

On the existence of meta-stable vacua in Klebanov-Strassler

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ABSTRACT: We solve for the complete space of linearized deformations of the Klebanov-Strassler background consistent with the symmetries preserved by a stack of anti-D3 branes smeared on the S^3 of the deformed conifold. We find that the only solution whose UV physics is consistent with that of a perturbation produced by anti-D3 branes must have a singularity in the infrared, coming from NS and RR three-form field strengths whose energy density diverges. If this singularity is admissible, our solution describes the backreaction of the anti-D3 branes, and is thus likely to be dual to the conjectured metastable vacuum in the Klebanov-Strassler field theory. If this singularity is not admissible, then our analysis strongly suggests that anti-D3 branes do not give rise to metastable Klebanov-Strassler vacua, which would have dramatic consequences for some string theory constructions of de Sitter space. Key to this result is a simple, universal form for the force on a probe D3-brane in our ansatz.

KEYWORDS: Flux compactifications, Gauge-gravity correspondence, AdS-CFT Correspondence, Superstring Vacua

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1 Introduction

Over the past few years the analysis of metastable vacua of supersymmetric field theories has begun to play an important role in phenomenological model building, as these vacua can bypass many of the familiar problems associated with old-fashioned dynamical supersymmetry breaking (DSB). Such meta-stable vacua were first found for massive $\mathcal{N} = 1$ $SU(N_c)$ SQCD in the free-magnetic phase [1], and soon thereafter in quite a large number of similar theories.

An obvious question posed by the ubiquitousness of metastable vacua in SQCD theories is whether such vacua also exist in the string-theory configurations whose low-energy physics yields these theories. These constructions involve D-branes and NS5 branes in type IIA string theory, and are generically referred to as MQCD [2]. Another obvious question is whether such metastable vacua exist in the type IIB supergravity backgrounds such as [3] that are dual to some of the SQCD theories via the gauge-gravity (AdS-CFT) correspondence.

For MQCD theories one can construct putative non-supersymmetric brane configurations whose low-energy physics matches that of the SQCD metastable vacuum. At $g_s = 0$ these brane configurations have the same asymptotics as the configuration describing the MQCD supersymmetric vacua. However, for $g_s \neq 0$ these non-supersymmetric configurations do not have the same asymptotics as the MQCD supersymmetric branes, and hence do not describe a metastable vacuum of a supersymmetric theory (where supersymmetry is broken dynamically), but rather a non-supersymmetric vacuum of a nonsupersymmetric theory [4]. The fact that the putative MQCD non-supersymmetric and supersymmetric configurations are not vacua of the same theory can also be ascertained by computing the tunneling probability between them: for $g_s \neq 0$ this probability is zero because the branes differ by an infinite amount, while for $g_s = 0$ this probability is zero because the branes have infinite mass.

The key reason for which the susy and non-susy brane configurations do not have the same asymptotics comes from the phenomenon of brane bending: essentially all MQCD brane descriptions of four-dimensional theories contain D4 branes ending on NS5 branes; since the end of D4 brane is codimension 2 inside the NS5 brane, it sources an NS5 world-volume field that is logarithmic at infinity, and that causes the NS5 brane to bend logarithmically. For the MQCD dual of $\mathcal{N} = 2$ SQCD, this bending encodes the logarithmic running of the coupling constant with the energy, characteristic of four-dimensional gauge theories [2]. The D4 branes in the deep infrared of the susy and non-susy MQCD brane configurations end on the NS5 branes in different fashions, and source fields of different kinds. Since these fields do not decay at infinity, but grow logarithmically, the tiny infrared difference is transposed into a log-growing difference in the UV, which is responsible for the susy and non-susy configurations not being vacua of the same theory.

It is also interesting to note that for three-dimensional gauge theories engineered using D3 branes and NS5 branes in type IIB string theory, this phenomenon does not happen. The D3 branes end in codimension-three sources on the NS5 branes, which source fields that decay like $1/r$ at infinity. This bending encodes the running of coupling constants in

three dimensions, which is linear in the inverse of the energy. Since the difference between the fields of the non-supersymmetric and supersymmetric brane configurations decays at infinity, the two do correspond to vacua of the same theory, and the non-supersymmetric configuration is really a 3D MQCD metastable vacuum.

Thus, the mechanism that ensures the absence of metastable vacua in four-dimensional MQCD appears to rely on just one ingredient: the existence of logarithmic modes that are sourced differently in the infrared for the BPS and non-BPS configurations. As these modes are present in four-dimensional but not in three dimensional gauge theories, it is only four-dimensional MQCD which does not have metastable vacua, three-dimensional MQCD does have such vacua.

In this paper we will analyze whether a similar mechanism is at work in gravity duals of $\mathcal{N} = 1$ four-dimensional gauge theories. As the coupling constants of these theories run logarithmically with the energy, all their gravity duals must contain fields that depend logarithmically on the AdS radius, at least asymptotically.¹ It is possible that a putative metastable configuration in the infrared couples to these logarithmic modes in different manner than the supersymmetric configuration; hence the non-BPS solution would differ in the ultraviolet from the BPS solution by one or more non-normalizable modes. If this indeed happens, then the non-BPS solution will not be dual to a metastable DSB vacuum of the supersymmetric field theory. Instead, it will describe a non-supersymmetric vacuum of a non-supersymmetric theory obtained by perturbing the Lagrangian of the supersymmetric theory. As a consequence, supersymmetry breaking will not be dynamical but explicit.

The best-known candidate for the IIB gravity dual of a metastable gauge theory vacuum has been proposed by Kachru, Pearson and Verlinde (KPV) [5], who argued that a collection of P anti-D3 branes placed at the bottom of the Klebanov-Strassler (KS) solution with M units of 3-form flux can decay into a BPS solution corresponding to the KS solution with $M - P$ D3 branes at the bottom of the throat. In particular, if $P = M$ this collection of anti-D3 branes could decay into the smooth, source-free KS solution. The argument presented by [5] was in the probe approximation and it important to extend this to a backreacted supergravity solution. Finding the backreacted solution would allow one to establish whether this configuration is indeed a metastable vacuum of a supersymmetric theory, or it is rather a non-supersymmetric vacuum of a non-supersymmetric theory. This necessity is particularly evident, given that in MQCD there exist non-backreacted brane configurations that are candidates for meta-stable vacua [4, 6, 7] but in fact break supersymmetry explicitly [4] once backreaction is taken into account.²

Note that this question is in no way settled by the results currently in the literature. For example the ultraviolet perturbation expansion around the Klebanov-Tseytlin [11] solution presented in [12] assumes *a-priori* that the anti-D3 branes source only a normalizable mode, and uses the consistency of the result (in particular they use the fact that the force

¹In the well-known Klebanov-Strassler solution [3] such a field is given by the integral of the NS B-field on the 2-cycle of the conifold.

²Worldsheet evidence in support of this MQCD analysis was given in [8] and furthermore, both the MQCD dual of the configuration of [5] (analyzed in [9]) as well as other duals of KPV-like configurations [10], suffer from the same problems in the UV, at least in certain regime of parameters.

on a probe D3 brane is not inconsistent with that calculated in [13]) to argue that such an assumption is correct. However as we will show in section 3, all $SU(2) \times SU(2) \times \mathbb{Z}_2$ -invariant first-order perturbations around the Klebanov-Strassler solution, whether normalizable or non-normalizable, give a force on a probe D3 brane that is either zero or has the precise behavior predicted in [13]. Hence this force calculation alone will not determine the normalizability or non-normalizability of the modes sourced by an anti-D3 brane. In addition, we will point out that subleading perturbations of the (singular) Klebanov-Tseytlin geometry do not in fact correspond to perturbations of the (regular) Klebanov-Strassler geometry and as such the Klebanov-Tseytlin geometry is an inappropriate arena for studying these issues.

One can argue that because an anti-D3 brane is a point source in a six-dimensional space, the fields sourced by such a brane should decay at infinity as $1/r^4$, and thus could not correspond to non-normalizable modes. As we will show in section 6.1 by explicit calculations, this intuition is not correct even under the weakest of assumptions about the backreacted solution sourced by anti-D3 branes, which must decay at least as strongly as $1/r^3$. Furthermore, as we have argued above, both the Klebanov-Strassler solution, as well as other backgrounds dual to four-dimensional gauge theories whose coupling constants run logarithmically with the energy, contain supergravity modes that behave asymptotically as $\log r$. If the anti-D3 brane in the deep infrared couples to such a mode, then the backreacted solution would differ from the BPS solution by a non-normalizable mode, and hence would not correspond to a metastable DSB vacuum.

It is also interesting to note that the only known fully-backreacted supergravity solution corresponding to a DSB metastable vacuum, constructed by Maldacena and Năstase [14], describes a vacuum of a $2+1$ dimensional gauge theory, for which none of the arguments presented above applies. As we have discussed above, these theories also have MQCD metastable DSB vacua, and their coupling constants are linear in the inverse of the energy, and hence do not correspond to bulk fields that grow in the UV and that could amplify the tiny infrared difference between the BPS and non-BPS solutions into a non-normalizable UV mode.

Our purpose in this paper is to establish the nature of the fields that are sourced by anti-D3 branes placed at the bottom of the Klebanov-Strassler solution, and to see whether these branes give rise to normalizable or non-normalizable modes. To do this in generality one would need to find the fully-backreacted solution corresponding to anti-D3 branes in the warped deformed conifold. Since these branes are point sources in the internal space, and since moreover they polarize (by the Myers effect [15]) into D5 or NS5 branes [5], the fields of such a non-supersymmetric solution would depend on three or four variables. Finding such a solution is clearly beyond hope with current technology.

One way to simplify the problem is to smear the anti-D3 branes on the S^3 of the deformed conifold, so that the resulting backreacted solution preserves the $SU(2) \times SU(2) \times \mathbb{Z}_2$ symmetry of the original solution. The ansatz for the most general solution preserving this isometry was written down by Papadopoulos and Tseytlin [16], who reduced the supergravity equations of motion to second-order ordinary differential equations for eight scalar functions of the “radial” variable τ of the KS solution.

One might object that this smearing washes down important features of the anti-D3 branes that give the putative metastable vacuum. Indeed, as discussed before, when these anti-D3 branes are together they acquire extra dipole moments by polarization, and hence couple to more supergravity fields than the smeared D3 branes. Furthermore, because of the broken supersymmetry one can argue that the anti-D3 branes cannot remain smeared on the S^3 , but will eventually condense into a single-center configuration. This condensation however takes place over a time that can be tuned to be parametrically large, and hence does not affect the fact that smeared anti-D3 branes give rise to consistent supergravity solution. Furthermore, as the smeared anti-D3 branes couple to *less* fields than the localized polarized ones, their potential for sourcing a potentially-dangerous mode that is non-normalizable in the ultraviolet will be much lower than for the localized and polarized anti-D3 branes. Hence, if the smeared branes give rise to non-normalizable modes, the localized branes will very likely do the same; however, if the smeared branes do not source non-normalizable modes, this does not exclude at all the possibility that the localized polarized branes will source them.

Even after smearing the anti-D3 branes, solving the system of eight coupled nonlinear differential equations is no easy task. To make the general problem tractable one must use the fact that the fields sourced by the anti-D3 branes are subdominant compared to the fields of the background, at least in the region far away from the tip. Hence, one can study these fields by performing a first-order perturbation expansion around the supersymmetric KS solution.

Fortunately, as shown in detail in [17], the second-order equations satisfied by the perturbations of the eight scalar functions of the PT ansatz around the KS solution, factorize into sixteen first-order equations of which eight form a closed system. Note that this factorization does not throw away any of the modes: both the eight second-order equations and the sixteen first-order ones have sixteen integration constants. In fact, as we will show in section 4.4, one of these integration constants is a gauge artifact and another one must be fixed to zero in order for the solutions to the sixteen equations to give a solution to Einstein's equations. Hence, the class of perturbative solutions that we construct depends on fourteen physically-relevant integration constants!

Before beginning it is worth discussing the implication of the potential existence of a non-normalizable mode sourced by anti-D3 branes at the bottom of the Klebanov-Strassler solution. Such anti-branes are part of the “staple diet” of string phenomenology and string cosmology constructions: they are a crucial ingredient in the KKLT construction of de Sitter space [18] and in the KKLMPT model for string theory inflation [13]. These constructions involve placing anti-D3 branes in a Klebanov-Strassler throat that is glued to an ambient Calabi-Yau. The anti-branes lift the energy of the AdS vacuum giving rise to de-Sitter vacua and can create a potential for a probe D3 brane that drives inflation. Furthermore, one generally assumes that the extra energy the branes bring is parametrically redshifted in the KS throat, and can give rise to mass hierarchies.

A non-normalizable mode would “climb” up all the way to the end of the Klebanov-Strassler throat, and will probably affect the gluing of this throat to the ambient Calabi-Yau. However, as this gluing is not understood, it is premature to say whether it will

be negatively affected by the non-normalizable mode. Nevertheless, if a non-normalizable mode is present, its energy will not be localized at the bottom of the KS throat, but rather up the throat, at the junction between this throat and the ambient Calabi-Yau. Hence, the extra energy brought about by an anti-D3 brane will contain both a redshifted contribution, coming from the bottom of the KS throat, and a non-redshifted contribution, from the junction region.

It would be interesting to explore whether this extra contribution to the energy could invalidate the KKLT construction of de Sitter vacua in string theory [18]. We would like to remind the reader that in this construction one adds anti-D3 branes to the bottom of a KS throat in order to lift the AdS vacuum to de Sitter. The contribution of these anti-D3 branes to the total energy needs to be small in order not to overrun the quantum corrections that are needed to stabilize the Kähler moduli. In [18] it was assumed that the contribution of these anti-D3 branes can be made exponentially small by placing them in an arbitrarily-long KS throat. Nevertheless, if anti-D3 branes source non-normalizable modes, the energy they bring cannot be made exponentially small by lengthening the throat. Hence, in some circumstances and for certain non-normalizable modes, this energy may be too large to allow the Kähler moduli to be stabilized. This without doubt negatively impacts the existence of large parts of the landscape of de Sitter vacua in string theory.

On the other hand, as we will show in section 3, the force felt by a probe D3 brane in the background sourced by anti-D3 branes does not depend at all on the presence of extra non-normalizable modes. It is always of the form $1/r^5$, exactly as needed in the KKLM model for inflation in string theory [13].

1.1 Strategy and results

Our goal is to identify the modes sourced by anti-D3 branes, smeared in an $SU(2) \times SU(2) \times \mathbb{Z}_2$ -invariant way at the tip of the warped deformed conifold. This involves solving the eight second-order coupled nonlinear differential equations that govern the metric and fluxes of the Papadopoulos-Tseytlin ansatz [16]. We simplify the problem by solving this system of equations perturbatively, treating the anti-D3 branes as a perturbation of the Klebanov-Strassler solution, and using the fact that the second-order equations factorize into sixteen first-order linear ones out of which form a closed system, that are relatively simpler to solve [17]. One of the main achievements of our paper is that we are able to solve these equations exactly in terms of integrals.

Having set up this rather heavy computational machinery, we then identify which parameters correspond to non-normalizable modes and which do not. Our first strategy is to enforce the correct UV boundary conditions appropriate for the addition of \overline{N} anti-D3 branes in the KS background and investigate the resulting IR behavior. This amounts to requiring that there are $-\overline{N}$ units of additional Maxwell charge, that there is a non-zero force on a probe D3-brane in the UV and the charge is the only non-normalizable mode which we allow for. We find that the resulting infrared geometry contains fields that diverge in a subtle manner which seems to be incompatible with anti-D3 brane sources.

