International Journal of Pure and Applied Mathematics
Volume 118 No. 4 2018, 1053-1067
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v118i4.18

ijpam.eu

ON THE EXTENDED CHEN'S AND PHAM'S SOFTWARE RELIABILITY MODELS. SOME APPLICATIONS

Nikolay Pavlov[1], Anton Iliev[2 §],
Asen Rahnev[3], Nikolay Kyurkchiev[4]

[1,2,3,4]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

Abstract: The Hausdorff approximation of the impulse function $\sigma^{**}(t)$ by sigmoidal functions based on the extended Chen's and Pham's cumulative functions are studied and an expression for the error of the best approximation is found. The received results are of independent significance in the study of issues related to neural networks and impulse technics. Using programming environment Mathematica we give results of many numerical examples which confirm the theory presented here. We give also real examples with data provided in [4] using extended Chen's software reliability model and extended Pham's deterministic software reliability model. Dataset included [5] Year 2000 compatibility modifications, operating system upgrade, and signaling message processing. Some direct comparisons are made.

AMS Subject Classification: 41A46
Key Words: three–parameters Chen's cumulative function (3Ccdf), four parameters Pham's cumulative function (4Pcdf), impulse function $\sigma^{**}(t)$, Hausdorff approximation, upper and lower bounds

1. Introduction

An important role within the hierarchical models in the procedure for quantifying the quality of software products is played by the so-called computational

Received: March 2, 2018
Revised: May 12, 2018
Published: May 23, 2018

§Correspondence author

method based on the theoretical and empirical dependencies (usually at an early stage in their development), statistical data accumulated during tests, exploitation and the accompaniment of the program product.

An important measure of reliability assessment (completeness, accuracy and consistency) is the so-called metric (asymptotic metrics).

There are two main approaches to testing: structured and functional.

Many studies have been devoted to this overarching theme. We will only note that, depending on the test data selected and the expected results, testing is divided into: deterministic testing and stochastic testing.

Detailed description of all elements in the area of debugging theory may be found in the following books [1]–[3].

For some degradation models with applications to reliability and survival analysis, see [6].

In the book [7], we pay particular attention to both deterministic approaches and probability models for debugging theories.

A Hausdorff metric was chosen to evaluate the test data which are fitted to the sigmoid models proposed in this book.

Some of the existing cumulative distributions (Gompertz–Makeham, Yamada-exponential, Yamada–Rayleigh, Yamada–Weibull, transmuted inverse exponential, transmuted Log-Logistic, Kumaraswamy–Dagum and Kumaraswamy Quasi Lindley) are considered in the light of modern debugging and test theories.

Some software reliability models, can be found in [8]–[32].

In a number of cases, these results have independent significance in the study of issues related to neural networks and impulse technics (see, for instance [33]–[39], [40]–[52]).

In this note we study the Hausdorff approximation of the impulse function $\sigma^{**}(t)$ by sigmoidal functions based on the extended Chen's [53] and Pham's cumulative functions.

We propose a software modules (intellectual properties) within the programming environment CAS Mathematica for the analysis.

The models have been tested with real-world data.

2. Preliminaries

The typical example of impulse function from antenna feeder technique has the following shape (see, Fig. 1):

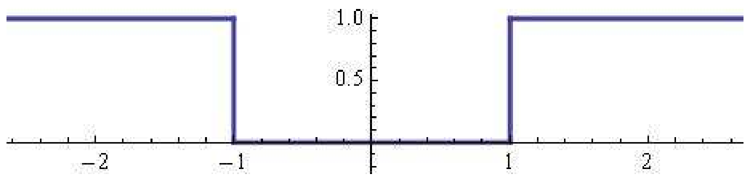

Figure 1: The signal of $\sigma^{**}(t)$ – type.

$$\sigma^{**}(t) = \begin{cases} 1, \ t \in [-\infty, -1) \cup (1, +\infty) \\ 0, \ t \in [-1, 1]. \end{cases} \tag{1}$$

Many probability distributions haven been introduced to analyze real datasets with bathtub failure rates.

Definition 1. The extended Chen [53] software reliability model is given as follows (see, Chaubey and Zhang [57]):

$$M(t) = \omega \left(1 - e^{\lambda(1-e^{t^\beta})}\right)^\alpha \tag{2}$$

where $\lambda > 0$, $\alpha > 0$, $\beta > 0$.

For other extensions of the Chen distribution, see [54] – [57].

