974)

$$\delta\rho/\rho = \begin{cases} Q(M/M_{\mathrm{D}})^{-(5+n)/6} & \text{for} \quad M > M_{\mathrm{J}} \\ Q(M/M_{\mathrm{D}})^{-(3+n)/6}\exp[-(M_{\mathrm{D}}/M)^{2/3}] & \text{for} \quad M < M_{\mathrm{J}} \end{cases} \tag{5}$$

where Q is constant and $M_J = 10^{17}(\Omega h^2)^{-2} M_\odot$ is the Jeans mass before recombination. For the cosmological model with $\Omega h^2 = 0.1$, the Silk and the Jeans masses are $M_D = 5 \times 10^{13} M_\odot$ and $M_J = 10^{19} M_\odot$, respectively.

These small fluctuations can form bound systems on mass scales $\gtrsim M_D$, which correspond to galaxy clusters in low Ω models, only if their amplitude at recombination is of the order of $(Z_f + 1)/(Z_{rec} + 1) \simeq 10^{-3}\Omega^{-1}$ (*cf.* Rees 1978, pp. 289–290). This condition yields the value of the constant Q in (5). In the present paper the initial spectrum is normalized so that the maximum value of $\delta\rho/\rho$ at Z_{rec} is $10^{-3}\Omega^{-1} = 10^{-2}$, thus

$$Q = 10^{-2}\,[4e/(n+3)]^{(n+3)/4}. \tag{6}$$

I consider initial spectra with $n = -1$, $n = 0$ and $n = 2$. The $n = -1$ spectrum has the virtue that the amplitude of the metric fluctuations is scale-independent (ZN, Section 23.9), while $n = 0$ corresponds to 'white noise' or a Poisson distribution of particles.

3 Equations of motion

In comoving coordinates, the equations of motion for the pressureless self-gravitating fluid are (Peebles 1971, pp. 213–225)

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{R}\frac{dR}{dt}\mathbf{v} + \frac{\nabla\phi}{R} = -\frac{(\mathbf{v}\cdot\nabla)\mathbf{v}}{R}, \tag{7}$$

$$\nabla^2\phi - 4\pi G\rho R^2 \mu = 0, \tag{8}$$

$$\frac{\partial \mu}{\partial t} + \frac{\nabla\cdot\mathbf{v}}{R} = -\frac{\nabla\cdot(\mathbf{v}\mu)}{R}. \tag{9}$$

Here $\mathbf{v}(\mathbf{x},t)$ is the peculiar velocity (i.e. velocity relative to an observer at fixed $\mathbf{x}$) and $\nabla\phi(\mathbf{x},t)/R$ is the gravitational acceleration at $\mathbf{x}$.

In the standard formulation of the gravitational instability theory, all non-linear terms on the right hand side of equations (7) and (9) are neglected (Peebles 1971; ZN). This is equivalent to the assumption that the growth of irregularities on a given scale is governed by the density irregularities and the matter currents on that same scale. In the present paper I consider weakly non-linear density fluctuations which can be treated in a second order perturbation calculation, in contrast to the 'strongly non-linear' fluctuations at $Z < Z_f$ when $\delta\rho/\rho > 1$ and the perturbation theory is inapplicable. Using this technique I investigate the evolution of the perturbations at $Z \gtrsim Z_f$, when their amplitudes are only slightly smaller than one, and the interaction between different scales may not be negligible.

The perturbation series for μ and $\mathbf{v}$ may be written as

$$\mu = \mu^{(1)} + \mu^{(2)} + \ldots; \quad \mathbf{v} = \mathbf{v}^{(1)} + \mathbf{v}^{(2)} + \ldots, \tag{10}$$

where $\mu^{(\beta)}$, $\mathbf{v}^{(\beta)}$ are of order β. Substituting (10) into the system of equations (7)–(9), then taking divergence of both sides of equation (7), using (8) and (9) to eliminate $\mathbf{v}$ and ρ from the left hand side of (7) and collecting terms of the same order, one finds

