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On the Existence of Best Proximity Points of Cyclic Contractions

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Abstract

In this paper, we shall give some results on existence of the best proximity point of cyclic φ-contractions in ordered metric spaces.

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1 Introduction

In 2003, Kirk et al. [5] generalized the Banach contraction principle by using two closed subsets of a complete metric space. Then, Petruşel [6] proved some results about periodic points of cyclic contraction maps. His results generalized the main result of Kirk. Later, in 2006, Eldered and Veeramani [3] proved some results about best proximity points of cyclic contraction maps. They raised a question about the existence of a best proximity point for a cyclic contraction map in a reflexive Banach space. In 2009, Al-Thagafi and Shahzad [1] gave a positive answer to this question. In fact, they solved

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the problem for cyclic φ-contraction maps. But they proved some results about best proximity point of weakly continuous cyclic contraction maps satisfying the proximal property on reflexive (and strictly convex) Banach spaces. In this way, they raised another question for cyclic φ-contraction maps. Recently, the authors have provided a positive answer to the question of Al-Thagafi and Shahzad.

Let (X, d) be a complete metric space. The well-known Banach contraction theorem assures us of a unique fixed point if $T : X \to X$ is a contraction. As a generalization of the Banach contraction principle, Kirk et al. proved the following fixed point result in 2003 (see [5]).

Theorem 1.1. *Let A and B be nonempty closed subsets of a complete metric space (X, d). Suppose that $T : A \cup B \to A \cup B$ is a map satisfying $T(A) \subseteq B$, $T(B) \subseteq A$ and there exists $k \in (0, 1)$ such that $d(Tx, Ty) \leq kd(x, y)$ for all $x \in A$ and $y \in B$. Then, T has a unique fixed point in $A \cap B$.*

Let A and B be nonempty subsets of a metric space (X, d) and let $T : A \cup B \to A \cup B$ such that $T(A) \subseteq B$ and $T(B) \subseteq A$. The map T is called a cyclic contraction if

$$d(Tx, Ty) \leq \alpha d(x, y) + (1 - \alpha)d(A, B)$$

for all $x \in A$ and $y \in B$, where $\alpha \in (0, 1)$ and $d(A, B) = \inf\{d(x, y) : x \in A, y \in B\}$. The map T is called a cyclic φ-contraction if $\varphi : [0, \infty) \to [0, \infty)$ is a strictly increasing map and

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)) + \varphi(d(A, B))$$

for all $x \in A$ and $y \in B$ (see [1]). Also, $x \in A \cup B$ is called a best proximity point if $d(x, Tx) = d(A, B)$. Note that a best proximity point x is a fixed point of T whenever $A \cap B \neq \emptyset$. Thus, it generalizes the notion of a fixed point in case when $A \cap B = \emptyset$. Recently, Anuradha and Veeramani provided the notion of proximal pointwise contraction maps (see [2]). They gave a result about best proximity points of proximal pointwise contraction maps whenever (A, B) is a nonempty weakly compact convex pair in a Banach space.

In this paper, we shall give some results about best proximity points of cyclic φ-contractions in ordered metric spaces. Note that a contractive map in an ordered metric space is not necessarily a contraction (see [4]).

Let X be a nonempty set and T a selfmap on X. We denote the set of all nonempty subsets of X by 2^X and the set of all invariant nonempty subsets of X by $I(T)$, that is

$$I(T) = \{Y \in 2^X : T(Y) \subseteq Y\}.$$

For each pair of sets X and Y and selfmaps $T : X \to X$ and $S : Y \to Y$, we define the selfmap $T \times S : X \times Y \to X \times Y$ by $T \times S(x, y) = (Tx, Sy)$. If $(X, \leq)$ is a partially ordered set, then we define

$$X_{\leq} = \{(x, y) \in X \times X : x \leq y \text{ or } y \leq x\}.$$

Let $(X, d, \leq)$ be an ordered metric space and $T : X \to X$ a selfmap on X. For each nonempty subset C of X and $x^* \in X$, we define