As we will discuss in detail in subsection (6.1.1) these singularities do not appear to have a distinct physical origin, as opposed to say singularities that come from explicit D-

brane sources. On the other hand, they are milder than the latter, as their divergent energy density integrates to a finite action. However, even such mild singularities must normally be excluded in order for the gauge/gravity duality to make sense.³ Whether this singularity is to be allowed or not gives rise to two very distinct conclusions. If the singularity is to be allowed, then we argue that we have found the correct non-supersymmetric linearized solution for anti-D3 branes in KS. If the singularity is to be deemed inadmissible then we argue that there is no solution which can interpolate between the KS boundary conditions in the UV and anti-D3 brane boundary conditions in the IR.

Our second strategy is to enforce anti-D3 brane boundary conditions in the infrared while disallowing the aforementioned singularities and examine the consequences for the ultraviolet. We show that the ultraviolet of this solution cannot match that of the KS solution with no non-normalizable modes turned on. As a consequence we argue that in this case the correct UV behavior is a large deformation of the KS boundary conditions.

There can of course only be one correct solution for anti-D3 branes in KS and since the two conclusions we have offered here differ so wildly from each other that it is of utmost importance to rigorously settle the issue of admissibility of the singularity which we have uncovered. We present various arguments for and against this singularity in the discussion section.

This paper is organized as follows: In section 2 we set up the perturbation theory following [17]. In section 3 we compute the force on a probe D3-brane in our supergravity ansatz and show that it has a simple universal form. In section 4 we solve for the modes and present their perturbative expansions. In section 5 we discuss the boundary conditions for D3 and anti-D3 branes in our backgrounds. In section 6 we attempt to construct the backreacted solution corresponding to a stack of smeared anti-D3 branes in the KS geometry. We conclude and discuss the implications of our results in section 7. Appendix A presents our conventions and appendix B shows the asymptotic expansion of the auxiliary fields.

2 Perturbations around a supersymmetric solution

To solve the IIB supergravity equations perturbatively around a supersymmetric solution, we will use the method developed by Borokhov and Gubser [17], who reduce the set of n second order equations for n fields ϕ^a depending on a single (radial) variable τ to $2n$ first order equations for ϕ^a and their “canonical conjugate variables” ξ_a . For this procedure to work, it is essential that the symmetries of the supergravity problem are sufficiently strong so as to just allow a dependence of the fields on a single, radial, variable.

Since we are perturbing around a known solution, the n linear second order equations from linearizing Einstein’s equation can of course be rewritten as a system of $2n$ first order equations without losing any information. While this can obviously always be done, the formalism of [17] allows eight of these equations to form a closed subsystem when perturbing

³The best example of an unphysical singularity whose action is finite is the negative-mass Schwarzschild solution.

around a supersymmetric solution. Related perturbation theory working directly with the second order equations appeared in [19].

2.1 The first order formalism

The starting point is the one dimensional Lagrangian

$$\mathcal{L} = -\frac{1}{2}G_{ab}\frac{d\phi^a}{d\tau}\frac{d\phi^b}{d\tau} - V(\phi) \tag{2.1}$$

which we require to have the simplifying property that it can be written in terms of a superpotential

$$\mathcal{L} = -\frac{1}{2}G_{ab}\left(\frac{d\phi^a}{d\tau} - \frac{1}{2}G^{ac}\frac{\partial W}{\partial\phi^c}\right)\left(\frac{d\phi^b}{d\tau} - \frac{1}{2}G^{bc}\frac{\partial W}{\partial\phi^c}\right) - \frac{1}{2}\frac{\partial W}{\partial\tau}, \tag{2.2}$$

where

$$V(\phi) = \frac{1}{8}G^{ab}\frac{\partial W}{\partial\phi^a}\frac{\partial W}{\partial\phi^b}. \tag{2.3}$$

The fields ϕ^a are expanded around their supersymmetric background value ϕ_0^a (which will correspond in our case to the Klebanov-Strassler solution [3])

$$\phi^a = \phi_0^a + \phi_1^a(X) + \mathcal{O}(X^2), \tag{2.4}$$

where X represents the set of perturbation parameters, and ϕ_1^a is linear in them. The supersymmetric solution $\phi^a = \phi_0^a$ satisfies the gradient flow equations

$$\frac{d\phi^a}{d\tau} - \frac{1}{2}G^{ab}\frac{\partial W}{\partial\phi^b} = 0, \tag{2.5}$$

while the deviation from the gradient flow equations for the perturbation ϕ_1^a is measured by the conjugate functions ξ_a , given by

$$\xi_a \equiv G_{ab}(\phi_0)\left(\frac{d\phi_1^b}{d\tau} - M^b{}_d(\phi_0)\phi_1^d\right), \quad M^b{}_d \equiv \frac{1}{2}\frac{\partial}{\partial\phi^d}\left(G^{bc}\frac{\partial W}{\partial\phi^c}\right). \tag{2.6}$$

The ξ_a are linear in the expansion parameters X , hence they are of the same order as the ϕ_1^a , and when all the ξ_a vanish the deformation is supersymmetric.

The main point of this construction is that the equations of motion reduce to a set of first order linear equations for (ξ_a, ϕ^a) :

$$\frac{d\xi_a}{d\tau} + \xi_b M^b{}_a(\phi_0) = 0, \tag{2.7}$$

$$\frac{d\phi_1^a}{d\tau} - M^a{}_b(\phi_0)\phi_1^b = G^{ab}\xi_b. \tag{2.8}$$

Note that equations (2.8) are just a rephrasing of the definition of the ξ_a in (2.6), while eqs. (2.7) imply the equations of motion [17].

2.2 Papadopoulos-Tseytlin ansatz for the perturbation

The KS background has $SU(2) \times SU(2) \times \mathbb{Z}_2$ symmetry. We are interested in a solution for the backreaction of a smeared set of anti-D3 branes and we have the liberty to smear these branes without breaking the \mathbb{Z}_2 symmetry (which exchanges the two copies of $SU(2)$). Furthermore one can see from the supergravity equations of motion that the anti-D3 branes and the fields they source do not create a source for the axion C_0 . So we are looking for a family of non-supersymmetric solutions with $SU(2) \times SU(2) \times \mathbb{Z}_2$ symmetry which are continuously connected to the KS solution. The most general ansatz consistent with these symmetries was proposed by Papadopoulos and Tseytlin (PT) [16]

$$ds_{10}^2 = e^{2A+2p-x} ds_{1,3}^2 + e^{-6p-x} d\tau^2 + e^{x+y} (g_1^2 + g_2^2) + e^{x-y} (g_3^2 + g_4^2) + e^{-6p-x} g_5^2, \quad (2.9)$$

where all functions depend on the variable τ . The fluxes and dilaton are

$$\begin{aligned} H_3 &= \frac{1}{2}(k-f)g_5 \wedge (g_1 \wedge g_3 + g_2 \wedge g_4) + d\tau \wedge (f'g_1 \wedge g_2 + k'g_3 \wedge g_4), \\ F_3 &= Fg_1 \wedge g_2 \wedge g_5 + (2P-F)g_3 \wedge g_4 \wedge g_5 + F'd\tau \wedge (g_1 \wedge g_3 + g_2 \wedge g_4), \\ F_5 &= \mathcal{F}_5 + *\mathcal{F}_5, \quad \mathcal{F}_5 = (kF + f(2P-F))g_1 \wedge g_2 \wedge g_3 \wedge g_4 \wedge g_5, \\ \Phi &= \Phi(\tau), \quad C_0 = 0. \end{aligned} \quad (2.10)$$

where P is a constant while f, k and F are functions of τ , and a prime denotes a derivative with respect to τ . There are several different conventions in the literature for the PT ansatz, we compare our conventions to those of others in appendix A.

We will denote the set of functions ϕ^a , $a = 1, \dots, 8$, in the following order

$$\phi^a = (x, y, p, A, f, k, F, \Phi). \quad (2.11)$$

The metric in the one dimensional Lagrangian (2.1) is

$$G_{ab}\phi'^a\phi'^b = e^{4p+4A} \left(x'^2 + \frac{1}{2}y'^2 + 6p'^2 - 6A'^2 + \frac{1}{4}e^{-\Phi-2x}(e^{-2y}f'^2 + e^{2y}k'^2 + 2e^{2\Phi}F'^2) + \frac{1}{4}\Phi'^2 \right)$$

and the superpotential is

$$W(\phi) = e^{4A-2p-2x} + e^{4A+4p} \cosh y + \frac{1}{2}e^{4A+4p-2x} (f(2P-F) + kF). \quad (2.12)$$

At this point it is worth emphasizing an important point regarding the choice of definition for the radial coordinate τ . If one redefines τ in the ten-dimensional ansatz, or equivalently in the Lagrangian (2.1), this will lead to a significant alteration in the equations of motion (2.7) and (2.8) since the metric G_{ab} and the connection M_b^a transform nontrivially. Moreover this also leads to an alteration in the definition of the ξ_a (2.6). Alternatively one might choose to redefine the radial coordinate after having derived the equations of motion (2.7) and (2.8) in which case the only transformation comes from the derivative with respect to the radial coordinate so essentially the other terms just get a multiplicative factor of $(\partial\tilde{\tau}/\partial\tau)^{-1}$. Of course both such redefinitions are completely legitimate, however they are not equivalent since the latter does not alter the definition of the ξ_a .

This is an important consideration since a judicious choice of radial variable can lead to a significant simplification in the equations of motion. This must be kept in mind when comparing the radial coordinates in [17] and [20]. The authors of [20] found a particularly natural choice such that the equations simplify somewhat and we will employ the same choice in the current work (see appendix A for more details).

2.3 The zero-th order solution: Klebanov-Strassler

Since we are expanding around the Klebanov-Strassler solution, we tabulate the form of the zeroth-order functions here:

$$\begin{aligned}
 e^{x_0} &= \frac{1}{4} h^{1/2} \left(\frac{1}{2} \sinh(2\tau) - \tau \right)^{1/3}, \\
 e^{y_0} &= \tanh(\tau/2), \\
 e^{6p_0} &= \frac{24 \left(\frac{1}{2} \sinh(2\tau) - \tau \right)^{1/3}}{h \sinh^2 \tau}, \\
 e^{6A_0} &= \frac{1}{3 \cdot 2^9} h \left(\frac{1}{2} \sinh(2\tau) - \tau \right)^{2/3} \sinh^2 \tau, \\
 f_0 &= -P \frac{(\tau \coth \tau - 1)(\cosh \tau - 1)}{\sinh \tau}, \\
 k_0 &= -P \frac{(\tau \coth \tau - 1)(\cosh \tau + 1)}{\sinh \tau}, \\
 F_0 &= P \frac{(\sinh \tau - \tau)}{\sinh \tau}, \\
 \Phi_0 &= 0
 \end{aligned} \tag{2.13}$$

where the function $h(\tau)$ related to the warp factor is given by the integral⁴

$$\begin{aligned}
 h &= e^{-4A_0 - 4p_0 + 2x_0} \\
 &= h_0 - 32P^2 \int_0^\tau \frac{t \coth t - 1}{\sinh^2 t} \left(\frac{1}{2} \sinh(2t) - t \right)^{1/3} dt.
 \end{aligned} \tag{2.14}$$

There is an important difference between the Klebanov-Tseytlin (KT) solution [11] and the Klebanov-Strassler (KS) one [3]. The KT solution has a warp factor

$$h_{KT}(r) = \frac{27\pi \left(N + aP^2 \ln(r/r_0) + aP^2/4 \right)}{4r^4}, \quad a = 3/(2\pi) \tag{2.15}$$

which has a naked singularity at some finite distance r_s where h becomes zero. The leading UV behavior of the KT solution does in fact agree with the KS solution with the identification $r = e^{\tau/3}$ but the subleading behavior of the KT solution is ambiguous due to ability to absorb a shift into the singularity. For this reason it is not reasonable to compare the subleading behavior of the KT and KS solution and in fact deformations of the KT solution (in particular the deformation proposed in [12]) which have subleading terms are not in general solutions of (2.7) and (2.8) with the ϕ_0^a of the KS solution.

⁴We have $h_0 = 32P^2 \int_0^\infty \frac{\tau \coth \tau - 1}{\sinh^2 \tau} \left(\frac{1}{2} \sinh(2\tau) - \tau \right)^{1/3} d\tau = 18.2373P^2$.

2.4 ξ_i equations

The first step is to solve the system of equations (2.7) for ξ_i . In [17] this system was solved perturbatively in the UV and IR separately. Since our goal in this work is to connect the UV and IR asymptotics, that analysis will not suffice for our purposes. In [20] this system was solved analytically with the assumption that $\xi_1 = \xi_3 = \xi_4 = 0$, which remarkably corresponds exactly to the non-supersymmetric deformation generated by a mass term for the gaugino in the dual field theory (often referred to as the $N = 0^*$ flow). This solution is still not general enough for our purposes, we need to lift all of these constraints and consider the most general solution space.

We find that to solve the ξ_i -equations in general, it is convenient to pass to the basis⁵

$$\tilde{\xi}_a \equiv (3\xi_1 - \xi_3 + \xi_4, \xi_2, -3\xi_1 + 2\xi_3 - \xi_4, -3\xi_1 + \xi_3 - 2\xi_4, \xi_5 + \xi_6, \xi_5 - \xi_6, \xi_7, \xi_8). \quad (2.16)$$

The equations in the order in which we solve them, are

$$\tilde{\xi}'_1 = e^{-2x_0} (2P f_0 - F_0(f_0 - k_0)) \tilde{\xi}_1 \quad (2.17)$$

$$\tilde{\xi}'_4 = -e^{-2x_0} (2P f_0 - F_0(f_0 - k_0)) \tilde{\xi}_1 \quad (2.18)$$

$$\tilde{\xi}'_5 = -\frac{1}{3} P e^{-2x_0} \tilde{\xi}_1 \quad (2.19)$$

$$\tilde{\xi}'_6 = -\tilde{\xi}'_7 - \frac{1}{3} e^{-2x_0} (P - F_0) \tilde{\xi}_1 \quad (2.20)$$

$$\tilde{\xi}'_7 = -\sinh(2y_0) \tilde{\xi}'_5 - \cosh(2y_0) \tilde{\xi}'_6 + \frac{1}{6} e^{-2x_0} (f_0 - k_0) \tilde{\xi}_1 \quad (2.21)$$

$$\tilde{\xi}'_8 = (P e^{2y_0} - \sinh(2y_0) F_0) \tilde{\xi}'_5 + (P e^{2y_0} - \cosh(2y_0) F_0) \tilde{\xi}'_6 + \frac{1}{2} (f_0 - k_0) \xi_7 \quad (2.22)$$

$$\tilde{\xi}'_3 = 3e^{-2x_0 - 6p_0} \tilde{\xi}'_3 + (5e^{-2x_0 - 6p_0} - e^{2x_0} (2P f_0 - F_0(f_0 - k_0))) \tilde{\xi}_1 \quad (2.23)$$

$$\begin{aligned} \tilde{\xi}'_2 = & \tilde{\xi}_2 \cosh y_0 + \frac{1}{3} \sinh y_0 (2\tilde{\xi}_1 + \tilde{\xi}_3 + \tilde{\xi}_4) \\ & + 4 \left((P e^{2y_0} - \cosh(2y_0) F_0) \tilde{\xi}'_5 + (P e^{2y_0} - \sinh(2y_0) F_0) \tilde{\xi}'_6 \right) \end{aligned} \quad (2.24)$$

where a prime indicates derivative with respect to τ .

