

# Applied Mathematics and Nonlinear Sciences

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## On the exact solutions to some system of complex nonlinear models

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### Abstract

In this manuscript, the application of the extended sinh-Gordon equation expansion method to the Davey-Stewartson equation and the (2+1)-dimensional nonlinear complex coupled Maccari system is presented. The Davey-Stewartson equation arises as a result of multiple-scale analysis of modulated nonlinear surface gravity waves propagating over a horizontal seabed and the (2+1)-dimensional nonlinear complex coupled Maccari equation describes the motion of the isolated waves, localized in a small part of space, in many fields such as hydrodynamic, plasma physics, nonlinear optics. We successfully construct some soliton, singular soliton and singular periodic wave solutions to these two nonlinear complex models. The 2D, 3D and contour graphs to some of the obtained solutions are presented.

**Keywords:** The extended ShEEM; Maccari's system; Davey-Stewartson equation; soliton solutions

**AMS 2010 codes:** 34A34.

## 1 Introduction

For the past two decades, the investigations of various travelling wave solutions to the nonlinear evolution equations have attracted the attentions of many scientist from all over the world. Nonlinear evolution equations (NLEEs) are used in describing many complex phenomena the arise on daily basis in the various fields of nonlinear sciences, such as; plasmas physics, quantum mechanics, biosciences, chemistry, water waves and so on. Various mathematical approaches have been formulated to tackle such type of problems, such as; the extended Conte's truncation method [1], the Hirota method [2], the local fractional Riccati differential equation method [3], the improved  $\tan(\varphi/2)$ -expansion method [4], the generalized algebraic method [5], the simplified Hirota's method [6], the extended Jacobi elliptic function expansion method [7], the tanh function method [8], the generalized Kudryashov method [9], the sine-cosine method [10], the complex hyperbolic function method [11], the spectral-homotopy analysis method [12], the improved Bernoulli sub-equation

function method [13], the modified  $\exp(-\phi(\xi))$ -expansion function method [14–16], sine-Gordon expansion method [17], the Adomian decomposition method [18], the Riccati equation method [19], the extended generalized Riccati equation mapping method [20] and many more other methods [21–51].

However, in this study, we present the application of the extended sinh-Gordon equation expansion method (ShGEEM) [52] to the Davey-Stewartson equation [53] and the (2+1)-dimensional nonlinear complex coupled Maccari system [54, 55].

The Davey-Stewartson equation reads

$$\begin{aligned} iu_t + a(u_{xx} + u_{yy}) + b|u|^2u - \alpha uv &= 0, \\ v_{xx} + v_{yy} - \beta(|u|^2)_{xx} &= 0. \end{aligned} \quad (1.1)$$

Eq. (1.1) arises as a result of multiple-scale analysis of modulated nonlinear surface gravity waves propagating over a horizontal seabed [53]. Eq. (1.1) may also be used in modelling long-wave, short-wave resonances and other patterns of propagating waves [56–58]. Various studies have been conducted on Eq. (1.1) [59–61].

The (2+1)-dimensional nonlinear complex coupled Maccari equation reads

$$\begin{aligned} iu_t + u_{xx} + uv &= 0, \\ v_t + v_y + (|u|^2)_x &= 0, \end{aligned} \quad (1.2)$$

where  $i = \sqrt{-1}$ .

Eq. (1.2) describes the motion of the isolated waves, localized in a small part of space, in many fields such as hydrodynamic, plasma physics, nonlinear optics etc. Eq. (1.2) was derived from the well known two-dimensional generalizations of the KdV equation [62, 63]. Several attempts by different scientists have been made to investigate Eq. (1.2) [64–71].

## 2 The Extended ShGEEM

In this sections, the general facts of the sinh-Gordon equation expansion method are presented.

To apply the ShGEEM, the following steps are followed:

**Step-1:** Consider the following nonlinear partial differential equation and the travelling wave transformation:

$$P(u_x, u^2u_{xx}, u_t, u_{xt}, \dots) = 0, \quad (2.1)$$

where  $P$  is a polynomial in  $u$ , the subscripts indicate the partial derivative of  $u$  with respect to  $x$  or  $t$ , and

$$u = \Psi(\eta), \quad \eta = x - ct, \quad (2.2)$$

respectively.