Definition 2. [5] The extended deterministic Pham's software reliability model is given as follows:

$$M_1(t) = N * \left(1 - \frac{\beta}{\beta + (at)^b}\right)^\alpha \tag{3}$$

where $a > 0$, $b > 0$, $\alpha > 0$, $\beta > 0$.

Definition 3. [58] The Hausdorff distance (the H–distance) $\rho(f, g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t, x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

3. Main Results

3.1. A Note on the Extended Chen's Function

We consider the following sigmoid:

$$M^*(t) = \left(1 - e^{\lambda(1-e^{t^\beta})}\right)^\alpha, \tag{4}$$

with $\omega = 1$, β - is an even number and

$$t_0 = \left(\ln\left(1 - \frac{1}{\lambda}\ln\left(1 - 2^{-\frac{1}{\alpha}}\right)\right)\right)^{\frac{1}{\beta}}; \quad M^*(t_0) = \frac{1}{2}. \tag{5}$$

The one–sided Hausdorff distance d between the function $\sigma^{**}(t)$ and the sigmoid ((4)–(5)) satisfies the relation

$$M^*(t_0 - d) = d. \tag{6}$$

The following theorem gives upper and lower bounds for d

Theorem 1. Let

$$p = \frac{1}{2},$$

$$q = -1 - \left(1 - \frac{1}{\lambda}\ln\left(1 - 2^{-\frac{1}{\alpha}}\right)\right)\left(1 - 2^{-\frac{1}{\alpha}}\right)\left(\frac{1}{2}\right)^{\frac{\alpha-1}{\alpha}}$$

$$\times\left(\ln\left(1 - \frac{1}{\lambda}\ln\left(1 - 2^{-\frac{1}{\alpha}}\right)\right)\right)^{\frac{\beta-1}{\beta}}\alpha\beta\lambda.$$

For the one–sided Hausdorff distance d between $\sigma^{**}(t)$ and the sigmoid ((4)–(5)) the following inequalities hold for:

$$-2.1q > e^{1.05}$$

$$d_l = \frac{1}{-2.1q} < d < \frac{\ln(-2.1q)}{-2.1q} = d_r. \tag{7}$$

Proof. Let us examine the function:

$$F(d) = M^*(t_0 - d) - d. \tag{8}$$

We see that $q < 0$. From $F'(d) < 0$ we conclude that function F is decreasing.

Consider the function

$$G(d) = p + qd. \tag{9}$$

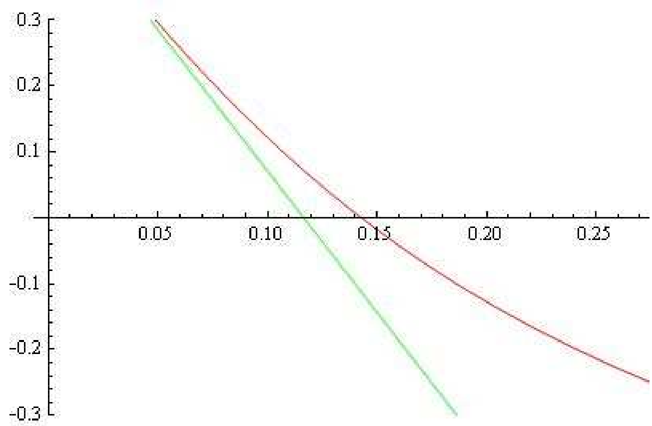

Figure 2: The functions $F(d)$ and $G(d)$.

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.
Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 2).
In addition $G'(d) < 0$.
Further, for $-2.1q > e^{1.05}$ we have $G(d_l) > 0$ and $G(d_r) < 0$.
This completes the proof of the theorem.

3.2. Numerical Examples

1. Hausdorff approximation of the impulse function $\sigma^{**}(t)$ by sigmoidal function based on the extended Chen's [53] cumulative function.

The model ((4)–(5)) for $\beta = 6$, $\alpha = 0.95$, $\lambda = 10$, $t_0 = 0.631995$ is visualized on Fig. 3.

From the nonlinear equation (6) and inequalities (7) we have: $d = 0.142801$, $d_l = 0.110996$ and $d_r = 0.243998$.

2. Application in the field of antenna–feeder technique.

After the substitution $t = kr\cos\theta + a$, where
– $k = \frac{2\pi}{l}$, l is the wave length;
– a is the phase difference;
– θ is the azimuthal angle;
– r is the distance between the emitters ($r = \frac{l}{2}$ is fixed),
the function $M^*(t)$ for $\lambda > 0$, $\alpha > 0$, $\beta > 0$ (or emitting chart of antenna factor) can be written in the form:

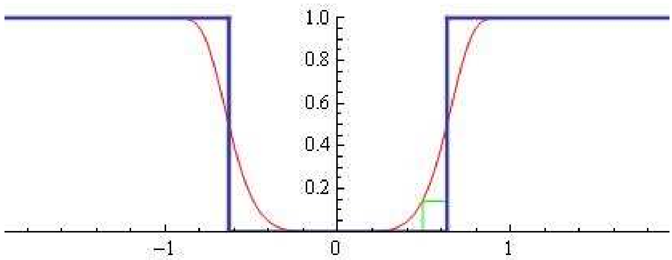

Figure 3: The model ((4)–(5)) for $\beta = 6$, $\alpha = 0.95$, $\lambda = 10$, $t_0 = 0.631995$; H–distance $d = 0.142801$, $d_l = 0.110996$, $d_r = 0.243998$.

$$M^*(\theta) = \left(1 - e^{\lambda(1-e^{(\pi\cos\theta+a)^\beta})}\right)^\alpha. \tag{10}$$

Typical emitting chart is visualized on Fig. 4.

Of course, the question of the practical realization of the activation functions which are generated as emitting charts remains open.

The mathematical apparatus proposed in the article can be successfully used for imitation and simulation of such charts.

3. Application in the field of debugging and test theory.

We give real examples with data provided in [4].

The operating time of the software is 167,900 days. 115 failures are detected for these days which contain 71 unique failures.

Table 1 shows the failures data which are united for each of the 13 months.

Dataset included [5] Year 2000 compatibility modifications, operating system upgrade, and signaling message processing.

The fitted model

$$M(t) = \omega\left(1 - e^{\lambda(1-e^{t^\beta})}\right)^\alpha$$

based on the data of Table 1 for the estimated parameters:

$$\omega = 110; \quad \lambda = 0.026895; \quad \beta = 0.816531; \quad \alpha = 0.457081$$

is plotted on Fig. 5.

In conclusion, we will note that the determination of compulsory in area of the Software Reliability Theory components, such as confidence intervals and confidence bounds, should also be accompanied by a serious analysis of the value of the best Hausdorff approximation - the subject of study in the present paper.

```
φ1[θ_] := (1 - Exp[λ * (1 - Exp[(Pi * Cos[θ] + a)^β])])^α
Manipulate[PolarPlot[(1 - Exp[λ * (1 - Exp[(Pi * Cos[θ] + a)^β])])^α ,
  {θ, -2 Pi, 2 Pi}], {β, 2, 10., Appearance → "Open"}, {α, 0.01, 10., Appearance → "Open"}
 , {λ, 0.01, 30, Appearance → "Open"}, {a, 0.01, 2 * Pi, Appearance → "Open"}]
```

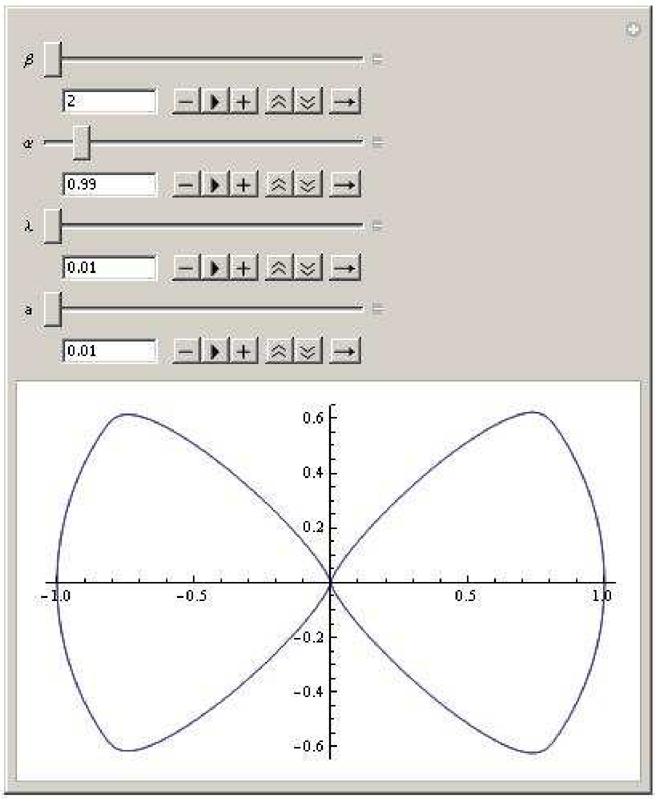

Figure 4: Typical emitting chart ($M^*(\theta)$) for $\beta = 2$; $a = 0.01$; $\alpha = 0.99$; $\lambda = 0.01$.

We hope that the results will be useful for specialists in this scientific area.

3.3. A Note on the Extended Deterministic Pham's Software Reliability Model

In this Section we study the Hausdorff approximation of the shifted Heaviside function

Month Index	System Days (Days)	System Days (Cumulative)	Failures	Cumulative Failures
1	961	961	7	7
2	4170	5131	3	10
3	8789	13,920	14	24
4	11,858	25,778	8	32
5	13,110	38,888	11	43
6	14,198	53,086	8	51
7	14,265	67,351	7	58
8	15,175	82,526	19	77
9	15,376	97,902	17	94
10	15,704	113,606	6	100
11	18,182	131,788	11	111
12	17,760	149,548	4	115
13	18,352	167,900	0	115

Table 1: Field failure data [4].

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0 \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{11}$$

by sigmoidal function based on the extended Pham's cumulative function

$$M_1^*(t) = N * \left(1 - \frac{\beta}{\beta + (at)^b}\right)^{\alpha} \tag{12}$$

with $N = 1, b = \beta$, without loosing of generality and

$$t_0 = \frac{1}{a}\left(\beta\left(\frac{1}{1 - \left(\frac{1}{2}\right)^{\frac{1}{\alpha}}} - 1\right)\right)^{\frac{1}{\beta}}; \quad M_1^*(t_0) = \frac{1}{2}. \tag{13}$$

The one–sided Hausdorff distance d between the Heaviside step function $h_{t_0}(t)$ and the sigmoid $M_1^*(t)$ satisfies the relation

$$M_1^*(t_0 + d_1) = 1 - d_1. \tag{14}$$

The following theorem gives upper and lower bounds for d_1

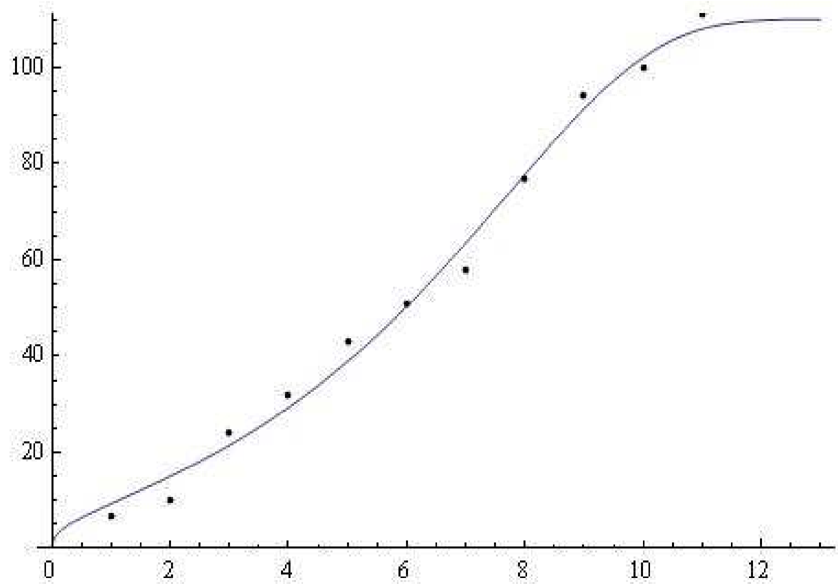

Figure 5: An example of the usage of dynamical and graphical representation for the function $M(t)$.

Theorem 2. Let

$$p_1 = -\frac{1}{2},$$

$$q_1 = 1 + \left(\beta\left(\frac{1}{1-\left(\frac{1}{2}\right)^{\frac{1}{\alpha}}} - 1\right)\right)^{\frac{\beta-1}{\beta}} \left(\frac{1}{2}\right)^{\frac{\alpha-1}{\alpha}} \left(1 - \left(\frac{1}{2}\right)^{\frac{1}{\alpha}}\right)^2 a\alpha.$$

For the one–sided Hausdorff distance d_1 between h_{t_0} and the sigmoid $M_1^*(t)$ the following inequalities hold for:

$$2.1q_1 > e^{1.05}$$

$$d_{l_1} = \frac{1}{2.1q_1} < d_1 < \frac{\ln(2.1q_1)}{2.1q_1} = d_{r_1}. \tag{15}$$

Proof. Let us examine the functions:

$$F_1(d_1) = M_1^*(t_0 + d_1) - 1 + d_1. \tag{16}$$

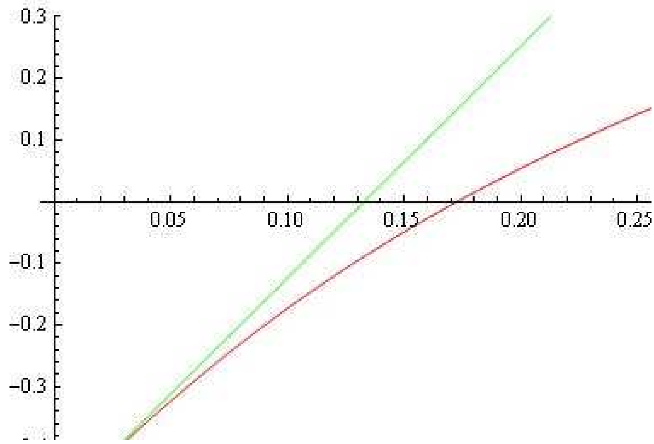

Figure 6: The functions $F_1(d_1)$ and $G_1(d_1)$.

$$G_1(d_1) = p_1 + q_1 d_1. \tag{17}$$

From Taylor expansion we obtain $G_1(d_1) - F_1(d_1) = O(d_1^2)$.

Hence $G_1(d_1)$ approximates $F_1(d_1)$ with $d_1 \to 0$ as $O(d_1^2)$ (see Fig. 6).

In addition $G'(d_1) > 0$.

Further, for $2.1q_1 > e^{1.05}$ we have $G_1(d_{l_1}) < 0$ and $G(d_{r_1}) > 0$.

This completes the proof of the theorem.

For the parameters of the function $M_1^*(t)$ at fixed t_0 we get $\alpha = 0.95$; $a = 3.9$; $\beta = 4$ and from the nonlinear equation (14) and inequalities (15) we have: $d_1 = 0.172737$, $d_{l_1} = 0.126554$ and $d_{r_1} = 0.261598$ (see, Fig. 7).

For other software reliability models, see [59]–[65]. A new class activation functions with application in the theory of impulse technics is being discussed in [52].

Acknowledgments

This work has been supported by the project FP17-FMI008 of Department for Scientific Research, Paisii Hilendarski University of Plovdiv.

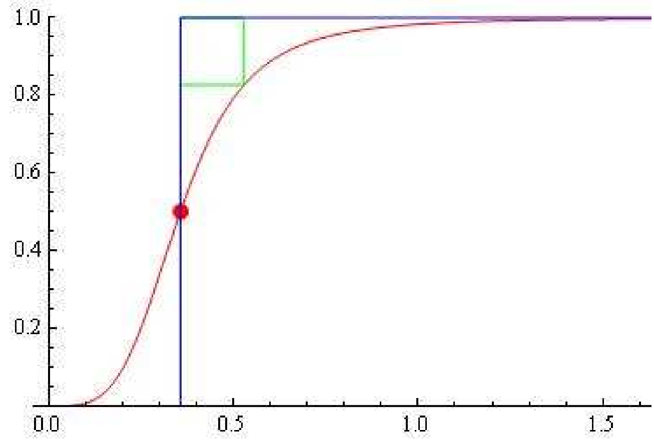

Figure 7: The function $M_1^*(t)$ for $\alpha = 0.95$; $a = 3.9$; $\beta = 4$; H–distance $d_1 = 0.172737$, $d_{l_1} = 0.126554$, $d_{r_1} = 0.261598$.

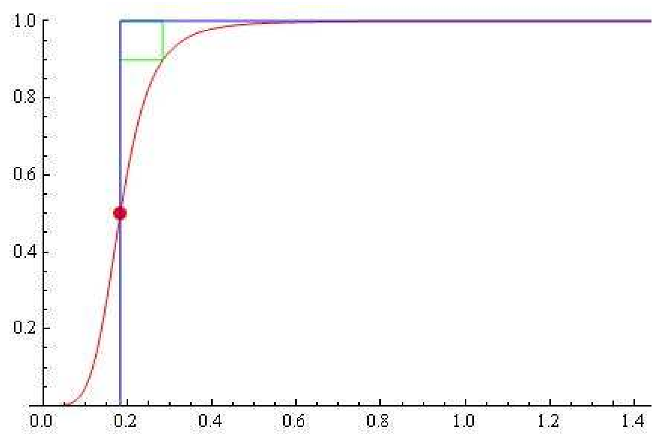

Figure 8: The function $M_1^*(t)$ for $\alpha = 0.99$; $a = 7.5$; $\beta = 5$; $t_0 = 0.183451$; H–distance $d_1 = 0.100876$, $d_{l_1} = 0.061107$, $d_{r_1} = 0.170802$.

References

[1] S. Yamada, Software Reliability Modeling: Fundamentals and Applications, Springer (2014).

[2] S. Yamada, Y. Tamura, *OSS Reliability Measurement and Assessment*, In: Springer Series

in Reliability Engineering (H. Pham, Ed.), Springer International Publishing Switzerland (2016).

[3] H. Pham, *System Software Reliability*, In: Springer Series in Reliability Engineering, Springer–Verlag London Limited (2006).

[4] D.R. Jeske, X. Zhang, Some successful approaches to software reliability modeling in industry, *J. Syst. Softw.*, 74 (2005), 85–99.

[5] K. Song, H. Pham, A Software Reliability Model with a Weibull Fault Detection Rate Function Subject to Operating Environments, *Appl. Sci.*, 7 (2017), 983; doi:10.3390/app7100983, 16 pp.

[6] *Advances in Degradation Modeling. Applications to reliability, survival analysis and finance* (M. Nikulin, N. Limnios, N. Balakrishnan, W. Kahle and C. Huber–Carol, Editors), Birkhauser (2010).

[7] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Some software reliability models: Approximation and modeling aspects*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-82805-0.

[8] K. Ohishi, H. Okamura, T. Dohi, Gompertz software reliability model: Estimation algorithm and empirical validation, *J. of Systems and Software*, 82 (2009), 535–543.

[9] D. Satoh, A discrete Gompertz equation and a software reliability growth model, *IEICE Trans. Inform. Syst.*, E83-D (2000), 1508–1513.

[10] D. Satoh, S. Yamada, Discrete equations and software reliability growth models, in: *Proc. 12th Int. Symp. on Software Reliab. and Eng.*, (2001), 176–184.

[11] S. Yamada, A stochastic software reliability growth model with Gompertz curve, *Trans. IPSJ*, 33 (1992), 964–969. (in Japanese)

[12] P. Oguntunde, A. Adejumo, E. Owoloko, On the flexibility of the transmuted inverse exponential distribution, *Proc. of the World Congress on Engineering*, July 5–7, 2017, London, 1 (2017).

[13] W. Shaw, I. Buckley, The alchemy of probability distributions: Beyond Gram–Charlier expansions and a skew–kurtotic–normal distribution from a rank transmutation map, (2009), *Research report*.

[14] M. Khan, Transmuted generalized inverted exponential distribution with application to reliability data, *Thailand Statistician*, 16 (2018), 14–25.

[15] A. Abouammd, A. Alshingiti, Reliability estimation of generalized inverted exponential distribution, *J. Stat. Comput. Simul.*, 79 (2009), 1301–1315.

[16] I. Ellatal, Transmuted generalized inverted exponential distribution, *Econom. Qual. Control*, 28 (2014), 125–133.

[17] N. Kyurkchiev, S. Markov, *Sigmoid functions: Some Approximation and Modelling Aspects*, LAP LAMBERT Academic Publishing, Saarbrucken (2015), ISBN 978-3-659-76045-7.

[18] A. Iliev, N. Kyurkchiev, S. Markov, A note on the new activation function of Gompertz type, *Biomath Communications*, 4 (2017).

[19] N. Kyurkchiev, A. Iliev, S. Markov, *Some techniques for recurrence generating of activation functions*, LAP LAMBERT Academic Publishing (2017), ISBN 978-3-330-33143-3.

[20] E. P. Virene, Reliability growth and its upper limit, in: *Proc. 1968, Annual Symp. on Realib.*, (1968), 265–270.

[21] S. Rafi, S. Akthar, Software Reliability Growth Model with Gompertz TEF and Optimal Release Time Determination by Improving the Test Efficiency, *Int. J. of Comput. Applications*, 7 (2010), 34–43.

[22] F. Serdio, E. Lughofer, K. Pichler, T. Buchegger, H. Efendic, Residua–based fault detection using soft computing techniques for condition monitoring at rolling mills, *Information Sciences*, 259 (2014), 304–320.

[23] S. Yamada, M. Ohba, S. Osaki, S–shaped reliability growth modeling for software error detection, *IEEE Trans, