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ON THE EXISTENCE OF SOME TYPES OF LP-SASAKIAN MANIFOLDS

Absos A. Shaikh, Kanak K. Baishya, and Sabina Eyasmin

Abstract. The object of the present paper is to provide the existence of LP-Sasakian manifolds with η-recurrent, η-parallel, ϕ-recurrent, ϕ-parallel Ricci tensor with several non-trivial examples. Also generalized Ricci recurrent LP-Sasakian manifolds are studied with the existence of various examples.

1. Introduction

In 1989 K. Matsumoto ([4]) introduced the notion of LP-Sasakian manifolds. Then I. Mihai and R. Rosca ([6]) introduced the same notion independently and obtained many interesting results. LP-Sasakian manifolds are also studied by U. C. De, K. Matsumoto and A. A. Shaikh ([2]), I. Mihai, U. C. De and A. A. Shaikh ([5]), A. A. Shaikh and S. Biswas ([8]) and others.

Recently A. A. Shaikh and K. K. Baishya ([7]) introduced the notion of LP-Sasakian manifolds with η-recurrent, ϕ-parallel and ϕ-recurrent Ricci tensor which generalizes the notion of η-parallel Ricci tensor, introduced by M. Kon ([3]) for a Sasakian manifold.

In the present paper the existence of such notions on LP-Sasakian manifolds are ensured by several non-trivial examples both in odd and even dimensions. Section 2 is concerned with basic identities of LP-Sasakian manifolds. Since the notion of Ricci η-recurrent is the generalization of Ricci η-parallelity, natural question arises does there exist LP-Sasakian manifolds with η-recurrent but not η-parallel Ricci tensor? The answer is affirmative as shown by several examples in section 3. In section 4, we obtain various examples of LP-Sasakian manifolds with (i) ϕ-parallel Ricci tensor, (ii) ϕ-recurrent but not ϕ-parallel Ricci tensor, (iii) ϕ-parallel but not η-parallel Ricci tensor. In ([1]) De et. al introduced the notion of generalized Ricci recurrent Riemannian manifolds. The last section deals with generalized Ricci recurrent LP-Sasakian manifolds and proved that such a manifold is Einstein and the associated 1-forms of the manifold are

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of opposite direction. Also the existence of generalized Ricci recurrent LP-Sasakian manifold is ensured by several non-trivial examples constructed with global vector fields.

2. LP-Sasakian manifolds

An n-dimensional differentiable manifold M is said to be an LP-Sasakian manifold ([7], [6]) if it admits a (1, 1) tensor field ϕ, a unit timelike contravariant vector field ξ, a 1-form η and a Lorentzian metric g which satisfy

$$\eta(\xi) = -1, \quad g(X,\xi) = \eta(X), \quad \phi^2 X = X + \eta(X)\xi, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \qquad \nabla_X \xi = \phi X, \tag{2.2}$$

$$(\nabla_X \phi)(Y) = g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.3}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g. It can be easily seen that in an LP-Sasakian manifold, the following relations hold :

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad \text{rank}\phi = n-1. \tag{2.4}$$

Again, if we put

$$\Omega(X,Y) = g(X, \phi Y)$$

for any vector field X,Y then the tensor field $\Omega(X,Y)$ is a symmetric (0, 2) tensor field ([4]). Also, since the vector field η is closed in an LP-Sasakian ([2],[4]) manifold, we have

$$(\nabla_X \eta)(Y) = \Omega(X,Y), \qquad \Omega(X,\xi) = 0 \tag{2.5}$$

for any vector field X and Y.

Let M be an n-dimensional LP-Sasakian manifold with structure (ϕ, ξ, η, g). Then the following relations hold ([7]) :

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.6}$$

$$S(X,\xi) = (n-1)\eta(X), \tag{2.7}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y) \tag{2.8}$$

for any vector field X, Y, Z where R is the curvature tensor of the manifold.

3. LP-Sasakian manifolds with η-recurrent Ricci tensor

Definition 3.1 ([7])**.** The Ricci tensor S of an LP-Sasakian manifold is said to be η-recurrent if it satisfies the following :

$$(\nabla_X S)(\phi Y, \phi Z) = A(X)S(\phi Y, \phi Z) \tag{3.1}$$

for all X, Y, Z where A is a non-zero 1-form such that $A(X) = g(X, \rho)$, ρ is the associated vector field of the 1-form A.

In particular, if the 1-form A vanishes then the Ricci tensor of the LP-Sasakian manifold is said to be η-parallel and this notion was first introduced by Kon ([3]) for Sasakian manifolds. Hence the notion of η-recurrent Ricci tensor generalizes the notion of η-parallel Ricci tensor.

In ([7]), A. A. Shaikh and K. K. Baishya also studied various properties of LP-Sasakian manifolds with η-recurrent Ricci tensor. We first construct an example of LP-Sasakian manifold with global vector fields whose Ricci tensor is η-parallel.

Example 3.1. We consider a 4-dimensional manifold $M = \{(x, y, z, u) \in R^4\}$, where (x, y, z, u) are the standard coordinates of R^4. Let $\{E_1, E_2, E_3, E_4\}$ be linearly independent global frame on M given by

$$E_1 = e^u \frac{\partial}{\partial x}, \quad E_2 = e^u \frac{\partial}{\partial y}, \quad E_3 = e^u \frac{\partial}{\partial z}, \quad E_4 = \frac{\partial}{\partial u}.$$

Let g be the Lorentzian metric defined by

$$g(E_1, E_3) = g(E_2, E_3) = g(E_1, E_4) = g(E_2, E_4) = g(E_3, E_4) = g(E_1, E_2) = 0,$$

$$g(E_1, E_1) = g(E_2, E_2) = g(E_3, E_3) = 1, \quad g(E_4, E_4) = -1.$$

Let η be the 1-form defined by $\eta(U) = g(U, E_4)$ for any $U \in \chi(M)$. Let ϕ be the (1, 1) tensor field defined by $\phi E_1 = -E_1$, $\phi E_2 = -E_2$, $\phi E_3 = -E_3$, $\phi E_4 = 0$. Then using the linearity of ϕ and g we have $\eta(E_4) = -1$, $\phi^2 U = U + \eta(U)E_4$ and $g(\phi U, \phi W) = g(U, W) + \eta(U)\eta(W)$ for any $U, W \in \chi(M)$. Thus for $E_4 = \xi$, (ϕ, ξ, η, g) defines a Lorentzian paracontact structure on M.

Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g. Then we have

$$[E_1, E_4] = -E_1, \quad [E_2, E_4] = -E_2, \quad [E_3, E_4] = -E_3.$$

Taking $E_4 = \xi$ and using Koszul formula for the Lorentzian metric g, we can easily calculate

$$\nabla_{E_1} E_4 = -E_1, \qquad \nabla_{E_2} E_2 = -E_4, \qquad \nabla_{E_3} E_4 = -E_3,$$

$$\nabla_{E_1} E_1 = -E_4, \qquad \nabla_{E_2} E_4 = -E_2, \qquad \nabla_{E_3} E_3 = -E_4.$$

From the above it can be easily seen that (ϕ, ξ, η, g) is an LP-Sasakian structure on M. Consequently $M^4(\phi, \xi, \eta, g)$ is an LP-Sasakian manifold. Using the above relations, we can easily calculate the non-vanishing components of the curvature tensor as follows :

$$R(E_1, E_3)E_1 = -E_3, \quad R(E_1, E_3)E_3 = E_1, \quad R(E_1, E_4)E_1 = -E_4,$$

$$R(E_1, E_4)E_4 = -E_1, \quad R(E_2, E_3)E_3 = E_2, \quad R(E_2, E_3)E_2 = -E_3,$$

$$R(E_2, E_4)E_2 = -E_4, \quad R(E_3, E_4)E_3 = -E_4, \quad R(E_3, E_4)E_4 = -E_3,$$

$$R(E_2, E_4)E_4 = -E_2, \quad R(E_1, E_2)E_2 = E_1, \quad R(E_1, E_2)E_1 = -E_2$$

and the components which can be obtained from these by the symmetry properties. From the above, we can easily calculate the non-vanishing components of the Ricci tensor S as follows :

$$S(E_1, E_1) = 1, \quad S(E_2, E_2) = 1, \quad S(E_3, E_3) = 1, \quad S(E_4, E_4) = -3.$$

Since $\{E_1, E_2, E_3, E_4\}$ forms a basis, any vector field $X, Y \in \chi(M)$ can be written as

$$X = a_1E_1 + b_1E_2 + c_1E_3 + d_1E_4$$

and

$$Y = a_2E_1 + b_2E_2 + c_2E_3 + d_2E_4,$$

where $a_i, b_i, c_i, d_i \in R^+$ (the set of all positive real numbers), $i = 1, 2$. This implies that

$$\phi X = -a_1E_1 - b_1E_2 - c_1E_3$$

and

$$\phi Y = -a_2E_1 - b_2E_2 - c_2E_3.$$

Hence

$$S(\phi X, \phi Y) = (a_1a_2 + b_1b_2 + c_1c_2) \neq 0.$$

By virtue of the above we have the following :

$$(\nabla_{E_i} S)(\phi X, \phi Y) = 0 \qquad \text{for} \quad i = 1,\ 2,\ 3,\ 4.$$

This leads to the following:

Theorem 3.1. *There exists an LP-Sasakian manifold (M^4, g) with η-parallel Ricci tensor.*

We now construct examples of LP-Sasakian manifolds with η-recurrent but not η-parallel Ricci tensor.

Example 3.2. We consider a 3-dimensional manifold $M = \{(x, y, z) \in R^3\}$, where (x, y, z) are the standard coordinates of R^3. Let $\{E_1, E_2, E_3\}$ be linearly independent global frame on M given by

$$E_1 = e^z \frac{\partial}{\partial x}, \qquad E_2 = e^{z-ax} \frac{\partial}{\partial y}, \quad E_3 = \frac{\partial}{\partial z},$$

where a is non-zero constant.

Let g be the Lorentzian metric defined by $g(E_1, E_3) = g(E_2, E_3) = g(E_1, E_2) = 0$, $g(E_1, E_1) = g(E_2, E_2) = 1$, $g(E_3, E_3) = -1$. Let η be the 1-form defined by $\eta(U) = g(U, E_3)$ for any $U \in \chi(M)$. Let ϕ be the (1, 1) tensor field defined by $\phi E_1 = -E_1$, $\phi E_2 = -E_2$, $\phi E_3 = 0$. Then using the linearity of ϕ and g we have $\eta(E_3) = -1$, $\phi^2 U = U + \eta(U)E_3$ and $g(\phi U, \phi W) = g(U, W) + \eta(U)\eta(W)$ for any $U, W \in \chi(M)$. Thus for $E_3 = \xi$, (ϕ, ξ, η, g) defines a Lorentzian paracontact structure on M.

Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g. Then we have

$$[E_1, E_2] = -ae^z E_2, \quad [E_1, E_3] = -E_1, \quad [E_2, E_3] = -E_2.$$

Taking $E_3 = \xi$ and using Koszul formula for the Lorentzian metric g, we can easily calculate

$$\nabla_{E_1}E_3 = -E_1, \qquad \nabla_{E_1}E_1 = -E_3, \qquad \nabla_{E_1}E_2 = 0,$$
$$\nabla_{E_2}E_3 = -E_2, \qquad \nabla_{E_3}E_2 = 0, \qquad \nabla_{E_2}E_1 = ae^zE_2,$$
$$\nabla_{E_3}E_3 = 0, \qquad \nabla_{E_2}E_2 = -ae^zE_1 - E_3, \qquad \nabla_{E_3}E_1 = 0.$$

From the above it can be easily seen that (ϕ, ξ, η, g) is an LP-Sasakian structure on M. Consequently $M^3(\phi, \xi, \eta, g)$ is an LP-Sasakian manifold. Using the above relations, we can easily calculate the non-vanishing components of the curvature tensor R as follows :

$$R(E_2, E_3)E_3 = -E_2, \quad R(E_1, E_3)E_3 = -E_1, \quad R(E_1, E_2)E_2 = (1 - a^2e^{2z})E_1,$$
$$R(E_2, E_3)E_2 = -E_3, \quad R(E_1, E_3)E_1 = -E_3, \quad R(E_1, E_2)E_1 = -(1-a^2e^{2z})E_2$$

and the components which can be obtained from these by the symmetry properties. From the above, we can easily calculate the non-vanishing components of the Ricci tensor S as follows :

$$S(E_1, E_1) = -(ae^z)^2, \quad S(E_2, E_2) = -(ae^z)^2, \quad S(E_3, E_3) = -2.$$

Since $\{E_1, E_2, E_3\}$ forms a basis, any vector field $X, Y \in \chi(M)$ can be written as

$$X = a_1E_1 + b_1E_2 + c_1E_3$$

and

$$Y = a_2E_1 + b_2E_2 + c_2E_3,$$

where $a_i, b_i, c_i \in R^+$ (the set of all positive real numbers), $i = 1, 2$. This implies that

$$\phi X = -a_1E_1 - b_1E_2$$

and

$$\phi Y = -a_2E_1 - b_2E_2.$$

Hence

$$S(\phi X, \phi Y) = -(a_1a_2 + b_1b_2)(ae^z)^2.$$

By virtue of the above we have the following :

$$(\nabla_{E_1}S)(\phi X, \phi Y) = 0,$$
$$(\nabla_{E_2}S)(\phi X, \phi Y) = -(a_1b_2 + a_2b_1)(ae^z)^3,$$
$$(\nabla_{E_3}S)(\phi X, \phi Y) = -2(a_1a_2 + b_1b_2)(ae^z)^2.$$

Let us now consider the 1-forms

$$A(E_1) = 0,$$
$$A(E_2) = \frac{(a_1b_2 + a_2b_1)}{(a_1a_2 + b_1b_2)}(ae^z),$$
$$A(E_3) = 2,$$

at any point $p \in M$. In our M^3, (3.1) reduces with these 1-forms to the following equations :

$$(\nabla_{E_i} S)(\phi X, \phi Y) = A(E_i) S(\phi X, \phi Y), \quad i = 1, \ 2, \ 3.$$

This implies that the manifold under consideration is an LP-Sasakian manifold with η-recurrent but not η-parallel Ricci tensor. This leads to the following :

Theorem 3.2. *There exists an LP-Sasakian manifold (M^3, g) with η-recurrent but not η-parallel Ricci tensor.*

Example 3.3. We consider a 4-dimensional manifold $M = \{(x, y, z, u) \in R^4\}$, where (x, y, z, u) are the standard coordinates of R^4. Let $\{E_1, E_2, E_3, E_4\}$ be linearly independent global frame on M given by

$$E_1 = ye^{-u}\frac{\partial}{\partial y}, \qquad E_2 = ye^{-u}\frac{\partial}{\partial x}, \quad E_3 = e^{-u}\frac{\partial}{\partial z}, \quad E_4 = \frac{\partial}{\partial u}.$$

Let g be the Lorentzian metric defined by

$$g(E_1, E_3) = g(E_2, E_3) = g(E_1, E_4) = g(E_2, E_4) = g(E_3, E_4) = g(E_1, E_2) = 0,$$

$$g(E_1, E_1) = g(E_2, E_2) = g(E_3, E_3) = 1, \quad g(E_4, E_4) = -1.$$

Let η be the 1-form defined by $\eta(U) = g(U, E_4)$ for any $U \in \chi(M)$. Let ϕ be the (1, 1) tensor field defined by $\phi E_1 = -E_1$, $\phi E_2 = -E_2$, $\phi E_3 = -E_3$, $\phi E_4 = 0$. Then using the linearity of ϕ and g we have $\eta(E_4) = -1$, $\phi^2 U = U + \eta(U)E_4$ and $g(\phi U, \phi W) = g(U, W) + \eta(U)\eta(W)$ for any $U, W \in \chi(M)$. Thus for $E_4 = \xi$, (ϕ, ξ, η, g) defines a Lorentzian paracontact structure on M.

Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g. Then we have

$$[E_1, E_2] = e^{-u}E_2, \quad [E_1, E_4] = E_1, \quad [E_2, E_4] = E_2, \quad [E_3, E_4] = E_3.$$

Taking $E_4 = \xi$ and using Koszul formula for the Lorentzian metric g, we can easily calculate

$$\nabla_{E_1} E_4 = E_1, \qquad \nabla_{E_2} E_2 = E_4 + e^{-u}E_1, \qquad \nabla_{E_2} E_1 = -e^{-u}E_2,$$

$$\nabla_{E_3} E_4 = E_3, \qquad \nabla_{E_1} E_1 = E_4, \qquad \nabla_{E_2} E_4 = E_2, \qquad \nabla_{E_3} E_3 = E_4.$$

From the above it can be easily seen that (ϕ, ξ, η, g) is an LP-Sasakian structure on M. Consequently $M^4(\phi, \xi, \eta, g)$ is an LP-Sasakian manifold. Using the above relations, we can easily calculate the non-vanishing components of the curvature tensor as follows :

$$R(E_1, E_3)E_1 = -E_3, \quad R(E_1, E_4)E_1 = -E_4,$$

$$R(E_1, E_4)E_4 = -E_1,$$

$$R(E_2, E_4)E_2 = -E_4, \quad R(E_3, E_4)E_3 = -E_4, \quad R(E_3, E_4)E_4 = -E_3,$$

$$R(E_2, E_4)E_4 = -E_2, \ R(E_1, E_2)E_2 = (1 - e^{-2u})E_1, \ R(E_1, E_2)E_1 = -(1 - e^{-2u})E_2$$

and the components which can be obtained from these by the symmetry properties. From the above, we can easily calculate the non-vanishing components of the Ricci tensor S as follows :

$$S(E_1,E_1)=(1-e^{-2u}),\ S(E_2,E_2)=(1-e^{-2u}),\ S(E_3,E_3)=1,\ S(E_4,E_4)=-3.$$

Since $\{E_1, E_2, E_3, E_4\}$ forms a basis, any vector field $X, Y \in \chi(M)$ can be written as

$$X = \alpha_1 E_1 + \alpha_2 E_2 + \alpha_3 E_3 + \alpha_4 E_4$$

and

$$Y = \beta_1 E_1 + \beta_2 E_2 + \beta_3 E_3 + \beta_4 E_4,$$

where $\alpha_i, \beta_i \in R^+$(the set of all positive real numbers), $i = 1,\ 2,\ 3,\ 4$. This implies that

$$\phi X = -\alpha_1 E_1 - \alpha_2 E_2 - \alpha_3 E_3$$

and

$$\phi Y = -\beta_1 E_1 - \beta_2 E_2 - \beta_3 E_3.$$

Hence

$$S(\phi X, \phi Y) = (\alpha_1\beta_1 + \alpha_2\beta_2)(1 - e^{-2u}) + \alpha_3\beta_3.$$

By virtue of the above we have the following :

$$\begin{aligned}
&(\nabla_{E_1} S)(\phi X, \phi Y) = 0,\\
&(\nabla_{E_2} S)(\phi X, \phi Y) = -(\alpha_1\beta_2 + \alpha_2\beta_1)e^{-3u},\\
&(\nabla_{E_3} S)(\phi X, \phi Y) = 0,\\
&(\nabla_{E_4} S)(\phi X, \phi Y) = 2(\alpha_1\beta_1 + \alpha_2\beta_2)e^{-2u}.
\end{aligned}$$

Let us now consider the 1-forms

$$\begin{aligned}
&A(E_1) = 0,\\
&A(E_2) = -\frac{(\alpha_1\beta_2 + \alpha_2\beta_1)e^{-3u}}{(\alpha_1\beta_1 + \alpha_2\beta_2)(1 - e^{-2u}) + \alpha_3\beta_3},\\
&A(E_3) = 0,\\
&A(E_4) = \frac{2(\alpha_1\beta_1 + \alpha_2\beta_2)e^{-2u}}{(\alpha_1\beta_1 + \alpha_2\beta_2)(1 - e^{-2u}) + \alpha_3\beta_3}
\end{aligned}$$

at any point $p \in M$. In our M^4, (3.1) reduces with these 1-forms to the following equations :

$$(\nabla_{E_i} S)(\phi X, \phi Y) = A(E_i) S(\phi X, \phi Y), \qquad i = 1,\ 2,\ 3,\ 4.$$

This implies that the manifold under consideration is an LP-Sasakian manifold with η-recurrent but not η-parallel Ricci tensor. This leads to the following :

Theorem 3.3. *There exists an LP-Sasakian manifold (M^4, g) with η-recurrent but not η-parallel Ricci tensor.*

4. LP-Sasakian manifolds with ϕ-recurrent Ricci tensor

Definition 4.1 ([7])**.** The Ricci tensor S of an LP-Sasakian manifold is said to be ϕ-recurrent if it satisfies

$$(\nabla_{\phi X} S)(\phi Y, \phi Z) = A(\phi X) S(\phi Y, \phi Z) \tag{4.1}$$

for all X, Y, Z where A is a non-zero 1-form.

In particular, if the 1-form A vanishes then the Ricci tensor of the LP-Sasakian manifold is said to be ϕ-parallel. We note that the condition of Ricci-ϕ-parallelity is much more weaker than Ricci-η-parallelity.

In ([7]), A. A. Shaikh and K. K. Baishya also studied several properties of LP-Sasakian manifolds with ϕ-recurrent Ricci tensor. We first construct an example of LP-Sasakian manifold with global vector fields whose Ricci tensor is ϕ-parallel.

Example 4.1. We consider a 4-dimensional manifold $M = \{(x, y, z, u) \in R^4\}$, where (x, y, z, u) are the standard coordinates of R^4. Let $\{E_1, E_2, E_3, E_4\}$ be linearly independent global frame on M given by

$$E_1 = e^{-u} \frac{\partial}{\partial x}, \quad E_2 = e^{-u} \frac{\partial}{\partial y}, \quad E_3 = e^{-u} \frac{\partial}{\partial z}, \quad E_4 = \frac{\partial}{\partial u}.$$

Let g be the Lorentzian metric defined by

$$g(E_1, E_3) = g(E_2, E_3) = g(E_1, E_4) = g(E_2, E_4) = g(E_3, E_4) = g(E_1, E_2) = 0,$$
$$g(E_1, E_1) = g(E_2, E_2) = g(E_3, E_3) = 1, \quad g(E_4, E_4) = -1.$$

Let η be the 1-form defined by $\eta(U) = g(U, E_4)$ for any $U \in \chi(M)$. Let ϕ be the (1, 1) tensor field defined by $\phi E_1 = -E_1$, $\phi E_2 = -E_2$, $\phi E_3 = -E_3$, $\phi E_4 = 0$. Then using the linearity of ϕ and g we have $\eta(E_4) = -1$, $\phi^2 U = U + \eta(U) E_4$ and $g(\phi U, \phi W) = g(U, W) + \eta(U)\eta(W)$ for any $U, W \in \chi(M)$. Thus for $E_4 = \xi$, (ϕ, ξ, η, g) defines a Lorentzian paracontact structure on M.

Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g. Then we have

$$[E_1, E_4] = E_1, \quad [E_2, E_4] = E_2, \quad [E_3, E_4] = E_3.$$

Taking $E_4 = \xi$ and using Koszul formula for the Lorentzian metric g, we can easily calculate

$$\nabla_{E_1} E_4 = E_1, \qquad \nabla_{E_2} E_2 = E_4, \qquad \nabla_{E_3} E_4 = E_3,$$
$$\nabla_{E_1} E_1 = E_4, \qquad \nabla_{E_2} E_4 = E_2, \qquad \nabla_{E_3} E_3 = E_4.$$