$$\left[\frac{\partial^2}{\partial t^2} + \frac{2}{R}\frac{dR}{dt}\frac{\partial}{\partial t} - 4\pi G\rho\right]\mu^{(1)} = 0, \tag{11}$$

$$\left[\frac{\partial^2}{\partial t^2} + \frac{2}{R}\frac{dR}{dt}\frac{\partial}{\partial t} - 4\pi G\rho\right]\mu^{(2)} = -\frac{\nabla}{R}\left[\left(\frac{\partial}{\partial t} + \frac{2}{R}\frac{dR}{dt}\right)\mathbf{v}^{(1)}\mu^{(1)} - \frac{(\mathbf{v}^{(1)}\cdot\nabla)\mathbf{v}^{(1)}}{R}\right]. \tag{12}$$

These equations taken together with the first-order continuity equation,

$$\frac{\partial \mu^{(1)}}{\partial t} + \frac{\nabla \cdot \mathbf{v}^{(1)}}{R} = 0 \tag{13}$$

define a complete system of three equations in three unknowns, i.e. all that is necessary to describe the time evolution of the density fluctuations to the second order in their amplitude.

4 The non-linear growth of small scale inhomogeneities

4.1 THE SHAPE OF THE SPECTRUM

Solving equations (11) and (13) and retaining only the growing modes in the solution (that is assuming $t \gg t_{\rm rec}$), one has

$$\mu^{(1)}(\mathbf{x},t) = \mu^{(1)}(\mathbf{x})(t/t_{\rm rec})^{2/3},$$
$$\mathbf{b}^{(1)}(\mathbf{k},t) = \frac{2}{3} a_{\mathbf{k}}^{(1)}\, i\mathbf{k}\, k^{-2} t^{1/3} t_{\rm rec}^{-4/3}. \tag{14}$$

Here

$$\mathbf{b}^{(1)}(\mathbf{k},t) \equiv (2\pi)^{-3} \int \mathbf{v}^{(1)}(\mathbf{x},t) \exp(-i\mathbf{k}\cdot\mathbf{x})\, d^3\mathbf{x}.$$

Similarly, $a_{\mathbf{k}}^{(1)} = a^{(1)}(\mathbf{k}, t_{\rm rec})$ is the inverse Fourier transform of the initial function $\mu^{(1)}(\mathbf{x})$. When the solution (14) is substituted into the Fourier-transformed equation (12), the result is

$$\left[\frac{\partial^2}{\partial t^2} + \frac{2}{3t}\frac{\partial}{\partial t} - \frac{2}{3t^2}\right] a^{(2)}(\mathbf{k},t) = \frac{1}{21} t^{-2/3} t_{\rm rec}^{-4/3} \int J(\mathbf{k},\mathbf{l},\mathbf{k}-\mathbf{l})\, a_{\mathbf{l}}^{(1)} a_{\mathbf{l}-\mathbf{k}}^{(1)}\, d^3\mathbf{l}, \tag{15}$$

where $a^{(2)}(\mathbf{k},t)$ is the inverse Fourier transform of $\mu^{(2)}(\mathbf{x},t)$, and

$$J(\mathbf{k},\mathbf{l},\mathbf{m}) \equiv 2k^2(\mathbf{l}\cdot\mathbf{m})(lm)^{-2} + 5(\mathbf{k}\cdot\mathbf{l})l^{-2}. \tag{16}$$

The quantities ρ and R in (12) were eliminated from (15) with the help of equation (1). The solution of equation (15) at $t \gg t_{\rm rec}$ has the form

$$a^{(2)}(\mathbf{k},t) = a_{\mathbf{k}}^{(2)}(t/t_{\rm rec})^{4/3}, \tag{17}$$

where

$$a_{\mathbf{k}}^{(2)} = \frac{1}{14} \int J(\mathbf{k},\mathbf{l},\mathbf{k}-\mathbf{l})\, a_{\mathbf{l}}^{(1)} a_{\mathbf{k}-\mathbf{l}}^{(1)}\, d^3\mathbf{l}. \tag{18}$$