$$E_{T,C}(x^*) = \{x \in C : \lim_{n\to\infty} T^{2n}x = x^*\}.$$

We say that X has the property (C) whenever for each monotone sequence $\{x_n\}$ in X with $x_n \to x$ for some $x \in X$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that every element of $\{x_{n_k}\}$ is comparable with x. Also, X is called regular whenever every bounded monotone sequence in X is convergent. We say that a selfmap $T : X \to X$ is orbitally continuous whenever for each $x \in X$ and sequence $\{n(i)\}_{i\geq 1}$ with $T^{n(i)}x \to a$ for some $a \in X$, we have $T^{n(i)+1}x \to Ta$. Here, $T^{m+1} = T(T^m)$.

2 Main Results

In this section, we shall state and prove some results about best proximity points of cyclic φ-contractions in ordered metric spaces. The authors proved the following result which is an extension of [3, Proposition 3.3] for cyclic φ-contraction maps (see [7]).

Theorem 2.1. *Let $\varphi : [0, \infty) \to [0, \infty)$ be a strictly increasing map. Also, let A and B be nonempty subsets of a metric space (X, d), $T : A \cup B \to A \cup B$ a cyclic φ-contraction map, $x_0 \in A \cup B$ and $x_{n+1} = Tx_n$ for all $n \geq 0$. Then, the sequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ are bounded.*

Now, we provide our results about best proximity points of cyclic contractions in ordered metric spaces.

Theorem 2.2. *Let $(X, d, \leq)$ be an ordered metric space, $A, B \in 2^X$ and T a decreasing selfmap on $A \cup B$ such that $T(A) \subseteq B$ and $T(B) \subseteq A$. Suppose that there exists $x_0 \in A$ such that $x_0 \leq T^2x_0 \leq Tx_0$ and*

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)) + \varphi(d(A, B))$$

for all $x \in A$ and $y \in B$ with $x \leq y$, where $\varphi : [0, \infty) \to [0, \infty)$ is a strictly increasing map. If $x_{n+1} = Tx_n$ and $d_n = d(x_{n+1}, x_n)$ for all $n \geq 0$, then $d_n \to d(A, B)$.

Proof. First note that we have

$$x_0 \leq x_2 \leq \cdots \leq x_{2n} \leq x_{2n+1} \leq \cdots \leq x_3 \leq x_1$$

for all $n \geq 1$. Thus, we obtain

$$0 \leq d_{n+1} \leq d_n - \varphi(d_n) + \varphi(d(A, B))$$

for all $n \geq 1$. Hence, the sequence $\{d_n\}$ is decreasing and bounded from below. If $d_{n_0} = 0$ for some n_0, then $d_n \to d(A, B) = 0$. Suppose that $d_n > 0$ for all $n \geq 1$ and $d_n \to t_0$ for some $t_0 \geq d(A, B)$. Since

$$\varphi(d(A,B)) \leq \varphi(d_n) \leq d_n - d_{n+1} + \varphi(d(A,B)),$$

we have $\varphi(d_n) \to \varphi(d(A,B))$. This implies that $\varphi(t_0) = \varphi(d(A,B))$. So, $t_0 = d(A,B)$ because φ is strictly increasing. □

Theorem 2.3. *Let $(X, d, \leq)$ be a regular ordered metric space, $B \in 2^X$, A a closed nonempty subset of X and T a decreasing selfmap on $A \cup B$ such that $T(A) \subseteq B$ and $T(B) \subseteq A$. Suppose that there exists $x_0 \in A$ such that $x_0 \leq T^2 x_0 \leq T x_0$ and*

$$d(Tx, Ty) \leq d(x,y) - \varphi(d(x,y)) + \varphi(d(A,B))$$

for all $x \in A$ and $y \in A \cup B$ with $x \leq y$, where $\varphi : [0, \infty) \to [0, \infty)$ is a strictly increasing map. If T is orbitally continuous or X has the property (C), then there exists $x \in A$ such that $d(x, Tx) = d(A, B)$.