2.5 ϕ^i Equations

Once the ξ_i equations are solved, one can insert the solutions in the ϕ^j equations, which we write here explicitly. We start by performing a linear field redefinition⁶

$$\tilde{\phi}_a = (x - 2p - 5A, y, x + 3p, x - 2p - 2A, f, k, F, \Phi). \quad (2.25)$$

where all the fields on the right hand side are the first order perturbations.

Then the system of equations is (in the order in which we will solve them)

$$\tilde{\phi}'_8 = -4e^{-4(A_0 + p_0)} \tilde{\xi}_8 \quad (2.26)$$

⁵The inverse is $\xi_a = \left(\frac{1}{3}(3\tilde{\xi}_1 + \tilde{\xi}_3 + \tilde{\xi}_4), \tilde{\xi}_2, \tilde{\xi}_1 + \tilde{\xi}_3, -\tilde{\xi}_1 - \tilde{\xi}_4, \frac{1}{2}(\tilde{\xi}_5 + \tilde{\xi}_6), \frac{1}{2}(\tilde{\xi}_5 - \tilde{\xi}_6), \tilde{\xi}_7, \tilde{\xi}_8 \right)$

⁶The inverse is $\phi^a - \phi_0^a = \left(\frac{1}{5}(-2\tilde{\phi}_1 + 2\tilde{\phi}_3 + 5\tilde{\phi}_4), \tilde{\phi}_2, \frac{1}{15}(2\tilde{\phi}_1 + 3\tilde{\phi}_3 - 5\tilde{\phi}_4), \frac{1}{3}(-\tilde{\phi}_1 + \tilde{\phi}_4), \tilde{\phi}_5, \tilde{\phi}_6, \tilde{\phi}_7, \tilde{\phi}_8 \right)$ and to avoid confusion we abuse notation at from now on put all indices down.

$$\tilde{\phi}'_2 = -\cosh y_0 \tilde{\phi}_2 - 2e^{-4(A_0+p_0)} \tilde{\xi}_2 \tag{2.27}$$

$$\tilde{\phi}'_3 = -3e^{-6p_0-2x_0} \tilde{\phi}_3 - \sinh y_0 \tilde{\phi}_2 - \frac{1}{6}e^{-4(A_0+p_0)}(9\tilde{\xi}_1 + 5\tilde{\xi}_3 + 2\tilde{\xi}_4) \tag{2.28}$$

$$\tilde{\phi}'_1 = 2e^{-6p_0-2x_0} \tilde{\phi}_3 - \sinh y_0 \tilde{\phi}_2 + \frac{1}{6}e^{-4(A_0+p_0)}(\tilde{\xi}_1 + 3\tilde{\xi}_4) \tag{2.29}$$

$$\tilde{\phi}'_5 = e^{2y_0}(F_0 - 2P)(2\tilde{\phi}_2 + \tilde{\phi}_8) + e^{2y_0} \tilde{\phi}_7 - 2e^{-4(A_0+p_0)+2(x_0+y_0)}(\tilde{\xi}_5 + \tilde{\xi}_6) \tag{2.30}$$

$$\tilde{\phi}'_6 = e^{-2y_0}(F_0(2\tilde{\phi}_2 - \tilde{\phi}_8) - \tilde{\phi}_7) - 2e^{-4(A_0+p_0)+2(x_0-y_0)}(\tilde{\xi}_5 - \tilde{\xi}_6) \tag{2.31}$$

$$\tilde{\phi}'_7 = \frac{1}{2}(\tilde{\phi}_5 - \tilde{\phi}_6 + (k_0 - f_0)\tilde{\phi}_8) - 2e^{-4(A_0+p_0)+2x_0} \tilde{\xi}_7 \tag{2.32}$$

$$\begin{aligned} \tilde{\phi}'_4 = & \frac{1}{5}e^{-2x_0}(f_0(2P - F_0) + k_0 F_0)(2\tilde{\phi}_1 - 2\tilde{\phi}_3 - 5\tilde{\phi}_4) + \frac{1}{2}e^{-2x_0}(2P - F_0)\tilde{\phi}_5 \\ & + \frac{1}{2}e^{-2x_0}F_0\tilde{\phi}_6 + \frac{1}{2}e^{-2x_0}(k_0 - f_0)\tilde{\phi}_7 - \frac{1}{3}e^{-4(A_0+p_0)}\tilde{\xi}_1 \end{aligned} \tag{2.33}$$

2.6 Imaginary (anti)-self duality of the three form flux

Before moving on to solving the first order system of equations, it is worthwhile considering the conditions for self duality of the three form flux. The complex three-form flux of the KS solution is well known to be imaginary self dual (as are all warped Calabi-Yau solutions of IIB supergravity [21, 22]) and one might be tempted to consider the possibility that the flux turned on by an anti-D3 brane is imaginary anti-self dual. It is important to note that since the metric is deformed at first order (as opposed to deformations of AdS_5 [23]), it is not consistent to consider the self duality of the flux with respect to the background metric. Instead, one should analyze the condition for self duality using the perturbed metric.

In our conventions, the condition for imaginary self duality is

$$\text{ISD} : e^\Phi * F_3 = -H_3, \tag{2.34}$$

and the condition for imaginary anti self duality is

$$\text{anti-ISD} : e^\Phi * F_3 = +H_3. \tag{2.35}$$

In terms of the PT ansatz, these conditions lead to

$$\begin{aligned} e^\Phi F e^{-2y} &= \mp k' \\ e^\Phi (2P - F) e^{2y} &= \mp f' \\ F' &= \mp \frac{1}{2} e^{-\Phi} (k - f). \end{aligned} \tag{2.36}$$

with the upper sign for the ISD condition.

Using eqs. (2.30)–(2.26) and the fact that the zeroth order three-form flux is imaginary self dual, the conditions that the first order flux be ISD become

$$\begin{aligned} (\tilde{\xi}_5 - \tilde{\xi}_6) &= 0, \\ (\tilde{\xi}_5 + \tilde{\xi}_6) &= 0, \\ \tilde{\xi}_7 &= 0. \end{aligned} \tag{2.37}$$

It is easy to see that these immediately lead to $\tilde{\xi}_1 = 0$ and force $(\tilde{\xi}_4, \tilde{\xi}_8)$ to be constant. Hence, the generic solution to the system of first-order equations [17] does not have purely ISD flux, contrary to recent claims in [24]. One can also see this by examining the small- τ expansion of the dilaton. ISD fluxes do not source the dilaton. Allowing for a delta-function source at the origin, the dilaton in an ISD background should be equal to

$$\phi = C_1 + C_2 \left(\frac{1}{\tau} + \frac{2\tau}{15} - \frac{\tau^3}{315} + \dots \right) \tag{2.38}$$

where the constant C_2 multiplies the free Green's function of the deformed conifold [24]. It turns out that the full expansion of the dilaton near the origin also contains even powers of τ . Setting those to zero and demanding the coefficient of the odd terms to be proportional to those in the Green's function (2.38) implies certain conditions on the integration constants ($X_1 = 0, X_5 = -X_6 = X_7$), which are actually weaker than (2.37).

It is interesting to note that all BPS solutions of (2.7), (2.8) have ISD three-form fluxes since for supersymmetric solutions all the ξ 's vanish. If one relaxes the \mathbb{Z}_2 symmetry in the PT ansatz then there exist BPS solutions with non-ISD flux [25, 26].

The conditions for the first order deformation to be anti-ISD are of course more involved:

$$\begin{aligned} (\tilde{\xi}_5 - \tilde{\xi}_6) &= e^{4(A_0+p_0)-2x_0} \left(F_0(2\tilde{\phi}_2 - \tilde{\phi}_8) + \tilde{\phi}_7 \right), \\ (\tilde{\xi}_5 + \tilde{\xi}_6) &= e^{4(A_0+p_0)-2x_0} \left((F_0 - 2P)(2\tilde{\phi}^2 + \tilde{\phi}^8) + \tilde{\phi}^7 \right), \\ \tilde{\xi}_7 &= -\frac{1}{2} e^{4(A_0+p_0)-2x_0} \left((\tilde{\phi}^6 - \tilde{\phi}^5) - \tilde{\phi}^8(k_0 - f_0) \right). \end{aligned} \tag{2.39}$$

One can try arguing that these conditions are necessary in order for the first-order perturbed solution to correspond to anti-D3 branes. However, as we will find in the next section there exists a much simpler and more physical signature of the presence of anti-D3 branes: the force on a probe D3 brane.

3 The force on a probe D3 brane

Before solving the above equations, we compute the force on a probe D3 brane in the perturbed solutions. As is well known, such a brane feels no force in the unperturbed Klebanov-Strassler solution. It appears at first glance that both the Dirac-Born-Infeld (DBI) and the Wess-Zumino (WZ) actions are rather complicated in the perturbed solution, however we show below that by using the first order equations of motion, most terms in the total action cancel and the final expression for the force on a probe D3 brane is quite simple.

The DBI action for the probe D3 brane is

$$\begin{aligned} V^{DBI} &= \sqrt{-g_{00}g_{11}g_{22}g_{33}} \\ &= e^{4A+4p-2x}, \end{aligned} \tag{3.1}$$

and in the first-order expansion, the derivative of this action with respect to τ is

$$\begin{aligned}
 F^{DBI} &= -\frac{dV^{DBI}}{d\tau} \\
 &= -e^{4A_0+4p_0-2x_0}(1+4A_1+4p_1-2x_1)(4A'_0+4p'_0-2x'_0+4A'_1+4p'_1-2x'_1) \\
 &= -\frac{dV_0^{DBI}}{d\tau} - e^{4A_0+4p_0-2x_0}(-2\tilde{\phi}_4(4A'_0+4p'_0-2x'_0)-2\tilde{\phi}'_4), \tag{3.2}
 \end{aligned}$$

where V_0^{DBI} is the DBI action in the unperturbed solution,

$$F_0^{DBI} = -e^{4A_0+4p_0-2x_0}(4A'_0+4p'_0-2x'_0), \tag{3.3}$$

and we have used the definition of $\tilde{\phi}_4$ in (2.25).

To compute the form of the WZ action one needs to know the value of the RR potential $C^{(4)}$ along the worldvolume of the brane, which we obtain by integrating the RR field strength $F^{(5)}$. It is however easier to compute instead the derivative of this action with respect to τ , which gives the force exerted on the D3 brane by the RR field:

$$\begin{aligned}
 F^{WZ} &= -\frac{dV^{WZ}}{d\tau} \\
 &= F_{0123\tau}^{(5)} \\
 &= -(kF + f(2P - F))e^{4A+4p-4x}.
 \end{aligned}$$

The zero-th order and first order WZ forces are

$$F_0^{WZ} = -(k_0F_0 + f_0(2P - F_0))e^{4A_0+4p_0-4x_0} \tag{3.4}$$

$$\begin{aligned}
 F_1^{WZ} &= -e^{4A_0+4p_0-4x_0} [(k_0F_0 + f_0(2P - F_0))(4A_1 + 4p_1 - 4x_1)] \\
 &\quad - e^{4A_0+4p_0-4x_0} [k_1F_0 + F_1(k_0 - f_0) + f_1(2P - F_0)]. \tag{3.5}
 \end{aligned}$$

It is not hard to check that the zeroth-order WZ and DBI contributions to the force cancel, as expected for a BPS D3 brane in the KS background.

Using this, as well as the equation of motion for $\tilde{\phi}'_4$ we get

$$\begin{aligned}
 F_1^{DBI} &= -e^{4A_0+4p_0-2x_0}(-2\tilde{\phi}_4(4A'_0+4p'_0-2x'_0)-2\tilde{\phi}'_4) \\
 &= 2e^{4A_0+4p_0-4x_0} \left[\frac{1}{5}(k_0F_0 + f_0(2P - F_0))(2\tilde{\phi}_1 - 2\tilde{\phi}_3 - 10\tilde{\phi}_4) \right] \\
 &\quad + e^{4A_0+4p_0-4x_0} [k_1F_0 + F_1(k_0 - f_0) + f_1(2P - F_0)] + \frac{2}{3}e^{-2x_0}\tilde{\xi}_1. \tag{3.6}
 \end{aligned}$$

From the definition of the $\tilde{\phi}^i$, we have that

$$2\tilde{\phi}_1 - 2\tilde{\phi}_3 - 10\tilde{\phi}_4 = 10(A_1 + p_1 - x_1),$$

thus the first and the second term of the Wess-Zumino force (3.5) cancel against corresponding terms from the DBI force (3.6). Hence to first order in perturbation theory, the only contribution to the force felt by a probe D3 brane comes from the term proportional to $\tilde{\xi}_1$:

$$F = F^{DBI} + F^{WZ} = \frac{2}{3}e^{-2x_0}\tilde{\xi}_1. \tag{3.7}$$

As we will show in detail in section 4, it is rather straightforward to compute the UV asymptotic expansion of $\tilde{\xi}_1$ by expanding the integrand in (4.1) for large values of τ :

$$\tilde{\xi}_1 = X_1 3(1 - 4\tau)e^{-4\tau/3} + \mathcal{O}(e^{-10\tau/3}) \quad (3.8)$$

By expanding in the ultraviolet e^{-2x_0} as well

$$e^{-2x_0} = \frac{8}{3P^2(4\tau - 1)} + \mathcal{O}(e^{-2\tau}), \quad (3.9)$$

we can see that the UV expansion of the force felt by a probe D3 brane in the first-order perturbed solution is always

$$F_\tau \sim X_1 e^{-4\tau/3} + \mathcal{O}(e^{-10\tau/3}). \quad (3.10)$$

Recalling that in the UV, τ is related to the canonical radial coordinate

$$r = e^{\tau/3}, \quad (3.11)$$

it follows that

$$F_r \sim \frac{X_1}{r^5} + \mathcal{O}\left(\frac{1}{r^{11}}\right) \quad (3.12)$$

and thus the potential goes like

$$V \sim C + \frac{X_1}{r^4}. \quad (3.13)$$

Having obtained this simple universal result, we pause to consider its physical implications. First, the force is largely independent of which modes perturb the KS solution and on whether these modes are normalizable or non-normalizable; it may be zero in certain solutions (such as [20], where $X_1 = 0$) but when non-zero it has a universal form (3.12). Out of the 14 physically-relevant $SU(2) \times SU(2) \times \mathbb{Z}_2$ -preserving perturbation parameters, only one enters in the force.

Second, this result agrees with that obtained in [13]. In that paper, the force felt by a probe anti-D3 brane in KS with D3 branes at the bottom was shown to scale like $1/r^5$ and to be linear in the D3 charge. It was also argued using Newton's third law that the same force should be felt by a probe D3-brane in KS with anti-D3 branes at the bottom. Our result implies therefore that in order to describe anti-D3 branes in the infrared, one *must* have a nontrivial $\tilde{\xi}_1$, and furthermore that the constant X_1 must be proportional to the number of anti-D3 branes.