Substituting Eq. (2.2) into Eq. (2.1), we get the following nonlinear ordinary differential equation (NODE):

$$Q(\Psi, \Psi', \Psi'', \Psi^2\Psi', \dots) = 0, \quad (2.3)$$

where  $Q$  is a polynomial in  $\Psi$  and the superscripts indicate the ordinary derivative of  $\Psi$  with respect to  $\eta$ .

**Step-2:** Eq. (2.3) is assumed to have solution of the form

$$\Psi(w) = \sum_{j=1}^m [B_j \sinh(w) + A_j \cosh(w)]^j + A_0, \tag{2.4}$$

where  $A_0, A_j, B_j$  ( $j = 1, 2, \dots, n$ ) are constants to be determine later and  $w$  is a function of  $\eta$  that satisfies the following ordinary differential equations:

$$w' = \sinh(w) \tag{2.5}$$

and

$$w' = \cosh(w) \tag{2.6}$$

To obtain the value of  $m$ , the homogeneous balance principle is used on the highest derivatives and highest power nonlinear term in Eq. (2.3).

Eqs. (2.5) and (2.6) have been extracted from the popularly known sinh-Gordon equation [52] given as

$$u_{xt} = \lambda \sinh(u). \tag{2.7}$$

Eq. (2.5) has the following solutions [52]:

$$\sinh(w) = \pm \operatorname{csch}(\eta) \quad \text{or} \quad \sinh(w) = \pm i \operatorname{sech}(\eta) \tag{2.8}$$

and

$$\cosh(w) = \pm \operatorname{coth}(\eta) \quad \text{or} \quad \cosh(w) = \pm \tanh(\eta), \tag{2.9}$$

where  $i = \sqrt{-1}$ .

Eq. (2.6) posses the following solutions [52]:

$$\sinh(w) = \tan(\eta) \quad \text{or} \quad \sinh(w) = -\cot(\eta) \tag{2.10}$$

and

$$\cosh(w) = \pm \sec(\eta) \quad \text{or} \quad \cosh(w) = \pm \csc(\eta). \tag{2.11}$$

**Step-3:** With fixed value of  $m$ , we substitute Eq. (2.4), its derivative along with Eq. (2.5) or (2.6) into Eq. (2.3) to obtain a polynomial equation in  $w'^s \sinh^i(w) \cosh^j(w)$  ( $s = 0, 1$  and  $i, j = 0, 1, 2, \dots$ ). We collect a set of over-determined nonlinear algebraic equations in  $A_0, A_j, B_j, c$  by setting the coefficients of  $w'^s \sinh^i(w) \cosh^j(w)$  to zero.

**Step-4:** The obtained set of over-determined nonlinear algebraic equations is then solved with aid of symbolic software to determine the values of the parameters  $A_0, A_j, B_j, c$ .

**Step-5:** Based on Eqs. (2.8), (2.9) and (2.10) and (2.11) solutions of Eq. (2.1) have the following forms:

$$\Psi(\eta) = \sum_{j=1}^m [\pm i B_j \operatorname{sech}(\eta) \pm A_j \tanh(\eta)]^j + A_0, \tag{2.12}$$

$$\Psi(\eta) = \sum_{j=1}^m [\pm B_j \operatorname{csch}(\eta) \pm A_j \operatorname{coth}(\eta)]^j + A_0. \tag{2.13}$$

$$\Psi(\eta) = \sum_{j=1}^m [\pm B_j \sec(\eta) + A_j \tan(\eta)]^j + A_0 \quad (2.14)$$

and

$$\Psi(\eta) = \sum_{j=1}^m [\pm B_j \csc(\eta) - A_j \cot(\eta)]^j + A_0. \quad (2.15)$$

### 3 Application

In this section, the application of the extended ShGEEM to the Davey-Stewartson equation and the (2+1)-dimensional nonlinear complex coupled Maccari system is presented.