The power spectrum $S(\mathbf{k},t)$ may be expanded in perturbation series

$$S(\mathbf{k},t) = S_{\mathbf{k}}^{(1)}(t/t_{\rm rec})^{4/3} + S_{\mathbf{k}}^{(2)}(t/t_{\rm rec})^{8/3} + \ldots, \tag{19}$$

where $S_{\mathbf{k}}^{(\beta)} \equiv \langle a_{\mathbf{k}}^{(\beta)} a_{-\mathbf{k}}^{(\beta)}\rangle$; $\beta = 1, 2$. The first term in (19), which dominates at $Z \gg Z_{\rm f}$, describes the evolution of the fluctuations in the linear regime, when the inhomogeneities on different scales grow independently of each other. At this stage the shape of the spectrum is preserved and the rate of growth of the density contrast is the same for all scales. The second term is expected to become comparable with the first-order term at redshifts close to $Z_{\rm f}$, when $\delta\rho/\rho \lesssim 1$. At this stage the interaction between different scales may lead to the change of the shape of the spectrum and a redistribution of binding energy between different scales.

The quantity $S_{\mathbf{k}}^{(1)}$ in (19) is defined by the initial spectrum. From equation (5) and (6) one finds

$$S_{\mathbf{k}}^{(1)} = (3\times10^{-4}/4\pi k_{\mathrm{D}}^{3})\,[4e/(n+3)]^{(n+3)/2} f(k/k_{\mathrm{D}}), \tag{20}$$

where

$$f(y) = \begin{cases} y^{n+2} & \text{for } y \leqslant 0.017, \\ y^{n}\exp(-2y^{2}) & \text{for } y > 0.017. \end{cases} \tag{21}$$

The wavenumber k_{D} in equation (20) corresponds to the damping mass, or to the comoving length scale $2\pi/k_{\mathrm{D}}(Z+1) = 24\,\mathrm{Mpc}\,(Z+1)^{-1}$. The wavenumber $0.017\,k_{\mathrm{D}}$ corresponds to the comoving Jeans mass $M_{\mathrm{J}} = 10^{19}M_{\odot}$.

Multiplying both sides of equation (18) by $a_{\mathbf{k}}^{(2)}$ and averaging, one has

$$\langle a_{\mathbf{k}}^{(2)} a_{-\mathbf{k}}^{(2)}\rangle \equiv S_{\mathbf{k}}^{(2)} = \frac{1}{14}\int J(\mathbf{k},\mathbf{l},\mathbf{k}-\mathbf{l})\,\langle a_{\mathbf{l}}^{(1)} a_{\mathbf{k}-\mathbf{l}}^{(1)} a_{-\mathbf{k}}^{(2)}\rangle\, d^{3}\mathbf{l} =$$

$$= (14)^{-2}\iint J(\mathbf{k},\mathbf{l},\mathbf{k}-\mathbf{l})J(\mathbf{k},\mathbf{m},\mathbf{k}-\mathbf{m})\,\langle a_{\mathbf{l}}^{(1)} a_{\mathbf{k}-\mathbf{l}}^{(1)} a_{-\mathbf{m}}^{(1)} a_{\mathbf{m}-\mathbf{k}}^{(1)}\rangle\, d^{3}\mathbf{l}\, d^{3}\mathbf{m}. \tag{22}$$

The assumption that the fluctuations are Gaussian gives (Yaglom 1962)

$$\langle a_{\mathbf{l}}^{(1)} a_{\mathbf{k}-\mathbf{l}}^{(1)} a_{-\mathbf{m}}^{(1)} a_{\mathbf{m}-\mathbf{k}}^{(1)}\rangle = S_{\mathbf{l}}^{(1)} S_{\mathbf{k}-\mathbf{l}}^{(1)}\,\{\delta^{3}(\mathbf{l}-\mathbf{m}) + \delta^{3}(\mathbf{k}-\mathbf{l}-\mathbf{m})\}, \tag{23}$$

and thus

$$S_{\mathbf{k}}^{(2)} = (14)^{-2}\int J(\mathbf{k},\mathbf{l},\mathbf{k}-\mathbf{l})\,\{J(\mathbf{k},\mathbf{l},\mathbf{k}-\mathbf{l}) + J(\mathbf{k},\mathbf{k}-\mathbf{l},\mathbf{l})\}\, S_{\mathbf{k}}^{(1)} S_{\mathbf{k}-\mathbf{l}}^{(1)}\, d^{3}\mathbf{l}. \tag{24}$$

Making use of equations (19), (20), (21) and (24) one is now able to describe the time development of the power spectrum $S(\mathbf{k},t)$ with a precision up to the second order in the perturbation theory.

The integral (24) was evaluated numerically for spectral indices $n = -1$, 0 and 2 and for wavenumbers $0.017 \leqslant k/k_{\mathrm{D}} \leqslant 17$, corresponding to mass scales $10^{19} \geqslant M/M_{\odot} \geqslant 10^{10}$. The curves, plotted in Figs 1–3 show the resulting spectra of the rms density fluctuations at $Z = Z_{\mathrm{f}} = 10$:

$$\delta\rho/\rho = \left\{\frac{4\pi k^{3}}{3}\,[S^{(1)}(\mathbf{k},t_{\mathrm{f}}) + S^{(2)}(\mathbf{k},t_{\mathrm{f}})]\right\}^{1/2} \tag{25}$$

Also shown for comparison are the spectra corresponding to the expectations of the linear theory,

$$(\delta\rho/\rho)_{\mathrm{linear}} = \left\{\frac{4\pi k^{3}}{3}\, S^{(1)}(\mathbf{k},t_{\mathrm{f}})\right\}^{1/2} \tag{26}$$

For all values of n considered here the differences between the linear and weakly non-linear theories are similar. In the linear theory the shape of the spectrum does not change in the epoch $Z_{\mathrm{rec}} \geqslant Z \geqslant Z_{\mathrm{f}}$, i.e. the matter distribution on scales $< M_{\mathrm{D}}$ is kept uniform, whereas, taking into account the second-order terms in the equations of motion results at $Z \gtrsim Z_{\mathrm{f}}$ in the appearance of short wave perturbations on scales $M_{\mathrm{D}} \gtrsim M \gtrsim 0.1\,M_{\mathrm{D}}$.

This phenomenon seems to have a simple physical explanation. The non-linear terms in equations (7) and (9) describe tidal interactions between the neighbouring inhomogeneities.

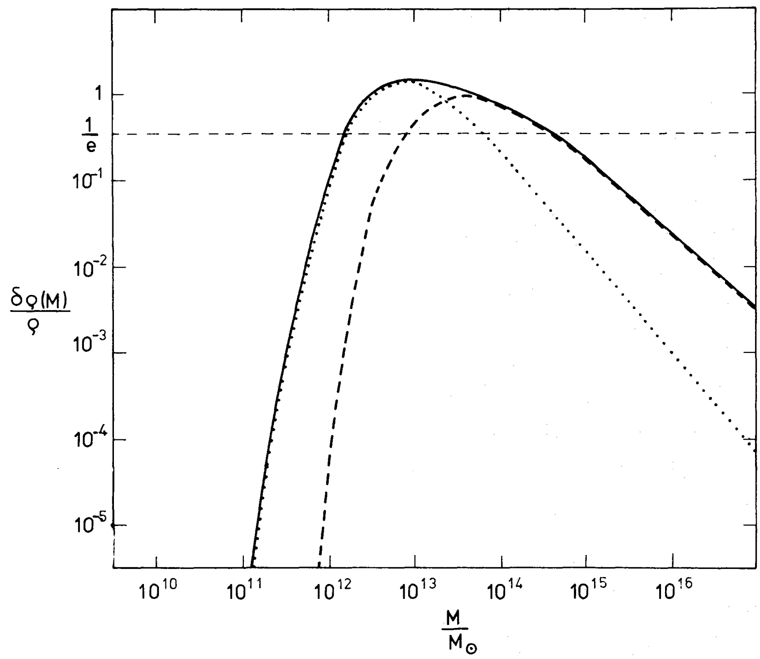

Figure 1. The spectrum of the rms density fluctuations at $Z = Z_f = 10$ for $n = 2$. The solid curve corresponds to the expectations of the weakly non-linear theory (equation 25). Also shown for comparison is the spectrum expected in the linear theory (dashed curve) and the second-order contribution to the spectrum, i.e. the quantity $(4\pi k^3 S^{(2)}(\mathbf{k}, t_f)/3)^{1/2}$, indicated by the dotted curve.