Proof. Define $x_{n+1} = T x_n$ for all $n \geq 0$. Again, note that

$$x_0 \leq x_2 \leq \cdots \leq x_{2n} \leq x_1$$

for all $n \geq 1$. Since X is regular and A is closed, there exists $x \in A$ such that $x_{2n} \to x$. Also, note that

$$d(A,B) \leq d(x_{2n}, Tx) = d(Tx_{2n-1}, Tx) \leq d(Tx_{2n-1}, Tx_{2n}) + d(Tx_{2n}, Tx)$$

for all $n \geq 1$. If T is orbitally continuous, then $d(Tx_{2n}, Tx) \to 0$. Hence,

$$d(x, Tx) = d(A,B)$$

because $d(Tx_{2n-1}, Tx_{2n}) \to d(A,B)$ by Theorem 2.1. Now, suppose that X has the property (C). Since $\{x_{2n}\}$ is a bounded and increasing sequence, there exists a subsequence $\{x_{2n_k}\}$ of $\{x_{2n}\}$ such that

$$x_{2n_1} \leq x_{2n_2} \leq \cdots \leq x_{2n_k} \leq \cdots \leq x.$$

Therefore,

$$\begin{aligned} d(A,B) &\leq d(x_{2n_k}, Tx) = d(Tx_{2n_k-1}, Tx) \\ &\leq d(Tx_{2n_k-1}, Tx_{2n_k}) + d(Tx_{2n_k}, Tx) \leq d(Tx_{2n_k-1}, Tx_{2n_k}) + d(x_{2n_k}, x) \end{aligned}$$

for all $k \geq 1$. This implies that $d(x, Tx) = d(A, B)$. □

The following is another example for a cyclic φ-contraction. Note that we should improve [1, Example 3] because T is not a cyclic φ-contraction in this example. For seeing this, it is sufficient that we put $x = \frac{-1}{2}$ and $y = \frac{1}{2}$. Then

$$\frac{2}{3} = d(Tx, Ty) > d(x, y) - \varphi(d(x, y)) + \varphi(d(A, B)) = \frac{1}{2}.$$

Now for improving, it is sufficient to replace the function φ by $\varphi(t) = \frac{t^2}{2(1+t)}$.

Example 2.4. Consider the Euclidian ordered metric space $X = \mathbb{R}$ with the usual norm. Suppose that $A = [-1, 0]$, $B = [0, 1]$ and $T : A \cup B \to A \cup B$ is defined by $Tx = \frac{-x}{3}$ for all $x \in A \cup B$. If $\varphi : [0, \infty) \to [0, \infty)$ is defined by $\varphi(t) = \frac{t}{2}$, then φ is strictly increasing and T is a cyclic φ-contraction map.

The following example shows that Theorem 2.3 may be applied in situations where [1, Theorem 8] does not work.

Example 2.5. Consider the regular ordered metric space $X = L^1([0, 1])$ with the norm $\|.\|_1$ and the order $f \leq g$ if and only if $f(t) \leq g(t)$ for almost all $t \in [0, 1]$. Suppose that $A = \{f \in X : -1 \leq f \leq 0\}$, $B = \{g \in X : 0 \leq g \leq 1\}$ and $T : A \cup B \to A \cup B$ is defined by $Tf = \frac{-f}{3}$ for all $f \in A \cup B$. If $\varphi : [0, \infty) \to [0, \infty)$ is defined by $\varphi(t) = \frac{t}{2}$, then φ is strictly increasing and T is a decreasing cyclic φ-contraction map. Note that A is closed and convex, T is orbitally continuous and $T0 = 0$. But X is not a reflexive Banach space.