1. Consider the Davey-Stewartson equation [53] given in (1.1).

Substituting the complex travelling wave transformation

$$u(x, y, t) = e^{i\theta} \Psi(\eta), \quad v(x, y, t) = V(\eta), \quad \eta = x + y + ct, \quad \theta = \sigma x + ny + rt \quad (3.1)$$

into (1.1), gives the following NODEs:

$$(-r - a(n^2 + \sigma^2))\Psi + b\Psi^3 - \alpha\Psi V + 2a\Psi'' = 0, \quad (3.2)$$

$$\beta(\Psi')^2 + \beta\Psi\Psi'' + V'' = 0, \quad (3.3)$$

from the real part, and the relation

$$c = -2a(n + \sigma). \quad (3.4)$$

Integrating Eq. (3.3) once, one can get

$$V = -\frac{\beta}{2}\Psi^2. \quad (3.5)$$

Substituting Eq. (3.5) into Eq. (3.2), we get

$$2(-r - a(n^2 + \sigma^2))\Psi + (2b + \alpha\beta)\Psi^3 + 4a\Psi'' = 0. \quad (3.6)$$

Balancing  $\Psi^3$  and  $\Psi''$ , we get  $m = 1$ .

With  $m = 1$ , Eqs. (2.4), (2.12), (2.13), (2.14) and (2.15) take the forms

$$\Psi(w) = B_1 \sinh(w) + A_1 \cosh(w) + A_0, \quad (3.7)$$

$$\Psi(\eta) = \pm i B_1 \operatorname{sech}(\eta) \pm A_1 \tanh(\eta) + A_0, \quad (3.8)$$

$$\Psi(\eta) = \pm B_1 \operatorname{csch}(\eta) \pm A_1 \operatorname{coth}(\eta) + A_0, \quad (3.9)$$

$$\Psi(\eta) = \pm B_1 \sec(\eta) + A_1 \tan(\eta) + A_0 \tag{3.10}$$

and

$$\Psi(\eta) = \pm B_1 \csc(\eta) - A_1 \cot(\eta) + A_0, \tag{3.11}$$

respectively.

Putting Eq. (3.7) and its second derivative along with Eq. (2.5) or (2.6) into Eq. (3.6), yields a polynomial in the power of hyperbolic functions. We collect a set of algebraic equations from the polynomial by equating each summations of the coefficients of the hyperbolic functions with the same power to zero. To obtain the values of the parameters involved, we simplify the set of the algebraic equations with aid of symbolic software. To get the new solutions to Eq. (1.1), we put the secured values of the parameters in each case into Eqs. (3.8), (3.9), (3.10) and (3.11), then into Eq. (3.1).

**Case-1:** When

$$A_0 = 0, A_1 = -\sqrt{\frac{2a}{-(2b + \alpha\beta)}}, B_1 = A_1, r = -a(1 + n^2 + \sigma^2),$$

we get the following solutions:

$$u_1(x, y, t) = \sqrt{\frac{2a}{-(2b + \alpha\beta)}} \left( \pm i \operatorname{sech}[x + y - 2a(n + \sigma)t] \pm \tanh[x + y - 2a(n + \sigma)t] \right) e^{i(\sigma x + n y - a(1 + n^2 + \sigma^2)t)}, \tag{3.12}$$

$$v_1(x, y, t) = \frac{a\beta}{2b + \alpha\beta} \left( \pm i \operatorname{sech}[x + y - 2a(n + \sigma)t] \pm \tanh[x + y - 2a(n + \sigma)t] \right)^2 \tag{3.13}$$

and

$$u_2(x, y, t) = \sqrt{\frac{2a}{-(2b + \alpha\beta)}} \tanh \left[ \frac{1}{2} (x + y - 2a(n + \sigma)t) \right] e^{i(\sigma x + n y - a(1 + n^2 + \sigma^2)t)}, \tag{3.14}$$

$$v_2(x, y, t) = \frac{a\beta}{2b + \alpha\beta} \tanh^2 \left[ \frac{1}{2} (x + y - 2a(n + \sigma)t) \right]. \tag{3.15}$$

**Case-2:** When

$$A_0 = 0, A_1 = 2\sqrt{\frac{2a}{-(2b + \alpha\beta)}}, B_1 = 0, r = -a(4 + n^2 + \sigma^2),$$

we get the following solutions:

$$u_3(x, y, t) = \pm 2 \sqrt{\frac{2a}{-(2b + \alpha\beta)}} \tanh[x + y - 2a(n + \sigma)t] e^{i(\sigma x + ny - a(4+n^2 + \sigma^2)t)}, \quad (3.16)$$

$$v_3(x, y, t) = \frac{4a\beta}{2b + \alpha\beta} \tanh^2[x + y - 2a(n + \sigma)t] \quad (3.17)$$

and

$$u_4(x, y, t) = \pm 2 \sqrt{\frac{2a}{-(2b + \alpha\beta)}} \coth[x + y - 2a(n + \sigma)t] e^{i(\sigma x + ny - a(4+n^2 + \sigma^2)t)}, \quad (3.18)$$

$$v_4(x, y, t) = \frac{4a\beta}{2b + \alpha\beta} \coth^2[x + y - 2a(n + \sigma)t]. \quad (3.19)$$

**Case-3:** When

$$A_0 = 0, A_1 = 0, B_1 = -2 \sqrt{\frac{2a}{-(2b + \alpha\beta)}}, r = -a(n^2 + \sigma^2 - 2),$$

we get the following solutions:

$$u_5(x, y, t) = \pm 2 \sqrt{\frac{2a}{2b + \alpha\beta}} \operatorname{sech}[x + y - 2a(n + \sigma)t] e^{i(\sigma x + ny - a(n^2 + \sigma^2 - 2)t)}, \quad (3.20)$$

$$v_5(x, y, t) = -\frac{4a\beta}{2b + \alpha\beta} \operatorname{sech}^2[x + y - 2a(n + \sigma)t] \quad (3.21)$$

and

$$u_6(x, y, t) = \pm 2 \sqrt{\frac{2a}{-(2b + \alpha\beta)}} \operatorname{csch}[x + y - 2a(n + \sigma)t] e^{i(\sigma x + ny - a(n^2 + \sigma^2 - 2)t)}, \quad (3.22)$$

$$v_6(x, y, t) = \frac{4a\beta}{2b + \alpha\beta} \operatorname{csch}^2[x + y - 2a(n + \sigma)t]. \quad (3.23)$$

**Case-4:** When

$$A_0 = 0, A_1 = -\sqrt{\frac{2a}{-(2b + \alpha\beta)}}, B_1 = A_1, r = -a(n^2 + \sigma^2 - 1),$$

we get the following solutions:

$$u_7(x,y,t) = \sqrt{\frac{2a}{-(2b+\alpha\beta)}} \left( \pm \sec[x+y-2a(n+\sigma)t] \pm \tan[x+y-2a(n+\sigma)t] \right) e^{i(\sigma x + n y - a(n^2 + \sigma^2 - 1)t)}, \tag{3.24}$$

$$v_7(x,y,t) = \frac{a\beta}{2b+\alpha\beta} \left( \pm \sec[x+y-2a(n+\sigma)t] \pm \tan[x+y-2a(n+\sigma)t] \right)^2 \tag{3.25}$$

and

$$u_8(x,y,t) = \sqrt{\frac{2a}{-(2b+\alpha\beta)}} \cot \left[ \frac{1}{2} (x+y-2a(n+\sigma)t) \right] e^{i(\sigma x + n y - a(n^2 + \sigma^2 - 1)t)}, \tag{3.26}$$

$$v_8(x,y,t) = \frac{a\beta}{2b+\alpha\beta} \cot^2 \left[ \frac{1}{2} (x+y-2a(n+\sigma)t) \right]. \tag{3.27}$$

2. Consider the (2+1)-dimensional nonlinear complex coupled Maccari equation [55] given in Eq. (1.2).

Substituting the complex wave transformation

$$u(x,y,t) = e^{i\theta} \Psi(\eta), \quad v(x,y,t) = V(\eta), \quad \eta = x+y+ct, \quad \theta = ax+by+rt \tag{3.28}$$

into Eq. (1.2), gives the following NODEs:

$$\Psi'' + \Psi V - (a^2 + r)\Psi = 0, \tag{3.29}$$

$$2\Psi\Psi' + (1+c)V' = 0 \tag{3.30}$$

from the real part and the relation

$$c = -2a \tag{3.31}$$

from the imaginary part.

Integrating Eq. (3.30) once, we obtain

$$V = -\frac{1}{1+c} \Psi. \tag{3.32}$$

Substituting Eq. (3.32) into Eq. (3.29), we have the following single NODE:

$$\Psi^3 + (1+c)(a^2+r)\Psi - (1+c)\Psi'', \tag{3.33}$$

Balancing the terms  $\Psi^3$  and  $\Psi''$  in Eq. (3.33), yields  $m = 1$ .

Proceedings as before, we obtained the following solutions for Eq. (1.2):

**Case-1:** When

$$A_0 = 0, A_1 = -\frac{\sqrt{1+\sqrt{-2-4r}}}{\sqrt{2}}, B_1 = A_1, a = -\sqrt{-\frac{1}{2}-r},$$

we get the following solutions:

$$u_1(x,y,t) = \frac{\sqrt{1+\sqrt{-2-4r}}}{\sqrt{2}} \left( \pm i \operatorname{sech} \left[ 2\sqrt{-\frac{1}{2}-r}t+x+y \right] \pm \tanh \left[ 2\sqrt{-\frac{1}{2}-r}t+x+y \right] \right) e^{i\left( rt - \sqrt{-\frac{1}{2}-r}x + by \right)}, \quad (3.34)$$

$$v_1(x,y,t) = -\frac{(1+\sqrt{-2-4r})}{2(1+2\sqrt{-\frac{1}{2}-r})} \left( \pm i \operatorname{sech} \left[ 2\sqrt{-\frac{1}{2}-r}t+x+y \right] \pm \tanh \left[ 2\sqrt{-\frac{1}{2}-r}t+x+y \right] \right)^2 \quad (3.35)$$

and

$$u_2(x,y,t) = \pm \frac{\sqrt{1+\sqrt{-2-4r}}}{\sqrt{2}} \left( \coth \left[ 2\sqrt{-\frac{1}{2}-r}t+x+y \right] \operatorname{csch} \left[ 2\sqrt{-\frac{1}{2}-r}t+x+y \right] \right) e^{i\left( rt - \sqrt{-\frac{1}{2}-r}x + by \right)}, \quad (3.36)$$

$$v_2(x,y,t) = -\frac{(1+\sqrt{-2-4r})}{2(1+2\sqrt{-\frac{1}{2}-r})} \left( \pm \coth \left[ 2\sqrt{-\frac{1}{2}-r}t+x+y \right] \pm \operatorname{csch} \left[ 2\sqrt{-\frac{1}{2}-r}t+x+y \right] \right)^2. \quad (3.37)$$

**Case-2:** When

$$A_0 = 0, A_1 = -\sqrt{2+4\sqrt{-2-r}}, B_1 = 0, a = -\sqrt{-2-r},$$

we get the following solutions:

$$u_3(x,y,t) = \pm \sqrt{2+4\sqrt{-2-r}} \tanh \left[ 2\sqrt{-2-r}t+x+y \right] e^{i\left( rt - \sqrt{-2-r}x + by \right)}, \quad (3.38)$$



$$v_3(x, y, t) = -\frac{(2 + 4\sqrt{-2-r})}{1 + 2\sqrt{-2-r}} \tanh^2 [2\sqrt{-2-r}t + x + y] \tag{3.39}$$

and

$$u_4(x, y, t) = \pm \sqrt{2 + 4\sqrt{-2-r}} \coth [2\sqrt{-2-r}t + x + y] e^{i(rt - \sqrt{-2-r}x + by)}, \tag{3.40}$$

$$v_4(x, y, t) = -\frac{(2 + 4\sqrt{-2-r})}{1 + 2\sqrt{-2-r}} \coth^2 [2\sqrt{-2-r}t + x + y]. \tag{3.41}$$

**Case-3:** When

$$A_0 = 0, A_1 = 0, B_1 = -\sqrt{2 + 4\sqrt{1-r}}, a = -\sqrt{1-r},$$

we get the following solutions:

$$u_5(x, y, t) = \pm \sqrt{2 + 4\sqrt{1-r}} \operatorname{sech} [2\sqrt{1-r}t + x + y] e^{i(rt - \sqrt{1-r}x + by)}, \tag{3.42}$$

$$v_5(x, y, t) = \frac{2 + 4\sqrt{1-r}}{1 + 2\sqrt{1-r}} \operatorname{sech}^2 [2\sqrt{1-r}t + x + y] \tag{3.43}$$

and

$$u_6(x, y, t) = \pm \sqrt{2 + 4\sqrt{1-r}} \operatorname{csch} [2\sqrt{1-r}t + x + y] e^{i(rt - \sqrt{1-r}x + by)}, \tag{3.44}$$

$$v_6(x, y, t) = -\frac{(2 + 4\sqrt{1-r})}{1 + 2\sqrt{1-r}} \operatorname{csch}^2 [2\sqrt{1-r}t + x + y]. \tag{3.45}$$