Thus the appearance of short wave components in the spectrum suggests that the clumps with masses $\gtrsim M_D$ are tidally disrupted into smaller units. Moreover, the shift of the cut-off in the spectrum from $M = M_D$ to a tenth of M_D seems to have a simple explanation. Consider a Fourier series representing $\mathbf{v}^{(1)}$, and assume that the coefficients for $k > k_D$ all vanish, i.e. there are only large scale matter currents in the first order. Substituting this expansion to the non-linear term on the right hand side of equations (7) one finds easily that these currents excite second-order currents on smaller scales, corresponding to $k_D \leqslant k \leqslant 2k_D$. Hence the cut-off is shifted from $M = M_D$ to $M = 2^{-3} M_D \simeq 0.1 M_D$.

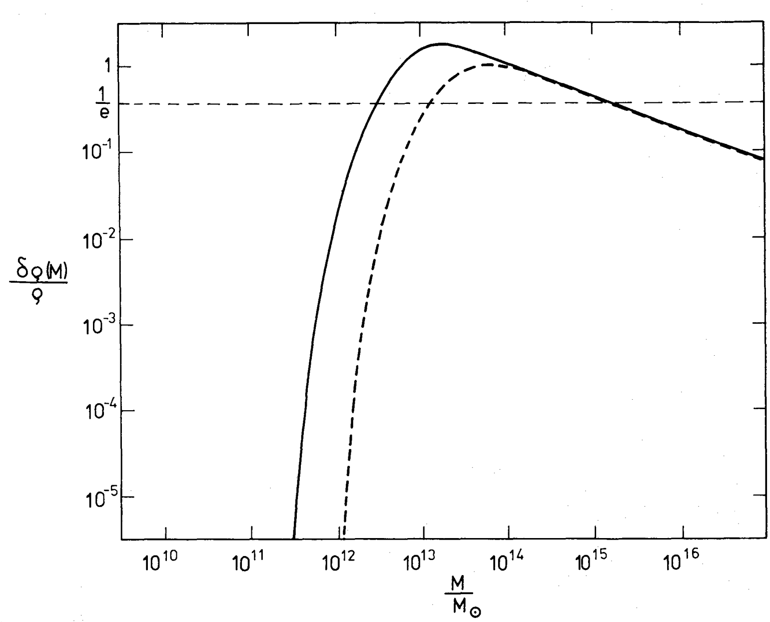

Figure 2. The weakly non-linear (solid curve) and linear (dashed curve) spectra of the rms density fluctuations at $Z = Z_f$ for $n = 0$.