From the above it can be easily seen that (ϕ, ξ, η, g) is an LP-Sasakian structure on M. Consequently $M^4(\phi, \xi, \eta, g)$ is an LP-Sasakian manifold. Using the above relations, we can easily calculate the non-vanishing components of the curvature tensor as follows :

$$R(E_1, E_3)E_1 = -E_3, \quad R(E_1, E_3)E_3 = E_1, \quad R(E_1, E_4)E_1 = -E_4,$$
$$R(E_1, E_4)E_4 = -E_1, \quad R(E_2, E_3)E_3 = E_2, \quad R(E_2, E_3)E_2 = -E_3,$$

$$R(E_2,E_4)E_2=-E_4, \quad R(E_3,E_4)E_3=-E_4, \quad R(E_3,E_4)E_4=-E_3,$$
$$R(E_2,E_4)E_4=-E_2, \quad R(E_1,E_2)E_2=E_1, \quad R(E_1,E_2)E_1=-E_2$$

and the components which can be obtained from these by the symmetry properties. From the components of R, we can easily calculate the non-vanishing components of the Ricci tensor S as follows :

$$S(E_1,E_1)=1, \quad S(E_2,E_2)=1, \quad S(E_3,E_3)=1, \quad S(E_4,E_4)=-3.$$

Since $\{E_1,E_2,E_3,E_4\}$ forms a basis of the LP-Sasakian manifold, any vector field $X, Y \in \chi(M)$ can be written as

$$X=a_1E_1+b_1E_2+c_1E_3+d_1E_4$$

and

$$Y=a_2E_1+b_2E_2+c_2E_3+d_2E_4,$$

where $a_i, b_i, c_i, d_i \in R^+$ (the set of all positive real numbers), $i=1,\ 2$. This implies that

$$\phi X=-a_1E_1-b_1E_2-c_1E_3$$

and

$$\phi Y=-a_2E_1-b_2E_2-c_2E_3.$$

Hence

$$S(\phi X,\phi Y)=(a_1a_2+b_1b_2+c_1c_2)\neq 0.$$

By virtue of the above we have the following :

$$(\nabla_{\phi E_i}S)(\phi X,\phi Y)=0 \qquad \text{for} \quad i=1,\ 2,\ 3.$$

Hence the Ricci tensor of the manifold under consideration is ϕ-parallel. Thus we can state the following:

Theorem 4.1. *There exists an LP-Sasakian manifold (M^4,g) with ϕ-parallel Ricci tensor.*

Example 4.2. We consider a 4-dimensional manifold $M=\{(x,y,z,u)\in R^4\}$, where (x,y,z,u) are the standard coordinates of R^4. Let $\{E_1,E_2,E_3,E_4\}$ be linearly independent global frame on M given by

$$E_1=e^u\frac{\partial}{\partial x}, \qquad E_2=e^{u-x}\frac{\partial}{\partial y}, \quad E_3=e^u\frac{\partial}{\partial z}, \qquad E_4=\frac{\partial}{\partial u}.$$

Let g be the Lorentzian metric defined by

$$g(E_1,E_3)=g(E_2,E_3)=g(E_1,E_4)=g(E_2,E_4)=g(E_3,E_4)=g(E_1,E_2)=0,$$
$$g(E_1,E_1)=g(E_2,E_2)=g(E_3,E_3)=1, \quad g(E_4,E_4)=-1.$$

Let η be the 1-form defined by $\eta(U)=g(U,E_4)$ for any $U\in\chi(M)$. Let ϕ be the (1, 1) tensor field defined by $\phi E_1=-E_1$, $\phi E_2=-E_2$, $\phi E_3=-E_3$, $\phi E_4=0$. Then using the linearity of ϕ and g we have $\eta(E_4)=-1$, $\phi^2U=U+\eta(U)E_4$ and $g(\phi U,\phi W)=g(U,W)+\eta(U)\eta(W)$ for any $U,W\in\chi(M)$. Thus for $E_4=\xi$, (ϕ,ξ,η,g) defines a Lorentzian paracontact structure on M.

Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g. Then we have

$$[E_1, E_2] = -e^u E_2, \quad [E_1, E_4] = -E_1, \quad [E_2, E_4] = -E_2, \quad [E_3, E_4] = -E_3.$$

Taking $E_4 = \xi$ and using Koszul formula for the Lorentzian metric g, we can easily calculate

$$\nabla_{E_1} E_4 = -E_1, \qquad \nabla_{E_2} E_2 = -E_4 - e^u E_1, \qquad \nabla_{E_2} E_1 = -e^u E_2,$$

$$\nabla_{E_3} E_4 = -E_3, \quad \nabla_{E_1} E_1 = -E_4, \quad \nabla_{E_2} E_4 = -E_2, \quad \nabla_{E_3} E_3 = -E_4.$$

From the above it can be easily seen that (ϕ, ξ, η, g) is an LP-Sasakian structure on M. Consequently $M^4(\phi, \xi, \eta, g)$ is an LP-Sasakian manifold. Using the above relations, we can easily calculate the non-vanishing components of the curvature tensor as follows :

$$R(E_2, E_3)E_3 = E_2, \quad R(E_1, E_3)E_3 = E_1, \quad R(E_1, E_4)E_1 = -E_4,$$

$$R(E_1, E_4)E_4 = -E_1, \quad R(E_2, E_3)E_2 = -E_3, \quad R(E_1, E_3)E_1 = -E_3,$$

$$R(E_2, E_4)E_2 = -E_4, \quad R(E_3, E_4)E_3 = -E_4, \quad R(E_3, E_4)E_4 = -E_3,$$

$$R(E_2, E_4)E_4 = -E_2, \ R(E_1, E_2)E_2 = (1-e^{2u})E_1, \ R(E_1, E_2)E_1 = -(1-e^{2u})E_2$$

and the components which can be obtained from these by the symmetry properties. From the above, we can easily calculate the non-vanishing components of the Ricci tensor S as follows :

$$S(E_1, E_1) = (1-e^{2u}), \ S(E_2, E_2) = (1-e^{2u}), \ S(E_3, E_3) = 1, \ S(E_4, E_4) = -3.$$