**Case-4:** When

$$A_0 = 0, A_1 = -\frac{\sqrt{1 + \sqrt{2-4r}}}{\sqrt{2}}, B_1 = 0, a = -\sqrt{\frac{1}{2} - r},$$

we get the following solutions:

$$u_7(x, y, t) = \frac{\sqrt{1 + \sqrt{2 - 4r}}}{\sqrt{2}} \left( \sec \left[ 2\sqrt{\frac{1}{2} - r} t + x + y \right] - \tan \left[ 2\sqrt{\frac{1}{2} - r} t + x + y \right] \right) e^{i \left( r t - \sqrt{\frac{1}{2} - r} x + b y \right)}, \quad (3.46)$$

$$v_7(x, y, t) = -\frac{(1 + \sqrt{2 - 4r})}{2(1 + 2\sqrt{\frac{1}{2} - r})} \left( \sec \left[ 2\sqrt{\frac{1}{2} - r} t + x + y \right] - \tan \left[ 2\sqrt{\frac{1}{2} - r} t + x + y \right] \right)^2 \quad (3.47)$$

and

$$u_8(x, y, t) = \frac{\sqrt{1 + \sqrt{2 - 4r}}}{\sqrt{2}} \left( \cot \left[ 2\sqrt{\frac{1}{2} - r} t + x + y \right] + \csc \left[ 2\sqrt{\frac{1}{2} - r} t + x + y \right] \right) e^{i \left( r t - \sqrt{\frac{1}{2} - r} x + b y \right)}, \quad (3.48)$$

$$v_8(x, y, t) = -\frac{(1 + \sqrt{2 - 4r})}{2(1 + 2\sqrt{\frac{1}{2} - r})} \left( \cot \left[ 2\sqrt{\frac{1}{2} - r} t + x + y \right] + \csc \left[ 2\sqrt{\frac{1}{2} - r} t + x + y \right] \right)^2. \quad (3.49)$$

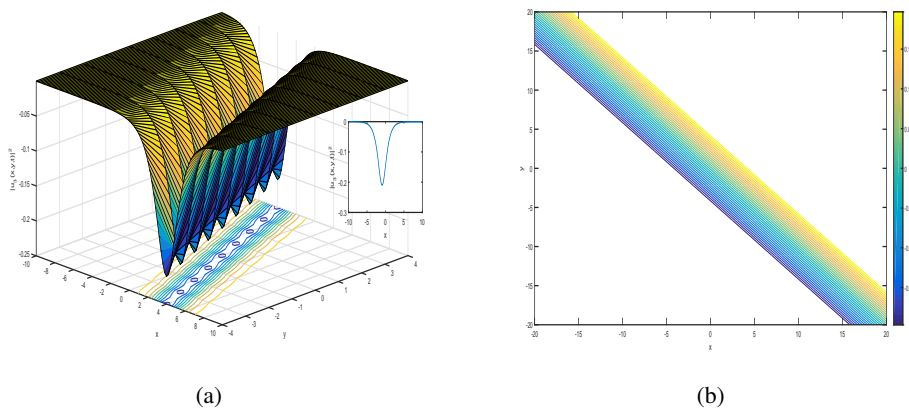


Figure 1 The (a) 3D, 2D surfaces (b) contour plot of Eq. (3.16).