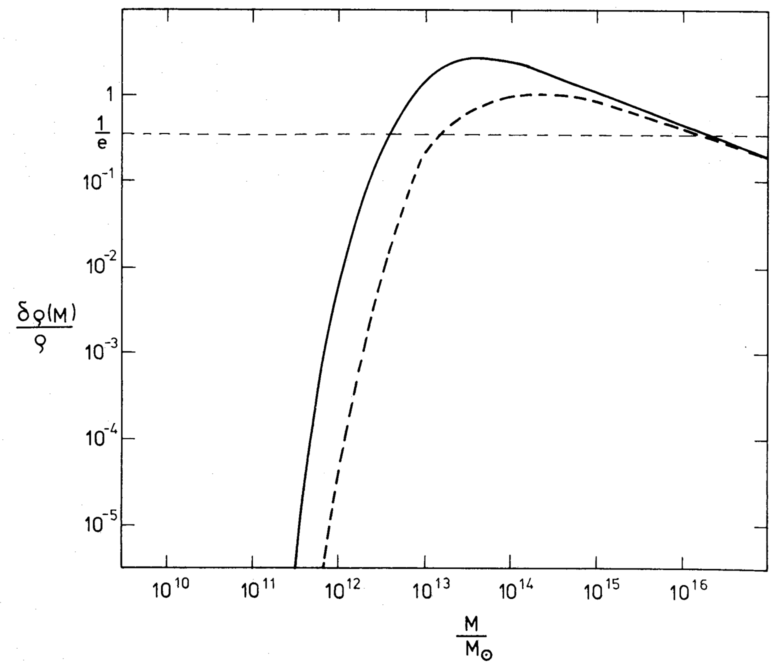

Figure 3. As Fig. 2, but for $n = -1$.

4.2 THE EVOLUTION OF THE CORRELATION FUNCTION

The results of the previous section can also be expressed in terms of the pair correlation function, defined by

$$\xi(r,t) \equiv \langle \mu(\mathbf{x}+\mathbf{r},t)\mu(\mathbf{x},t)\rangle = 4\pi\int_0^\infty S(\mathbf{k},t)\frac{\sin(kr)}{kr}k^2\,dk =$$

$$= \xi^{(1)}(r)(t/t_{\rm rec})^{2/3} + \xi^{(2)}(r)(t/t_{\rm rec})^{4/3}, \tag{27}$$

where

$$\xi^{(\beta)}(r) = 4\pi\int_0^\infty S_{\mathbf{k}}^{(\beta)}\frac{\sin(kr)}{kr}k^2\,dk.$$

It is well known (ZN, Section 12.2) that if there is any preferred scale in the distribution of matter, then it corresponds roughly to the smallest value of r for which ξ vanishes. In particular, when the power spectrum has the shape of a delta function, centred at $k = k_{\rm D}$, the correlation function vanishes at $r = \pi/k_{\rm D}$. In this case there are only perturbations with masses $M_{\rm D}$ and comoving diameters $2\pi R/k_{\rm D}$. Thus $r \equiv \pi/k_{\rm D}$ corresponds to the 'typical radius' of an inhomogeneity. For more realistic shapes of the power spectrum, when S is smoothly distributed around $k = k_{\rm D}$, one expects the correlation function to vanish somewhere in the neighbourhood of $r = \pi/k_{\rm D}$. For the spectrum (20) with $n = 2$ one obtains

$$\xi^{(1)}(r) = 4.1\times 10^{-5}(12 - k_{\rm D}^2 r^2)\exp(-k_{\rm D}^2 r^2/8). \tag{28}$$

This function vanishes at

$$r = r_{\rm D} \simeq 3.46/k_{\rm D}. \tag{29}$$

The second-order contribution to the correlation function may be calculated by a numerical evaluation of the integral $\xi^{(2)}(r)$, where $S_{\mathbf{k}}^{(2)}$ is given by the equations (24), (20) and (21)