We would also like to note that in the ultraviolet the mode (in the ϕ_i) proportional to X_1 is a normalizable mode decaying as r^{-8} , in fact this is the most convergent of the 14 modes. The necessity for having such a mode in order to find the r^{-5} force predicted by [13] was discussed in [12], although the mode itself was not identified. However, the force analysis alone in no way supports or disfavors the possibility that an anti-D3 brane sources a non-normalizable mode. The force has exactly the same r -dependence regardless of whether non-normalizable modes are turned on or not. Hence, the cancelation of terms in the force up to order $1/r^4$ is universal.

4 The space of solutions

In this section we find the generic solution to the system (2.17)–(2.33), which depends on sixteen integration constants (although only fourteen are physical). As we will see, the full solution involves integrals that cannot be done explicitly but these integrals can be expanded both in the UV and IR, giving rise to sixteen UV and sixteen IR integration constants. Having the expression for the full solution permits one to relate numerically the UV and IR integration constants.

The first equation (2.17) is solved by

$$\tilde{\xi}_1 = X_1 \exp\left(\int_0^\tau d\tau' e^{-2x_0} [2P f_0 - F_0(f_0 - k_0)]\right) \quad (4.1)$$

and this integral cannot be done explicitly. In a cruel twist, having argued above that the integration constant X_1 is the only mode which contributes to the force on a D3 brane and is thus the signature of the anti-D3 brane background, we have immediately found that as long as X_1 is nonzero there is no analytic expression for any of the ξ_i modes. On the other hand, when $X_1 = 0$, we can fully solve the system of equations for the ξ_i analytically. We proceed by initially setting X_1 to zero, finding an analytic expression for all the other ξ_i , and then building on top of this, the integral expressions for the full solution with nonzero X_1 .

4.1 Solving the ξ_i equations for $X_1 = 0$

Here we solve the ξ_i equations for the choice

$$X_1 = 0 \Rightarrow \tilde{\xi}_1 = 0 \quad (4.2)$$

and later we will find integral expressions for the full solution including $\tilde{\xi}_1$. Having set $\tilde{\xi}_1 = 0$, we then immediately read off that $(\tilde{\xi}_4, \tilde{\xi}_5)$ are constant

$$\tilde{\xi}_4 = X_4, \quad \tilde{\xi}_5 = X_5. \quad (4.3)$$

We now have a pair of coupled o.d.e's for $(\tilde{\xi}_6, \tilde{\xi}_7)$

$$\tilde{\xi}_6' = -\tilde{\xi}_7, \quad (4.4)$$

$$\tilde{\xi}_7' = -\sinh(2y_0)X_5 - \cosh(2y_0)\tilde{\xi}_6. \quad (4.5)$$

Doing the derivative of (4.4) and using (4.5) one easily determines the solution

$$\tilde{\xi}_6 = \frac{(1 + 2\tau)X_5 + X_6 - 2\tau X_7 + X_7 \sinh(2\tau)}{2 \sinh \tau}, \quad (4.6)$$

$$\tilde{\xi}_7 = \frac{((1 + 2\tau)X_5 + X_6 - 2\tau X_7) \cosh \tau - (2X_5 + X_7(\cosh 2\tau - 3)) \sinh \tau}{2 \sinh^2 \tau}. \quad (4.7)$$

To ease notation the two integration constants appearing in the second order equation for $\tilde{\xi}_6$ have been denoted X_6 and X_7 , but one should keep in mind that they both appear in $\tilde{\xi}_6$ and $\tilde{\xi}_7$. To compare to the solution in [20] one sets $X_7 = -X_6 = X_5$.

We then directly integrate (2.22) and (2.23) to find

$$\begin{aligned} \tilde{\xi}_8 = & (1 - e^{2\tau})^{-3} \left(2e^{3\tau} (P((-1 + 2\tau + 4\tau^2)X_5 + X_7 + 2\tau(X_6 - 2\tau X_7))) \cosh \tau \right. \\ & + P(X_5 - X_7) \cosh 3\tau + (3X_8 + P(X_5(1 - 5\tau) - 2X_6 + (-3 + 5\tau)X_7)) \sinh \tau \\ & \left. - (X_8 + P(1 + \tau)(X_5 - X_7)) \sinh 3\tau \right), \end{aligned} \quad (4.8)$$

$$\tilde{\xi}_3 = 2X_3(\sinh 2\tau - 2\tau). \quad (4.9)$$

Finally we insert the result for $\tilde{\xi}_3$ into (2.24) and solve the o.d.e. to find

$$\begin{aligned} \tilde{\xi}_2 = & \frac{1}{6} \left((2X_4 + 4\tau(-4X_3 + 3PX_7)) \cosh \tau + 2(6X_2 - X_4) \sinh \tau \right) \\ & + \frac{P \cosh \tau}{2 \sinh^2 \tau} \left((2(X_5 - X_7) - (X_5(1 + 4\tau) + X_6 - 4\tau X_7)) \right. \\ & \left. + \frac{P\tau}{2 \sinh^3 \tau} (X_5(1 + 2\tau) + X_6 - 2\tau X_7) \right). \end{aligned} \quad (4.10)$$

So far we have introduced 8 integration constants

$$(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) \quad (4.11)$$

however the the zero-energy condition is a constraint amongst these

$$-X_4 + 6X_2 - 3PX_5 - 4X_3 + 9PX_7 = 0. \quad (4.12)$$

This leaves 7 independent constants, but for the time being we will keep the 8 constants since this will provide a clearer picture of the UV asymptotics.

4.2 Solution for $X_1 \neq 0$, UV and IR expansions

When $\tilde{\xi}_1$ is nonzero, one can find integral expressions for the solution, but the integrals cannot be performed explicitly since $\tilde{\xi}_1$ itself, given in (4.1), cannot be found explicitly. $\tilde{\xi}_1$ appears on the right hand side in all of the $\tilde{\xi}$ equations (except for the equation for $\tilde{\xi}_8$, but the right hand side of (2.22) contains $\tilde{\xi}_{5,6,7}$ which are modified by $\tilde{\xi}_1$) and therefore all $\tilde{\xi}$'s get an extra contribution proportional to X_1 . A simple way to understand this extra contribution, is to recall the following method to solve a system of coupled o.d.e's, which will in fact be used to solve the equations for ϕ . Given the system

$$g'_i = A_i^j g_j + b_i \quad (4.13)$$

where $i = 1, \dots, n$, and g, A, b are functions of a single variable τ , one first finds the solutions $g_{h,i}^j$ of the homogeneous equations (again $j = 1, \dots, n$). Then one constructs the ansatz for the inhomogeneous solution by a linear combination of the homogeneous solutions where the coefficients are promoted to functions, and then it is easy to see that one has

$$g_i = \sum_j g_{h,i}^j \lambda_j(\tau), \quad \lambda_j = \int (g_h^{-1})_j^i b_i. \quad (4.14)$$

The integration constant here corresponds to adding any amount of the homogeneous solution to g_i . For the simple case $n = 1$, that will appear repeatedly in the following, we have

$$g(\tau) = \lambda(\tau)g_h(\tau), \quad g_h = e^{\int A}, \quad \lambda = \int_{\tau_0}^{\tau} \frac{b(\tau')}{g_h(\tau')} d\tau', \quad (4.15)$$

where the integration constant is related to the choice of τ_0 .

Adding $\tilde{\xi}_1$ on the right hand side of (2.18)–(2.24) amounts to the change $b_i \rightarrow b_i + X_1 f_i(\tau) \equiv b_i^0 + X_1 b_i^1$ where b_i^1 contains ξ_1 .⁷ This implies

$$\lambda_i \rightarrow \lambda_i + X_1 \int_0^{\tau} d\tau' (g_h^{-1})_i^j b_j^1 \equiv \lambda_i^0 + X_1 \lambda_i^1, \quad g_i \rightarrow g_i + X_1 g_{h,i}^j \lambda_j^1. \quad (4.16)$$

Note that in the expression for λ we have chosen to absorb the n overall integration constants in λ_i^0 and fixed the limits of the integrand in λ_i^1 .

The integrands can always be expanded around the IR and UV, and the integrals can be performed in these limits. Expanding (4.1) for small τ amounts to expanding the integrand in a series of powers of τ , doing the integral explicitly and then expanding the exponential again.⁸ This gives

$$\tilde{\xi}_1^{IR} = X_1^{IR} \left(1 - \frac{16 \left(\frac{2}{3}\right)^{1/3} P^2 \tau^2}{3 h_0} + \mathcal{O}(\tau^4) \right). \quad (4.17)$$

The contribution from $\tilde{\xi}_1$ on the rest of the $\tilde{\xi}$'s can be obtained similarly by expanding the integrands in λ_i^a . We give the IR behavior of all the ξ 's in appendix B.1.

The UV behavior is slightly more tricky. On one hand, the expansion is in powers of $e^{-\tau/3} \sim 1/r$, where at each power there are terms proportional to $\tau^n \sim (\log r)^n$ (typically only $n = 0$ and $n = 1$ appear). Fortunately all these terms can easily be integrated. On the other hand, expressions like (4.1) are split into

$$\begin{aligned} \tilde{\xi}_1 &= X_1 \exp \left(\int_0^{\tau} d\tau' f(\tau') \right) = X_1^{IR} \exp \left(\int_0^{\infty} d\tau' f(\tau') \right) \exp \left(\int_{\infty}^{\tau} d\tau' f(\tau') \right) \\ &\equiv X_1^{UV} \exp \left(\int_{\infty}^{\tau} d\tau' f(\tau') \right) \end{aligned} \quad (4.18)$$

where the integrand can now be expanded around large τ , and the integral can be done analytically. The integral from zero to infinity giving the relation between X_1^{UV} and $X_1 \equiv X_1^{IR}$ can in principle be done numerically. This gives

$$\tilde{\xi}_1^{UV} = X_1^{UV} e^{-4\tau/3} (3 - 12\tau) + \mathcal{O}(e^{-10\tau/3}) \quad (4.19)$$

⁷The indices 0 and 1 in b are used to distinguish between the expression without $\tilde{\xi}_1$ and that purely due to $\tilde{\xi}_1$. They should not be confused with zero and first order in perturbation theory (they are both first order).

⁸In general, if for a given expression to be integrated in the IR there are negative powers of τ showing up in the expansion, we subtract the infinite contribution from the lower limit of the integral, namely we replace $\int_0^{\tau} f_n t^{-n} \rightarrow \int^{\tau} f_n t^{-n} = f_n \frac{t^{-n+1}}{-n+1}$ (for $n \neq 1$).

For the other $\tilde{\xi}$'s, one uses the same expansions to obtain the UV behavior of λ_i^1 . We get from (4.16)

$$X_1 \lambda_i^1 = X_1 \left(\int_0^\infty d\tau' (g_h^{-1})_i{}^j b_j^1 + \int_\infty^\tau d\tau' (g_h^{-1})_i{}^j b_j^1 \right) = X_1 \left(k_i + \int_\infty^\tau d\tau' (g_h^{-1})_i{}^j b_j^1 \right), \quad (4.20)$$

where k_i are some constants that can be obtained numerically, and in the remaining integral one expands the integrand for large τ and performs the integral analytically. The contributions $X_1 k_i$ shift the constants of integration in λ_i^0 . We therefore define

$$X_a^{UV} \equiv X_a^{IR} + X_1 k_a. \quad (4.21)$$

We give the UV expansion of all the ξ 's in appendix B.2.

4.3 Solving the ϕ^i equations

4.3.1 The space of solutions

Here we show how to solve the system of ϕ^i equations (2.26)–(2.33), assuming $X_1 = 0$. First we note that $\tilde{\phi}_8$ is given by the following integral, which however cannot be solved analytically

$$\tilde{\phi}_8 = -64 \int d\tau \frac{\tilde{\xi}_8}{(\frac{1}{2} \sinh(2\tau) - \tau)^{2/3}} + Y_8^{IR}, \quad (4.22)$$

where $\tilde{\xi}_8$ is given in (4.8).

The equation for $\tilde{\phi}^2$ is solved using (4.15), which implies

$$\tilde{\phi}_2 = \frac{\tilde{\lambda}^2(\tau)}{\sinh \tau}, \quad \tilde{\lambda}^2 = -32 \int d\tau \frac{\sinh \tau \tilde{\xi}_2}{(\frac{1}{2} \sinh(2\tau) - \tau)^{2/3}} + Y_2^{IR} \quad (4.23)$$

where $(\sinh \tau)^{-1}$ is the solution to the homogeneous equation, and $\tilde{\xi}_2$ is given in (4.10). The same trick is used for $\tilde{\phi}^3$, which is given by

$$\begin{aligned} \tilde{\phi}_3 &= \frac{\tilde{\lambda}_3(\tau)}{\sinh(2\tau) - 2\tau}, \quad (4.24) \\ \tilde{\lambda}_3 &= \int d\tau (\sinh(2\tau) - 2\tau) \left(\frac{\tilde{\phi}_2}{\sinh \tau} - \frac{16 (\sinh(2\tau) - 2\tau) 5\tilde{X}_3 + \tilde{X}_4}{3 (\frac{1}{2} \sinh(2\tau) - \tau)^{2/3}} \right) + Y_3^{IR}. \end{aligned}$$

We obtain an integral expression for $\tilde{\phi}^1$:

$$\tilde{\phi}_1 = \int d\tau \left(\frac{8 \sinh^2 \tau \tilde{\phi}_3}{3 \sinh(2\tau) - 2\tau} + \frac{\tilde{\phi}_2}{\sinh \tau} + \frac{32 \tilde{X}_4}{9 (\frac{1}{2} \sinh(2\tau) - \tau)^{2/3}} \right) + Y_1^{IR}. \quad (4.25)$$

and then using (4.14), we find that the fluxes $(\tilde{\phi}_5, \tilde{\phi}_6, \tilde{\phi}_7) = (f, k, F)$ are given by

$$\begin{pmatrix} \tilde{\phi}_5 \\ \tilde{\phi}_6 \\ \tilde{\phi}_7 \end{pmatrix} = \begin{pmatrix} 2\tau - \sinh \tau - \tanh(\tau/2) - \frac{\tau}{2 \cosh^2(\tau/2)} & \frac{1}{2 \cosh^2(\tau/2)} & 1 \\ 2\tau + \sinh \tau - \coth(\tau/2) + \frac{\tau}{2 \sinh^2(\tau/2)} & -\frac{1}{2 \sinh^2(\tau/2)} & 1 \\ \frac{\tau}{\sinh \tau} - \cosh \tau & -\frac{1}{\sinh \tau} & 0 \end{pmatrix} \begin{pmatrix} \lambda_5 \\ \lambda_6 \\ \lambda_7 \end{pmatrix}. \quad (4.26)$$

This 3×3 matrix contains the solutions to the homogeneous equations, and the derivatives of the functions λ_a are given by

$$\begin{pmatrix} \lambda'_5 \\ \lambda'_6 \\ \lambda'_7 \end{pmatrix} = \begin{pmatrix} -\frac{1}{4 \sinh \tau} & \frac{1}{4 \sinh \tau} & -\frac{\cosh \tau}{2 \sinh^2 \tau} \\ \frac{1}{4} \left(\cosh \tau - \frac{\tau}{\sinh \tau} \right) & -\frac{1}{4} \left(\cosh \tau - \frac{\tau}{\sinh \tau} \right) & -\frac{(\sinh(3\tau) + 4\tau \cosh \tau - 7 \sinh \tau)}{8 \sinh^2 \tau} \\ \frac{1}{2} \left(1 + \frac{\tau}{\sinh \tau} \right) & \frac{1}{2} \left(1 - \frac{\tau}{\sinh \tau} \right) & \frac{1}{\sinh \tau} \left(-1 + \tau \frac{\cosh \tau}{\sinh \tau} \right) \end{pmatrix} \begin{pmatrix} b_5 \\ b_6 \\ b_7 \end{pmatrix} \quad (4.27)$$

where b_5, b_6, b_7 are the rhs of (2.30), (2.31) and (2.32) respectively, and the 3×3 matrix is the inverse of the one in (4.26). We will call the constants of integration in these equations Y_5, Y_6, Y_7 , even though the three functions depend on the three of them (i.e. the integration constant in $\tilde{\phi}_5$ is not just Y_5 , but a combination of $Y_{5,6,7}$).

Finally, the equations for $\tilde{\phi}^4$ is solved using the same trick, namely (4.15) except that the solution to the homogeneous equation can be found only as an integral expression. Formally, it is

$$\tilde{\phi}_4 = \lambda_4 \tilde{\phi}_{4h} \quad , \quad \lambda_4 = \int \frac{b_4}{\tilde{\phi}_{4h}} + Y_4^{IR} \quad (4.28)$$

where b_4 is the right hand side of (2.33) (setting $\tilde{\phi}^4$ to zero) and

$$\tilde{\phi}_{4h} = \exp \left[8 P^2 \int d\tau \frac{\tau \sinh(3\tau) - \cosh(3\tau) + 5\tau \sinh \tau - \cosh \tau (4\tau^2 - 1)}{h(\tau) \sinh^3 \tau (\cosh \tau \sinh \tau - \tau)^{2/3}} \right]. \quad (4.29)$$

4.3.2 IR behavior

We now give the IR expansions of the ϕ^i . We write the divergent terms and the constant terms since terms which are regular in the IR will not provide any constraints on our solution space. Here we have used the zero energy condition (4.32) to substitute for X_2^{IR} .

$$\begin{aligned} \tilde{\phi}_8 &= \frac{16}{\tau} (2/3)^{1/3} (6X_8^{IR} + P(5X_5^{IR} - X_6^{IR} - 6X_7^{IR})) + Y_8^{IR} + \mathcal{O}(\tau) \\ \tilde{\phi}_2 &= \frac{1}{\tau} Y_2^{IR} + \frac{\log \tau}{\tau} 16(2/3)^{1/3} \left(- \left(\frac{1}{3} + 16(2/3)^{1/3} \frac{P^2}{h_0} \right) X_1^{IR} \right. \\ &\quad \left. + (-2X_4^{IR} + PX_5^{IR} + PX_6^{IR}) \right) + \mathcal{O}(\tau) \\ \tilde{\phi}_3 &= \frac{1}{\tau^3} \frac{3Y_3^{IR}}{4} + \frac{1}{\tau} \left(\frac{Y_2^{IR}}{2} - \frac{3Y_3^{IR}}{20} + \frac{X_1^{IR} 64(2/3)^{2/3} P^2}{h_0} - P(X_5^{IR} + X_6^{IR}) 4(2/3)^{1/3} \right) \\ &\quad + \frac{\log \tau}{\tau} \left(-X_1^{IR} (8(2/3)^{1/3}/3 + (2/3)^{2/3} 128P^2/h_0) + \right. \\ &\quad \left. 8(2/3)^{1/3} (-2X_4^{IR} + P(X_5^{IR} + X_6)) \right) + \mathcal{O}(\tau) \end{aligned}$$

$$\begin{aligned}
 \tilde{\phi}_1 &= -\frac{1}{\tau^3} \frac{Y_3^{IR}}{2} + \frac{1}{\tau} \left(-2Y_2^{IR} + \frac{Y_3^{IR}}{10} + X_1^{IR} \left(10(2/3)^{4/3} + \frac{2^{2/3} 3^{1/3} 128 P^2}{h_0} \right) \right. \\
 &\quad \left. + 4(2/3)^{1/3} (10X_4^{IR} - 6P(X_5^{IR} + X_6^{IR})) \right) \\
 &\quad + \frac{\log \tau}{\tau} \left(X_1^{IR} \left(16(2/3)^{4/3} + \frac{(2/3)^{2/3} 512 P^2}{h_0} \right) \right. \\
 &\quad \left. + X_4^{IR} 64(2/3)^{1/3} - (X_5^{IR} + X_6^{IR} 32(2/3)^{1/3}) \right) + Y_1^{IR} + \mathcal{O}(\tau) \\
 \tilde{\phi}_5 &= \frac{Y_6^{IR}}{2} + Y_7^{IR} + X_1^{IR} \frac{16P}{3} \left(\frac{16(2/3)^{2/3} P^2}{h_0} - (2/3)^{1/3} \right) - X_4^{IR} 16(2/3)^{1/3} P + \mathcal{O}(\tau) \\
 \tilde{\phi}_6 &= \frac{1}{\tau^2} \left(-2Y_6^{IR} + (2/3)^{1/3} 32P X_1^{IR} - (4/3) h_0 (X_5^{IR} + X_6^{IR}) \right) \\
 &\quad + \left(\frac{Y_6^{IR}}{6} + Y_7^{IR} + X_4^{IR} (2/3)^{1/3} 16P \right. \\
 &\quad \left. - \left(\frac{2h_0}{9} + \frac{64}{3} (1/3)^{1/3} P^2 \right) X_5^{IR} - 32(2/3)^{1/3} P X_8^{IR} \right. \\
 &\quad \left. - \frac{2P}{3} Y_2^{IR} - \left(\frac{2h_0}{9} + \frac{32}{3} (1/3)^{1/3} P^2 \right) X_6^{IR} + 32(2/3)^{1/3} P^2 (X_6^{IR}/3 + X_7^{IR}) \right) \\
 &\quad + \log \tau \left(X_1^{IR} \left(\frac{(2/3)^{1/3} 32P}{9} + \frac{512(2/3)^{2/3} P^3}{3h_0} \right) \right. \\
 &\quad \left. + \frac{32(2/3)^{1/3} P}{3} (2X_4^{IR} - P^2(X_5^{IR} + X_6^{IR})) \right) \\
 &\quad + \mathcal{O}(\tau) \\
 \tilde{\phi}_7 &= \frac{1}{\tau} \left(-Y_6^{IR} + \frac{h_0}{3} (X_5^{IR} + X_6^{IR}) \right) + \mathcal{O}(\tau) \\
 \tilde{\phi}_4 &= \frac{1}{\tau} \left(-\frac{16(2/3)^{1/3} P Y_6^{IR}}{h_0} - \frac{2^{1/3} 3^{2/3} 8 P Y_7^{IR}}{h_0} \right. \\
 &\quad \left. + X_1^{IR} \left(8(2/3)^{1/3} + 64(2/3)^{2/3} P^2/h_0 - \frac{4096 P^4}{3h_0^2} \right) \right. \\
 &\quad \left. + X_4^{IR} \frac{256(3/2)^{1/3} P^2}{h_0} + 8(2/3)^{4/3} (X_5^{IR} + X_6^{IR}) - \frac{16(2/3)^{1/3} P^2}{3h_0} Y_3^{IR} \right) \\
 &\quad + Y_4^{IR} + \mathcal{O}(\tau)
 \end{aligned}$$

4.3.3 UV behavior

Here we provide the UV asymptotics for the $\tilde{\phi}^i$ including terms which fall off as $e^{-4\tau/3}$ and slower. However as shown in the table 1, certain modes have leading behavior in the UV

which is more convergent than this.

$$\begin{aligned}
 \tilde{\phi}_8 &= Y_8^{UV} + 2^{1/3}24e^{-4\tau/3}(P(3+4\tau)(X_5^{UV} - X_7^{UV}) + 4X_8^{UV}) + \mathcal{O}(e^{-10\tau/3}) \\
 \tilde{\phi}_2 &= -2^{1/3}48e^{-\tau/3}(2X_2^{UV} - (-3+2\tau)(2X_3^{UV}/3 - PX_7^{UV})) + 2e^{-\tau}Y_2^{UV} + \mathcal{O}(e^{-7\tau/3}) \\
 \tilde{\phi}_3 &= -2^{1/3}10X_3^{UV}e^{2\tau/3} \\
 &\quad - e^{-4\tau/3}2^{10/3}\left(36X_2^{UV} + 2X_4^{UV} + X_3^{UV}\left(112 - \frac{137}{3}\tau\right) - 36PX_7^{UV}(3-\tau)\right) \\
 &\quad + \mathcal{O}(e^{-2\tau}) \\
 \tilde{\phi}_1 &= Y_1^{UV} - 2^{1/3}20X_3^{UV}e^{2\tau/3} + 2^{1/3}4(108X_2^{UV} + X_4^{UV} + 176X_3^{UV} - 189PX_7^{UV})e^{-4\tau/3} \\
 &\quad - e^{-4\tau/3}\tau 2^{1/3}16\left(\frac{79}{3}X_3^{UV} - 27PX_7^{UV}\right) + \mathcal{O}(e^{-2\tau}) \\
 \tilde{\phi}_5 &= -\frac{Y_5^{UV}}{2}e^\tau + (-Y_5^{UV} + Y_7^{UV}) + (2Y_5^{UV} - PY_8^{UV})\tau \\
 &\quad + 2^{1/3}36(2PX_2^{UV} + 7PX_3^{UV} + 7P^2X_7^{UV})e^{-\tau/3} - \frac{144PX_3^{UV}}{3}e^{-\tau/3}\tau \\
 &\quad + \frac{1}{2}(5Y_5^{UV} + 4Y_6^{UV} + 2PY_2^{UV} - 2PY_8^{UV})e^{-\tau} + \frac{1}{2}(-4Y_5^{UV} + 2PY_8^{UV})e^{-\tau}\tau \\
 &\quad + 2^{1/3}72P(-4X_2^{UV} + X_8^{UV} - 5X_3^{UV} + 2P(X_5^{UV} + X_6^{UV}))e^{-4\tau/3} \\
 &\quad - 2^{1/3}72P\left(4X_2^{UV} - 2PX_5^{UV} - \frac{2X_3^{UV}}{3} + PX_7^{UV}\right)e^{-4\tau/3}\tau \\
 &\quad - 2^{1/3}72P\left(-\frac{8X_3^{UV}}{3} + 4PX_7^{UV}\right)e^{-4\tau/3}\tau^2 + \mathcal{O}(e^{-2\tau}) \\
 \tilde{\phi}_6 &= +\frac{Y_5^{UV}}{2}e^\tau + (-Y_5^{UV} + Y_7^{UV}) + (2Y_5^{UV} - PY_8^{UV})\tau \\
 &\quad - 2^{1/3}36(2PX_2^{UV} + 7PX_3^{UV} + 7P^2X_7^{UV})e^{-\tau/3} + \frac{144PX_3^{UV}}{3}e^{-\tau/3}\tau \\
 &\quad - \frac{1}{2}(5Y_5^{UV} + 4Y_6^{UV} + 2PY_2^{UV} - 2PY_8^{UV})e^{-\tau} - \frac{1}{2}(-4Y_5^{UV} + 2PY_8^{UV})e^{-\tau}\tau \\
 &\quad + 2^{1/3}72P(-4X_2^{UV} + X_8^{UV} - 5X_3^{UV} + 2P(X_5^{UV} + X_6^{UV}))e^{-4\tau/3} \\
 &\quad - 2^{1/3}72P\left(4X_2^{UV} - 2PX_5^{UV} - \frac{2X_3^{UV}}{3} + PX_7^{UV}\right)e^{-4\tau/3}\tau \\
 &\quad - 2^{1/3}72P\left(-\frac{8X_3^{UV}}{3} + 4PX_7^{UV}\right)e^{-4\tau/3}\tau^2 + \mathcal{O}(e^{-2\tau}) \\
 \tilde{\phi}_7 &= -\frac{Y_5^{UV}}{2}e^\tau + 2^{1/3}36P(-6X_2^{UV} - (9-4\tau)X_3^{UV} + (10-4\tau)PX_7^{UV})e^{-\tau/3} \\
 &\quad + \left(\frac{1}{2}(4\tau-1)Y_5^{UV} - 2Y_6^{UV} + PY_2 + P(\tau-1)Y_8^{UV}\right)e^{-\tau} + \mathcal{O}(e^{-7\tau/3})
 \end{aligned}$$

dim Δ	non-norm/norm	int. constant
8	r^4/r^{-8}	Y_4/X_1
7	r^3/r^{-7}	Y_5/X_6
6	r^2/r^{-6}	X_3/Y_3
5	r/r^{-5}	---
4	r^0/r^{-4}	$Y_7, Y_8, Y_1/X_5, X_4, X_8$
3	r^{-1}/r^{-3}	$X_2, X_7/Y_6, Y_2$
2	r^{-2}/r^{-2}	---

Table 1. The UV behavior of all sixteen modes in the $SU(2) \times SU(2) \times \mathbb{Z}_2$ symmetric deformation of the Klebanov-Strassler solution.

$$\begin{aligned}
 \tilde{\phi}_4 = & \frac{Y_4^{UV}}{3(4\tau - 1)} e^{4\tau/3} + \frac{2^{1/3} 16 X_3^{UV} (2\tau + 1)}{(4\tau - 1)} e^{2\tau/3} \\
 & + \frac{-4Y_7^{UV} + (3 + 4\tau)Y_8^{UV}}{2P(4\tau - 1)} - \frac{1}{P} Y_5^{UV} \\
 & - \frac{2Y_1^{UV}}{5} + \frac{32Y_4^{UV} (12 - 85\tau + 25\tau^2)}{1125(1 - 4\tau)^2} e^{-2\tau/3} \\
 & + \frac{2^{1/3}}{4\tau - 1} \left(-18P(11 + 8\tau)X_5^{UV} + 72(1 + 8\tau)X_2^{UV} + 2(-5 + 8\tau)X_4^{UV} - 72X_8^{UV} \right. \\
 & \quad \left. + 9P(95 - 56\tau + 80\tau^2)X_7^{UV} \right. \\
 & \quad \left. - \frac{64X_3(-3279 + 17012\tau - 20785\tau^2 + 14900\tau^3)}{375(4\tau - 1)} \right) e^{-4\tau/3} + \mathcal{O}(e^{-2\tau})
 \end{aligned}$$

To understand the holographic physics of the $\tilde{\phi}^i$ modes it is useful to tabulate the leading UV behavior coming from each mode. For each local operator \mathcal{O}_i of quantum dimension Δ in the field theory, the well known holographic dictionary [27, 28] relates two modes in dual AdS space, one normalizable and one non-normalizable. These two gravity modes are dual respectively to the vacuum expectation value (VEV) $\langle 0|\mathcal{O}_i|0\rangle$ and the deformation of the action $\delta S \sim \int d^d x \mathcal{O}_i$:

$$\begin{aligned}
 \text{normalizable modes} & \sim r^{-\Delta} \leftrightarrow \text{field theory VEV's} \\
 \text{non-normalizable modes} & \sim r^{\Delta-4} \leftrightarrow \text{field theory deformations of the action,}
 \end{aligned}$$

and we recall that we have in the UV $r = e^{\tau/3}$. For backgrounds like Klebanov-Strassler which are asymptotically AdS only up to certain logarithm terms, it is expected that this dictionary still holds. In table 1 we have summarized which integration constants correspond to normalizable and non-normalizable modes. It is very interesting to note that in all cases a normalizable/non-normalizable pair consists of one BPS mode and one non-BPS mode.

It perhaps bears repeating here that the X_i are integration constants for the ξ_i modes and break supersymmetry, while the Y_i are integration constants for the modes ϕ^i . One key

result from this table which cannot be gleaned from the field expansions we have provided, is that the mode ξ_1 , whose integration constant is X_1 and which is the only mode responsible for the force on a probe D3-brane, is the most convergent mode in the UV.

4.4 The zero energy condition and gauge freedom

In addition to the first-order equations of motion (2.5) there is a constraint, coming from the $R_{\tau\tau}$ component of Einstein's equation:

$$\frac{1}{2}G_{ab} \frac{d\phi^a}{d\tau} \frac{d\phi^b}{d\tau} = V(\phi) \tag{4.30}$$

which must be enforced in order for the solutions of our system of equations to correspond to solutions of supergravity. This constraint is imposed after fixing the gauge freedom associated with redefining the radial coordinate. When linearizing the PT ansatz around the KS background this becomes a constraint on just the ξ_a

$$\xi_a \frac{d\phi_0^a}{d\tau} = 0. \tag{4.31}$$

In addition, the common rescaling of the coordinates (t, x_1, x_2, x_3) can be used to eliminate the integration constant in the metric function A (since $A = \frac{1}{3}(-\tilde{\phi}_1 + \tilde{\phi}_4)$ in the UV this integration constant is Y_1^{UV}). Hence the linearized solution depends on an even number of physically-relevant parameters (fourteen), consistent with the expectation from the AdS-CFT correspondence.

On our solution to (2.7) and (2.8), we find that the zero energy condition is a linear algebraic relation amongst the integration constants

$$-6X_2 + 4X_3 + X_4 + 3X_5 - 9X_7 = 0 \tag{4.32}$$

(note that X_1 does not enter in the condition). Using this to eliminate X_4 or X_5 then leaves just two normalizable and two non-normalizable modes with $\Delta = 4$ in table 1.

5 Boundary conditions for three-branes

Within the space of solutions we have derived in section 4 we are interested in finding the modes which might result from the backreaction of a collection of smeared anti-D3 branes. These branes are placed at $\tau = 0$ and smeared on the finite size S^3 , and for describing them it is necessary to carefully impose the correct infrared boundary conditions.

The gravity solution for a stack of localized D3-branes in flat space has a warp factor $h(r) = 1 + Q/r^4$ and as $r \rightarrow 0$ the full solution is smooth due to the infinite throat. However when these branes are smeared in n -dimensions, the warp factor will scale as r^{-4+n} as $r \rightarrow 0$ since it is now the solution to a wave equation in dimension $d = 6 - n$. This is the IR boundary condition we will impose on the metric.

In addition we must impose boundary conditions on the various fluxes. This is relatively straightforward for D3-branes in flat space, where the energy from $F^{(5)}$ cancels against that from the curvature. In the presence of other types of flux, the IR boundary

conditions are slightly more subtle. When the background is on-shell, contributions to the stress tensor from all types of flux taken together cancel the energy from the curvature: this is the basic nature of Einstein’s equation but this is too loose a criterion to signal the presence of D3 branes. The correct boundary conditions for D3-branes should require the dominant contribution to the stress-energy tensor to come from the $F^{(5)}$ flux.

5.1 BPS three-branes

The Klebanov-Strassler background is a smooth solution and has no D3 Page charge. The D3 and D5 Page charges are

$$\begin{aligned} Q_{D3}^{Page} &= \frac{1}{(4\pi^2)^2} \int_{\mathcal{M}_5} \left(\mathcal{F}^{(5)} - B^{(2)} \wedge F^{(3)} + \frac{1}{2} B^{(2)} \wedge B^{(2)} \wedge F^{(1)} \right) \\ Q_{D5}^{Page} &= \frac{1}{4\pi^2} \int_{S^3} \left(F^{(3)} - B^{(2)} \wedge F^{(1)} \right) \end{aligned}$$

and evaluated on the KS background these are

$$Q_{D3} = 0, \quad Q_{D5} = P.$$

It is trivial to obtain the BPS solution in which smeared D3 branes are added at the tip of the deformed conifold. In the PT ansatz one can add D3-brane sources by shifting $\mathcal{F}^{(5)}$ by a constant multiple of the volume form on $T^{1,1}$. This shifts the D3 Page charge, as well as $*\mathcal{F}^{(5)}$, in such a way that the warp factor becomes singular at the tip.⁹

The integral that gives the warp factor remains of the same form as in the original KS solution,

$$h(\tau) \sim \int^\tau \frac{f_0(2P - F_0) + k_0 F_0}{\sinh^2 \tau' K^2(\tau')} d\tau' \tag{5.1}$$

with $K(\tau) = \frac{(\sinh 2\tau - 2\tau)^{1/3}}{2^{1/3} \sinh \tau}$. Shifting $\mathcal{F}^{(5)}$ by a constant corresponds to shifting f_0 and k_0 by equal amounts. Under a shift of $N/(2P)$ the D3 Page charge shifts by

$$Q_{D3} \rightarrow Q_{D3} + N \tag{5.2}$$

while Q_{D5} remains invariant. This introduces in the IR a $1/\tau$ singularity in $h(\tau)$, whose physical interpretation is obvious: we have smeared BPS D3-branes (whose harmonic function diverges as $1/r^4$ near the sources) on the S^3 of the warped deformed conifold.

In order to “calibrate” our perturbation-theory machinery it is interesting to see how this BPS solution can be obtained in an expansion around the BPS KS background. First, all the ξ_i fields, that correspond to modes that break supersymmetry at first order, should be set to zero, and hence $X_i = 0$. Furthermore, since ξ_i are zero, $Y_2^{IR} = Y_2^{UV}$, and requiring no divergent terms in the IR of $\tilde{\phi}_2$ sets these to zero. The same thing happens with Y_3 and Y_1 (for the latter we require no constant term in the UV of $\tilde{\phi}_1$). The constant Y_8 has a very clear meaning: it corresponds to a shift in the dilaton. Note that this shifts $\tilde{\phi}_5, \tilde{\phi}_6$ and $\tilde{\phi}_4$

⁹Note that one can also shift the D3 Page charge by an integer multiple of P , by a large gauge transformation of B_2 . However, this will not affect $*\mathcal{F}^{(5)}$ or the warp factor, and hence will leave the physics invariant.

as expected from the exact KS solution.¹⁰ We consider perturbation where the dilaton is not shifted at infinity and hence set $Y_8^{IR} = Y_8^{UV} = 0$. The fields $\tilde{\phi}_5, \tilde{\phi}_6$ and $\tilde{\phi}_7$ then satisfy the homogeneous equation, and their solution is obtained from (4.26) by setting $\lambda_{5,6,7}$ to $Y_{5,6,7}$. Requiring no exponentially divergent terms in the UV and no divergencies in the IR set respectively $Y_5 = 0$ and $Y_6 = 0$. The only non-zero integration constant one can have is therefore Y_7 .

Hence, the perturbation corresponding to adding N BPS D3 branes is obtained by just setting $Y_7 \sim N$. Note that this perturbation causes the warp factor $\tilde{\phi}_4$ to diverge in the infrared as N/τ , exactly as one expects from the full BPS solution. All the other $\tilde{\phi}_i$ change by subleading terms, except $\tilde{\phi}_5$ and $\tilde{\phi}_6$, which as argued above shift by N .

Note that in the UV Y_7 multiplies a non-normalizable mode corresponding to the fact that we have changed the rank of the gauge group of the dual field theory by adding D3-branes:

$$\tilde{\phi}^4 = -\frac{2Y_7}{3P(4\tau - 1)} + \dots$$

Hence the new warp factor is

$$\begin{aligned} h &= e^{4A+4p-2x} = P^2 e^{-4\tau/3} (4\tau - 1) (1 - 2\tilde{\phi}^4) \\ &= P^2 e^{-4\tau/3} (4\tau - 1) + \frac{4Y_7 P}{3} e^{-4\tau/3} + \dots, \end{aligned}$$

and we can see that Y_7 multiplies a $1/r^4$ term, as one expects for pure (non-fractional) D3 branes.

5.2 Anti-BPS 3-branes in anti-Klebanov-Strassler

The anti-KS solution is obtained from the KS solution by flipping the sign of $H^{(3)}$ and $\mathcal{F}^{(5)}$ but not of $F^{(3)}$. This flips the signs of k_0 and f_0 in the expression for $H^{(3)}$ and has the same D5 brane Page charge as the KS solution, namely $Q_{D5}^{Page} = P$. It is rather straightforward to see that this solution is also supersymmetric but it preserves different Killing spinors than the original KS solution. With the addition of \overline{N} anti-D3 branes smeared on the S^3 the warp factor h again diverges in the infrared as $\frac{\overline{N}}{\tau}$, and the functions f and k are equal and approach $-\overline{N}$. The D3 brane page charge is $Q_{D3}^{Page} = -\overline{N}$.

The KS solution with D3 branes at the bottom can be thought of as coming from first taking the Ricci-flat deformed conifold with a topological $F^{(3)}$ flux, putting D3 branes in, and backreacting this configuration. The orientation of $\mathcal{F}^{(5)}$ (and subsequently that of $H^{(3)}$) is determined by the orientation of these D3 branes. In a similar fashion, one can obtain the anti-KS solution with anti-D3 branes by starting from the same Ricci-flat deformed conifold with the same topological $F^{(3)}$ flux and putting anti-D3 branes in; this backreacted system has $\mathcal{F}^{(5)}$ and $H^{(3)}$ of opposite orientation compared to the solution in section 5.1. Hence, for one orientation one obtains BPS D3 branes in the KS solution, while for the opposite orientation one obtains the anti-BPS 3-branes in anti-KS.

¹⁰The NSNS B-field has a factor of g_s which we have set for simplicity to 1 in eq. (2.13). Similarly the integral that gives h in eq. (2.14) contains a factor of g_s^2 ; shifting g_s in the BPS solution changes $\tilde{\phi}_4$ by a term proportional to $\tau/(4\tau - 1)$, exactly as in our UV expansions.

Note that if one thinks about the solutions in this way, it does not make sense to a-priori fix the sign of the charge of the “fractional” D3 branes. The only quantity whose orientation is fixed is the integral of $F^{(3)}$ on the 3-cycle. Hence, from this perspective, changing the orientation of the D3 brane sources reverses the sign of $\mathcal{F}^{(5)}$ throughout the solution, and hence transforms the “fractional” D3 branes into “fractional” anti-D3 branes.

5.3 Anti-BPS 3-branes in Klebanov-Strassler

If one now inserts anti-D3 branes in the BPS Klebanov-Strassler solution, one expects the physics at the tip to be dominated by the anti-D3 branes sources, and in addition to have an extra D3-charge dissolved in flux, as expected from the brane-flux annihilation process [5]. As a result, we need to consider how branes and fluxes contribute to the Maxwell charge. Recall that the Maxwell charge for D3-branes is defined as

$$Q_{D3}^{Max} = \frac{1}{(4\pi^2)^2} \int_{T_\infty^{11}} \mathcal{F}^{(5)}, \tag{5.3}$$

where we have indicated that the integral is over the T^{11} slice at infinity. For the zero-th order KS solution, the Maxwell charge is not constant, it measures the running of the rank of dual gauge groups and thus is a rough measure of the degrees of freedom. However we are concerned only with the *additional* Maxwell charge which we will add to the background and thus we will abuse terminology and refer to this additional charge as Q_{D3}^{Max} . In other words we have normalized the background Maxwell D3 charge to zero.

As is clear from the Bianchi identity for $F^{(5)}$, Q_{D3}^{Max} gets contributions from branes and flux combined

$$d\mathcal{F}^{(5)} = H^{(3)} \wedge F^{(3)} + \sum_i \delta(x - x_i) \tag{5.4}$$

where the second term comes from explicit D3-brane sources. Integrating (5.3) by parts yields

$$Q_{D3}^{Max} = q_b + q_f \tag{5.5}$$

where

$$q_b = \frac{1}{(4\pi^2)^2} \int_{T_0^{11}} F^{(5)}, \tag{5.6}$$

$$q_f = \frac{1}{(4\pi^2)^2} \int_{M_6} H^{(3)} \wedge F^{(3)} \tag{5.7}$$

are the charges from the explicit D3-brane sources and flux respectively. Note that Q_{D3}^{Max} is gauge invariant (unlike the Page charge) but is not necessarily quantized.¹¹

6 Constructing the anti-D3 brane solution

As advertized in section 1.1, we have two strategies to look for the “anti-D3 brane in KS” solution. The first is to impose UV boundary conditions, perturb the KS solution, and try

¹¹A very nice description of the various charges in type II string theory can be found in [29–31].

to match the expected boundary conditions in the infrared. The second is to impose the IR boundary conditions (which make the near-tip solution to be that of anti-BPS 3-branes in anti-Klebanov-Strassler), and to try to obtain the UV KS boundary conditions as a non-normalizable perturbation of the anti-KS solution.

6.1 UV → IR, X_1 is non zero

The UV boundary conditions we wish to enforce on our prospective new solutions are that $\mathcal{Q}_{D3}^{Max} = \bar{N}$ and that there should be a non-zero force on a probe D3-brane. In our ansatz the first boundary condition requires a careful analysis of

$$\mathcal{F}_5 = (kF + f(2P - F)) \text{vol}_{T^{11}} \tag{6.1}$$

in the UV. In addition the brane probe calculation of section 3 demonstrates that the force on a probe D3 brane depends only on X_1^{UV} . Since X_1^{IR} is proportional to X_1^{UV} (see eq. (4.18)), demanding a non-zero force in the UV will imply

$$X_1^{UV} \neq 0 \Rightarrow X_1^{IR} \neq 0. \tag{6.2}$$

In addition, we also demand very divergent UV non-normalizable modes to be absent, and hence set

$$Y_5^{UV} = Y_4^{UV} = 0. \tag{6.3}$$

These are the only UV conditions that we will impose so in principle all the other non-normalizable modes could be turned on.

Now we need to extract from this abstract discussion the explicit UV and IR behavior for the modes in our deformation space.¹² The total Maxwell charge is evaluated in the UV by demanding

$$(\tilde{\phi}_6 - \tilde{\phi}_5)F_0 + (k_0 - f_0)\tilde{\phi}_7 + 2P\tilde{\phi}_5 = \mathcal{Q}_{D3}^{Max} + \mathcal{O}(e^{-\tau/3}). \tag{6.4}$$

Using (2.13), (6.3) and the UV expansions in section 4.3.3 we find that

$$\begin{aligned} \mathcal{Q}_{D3}^{Max} &= P(\tilde{\phi}_5^{UV} + \tilde{\phi}_6^{UV}) \\ &= 2PY_7^{UV}. \end{aligned} \tag{6.5}$$

In the IR, we want the $\tilde{\phi}^i$ to contain no divergent terms except those coming from q_b . From (5.6) and (2.10) we immediately find that,

$$(\tilde{\phi}_6 - \tilde{\phi}_5)F_0 + (k_0 - f_0)\tilde{\phi}_7 + 2P\tilde{\phi}_5 \sim q_b + \mathcal{O}(\tau). \tag{6.6}$$

From the zero-th order expressions (2.13) we see that

$$F_0 \sim \mathcal{O}(\tau^2), \quad f_0 \sim \mathcal{O}(\tau^3), \quad k_0 \sim \mathcal{O}(\tau) \tag{6.7}$$

so that disallowing divergences in $(\tilde{\phi}_5, \tilde{\phi}_6, \tilde{\phi}_7)$ we find the constraint

$$2P\tilde{\phi}_5 \sim q_b + \mathcal{O}(\tau). \tag{6.8}$$

¹²Recall that in our notation $(\tilde{\phi}_5, \tilde{\phi}_6, \tilde{\phi}_7) = (f_1, k_1, F_1)$, the perturbations to (f_0, k_0, F_0) .