Theorem 2.6. *Let $(X, d, \leq)$ be an ordered metric space, $A, B \in 2^X$ and T a selfmap on $A \cup B$ such that $T(A) \subseteq B$, $T(B) \subseteq A$ and $((A \times B) \cup (B \times A)) \cap X_{\leq} \in I(T \times T)$. Suppose that there exists $x_0 \in A$ such that $(x_0, Tx_0) \in X_{\leq}$ and*

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)) + \varphi(d(A, B))$$

for all $x \in A$ and $y \in B$ with $(x, y) \in X_{\leq}$, where $\varphi : [0, \infty) \to [0, \infty)$ is a strictly increasing map. If $x_{n+1} = Tx_n$ and $d_n = d(x_{n+1}, x_n)$ for all $n \geq 0$, then

$$d_n \to d(A, B).$$

Proof. First note that we have

$$d(T^{2n+1}x_0, T^{2n}x_0) \leq d(T^{2n}x_0, T^{2n-1}x_0) - \varphi(d(T^{2n}x_0, T^{2n-1}x_0)) + \varphi(d(A, B))$$

for all $n \geq 1$. Thus, we obtain

$$0 \leq d_{n+1} \leq d_n - \varphi(d_n) + \varphi(d(A, B))$$

for all $n \geq 1$. Hence, the sequence $\{d_n\}$ is decreasing and bounded from below. If $d_{n_0} = 0$ for some n_0, then $d_n \to d(A, B) = 0$. Suppose that $d_n > 0$ for all $n \geq 1$ and $d_n \to t_0$ for some $t_0 \geq d(A, B)$. Since

$$\varphi(d(A, B)) \leq \varphi(d_n) \leq d_n - d_{n+1} + \varphi(d(A, B)),$$

we have $\varphi(d_n) \to \varphi(d(A, B))$. This implies that $\varphi(t_0) = \varphi(d(A, B))$. So, $t_0 = d(A, B)$ because φ is strictly increasing. □

Theorem 2.7. *Let* $(X, d, \leq)$ *be an ordered metric space,* $A, B \in 2^X$ *and* T *a selfmap on* $A \cup B$ *such that* $T(A) = B$, $T(B) \subseteq A$ *and* $((A \times B) \cup (B \times A)) \cap X_{\leq} \in I(T \times T)$. *Suppose that for each* $x, y \in A$ *there exists* $z \in A$ *such that* $(x, z), (y, z) \in X_{\leq}$. *Also, suppose that there exist* $x_0, x^* \in A$ *such that* $x_0 \in E_{T,A}(x^*)$, $(x_0, Tx_0) \in X_{\leq}$ *and*

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)) + \varphi(d(A, B))$$

for all $x \in A$ *and* $y \in B$ *with* $(x, y) \in X_{\leq}$, *where* $\varphi : [0, \infty) \to [0, \infty)$ *is a strictly increasing map. Also, suppose that* $y \in A$, $(x, y) \in X_{\leq}$ *and* $x \in E_{T,A}(x^*)$ *imply that* $y \in E_{T,A}(x^*)$. *Then,* $E_{T,A}(x^*) = A$ *and the following statement holds:*

$$E_{T,B}(Tx^*) = B \text{ and } d(x^*, Tx^*) = d(A, B) \Leftrightarrow T \text{ is orbitally continuous}.$$

Proof. Let $x \in A$. If $(x_0, x) \in X_{\leq}$, then $x \in E_{T,A}(x^*)$. If $(x_0, x) \notin X_{\leq}$, then there exists $z \in A$ such that $(x_0, z) \in X_{\leq}$ and $(x, z) \in X_{\leq}$. Hence, $x \in E_{T,A}(x^*)$. Thus, $E_{T,A}(x^*) = A$.

Now, suppose that T is orbitally continuous and $y \in B$. Choose $x' \in A$ such that $Tx' = y$. Since $E_{T,A}(x^*) = A$, $T^{2n}x' \to x^*$ and so $T^{2n+1}x' \to Tx^*$. Hence, we have $T^{2n}y \to Tx^*$. Thus, $E_{T,B}(Tx^*) = B$. If $d(x^*, Tx^*) \neq d(A, B)$, then $\{d(T^{2n+1}x_0, T^{2n}x_0)\}$ is a decreasing sequence because $(x_0, Tx_0) \in X_{\leq}$. By Theorem 2.2, $d(T^{2n+1}x_0, T^{2n}x_0) \downarrow d(A, B)$. Choose a natural number n such that

$$d(A, B) \leq d(T^{2n+1}x_0, T^{2n}x_0) < d(x^*, Tx^*).$$

Put $x = T^{2n}x_0$ and $y = T^{2n+1}x_0$. Since $(x, y) \in X_{\leq}$, $(Tx, Ty) \in X_{\leq}$ and so $\{d(T^{2n}x, T^{2n}y)\}$ is a decreasing sequence and $d(T^{2n}x, T^{2n}y) \downarrow d(x^*, Tx^*)$. Hence, $d(x^*, Tx^*) \leq d(T^{2n+1}x_0, T^{2n}x_0) < d(x^*, Tx^*)$ which is a contradiction. Therefore, $d(x^*, Tx^*) = d(A, B)$. Now, suppose that $d(x^*, Tx^*) = d(A, B)$, $E_{T,B}(Tx^*) = B$, $x \in A \cup B$ and $T^{n(i)}x \to a$ for some $a \in A \cup B$. We shall show that $T^{n(i)+1}x \to Ta$. Put $A' = A \cap \{T^{n(i)}x\}$ and $B' = B \cap \{T^{n(i)}x\}$.

Case I. Let $d(A, B) = 0$. First suppose that $A' = \{T^{n_1(i)}x\}$ and $B' = \{T^{n_2(i)}x\}$ are subsequences of $\{T^{n(i)}x\}$. Since $\{T^{n_1(i)}x\}$ is a subsequence of $\{T^{2n}x\}$, $T^{n_1(i)}x \to x^*$. Also, we have $T^{n_1(i)+1}x \to Tx^*$ because $Tx \in B$ and $E_{T,B}(Tx^*) = B$. Since $\{T^{n_1(i)}x\}$ is a subsequence of $\{T^{n(i)}x\}$ and $T^{n(i)}x \to a$, $T^{n_1(i)}x \to a$. Thus, $a = x^*$ and so $a = x^* = Ta = Tx^*$. Since $\{T^{n_2(i)}x\}$ is a subsequence of $\{T^{2n+1}x\} = \{T^{2n}(Tx)\}$, $Tx \in$

B and $E_{T,B}(Tx^*) = B$, $T^{n_2(i)}x \to Tx^*$. Also, we have $T^{n_2(i)+1}x \to x^*$ because $T^2x \in A$, $E_{T,A}(x^*) = A$ and $\{T^{n_2(i)}x\}$ is a subsequence of $\{T^{2n+2}x\} = \{T^{2n}(T^2x)\}$. Hence, $T^{n(i)+1}x \to Ta$. Now, suppose that $B' = \{t_1, \cdots, t_k\}$ is finite. By using a similar argument, we have $T^{n_1(i)}x \to x^*$, $T^{n_1(i)+1}x \to Tx^*$ and $a = x^* = Ta = Tx^*$. Since $\{T^{n(i)+1}x\} = \{T^{n_1(i)+1}x\} \cup \{Tt_1, \cdots, Tt_k\}$, $T^{n(i)+1}x \to Ta$. If $A' = \{s_1, \cdots, s_m\}$ is finite, then $B' = \{T^{n_2(i)}x\}$ is a subsequence of $\{T^{n(i)}x\}$ and so $T^{n_2(i)}x \to a$. By using a similar argument, we have $T^{n_2(i)}x \to Tx^*$ and $T^{n_2(i)+1}x \to x^*$. Thus, $a = x^* = Ta = Tx^*$. Since $\{T^{n(i)+1}x\} = \{T^{n_2(i)+1}x\} \cup \{Ts_1, \cdots, Ts_m\}$, we have $T^{n(i)+1}x \to Ta$.

Case II. Let $d(A, B) > 0$. We claim that A' or B' is finite. In fact, if A' and B' are infinite, then similar to the above case we have $T^{n_1(i)}x \to x^*$ and $T^{n_2(i)}x \to Tx^*$. Since $\{T^{n_1(i)}x\}$ and $\{T^{n_2(i)}x\}$ are subsequences of $\{T^{n(i)}x\}$ and $T^{n(i)}x \to a$, we obtain $a = x^* = Tx^*$. So, $d(A, B) = d(x^*, Tx^*) = 0$ which is a contradiction. Now, suppose that $B' = \{t_1, \cdots, t_k\}$ is finite. By using a similar argument in case I, we have $T^{n_1(i)}x \to x^*$, $T^{n_1(i)+1}x \to Tx^*$ and $a = x^*$. Since $\{T^{n(i)+1}x\} = \{T^{n_1(i)+1}x\} \cup \{Tt_1, \cdots, Tt_k\}$, $T^{n(i)+1}x \to Ta$. If $A' = \{s_1, \cdots, s_m\}$ is finite, then $B' = \{T^{n_2(i)}x\}$ is a subsequence of $\{T^{n(i)}x\}$ and so $T^{n_2(i)}x \to a$. By using a similar argument as in case I, we have $T^{n_2(i)}x \to Tx^*$. Thus, $a = Tx^*$. Also, we have $T^{n_2(i)+1}x \to x^*$ because $T^2x \in A$, $E_{T,A}(x^*) = A$ and $\{T^{n_2(i)}x\}$ is a subsequence of $\{T^{2n+2}x\} = \{T^{2n}(T^2x)\}$. Now, we show that $Ta = x^*$. In fact, $(x^*, x^*) \in X_{\leq}$ and

$$d(x^*, T^2x^*) \leq d(T^{2n}x^*, x^*) + d(T^{2n}x^*, T^2x^*).$$

Hence, by using the assumptions, we have

$$d(T^{2n}x^*, T^2x^*) \leq d(T^{2n-2}x^*, x^*).$$

Thus, $d(x^*, T^2x^*) \leq d(T^{2n}x^*, x^*) + d(T^{2n-2}x^*, x^*)$. Since $E_{T,A}(x^*) = A$ and $x^* \in A$, $T^{2n}x^* \to x^*$ and $T^{2n-2}x^* \to x^*$. Hence, $x^* = T^2x^*$. Since $a = Tx^*$, $Ta = x^*$. Thus, $T^{n_2(i)+1}x \to Ta$. Since $\{T^{n(i)+1}x\} = \{T^{n_2(i)+1}x\} \cup \{Ts_1, \cdots, Ts_m\}$, we have $T^{n(i)+1}x \to Ta$. This completes the proof. $\square$

The following example shows that the assumption

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)) + \varphi(d(A, B))$$

for all $x \in A$ and $y \in B$ with $(x, y) \in X_{\leq}$, does not imply the following assumption:

$$y \in A, (x, y) \in X_{\leq}, x \in E_{T,A}(x^*) \Rightarrow y \in E_{T,A}(x^*).$$

Example 2.8. Consider the subsets

$$A = \{x_1 = (6, 3), x_2 = (1, 3)\} \text{ and } B = \{y_1 = (2, 0), y_2 = (0, 4)\}$$

of $\mathbb{R}^2$ via the following order:

$$(a, b) \leq (c, d) \Leftrightarrow a \leq c \text{ and } b \leq d.$$

Define $T : A \cup B \to A \cup B$ by $Tx_1 = y_2$, $Tx_2 = y_1$, $Ty_1 = x_2$, $Ty_2 = x_1$. Note that $x_2 \leq x_1$ and $y_1 \leq x_1$, and other elements are not comparable. Also, we have $d(Tx_1, Tx_2) = d(x_2, y_2) = d(A, B) = \sqrt{2}$ and $d(x_1, y_1) = \sqrt{25}$. Consider the map $\varphi : [0, \infty) \to [0, \infty)$ by $\varphi(x) = \dfrac{x}{2}$. Then, we have

$$d(Tx_1, Ty_1) \leq d(x_1, y_1) - \varphi(d(x_1, y_1)) + \varphi(d(A, B)),$$

while $T^{2n}x_1 \to x_1$ and $T^{2n}x_2 \to x_2$.

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