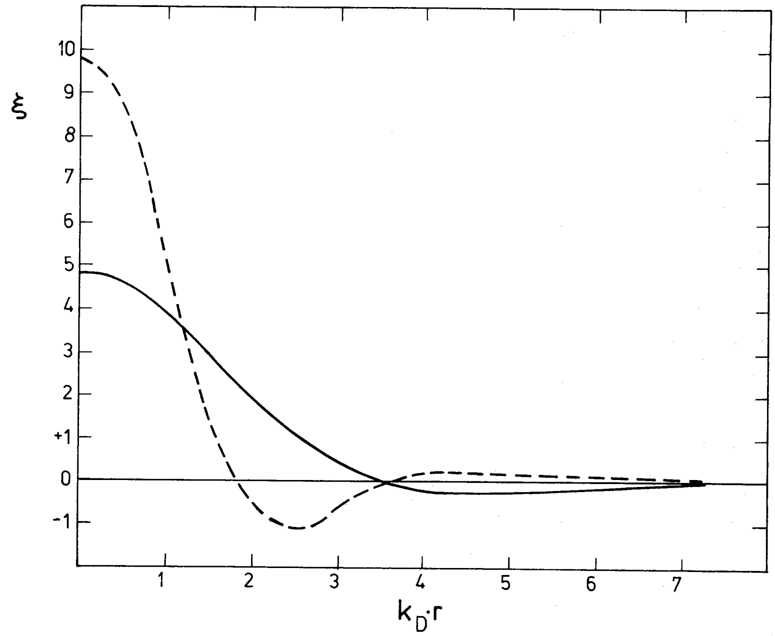

Figure 4. The functions $\xi^{(1)}(r)(t_f/t_{rec})^{2/3}$ and $\xi^{(2)}(r)(t_f/t_{rec})^{4/3}$ for $n = 2$, indicated respectively by the solid and dashed curve.

with $n = 2$. The result of this calculation is shown in Fig. 4. The function $\xi^{(2)}(r)$ vanishes at

$$r = 1.79/k_D \simeq r_D/2. \tag{30}$$

Hence the typical length scale of the fragments produced by tidal disruption of large scale protostructures is approximately $r_D/2$, in full agreement with the simplified picture of the fragmentation process given at the end of Section 4.1.

It is interesting also to compare the time evolution of the correlation function given by equation (27) with that from linear theory. The evolution of both (for the $n = 2$ case) is shown in Figs 5 and 6.

5 Discussion

Taking into account second-order effects in the gravitational instability theory, I have shown that the non-linear interaction between long wave adiabatic perturbations with wavenumbers $k \leqslant k_D$ acts as a source of short wave harmonics with wavenumbers in the range $k_D \leqslant k \leqslant 2k_D$. Using arguments similar to those at the end of Section 4.1 one may show that the

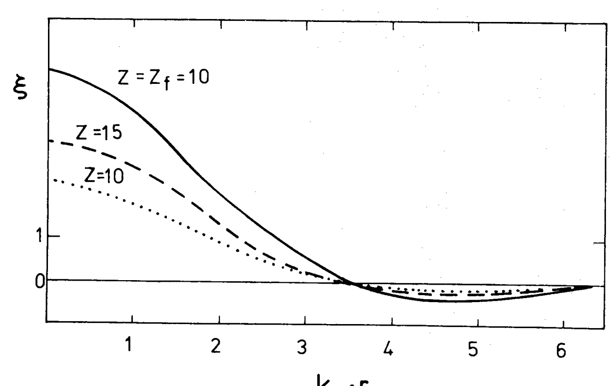

Figure 5. The time evolution of the linear correlation function ($n = 2$). Note that the typical length scale of the inhomogeneities is the same for all values of Z.

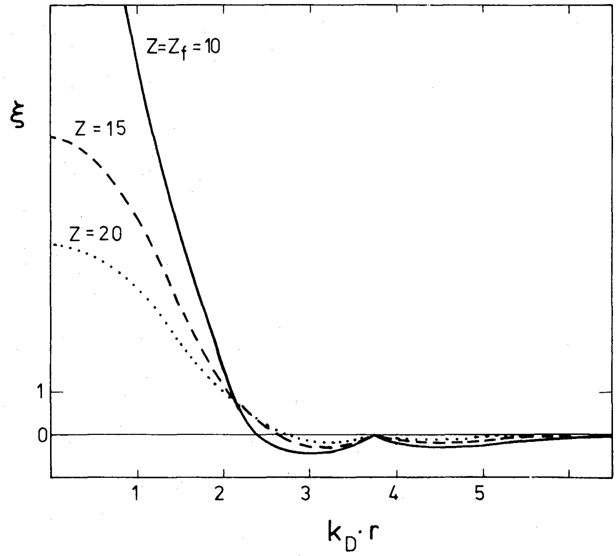

Figure 6. The time evolution of the weakly non-linear correlation function for $n = 2$. The typical length scale decreases with time.

higher-order effects reduce the cut-off in the spectrum progressively (i.e. third-order effects will shift the cut-off to $k = 3k_{\mathrm{D}}$, etc.). One possible interpretation of this phenomenon is that we deal with tidally induced fragmentation of large scale inhomogeneities into smaller units.

However, as it was pointed out by Doroshkevich & Zeldovich (1975), such an interpretation is correct only if the phases of the higher-order harmonics are statistically independent from the phases of the first-order perturbations that created them. In the case of phase coherence the breakdown of the spectrum to short waves is produced by the steepening of the density profile inside of the inhomogeneities rather than by the formation of distinct substructures on mass scales $< M_{\mathrm{D}}$. Thus the validity of assumption (iii) in Section 2 plays a crucial role for the interpretation of the results obtained in this paper.

It seems hopeless at present to prove or disprove on purely theoretical grounds the validity of the random phase approximation in this context. It is probably more plausible to treat this approximation as a statistical hypothesis and to test its validity indirectly, e.g. by comparing the predictions based on the random phase approximation with the results of numerical simulations. The results of two-dimensional numerical simulations of the gravitational instability process, published recently by Doroshkevich *et al.* (1980) seem to confirm the results obtained in the present paper. This is seen when one compares their Fig. 7(a) and (b), showing two-dimensional distributions of particles obtained (a) according to the approximate theory and (b) as a result of the numerical experiment (in both cases the initial conditions were identical and the distribution of points is given for the same time). In their Fig. 7(a) there is only one preferred scale of clustering (the matter distribution on mass scales $< M_{\mathrm{D}}$ remains uniform), whereas in their Fig. 7(b) there appear to be density inhomogeneities on a scale much smaller than M_{D}. These are seen as compact clusters of particles in regions where the tidal interactions between the neighbours appear to dominate over their self-gravity, and what is actually seen in Fig. 7(b) may be just the process of tidal fragmentation of large scale inhomogeneities into smaller units. However, it might also be a consequence of discreteness effects in the initial distribution of particles of the numerical experiment.

If the random phase approximation is valid one may expect serious departures from the Doroshkevich *et al.* (1974; *cf.* also Zeldovich 1978) scenario for galaxy formation. In the $\Omega h^2 = 0.1$ cosmological model, favoured by these authors, the Silk mass is of the order of the masses of clusters of galaxies. After recombination perturbations on scales $\geqslant M_D$, which survived damping, start to grow. It is assumed that their growth is well described by the linear gravitational instability theory up to the redshift $Z_f = \Omega^{-1}$, when $\delta\rho/\rho = 1$. At $Z = Z_f$ the inhomogeneities cease to expand with the Hubble flow and start to contract in aspherical manner. An approximate heuristic theory is used to describe their behaviour in the non-linear stage of their collapse ($Z < Z_f$, $\delta\rho/\rho > 1$). This approximate theory does not describe the interactions between the neighbouring perturbations, but only the self-gravity of each perturbation alone. The matter distribution on mass scales $<M_D$ is kept uniform (simply because all tidal effects are neglected). The presumed failure of gas to fragment leads to the formation of shock fronts and caustic surfaces on scales $\gtrsim M_D$ and all proto-cluster clouds are compressed into thin layers called 'pancakes'. The infall energy is thermalized and radiated away. Galaxies form due to the fragmentation process, caused by the thermal instability, which takes place after the collapse is completed. Thus galaxies form later than their clusters.

The above picture changes when the tidal processes are taken into account. In this case, large scale inhomogeneities start to fragment at $Z > Z_f$, before the beginning of their collapse and this prevents large scale shock formation. What one might expect to happen at $Z < Z_f$, is something between the N-body, or 'stellar-dynamical' picture (when the collapse of a protocluster ends with the violent relaxation) and the 'purely hydrodynamical' picture of the pancake theory. The fragmentation may be complete enough to avoid large scale shock formation but not so complete to prevent cloud–cloud collisions between the fragments. The large scale cellular structure of the matter distribution in the Universe, expected in the standard version of the pancake theory (Zeldovich 1978) is probably weakened. The resulting large scale distribution of matter in the Universe may then correspond to a compromise between the hierarchical clustering (*cf.* Rees 1978) and the pancake picture. Accurate N-body calculations are necessary to judge finally whether the phases are random or coherent and whether tidal interaction really leads to fragmentation.

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