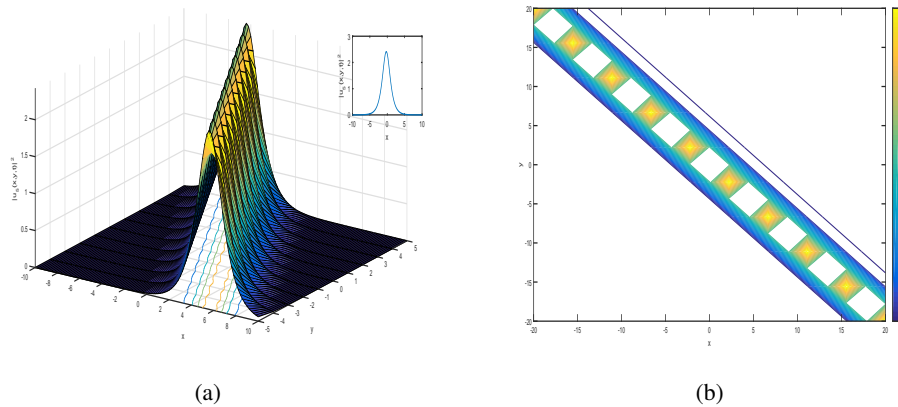


Figure 2 The (a) 3D, 2D surfaces (b) contour plot of Eq. (3.20).

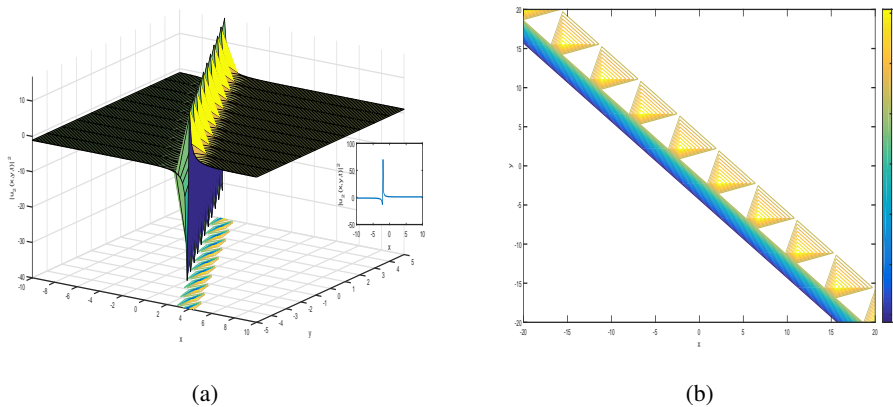


Figure 3 The (a) 3D, 2D surfaces (b) contour plot of Eq. (3.36).

#### 4 Conclusion

In this study, we successfully constructed some soliton, singular soliton and singular periodic wave solutions to the Davey-Stewartson equation and the (2+1)-dimensional nonlinear complex coupled Maccari system by using the extended sinh-Gordon equation expansion method. Under the choice of suitable parameters, the 2D, 3D and contour graphs to some of the obtained solutions are presented. The reported results in this study have some physical meanings, for instance; the hyperbolic tangent arises in the calculation of magnetic moment and rapidity of special relativity, the hyperbolic secant arises in the profile of a laminar jet, and hyperbolic cotangent arises in the Langevin function for magnetic polarization [72]. In order to have clear and good understanding of the physical properties of the reported topological, non-topological, singular solitons and singular periodic wave solutions, under the choice of the suitable values of parameters, the 3D, 2D and the contour graphs are plotted. The perspective view of the topological Eq. (3.16), non-topological Eq. (3.20) and mixed singular solitons Eq. (3.36) can be seen in the 3D graphs which appear in the (a) parts of figs. 1, 2 and 3, respectively. The propagation pattern of the wave along the  $x$ -axis for Eq. is illustrated in the 2D graphs which is located at the top right corner of the (a) parts of figs. 1, 2 and 3. The contour graphs is an alternative of the 3D plots. The the contour graph in the (b) part of fig. 1 illustrates the unstable propagation of the exact topological soliton and contour graphs

in the (b) parts of fig. 2 illustrates the stable propagation of the exact fundamental non-topological soliton. The extended sinh-Gordon equation expansion method is powerful and efficient mathematical approach that can be used for investigating various nonlinear physical models. To the best of our knowledge the applications of the extended sinh-Gordon equation expansion method to the Davey-Stewartson equation and the (2+1)-dimensional nonlinear complex coupled Maccari system have not been submitted to the literature beforehand.

### Conflict of Interests

The authors declare that they have no conflict of interests.

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