Since $\{E_1, E_2, E_3, E_4\}$ forms a basis of the LP-Sasakian manifold, any vector field $X, Y \in \chi(M)$ can be written as

$$X = \alpha_1 E_1 + \alpha_2 E_2 + \alpha_3 E_3 + \alpha_4 E_4 \quad \text{and} \quad Y = \beta_1 E_1 + \beta_2 E_2 + \beta_3 E_3 + \beta_4 E_4, \tag{4.2}$$

where $\alpha_i, \beta_i \in R^+$(the set of all positive real numbers), $i = 1, 2, 3, 4$. This implies that

$$\phi X = -\alpha_1 E_1 - \alpha_2 E_2 - \alpha_3 E_3$$

and

$$\phi Y = -\beta_1 E_1 - \beta_2 E_2 - \beta_3 E_3.$$

Hence

$$S(\phi X, \phi Y) = (\alpha_1 \beta_1 + \alpha_2 \beta_2)(1 - e^{2u}) + \alpha_3 \beta_3.$$

By virtue of the above we have the following :

$$\begin{aligned}
&(\nabla_{\phi E_1} S)(\phi X, \phi Y) = 0, \\
&(\nabla_{\phi E_2} S)(\phi X, \phi Y) = (\alpha_1 \beta_2 + \alpha_2 \beta_1) e^{3u}, \\
&(\nabla_{\phi E_3} S)(\phi X, \phi Y) = 0.
\end{aligned}$$

Let us now consider the 1-forms

$$A(E_1) = 0,$$
$$A(E_2) = \frac{(\alpha_1\beta_2 + \alpha_2\beta_1)e^{3u}}{(\alpha_1\beta_1 + \alpha_2\beta_2)(1 - e^{2u}) + \alpha_3\beta_3},$$
$$A(E_3) = 0$$

at any point $p \in M$. In our M^4, (4.1) reduces with these 1-forms to the following equations :

$$(\nabla_{\phi E_i} S)(\phi X, \phi Y) = A(E_i)S(\phi X, \phi Y), \quad i = 1, 2, 3.$$

This implies that the Ricci tensor of the manifold under consideration is ϕ-recurrent but not ϕ-parallel. This leads to the following :

Theorem 4.2. *There exists an LP-Sasakian manifold (M^4, g) with ϕ-recurrent Ricci tensor but not ϕ-parallel.*

However, since $\{E_1, E_2, E_3, E_4\}$ is a basis of M^4, if we consider the vector fields $X, Y \in \chi(M)$ in (4.2) such that $\alpha_2 = k\alpha_1$ and $\beta_2 = -k\beta_1$ where $k \in R - \{-1, 0, 1\}$, then we have

$$\phi X = -\alpha_1 E_1 - k\alpha_1 E_2 - \alpha_3 E_3$$

and

$$\phi Y = -\beta_1 E_1 + k\beta_1 E_2 - \beta_3 E_3.$$

Consequently, we get

$$S(\phi X, \phi Y) = (1 - k^2)(1 - e^{2u})\alpha_1\beta_1 + \alpha_3\beta_3 \neq 0,$$
$$(\nabla_{\phi E_i} S)(\phi X, \phi Y) = 0, \quad i = 1, 2, 3, 4$$

and

$$\begin{aligned}(\nabla_{E_i} S)(\phi X, \phi Y) &= 0 \qquad i = 1, 2, 3,\\ &= -2(1 - k^2)e^{2u}\alpha_1\beta_1 \quad \text{for} \quad i = 4,\end{aligned}$$

for all $X, Y \in \chi(M)$ and hence the Ricci tensor S of M^4 is ϕ-parallel but not η-parallel. This leads to the following :

Theorem 4.3. *There exists an LP-Sasakian manifold (M^4, g) with ϕ-parallel Ricci tensor but not η-parallel.*

5. Generalized Ricci recurrent LP-Sasakian manifolds

Definition 5.1 ([1])**.** An LP-Sasakian manifold is said to be generalized Ricci recurrent if its Ricci tensor S of type (0, 2) satisfies the condition

$$(\nabla_X S)(Y, Z) = A(X)S(Y, Z) + B(X)g(Y, Z), \tag{5.1}$$

where A and B are two non-zero 1-forms such that $A(X) = g(X, P)$ and $B(X) = g(X, L)$, P and L being associated vector fields of the 1-form.

Theorem 5.1. *In a generalized Ricci recurrent LP-Sasakian manifold the associated 1-forms are linearly dependent and the vector fields of the associated 1-forms are of opposite direction.*

Proof. In a generalized Ricci recurrent LP-Sasakian manifold we have the relation (5.1). Setting $Z = \xi$ in (5.1) we have

$$(\nabla_X S)(Y, \xi) = [(n-1)A(X) + B(X)]\eta(Y). \tag{5.2}$$

Again

$$(\nabla_X S)(Y, \xi) = \nabla_X S(Y, \xi) - S(\nabla_X Y, \xi) - S(Y, \nabla_X \xi),$$

which yields by virtue of (2.1), (2.2) and (2.5) that

$$(\nabla_X S)(Y, \xi) = (n-1)g(X, \phi Y) - S(\phi X, Y). \tag{5.3}$$

From (5.2) and (5.3) it follows that

$$[(n-1)A(X) + B(X)]\eta(Y) = (n-1)g(X, \phi Y) - S(\phi X, Y). \tag{5.4}$$

Replacing Y by ξ in (5.4) we obtain

$$(n-1)A(X) + B(X) = 0. \tag{5.5}$$

This proves the Theorem. □

Theorem 5.2. *A generalized Ricci recurrent LP-Sasakian manifold is Einstein.*

Proof. In a generalized Ricci recurrent LP-Sasakian manifold we have the relation (5.4). Hence setting $Y = \phi Y$ in (5.4) and then using (2.8) we have

$$S(X, Y) = (n-1)g(X, Y). \tag{5.6}$$

This proves the Theorem. □

Example 5.1. We consider a 3-dimensional manifold $M = \{(x, y, z) \in R^3\}$, where (x, y, z) are the standard coordinates of R^3. Let $\{E_1, E_2, E_3\}$ be linearly independent global frame on M given by

$$E_1 = e^z \frac{\partial}{\partial y}, \qquad E_2 = e^z\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right), \quad E_3 = \frac{\partial}{\partial z}.$$

Let g be the Lorentzian metric defined by $g(E_1, E_3) = g(E_2, E_3) = g(E_1, E_2) = 0$, $g(E_1, E_1) = g(E_2, E_2) = 1, g(E_3, E_3) = -1$. Let η be the 1-form defined by $\eta(U) = g(U, E_3)$ for any $U \in \chi(M)$. Let ϕ be the (1, 1) tensor field defined by $\phi E_1 = -E_1$, $\phi E_2 = -E_2$, $\phi E_3 = 0$. Then using the linearity of ϕ and g we have $\eta(E_3) = -1$, $\phi^2 U = U + \eta(U)E_3$ and $g(\phi U, \phi W) = g(U, W) + \eta(U)\eta(W)$ for any $U, W \in \chi(M)$. Thus for $E_3 = \xi$, (ϕ, ξ, η, g) defines a Lorentzian paracontact structure on M.

Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g. Then we have

$$[E_1, E_2] = 0, \quad [E_1, E_3] = -E_1, \quad [E_2, E_3] = -E_2.$$

Taking $E_3 = \xi$ and using Koszul formula for the Lorentzian metric g, we can easily calculate

$$\nabla_{E_1}E_3 = -E_1, \qquad \nabla_{E_1}E_1 = -E_3, \qquad \nabla_{E_1}E_2 = 0,$$
$$\nabla_{E_2}E_3 = -E_2, \qquad \nabla_{E_2}E_2 = -E_3, \qquad \nabla_{E_2}E_1 = 0,$$
$$\nabla_{E_3}E_3 = 0, \qquad \nabla_{E_3}E_2 = 0, \qquad \nabla_{E_3}E_1 = 0.$$

From the above it can be easily seen that (ϕ, ξ, η, g) is an LP-Sasakian structure on M. Consequently $M^3(\phi, \xi, \eta, g)$ is an LP-Sasakian manifold. Using the above relations, we can easily calculate the non-vanishing components of the curvature tensor as follows :

$$R(E_2, E_3)E_3 = -E_2, \quad R(E_1, E_3)E_3 = -E_1, \quad R(E_1, E_2)E_2 = -E_1,$$
$$R(E_2, E_3)E_2 = -E_3, \quad R(E_1, E_3)E_1 = -E_3, \quad R(E_1, E_2)E_1 = E_2$$

and the components which can be obtained from these by the symmetry properties. From the above, we can easily calculate the non-vanishing components of the Ricci tensor S as follows :

$$S(E_3, E_3) = -2.$$

Since $\{E_1, E_2, E_3\}$ forms a basis of the LP-Sasakian manifold, any vector field $X, Y \in \chi(M)$ can be written as

$$X = a_1E_1 + b_1E_2 + c_1E_3$$

and

$$Y = a_2E_1 + b_2E_2 + c_2E_3,$$

where $a_i, b_i, c_i \in R^+$ (the set of all positive real numbers), $i = 1, 2$. Hence

$$S(X, Y) = -2c_1c_2 \quad \text{and} \quad g(X, Y) = a_1a_2 + b_1b_2 - c_1c_2.$$

By virtue of the above we have the following :

$$(\nabla_{E_1}S)(X, Y) = -2(a_1c_2 + a_2c_1), \qquad (\nabla_{E_2}S)(X, Y) = -2(b_1c_2 + b_2c_1),$$
$$\text{and} \quad (\nabla_{E_3}S)(X, Y) = 0.$$

Consequently, the manifold under consideration is not Ricci symmetric. Let us now consider the 1-forms

$$A(E_1) = \frac{(a_1c_2 + a_2c_1)}{(a_1a_2 + b_1b_2)}, \qquad B(E_1) = \frac{-2(a_1c_2 + a_2c_1)}{(a_1a_2 + b_1b_2)},$$
$$A(E_2) = \frac{(b_1c_2 + b_2c_1)}{(a_1a_2 + b_1b_2)}, \qquad B(E_2) = \frac{-2(b_1c_2 + b_2c_1)}{(a_1a_2 + b_1b_2)},$$
$$A(E_3) = 0, \qquad B(E_3) = 0$$

at any point $x \in M$. From (5.1) we have

$$(5.7) \qquad (\nabla_{E_i}S)(X, Y) = A(E_i)S(X, Y) + B(E_i)g(X, Y), \quad i = 1, \ 2, \ 3.$$

It can be easily shown that the manifold with these 1-forms satisfies the relation (5.7). Hence the manifold under consideration is a generalized Ricci recurrent

LP-Sasakian manifold which is neither Ricci-symmetric nor Ricci-recurrent. This leads to the following :

Theorem 5.3. *There exists a generalized Ricci recurrent LP-Sasakian manifold* (M^3, g) *which is neither Ricci-symmetric nor Ricci-recurrent.*

Example 5.2. We consider a 4-dimensional manifold $M = \{(x, y, z, u) \in R^4\}$, where (x, y, z, u) are the standard coordinates of R^4. Let $\{E_1, E_2, E_3, E_4\}$ be linearly independent global frame on M given by

$$E_1 = u\frac{\partial}{\partial y}, \quad E_2 = u(\frac{\partial}{\partial x} + z\frac{\partial}{\partial z}), \quad E_3 = u\frac{\partial}{\partial z}, \quad E_4 = \frac{\partial}{\partial u}.$$

Let g be the Lorentzian metric defined by

$$g(E_1, E_3) = g(E_2, E_3) = g(E_1, E_4) = g(E_2, E_4) = g(E_3, E_4) = g(E_1, E_2) = 0,$$

$$g(E_1, E_1) = g(E_2, E_2) = g(E_3, E_3) = 1, \quad g(E_4, E_4) = -1.$$

Let η be the 1-form defined by $\eta(U) = g(U, E_4)$ for any $U \in \chi(M)$. Let ϕ be the (1, 1) tensor field defined by $\phi E_1 = -E_1$, $\phi E_2 = -E_2$, $\phi E_3 = -E_3$, $\phi E_4 = 0$. Then using the linearity of ϕ and g we have $\eta(E_4) = -1$, $\phi^2 U = U + \eta(U)E_4$ and $g(\phi U, \phi W) = g(U, W) + \eta(U)\eta(W)$ for any $U, W \in \chi(M)$. Thus for $E_4 = \xi$, (ϕ, ξ, η, g) defines a Lorentzian paracontact structure on M.

Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g. Then we have

$$[E_2, E_3] = -uE_3, \quad [E_1, E_4] = -E_1, \quad [E_2, E_4] = -E_2, \quad [E_3, E_4] = -E_3.$$

Taking $E_4 = \xi$ and using Koszul formula for the Lorentzian metric g, we can easily calculate

$$\nabla_{E_1} E_4 = -E_1, \qquad \nabla_{E_2} E_4 = -E_2, \qquad \nabla_{E_3} E_3 = -E_4 - uE_2,$$

$$\nabla_{E_3} E_4 = -E_3, \quad \nabla_{E_1} E_1 = -E_4, \quad \nabla_{E_2} E_1 = -uE_2, \quad \nabla_{E_2} E_3 = -E_4.$$

From the above it can be easily seen that (ϕ, ξ, η, g) is an LP-Sasakian structure on M. Consequently $M^4(\phi, \xi, \eta, g)$ is an LP-Sasakian manifold. Using the above relations, we can easily calculate the non-vanishing components of the curvature tensor as follows :

$$R(E_1, E_2)E_1 = -E_2, \quad R(E_1, E_2)E_2 = E_1, \quad R(E_1, E_3)E_1 = -E_3,$$

$$R(E_1, E_3)E_3 = E_1, \quad R(E_1, E_4)E_1 = -E_4, \quad R(E_1, E_4)E_4 = -E_1,$$

$$R(E_2, E_4)E_2 = -E_4, \quad R(E_3, E_4)E_3 = -E_4, \quad R(E_3, E_4)E_4 = -E_3,$$

$$R(E_2, E_4)E_4 = -E_2, \; R(E_2, E_3)E_3 = (1-u^2)E_2, \; R(E_2, E_3)E_2 = -(1-u^2)E_3$$

and the components which can be obtained from these by the symmetry properties. From the above, we can easily calculate the non-vanishing components of the Ricci tensor S as follows :

$$S(E_1, E_1) = 1, \; S(E_2, E_2) = (1-u^2), \; S(E_3, E_3) = (1-u^2), \; S(E_4, E_4) = -3.$$

Since $\{E_1, E_2, E_3\}$ forms a basis of the LP-Sasakian manifold, any vector field $X, Y \in \chi(M)$ can be written as

$$X = a_1E_1 + b_1E_2 + c_1E_3 + d_1E_4$$

and

$$Y = a_2E_1 + b_2E_2 + c_2E_3 + d_2E_4,$$

where $a_i, b_i, c_i, d_i \in R^+$(the set of all positive real numbers), $i = 1,\ 2$. Hence

$$S(X, Y) = (a_1a_2 - 3d_1d_2) + (b_1b_2 + c_1c_2)(1 - u^2)$$

and

$$g(X, Y) = a_1a_2 + b_1b_2 + c_1c_2 - d_1d_2.$$

By virtue of the above we have the following :

$$\begin{aligned}
(\nabla_{E_1}S)(X, Y) &= -2(a_1d_2 + a_2d_1)\\
(\nabla_{E_2}S)(X, Y) &= -2(b_1d_2 + b_2d_1)\\
(\nabla_{E_3}S)(X, Y) &= -(u^2 + 2)(c_1d_2 + c_2d_1)\\
(\nabla_{E_4}S)(X, Y) &= -2u^2(b_1b_2 + c_1c_2).
\end{aligned}$$

This implies that the manifold under consideration is not Ricci symmetric. Let us now consider the 1-forms

$$\begin{aligned}
A(E_1) &= \frac{2(a_1d_2 + a_2d_1)}{2a_1a_2 + (u^2 + 2)(b_1b_2 + c_1c_2)},\\
B(E_1) &= -\frac{6(a_1d_2 + a_2d_1)}{2a_1a_2 + (u^2 + 2)(b_1b_2 + c_1c_2)},\\
A(E_2) &= \frac{2(b_1d_2 + b_2d_1)}{2a_1a_2 + (u^2 + 2)(b_1b_2 + c_1c_2)},\\
B(E_2) &= -\frac{6(b_1d_2 + b_2d_1)}{2a_1a_2 + (u^2 + 2)(b_1b_2 + c_1c_2)},\\
A(E_3) &= -\frac{(u^2 + 2)(c_1d_2 + c_2d_1)}{2a_1a_2 + (u^2 + 2)(b_1b_2 + c_1c_2)},\\
B(E_3) &= \frac{3(u^2 + 2)(c_1d_2 + c_2d_1)}{2a_1a_2 + (u^2 + 2)(b_1b_2 + c_1c_2)},\\
A(E_4) &= \frac{2u^2(b_1b_2 + c_1c_2)}{2a_1a_2 + (u^2 + 2)(b_1b_2 + c_1c_2)},\\
B(E_4) &= -\frac{6u^2(b_1b_2 + c_1c_2)}{2a_1a_2 + (u^2 + 2)(b_1b_2 + c_1c_2)}
\end{aligned}$$

at any point $x \in M$. From (5.1) we have

$$(\nabla_{E_i}S)(X, Y) = A(E_i)S(X, Y) + B(E_i)g(X, Y), \quad i = 1,\ 2,\ 3,\ 4. \tag{5.8}$$

It can be easily shown that the manifold with the 1-forms under consideration satisfies the relation (5.8). Hence the manifold under consideration is a generalized Ricci recurrent LP-Sasakian manifold which is neither Ricci-symmetric nor Ricci-recurrent. This leads to the following :

Theorem 5.4. *There exists a generalized Ricci recurrent LP-Sasakian manifold* (M^4, g) *which is neither Ricci-symmetric nor Ricci-recurrent.*

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Absos A. Shaikh
Department of Mathematics
University of Burdwan, Golapbag
Burdwan-713104, West Bengal, India
E-mail address: aask2003@yahoo.co.in

Kanak K. Baishya
Department of Mathematics
University of Burdwan, Golapbag
Burdwan-713104, West Bengal, India

Sabina Eyasmin
Department of Mathematics
University of Burdwan, Golapbag
Burdwan-713104, West Bengal, India