Then on physical grounds we demand that in the IR the singularity in the Einstein tensor (due to the singularity in the warp factor) is commensurate with the singularity in the energy density from $F^{(5)}$ (recall that $\tilde{\phi}_4$ is the warp factor):

$$\tilde{\phi}_4 \sim |q_b|/\tau + \mathcal{O}(\tau^0). \tag{6.9}$$

So this divergence is allowed since it is clearly coming from the anti-D3 brane sources.

However we must eliminate other divergences. We can see from the equations for $\tilde{\phi}^2$ and $\tilde{\phi}^3$ that this implies

$$Y_2^{IR} = Y_3^{IR} = 0. \tag{6.10}$$

Note that unlike the integration constant X_1 , whose values in the UV and IR expansions are proportional to each other, all the other 13 integration constants differ the UV and IR by an additive constant, as reviewed in section 4.2. Hence, the fact that Y_2^{IR} must be set to zero implies that Y_2^{UV} will be a very nontrivial function of all the X_i , which is highly unlikely to be zero unless all the X_i are zero and we have a BPS solution. Hence, any prospective anti-D3 solution will source modes that behave in the UV as $1/r^3$, contrary to what one might expect based on codimension-counting.

The absence of divergences in $\tilde{\phi}^7$ further implies that

$$Y_6^{IR} = \frac{h_0}{3}(X_5^{IR} + X_6^{IR}) \tag{6.11}$$

and the absence of $1/\tau$ -divergences in $\tilde{\phi}^6$ implies

$$(X_5^{IR} + X_6^{IR}) = \frac{16(2/3)^{1/3} P X_1^{IR}}{h_0}. \tag{6.12}$$

Furthermore the absence of a log-divergent mode in $\tilde{\phi}^6$ combined with the equations above also implies

$$X_4^{IR} = -\frac{X_1^{IR}}{6}. \tag{6.13}$$

The absence of a divergent term in $\tilde{\phi}^8$ imposes a relation between X_8 and the other variables:

$$X_8^{IR} = -\frac{P}{6}(5X_5^{IR} - X_6^{IR} - 6X_7^{IR}). \tag{6.14}$$

Furthermore, upon using (6.12) one can see that the $\frac{1}{\tau}$ term in $\tilde{\phi}^4$ depends only on X_1^{IR} and Y_7^{IR}

$$\tilde{\phi}_4 = \frac{1}{\tau} \left(-\frac{2^{10/3} 3^{2/3} P}{h_0} Y_7^{IR} + \frac{2^{4/3}}{3^{1/3}} X_1^{IR} - \frac{4096 P^4}{3 h_0^2} X_1^{IR} \right) + Y_4^{IR} + \mathcal{O}(\tau) \tag{6.15}$$

Upon using these conditions above, the expansions of $\tilde{\phi}_5$ becomes

$$\tilde{\phi}_5 = Y_7^{IR} + \frac{256 P^3}{3 h_0} \left(\frac{2}{3} \right)^{2/3} X_1^{IR} + 4(2/3)^{1/3} P X_1^{IR} \tau^2 + \mathcal{O}(\tau^3). \tag{6.16}$$

Note however that the values for (Y_7^{IR}, X_1^{IR}) have already been fixed in terms of \bar{N} : firstly, (6.5) fixes Y_7^{UV} in terms of the number of initial anti-D3 branes (\bar{N}) put into

the system, which is in turn related to Y_7^{IR} by an additive constant. Secondly, eqs. (6.9) and (6.15) give X_1^{IR} in terms of Y_7^{IR} and q_b . One can then use (6.8) and (6.16) to obtain X_1^{IR} in terms of Y_7^{IR} . At this point we have computed all the IR quantities including q_b in terms of \bar{N} and one might declare this to be the solution for anti-D3 branes in the KS background that we have been seeking. However, the singularity analysis is more subtle than that presented above.

6.1.1 IR singularities

Recall that the NS and RR three forms in our ansatz are

$$\begin{aligned} H_3 &= \frac{1}{2}(k - f)g_5 \wedge (g_1 \wedge g_3 + g_2 \wedge g_4) + d\tau \wedge (f'g_1 \wedge g_2 + k'g_3 \wedge g_4), \\ F_3 &= Fg_1 \wedge g_2 \wedge g_5 + (2P - F)g_3 \wedge g_4 \wedge g_5 + F'd\tau \wedge (g_1 \wedge g_3 + g_2 \wedge g_4), \end{aligned}$$

and in the IR the unperturbed metric is regular and is given by

$$ds_{10}^2 \sim \alpha_1 ds_4^2 + \alpha_2 \left(\frac{1}{2}d\tau^2 + \left(g_3^2 + g_4^2 + \frac{1}{2}g_5^2 \right) + \frac{\tau^2}{4}(g_1^2 + g_2^2) \right).$$

for some numerical constants (c_1, c_2) . From this we observe that if

$$(k - f) \sim a_0 + \mathcal{O}(\tau) \tag{6.17}$$

then the energy density has a term that divergences as

$$H_3^2 \sim \frac{a_0^2}{\tau^2}. \tag{6.18}$$

Furthermore, if

$$f \sim b_0 + b_1\tau + b_2\tau^2 + \mathcal{O}(\tau^3) \tag{6.19}$$

then there is a singularity in the energy density of the form

$$H_3^2 \sim \frac{b_1^2}{\tau^4} + \frac{b_2^2}{\tau^2}. \tag{6.20}$$

The singularities from a_0 and b_2 are somewhat more benign than that from b_1 , since their divergent energy density integrates to a finite action¹³ (recall that $\sqrt{G} \sim \tau^2 + \mathcal{O}(\tau^4)$)

$$\left| \int d^{10}x \sqrt{G} H_3^2 \right| < \infty. \tag{6.21}$$

The b_1 term in (6.19) however would result in a divergent action. Similarly, the IR expansion of F_3 :

$$\tilde{\phi}_\tau = c_0 + c_1\tau + \mathcal{O}(\tau^2) \tag{6.22}$$

gives a singularity in the energy density of order

$$F_3^2 \sim \frac{c_0^2}{\tau^4} + \frac{c_1^2}{\tau^2} \tag{6.23}$$

¹³We are very grateful to Igor Klebanov for sharing this observation with us.

and so we see that both the RR and NS three-form field strengths have potentially inadmissible singularities.

Using the boundary conditions from section 6.1 we find that

$$\tilde{\phi}_5 - \tilde{\phi}_6 = -\frac{16P}{3} \left(\frac{2}{3}\right)^{1/3} X_1^{IR} + \mathcal{O}(\tau), \tag{6.24}$$

$$\tilde{\phi}_7 = \frac{8P}{3} \left(\frac{2}{3}\right)^{1/3} X_1^{IR} \tau + \mathcal{O}(\tau^2). \tag{6.25}$$

The difference $\tilde{\phi}_5 - \tilde{\phi}_6$ has a nonzero constant in its IR expansion ($a_0 \neq 0$), $\tilde{\phi}_5$ has a nonzero quadratic term ($b_2 \neq 0$) and $\tilde{\phi}_7$ has a nonzero linear term ($c_1 \neq 0$), all of which are proportional to X_1^{IR} and lead to an energy density which diverges as τ^{-2} . However, both the linear term in $\tilde{\phi}_5$ and the constant term in $\tilde{\phi}_7$ are zero ($b_1 = c_0 = 0$) and thus there are no singular energy densities that scale like τ^{-4} , are hence no infinite-action divergencies.

Our analysis establishes that the only solution whose ultraviolet is consistent with that of would-be anti-D3 branes in the Klabanov-Strassler solution must have finite-action divergences in the infrared, both in the RR and the NS-NS three-form field strengths. The only way to eliminate these divergencies is by setting $X_1^{IR} = 0$, which in turn implies that $X_1^{UV} = 0$ and is thus in contradiction with the UV boundary condition (6.2) necessary in order to have a nonzero force on a probe D3 brane.

Clearly, finite-action divergencies are better than infinite-action divergencies, but it is not clear whether in the context of the AdS-CFT duality even such finite-action divergencies should be allowed. The best example of a finite-action singularity that must be excluded from the spectrum of any consistent theory of two derivative gravity is the negative-mass Schwarzschild solution.¹⁴

Furthermore, this singularity does not appear a-priori to have any direct connection with the infrared physics of probe anti-D3 branes in the KS solution [5], and the brane-flux annihilation that occurs via the polarization of these anti-D3 branes into NS5 branes wrapping an S^2 inside the S^3 : First, brane-flux annihilation results in the non-trivial division of Q_{D3}^{Max} into (q_b, q_f) and this in turn is related to the IR and UV values of f , and not to the IR value of $k - f$. As such, this singularity could not have been predicted as a consequence of brane-flux annihilation. Second, the NS5 brane that mediates brane-flux annihilation couples magnetically with an NS-NS three-form field strength, while one of the fields whose energy density diverges in our solution is a RR three-form field strength ($c_1 \neq 0$). Third, in the probe analysis of brane-charge annihilation, the amount of charge dissolved in fluxes depends nonlinearly on the amount of anti-D3 branes, while in our solution both the anti-D3 charge and the coefficient of the divergent terms are proportional to X_1^{IR} .

Hence, given the absence of a microscopic explanation for this singularity, and given the absence of any criterion under which this singularity would be acceptable while the negative-mass Schwarzschild singularity would be not, it is fair to say that if this singularity

¹⁴We are very grateful to Gary Horowitz for pointing this out.

had appeared all by itself in a fully-backreacted solution, it should have been deemed unacceptable.

However, as we have seen in the previous section, this singularity does not appear all by itself: the infrared geometry also contains metric and five-form fields whose energy density computed in first-order perturbation theory diverges as τ^{-4} . These fields have an obvious microscopic interpretation: they are sourced by a smeared distribution of anti-D3 branes, the metric and five-form divergences are commensurate, as typical for D-branes, and we expect this singularity to make perfect sense in string theory. Hence, it is possible that upon taking into account the string-theoretic resolution of the singularity sourced by the smeared D3-branes, the weaker finite-action singularity may become benign.

6.2 IR \rightarrow UV

We now start from a solution that eliminates the IR singularities of the previous section, and argue that the UV expansion of this solution cannot match the UV expansion of the KS solution with none but the charge non-normalizable mode turned on. This problem is equivalent to starting from the BPS D3 branes in the IR and arguing that one cannot match the anti-KS solution in the UV.

If we are to match the boundary conditions near the anti-branes, eqs. (6.10)–(6.14), the fields $\tilde{\phi}^5$ and $\tilde{\phi}^6$ must be equal, and using (6.16) this implies

$$X_1^{IR} = X_1^{UV} = 0.$$

Note that in the absence of X_1 we have already solved exactly the equations (2.7) for the ξ^i in section 4.1, and thus the values of the X_i in the UV and in the IR must be the same. The infrared regularity equations in the previous section imply:

$$\begin{aligned} X_5 + X_6 &= X_4 = 0, \\ Y_2^{IR} = Y_3^{IR} = Y_6^{IR} &= 0, \end{aligned} \tag{6.26}$$

and therefore

$$X_8 = -P(X_5 - X_7).$$

Furthermore, since we do not want divergent and non-normalizable $1/r$ modes in the UV,

$$X_3 = X_2 = X_7 = 0.$$

However, in deriving the infrared boundary conditions in the previous subsection we have used the zero energy condition (4.12) to express X_2 as a function of X_3, X_4, X_5 and X_7 , and setting it to zero combined with the other equations implies

$$X_5 = 0$$

and therefore all X 's must vanish. Hence, if one tries to match all the correct IR boundary conditions and avoid un-physical non-normalizable modes, one must set all the ξ_i to zero.

One can now proceed exactly as in section 4.3 to establish that $\tilde{\phi}_8$ is a constant, and use equations (4.23), (4.24) and (4.25) together with (6.26) to show that $\tilde{\phi}^2 = \tilde{\phi}^3 = 0$, and

that $\tilde{\phi}^1$ must be a constant. The next step is to solve the system (4.26) exactly, and match the infrared and ultraviolet integration constants. One finds that

$$\begin{aligned}
 \tilde{\phi}^5 &= \frac{1}{2} \operatorname{sech}^2 \frac{\tau}{2} \left(\cosh \tau (2Y_5 \tau - Y_5 \sinh \tau + Y_7 - PY_8 \tau) \right. \\
 &\quad \left. + \sinh \tau (PY_8 - 2Y_5) + Y_5 \tau + Y_6 + Y_7 \right), \\
 \tilde{\phi}^6 &= -\frac{1}{2} \operatorname{csch}^2 \frac{\tau}{2} \left(-\cosh \tau (2Y_5 \tau + Y_5 \sinh \tau + Y_7 - PY_8 \tau) \right. \\
 &\quad \left. + \sinh \tau (PY_8 - 2Y_5) + Y_5 \tau + Y_6 + Y_7 \right), \\
 \tilde{\phi}^7 &= -Y_5 \cosh \tau + (Y_5 \tau - Y_6) \operatorname{csch} \tau .
 \end{aligned} \tag{6.27}$$

Comparing to the expansions in sections 4.3.2 and 4.3.3 we get

$$Y_5 = Y_5^{UV} = Y_5^{IR} + \frac{PY_8}{6}, \quad Y_6 = Y_6^{IR} = Y_6^{UV}, \quad Y_7 = Y_7^{IR} = Y_7^{UV} . \tag{6.28}$$

Requiring no divergencies in the UV and IR sets

$$Y_5 = Y_6 = 0, \tag{6.29}$$

Importantly, we see that the sign of Y_7 , which is proportional to the effective D3-brane charge (recall that Y_8 just corresponds to a shift in the dilaton), cannot change from the IR to the UV. This implies that this solution has positive D3 brane charge throughout. Hence, the solution that preserves D3 boundary conditions in the infrared cannot match a solution with anti-KS boundary conditions in the UV and viceversa.

7 Discussion

With a view towards identifying the modes sourced by anti-D3 branes, we have computed the spectrum of linearized fluctuations around the Klebanov-Strassler solution which respect the symmetries of the original background. Finding such solutions, with anti-D3 brane boundary conditions in the infrared and Klebanov-Strassler boundary conditions in the ultraviolet has been a key unresolved step in many constructions of de Sitter vacua in string theory. Much work has been done using this mechanism to show that large numbers of such vacua exist.

We have found in our spectrum a single solution where some of the anti-D3 charge is dissolved in flux, supersymmetry is broken (as suggested in [5]) and there is a force on a probe D3 brane of the same form as the one computed in [13]. While this is all somewhat encouraging, the solution we find must necessarily have an infrared singularity, coming from the fact that the energy densities of both the RR and NS three form fluxes diverge quadratically at the origin.

We have discussed this singularity in great detail in section 6.1, and have seen that as far as first-order perturbation theory is concerned, this singularity is better behaved than other possible IR singularities - in particular its divergent energy density integrates

to a finite action. However, even if the finiteness of the action makes this singularity more acceptable than others, it cannot be used as a criterion to establish that this singularity is physical. For example, the negative-mass Schwarzschild solution also has a finite-action singularity, and yet this solution is unphysical, and must be eliminated from the bulk spectrum in order for the gauge/gravity duality to make sense. Indeed, the breakdown of general relativity at a singularity does not come from the diverging action but rather from the fact that the derivative expansion in the effective action for gravity does not provide a good perturbation theory. Hence, divergent energy densities in the RR and NS-NS fields are as unphysical as a divergent Ricci scalar, and indicate that perturbation theory has broken down.

We have also tried to find a microscopic interpretation for this singularity, to see whether the divergent fields can be explained as coming from D-brane or NS5-brane sources that one may expect to find in the infrared, and hence whether stringy corrections could resolve this singularity. We have argued that the existence of this singularity is neither a direct consequence of the expected brane-flux annihilation, nor of the polarization of anti-D3 branes into NS5 branes in the infrared. From such arguments, the most direct conclusion would be that this finite-energy singularity should be treated as other such singularities in gravity and must be removed.

On the other hand there is one reason that we should persist with this solution and perform further tests on it despite the singularity. It may be that the finite-action singularity is not a *signal* of the breakdown of general relativity but rather a *consequence*. It is clear that one can never completely capture the physics of the solution near the anti-D3 branes by doing perturbation theory around the original KS background, since in that region the energy in the fields sourced by the explicit anti-D3 brane sources diverges. This is similar to the fact that one cannot treat an electron in a background electric field as a perturbation, since near the electron its electric field always dominates. The energy density of the fields sourced by the explicit anti-D3 brane sources in the IR diverges quartically in the IR, whereas the energy density of the three forms diverges quadratically. It may be that the stringy resolution of the anti-D3 brane singularities, or even their full supergravity backreaction, will also resolve the milder finite-action singularity we have found and render the whole solution physical.

It is clearly crucial to establish whether this finite-action singularity is indeed resolved this way and could be acceptable from the point of view of string theory, or whether it is not acceptable and must be removed.

If the finite-action singularity is physical, than our analysis has found a first-order backreacted solution that has anti-D3 brane charge at the bottom of the Klebanov-Strassler solution. The objects with anti-D3 brane charge source modes that in the ultraviolet can be treated as a perturbation of the Klebanov-Strassler solution, and it is very likely that this solution represents the backreaction of the smeared system of anti-D3 branes considered by Kachru Pearson and Verlinde in [5]. Our analysis does not fully establish that these anti-D3 branes source only normalizable modes - to prove this one needs to relate the UV and IR integration constants, which is quite nontrivial [32]. If this is so, then this solution would represent the supergravity dual of the conjectured metastable vacuum of

the Klebanov-Strassler field theory, and would be the first supergravity solution dual to a metastable vacuum of a four-dimensional theory. By exploring the vacuum expectation values of various operators, this solution would allow one to characterize very precisely the physics of this metastable vacuum, and would quite likely increase our understanding of gauge theory metastable vacua in general.

On the other hand, if this singularity is not physical, our analysis establishes that the solution corresponding to backreacting anti-D3 branes at the bottom of the Klebanov-Strassler background cannot be treated in the UV as a perturbation around the BPS vacuum. Hence, these anti-D3 branes source non-normalizable modes (other than the charge) whose energy is at least as strong as that of the original KS solution.

While this conclusion might seem at first glance unexpected, it is in fact not once one realizes that some of the modes may have two-dimensional dynamics, and a perturbation in such modes does not decay at infinity. Indeed, as we have argued in the Introduction, all supergravity solutions dual to four-dimensional theories with a running coupling constant must have bulk modes that depend logarithmically on the radial direction, coming from the fact that the coupling constants of these theories run logarithmically with the energy. Hence, it is quite natural to expect the anti-D3 branes to couple to those modes, and hence to source perturbations that grow logarithmically with the radius. It is not hard to see that the energy of such logarithmic modes is as strong as that of the unperturbed solution.

Furthermore, this behavior would mimic exactly what happens when one tries constructing an MQCD metastable vacuum [4]. First, if one tries to obtain this vacuum by perturbation theory around the BPS one, one also finds modes that diverge in the infrared, and cannot match the correct boundary conditions. This is a first hint that the fully-backreacted metastable vacuum might not exist, and is confirmed by the subsequent backreacted calculation. Furthermore, if one tries to find the solution whose infrared has regular boundary conditions, one finds a solution that differs from the BPS one by a logarithmically-growing mode, and that cannot be obtained as a perturbation of the latter. This is again a reflection of the fact that this mode has two-dimensional dynamics, and once excited in the infrared it does not decay at infinity.

If one takes the analogy with the MQCD construction of metastable vacua one step further, one can also identify an obvious candidate for the full solution consistent with anti-D3 branes in the infrared and no finite-action singularity: the anti-D3 branes in the anti-Klebanov-Strassler solution. Much like the MQCD IR-compatible solution,¹⁵ this solution differs from the BPS one by log-growing modes, it is not a perturbation of the latter and is BPS by itself. If this is indeed *the* solution for the back-reacted anti-D3 branes, this would imply that the sign of the charge dissolved in flux in the KS background must always have the same sign of the D3 branes at the bottom, and changes sign as one changes the orientation of these branes.

If the finite-action singularity is excluded, or if it is allowed but the anti-D3 branes source modes that are nonnormalizable, this would strongly suggest that the dual gauge theory does not have a metastable vacuum. Though unexpected, this result would be

¹⁵Given by eq. (3.22) in [4].

consistent with the fact that a careful analysis of the vacuum structure of this theory [33] has not found such a vacuum. Furthermore, as we have explained in the Introduction, this would probably affect the KKLT construction of deSitter vacua in string theory, and the existence of a landscape of such vacua in general, which in turn may invalidate many string phenomenology and string cosmology constructions.

It is fair to say that neither our analysis, nor any in the literature definitively show that the finite-action singularity is unphysical, and hence rule out the existence of KS anti-D3 metastable vacua. On the other hand, we have not been able to find any microscopic arguments or singularity-resolution arguments of why this singular behavior in the fields may be acceptable, which would establish that the solution we have constructed is the supergravity dual of the KS metastable vacuum, and implicitly prove that such a vacuum exists.

Our analysis crystalizes and reduces the complicated problem of the existence of KS metastable vacua to the simpler problem of whether the finite-action singularity is physically acceptable or not. Establishing whether this singularity is physical would ideally come from a microscopic understanding of the infrared physics or from a near-brane analysis of the fully backreacted, localized solution (for example extending the analysis of [34]). Any developments on these issues would clearly be very exciting.

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A Conventions

We use the somewhat unsightly notation ξ_a [20] to indicate that we refer to ξ_a as defined in reference [20]. This is to try to keep straight the different conventions which appear in the various papers on the subject. The relation to the variables used in other papers is the following. The angular variables $\epsilon_1, \epsilon_2, \epsilon_3, e_1, e_2$ of [16] are related to g_i by

$$\epsilon_1 = \frac{1}{\sqrt{2}}(-g_2 + g_4), \quad \epsilon_2 = \frac{1}{\sqrt{2}}(-g_1 + g_3), \quad e_1 = \frac{1}{\sqrt{2}}(g_2 + g_4), \quad e_2 = \frac{1}{\sqrt{2}}(g_1 + g_3), \quad \epsilon_3 = g_5. \tag{A.1}$$

Our radial variable τ is related to the radial variables in [3, 12, 16, 17, 20] by

$$d\tau = d\tau_{[20]} = e^{4p_1} d\tau_{[17]} = e^{4p_1} d\tau_{[3]} = e^{4(p_0+p_1)} du_{[16]} = \sqrt{6} e^{3p_0+x_0+4p_1} d \ln r_{[12]}. \tag{A.2}$$

To zeroth order in perturbation this last relation reads in the UV $r = e^{\tau/3}$.

Our metric functions x, y, p, A are related to q, y, p, Y used in [20] and a, b ($y = 0$ and in the solution $c = 0$) in [12] (relations to the latter are only valid in the UV) by

$$\begin{aligned} x &= 3q + 2p_{[20]} = -2a - \ln 6, & y &= y_{[20]}, \\ p &= p_{[20]} - q = \frac{1}{3}(2a - b + \ln(3\sqrt{6})), & A &= Y = a + \ln r. \end{aligned} \quad (\text{A.3})$$

The functions f, k, F in the 3-form fluxes are related by

$$\begin{aligned} f &= -2Pf_{[20]} = 2f_{[17]} = 6k_{[12]}, \\ k &= -2Pk_{[20]} = 2k_{[17]} = 6k_{[12]}, \\ F &= 2PF_{[20]} = 2F_{[17]}. \end{aligned} \quad (\text{A.4})$$

B Behavior of $\tilde{\xi}$

B.1 IR behavior of $\tilde{\xi}$

The IR behavior of $\tilde{\xi}_a$ is the following

$$\begin{aligned} \tilde{\xi}_1 &= X_1^{IR} \left(1 - \frac{16 \left(\frac{2}{3}\right)^{1/3} P^2 \tau^2}{3 h_0} \right) + \mathcal{O}(\tau^4) \\ \tilde{\xi}_4 &= X_4^{IR} + \frac{16 X_1^{IR} \left(\frac{2}{3}\right)^{1/3} P^2 \tau^2}{3 h_0} + \mathcal{O}(\tau^4) \\ \tilde{\xi}_5 &= \frac{8 X_1^{IR} \left(\frac{2}{3}\right)^{1/3} P}{h_0 \tau} + X_5^{IR} + \frac{16 X_1^{IR} \left(\frac{2}{3}\right)^{1/3} P^2 \tau}{15 h_0} + \mathcal{O}(\tau^4) \\ \tilde{\xi}_6 &= \frac{X_5^{IR} + X_6^{IR}}{2\tau} + X_5^{IR} + \left(\frac{56 X_1^{IR} \left(\frac{2}{3}\right)^{1/3} P^2}{15 h_0} - \frac{X_5^{IR} + X_6^{IR}}{12} \right) \tau + \mathcal{O}(\tau^2) \\ \tilde{\xi}_7 &= \left(-\frac{8 X_1^{IR} \left(\frac{2}{3}\right)^{1/3} P^2}{h_0} + \frac{X_5^{IR} + X_6^{IR}}{2} \right) \frac{1}{\tau^2} \\ &\quad + \left(-\frac{4 X_1^{IR} \left(\frac{2}{3}\right)^{1/3} P^2}{3 h_0} + \frac{X_5^{IR} + X_6^{IR}}{12} \right) + \frac{X_5^{IR} - 4 X_7^{IR}}{3} \tau + \mathcal{O}(\tau^2) \\ \tilde{\xi}_8 &= X_8^{IR} + \frac{1}{6} P (5 X_5^{IR} - X_6^{IR} - 6 X_7^{IR}) \\ &\quad + P \left(\frac{32 X_1^{IR} \left(\frac{2}{3}\right)^{1/3} P^2}{45 h_0} + \frac{X_5^{IR} + X_6^{IR}}{15} \right) \tau^2 + \frac{2}{9} P (X_5^{IR} - X_7^{IR}) \tau^3 + \mathcal{O}(\tau^4) \\ \tilde{\xi}_3 &= -\frac{5}{3} X_1^{IR} + \frac{16 X_1^{IR} \left(\frac{2}{3}\right)^{1/3} P^2 \tau^2}{3 h_0} + \frac{8}{3} X_3^{IR} \tau^3 + \mathcal{O}(\tau^4) \end{aligned} \quad (\text{B.1})$$

$$\begin{aligned} \tilde{\xi}_2 = & \left(\frac{X_1^{IR}}{9} + \frac{16X_1^{IR} \left(\frac{2}{3}\right)^{1/3} P^2}{3h_0} + \frac{X_4^{IR} - P(X_5^{IR} + X_6^{IR})}{3} \right) \\ & + \left(-\frac{1}{3}X_4^{IR} + 2X_2^{IR} - PX_5^{IR} - \frac{4}{3}X_3^{IR} + 3PX_7^{IR} \right) \tau \\ & + \left(\frac{X_1^{IR}}{18} + \frac{56X_1^{IR} \left(\frac{2}{3}\right)^{1/3} P^2}{15h_0} + \frac{X_4^{IR} + \frac{3}{5}P(X_5^{IR} + X_6^{IR})}{6} \right) \tau^2 + \mathcal{O}(\tau^3) \end{aligned}$$

B.2 UV behavior of $\tilde{\xi}$

The UV behavior of $\tilde{\xi}_a$ is the following

$$\begin{aligned} \tilde{\xi}_1 &= X_1^{UV} e^{-4\tau/3} (3 - 12\tau) + \mathcal{O}(e^{-10\tau/3}) \\ \tilde{\xi}_4 &= X_4^{UV} - X_1^{UV} (3 - 12\tau) e^{-4\tau/3} + \mathcal{O}(e^{-10\tau/3}) \\ \tilde{\xi}_5 &= X_5^{UV} - \frac{2}{P} X_1^{UV} e^{-4\tau/3} + \mathcal{O}(e^{-10\tau/3}) \\ \tilde{\xi}_6 &= \frac{1}{2} X_7^{UV} e^\tau + \left(2\tau(X_5^{UV} - X_7^{UV}) + X_5^{UV} + X_6^{UV} + \frac{1}{2} X_7^{UV} \right) e^{-\tau} + \mathcal{O}(e^{-3\tau}) \\ \tilde{\xi}_7 &= -\frac{1}{2} X_7^{UV} e^\tau + \left(2\tau(X_5^{UV} - X_7^{UV}) - X_5^{UV} + X_6^{UV} + \frac{5}{2} X_7^{UV} \right) e^{-\tau} + \mathcal{O}(e^{-3\tau}) \quad (\text{B.2}) \\ \tilde{\xi}_8 &= X_8^{UV} + P(X_5^{UV} - X_7^{UV})\tau + \frac{3}{2} X_1^{UV} e^{-4\tau/3} \\ & \quad + 2P \left(-2(X_5^{UV} - X_7^{UV})\tau^2 + (3X_5^{UV} - X_6^{UV} - 4X_7^{UV})\tau + X_6 + X_7^{UV} \right) e^{-2\tau} + \mathcal{O}(e^{-3\tau}) \\ \tilde{\xi}_3 &= X_3^{UV} (e^{2\tau} - 4\tau - e^{-2\tau}) + \frac{3}{25} X_1^{UV} e^{-4\tau/3} (-23 + 140\tau) + \mathcal{O}(e^{-10\tau/3}) \\ \tilde{\xi}_2 &= \left(\left(-\frac{2}{3} X_3^{UV} + PX_7^{UV} \right) \tau + X_2^{UV} \right) e^\tau \\ & \quad + \left(\left(-\frac{2}{3} X_3^{UV} + 5PX_7^{UV} - 4PX_5^{UV} \right) \tau - X_2^{UV} + \frac{1}{3} X_4^{UV} \right. \\ & \quad \left. + P(X_5^{UV} - X_6^{UV} - 2X_7^{UV}) \right) e^{-\tau} + \mathcal{O}(e^{-3\tau}) \end{aligned}$$

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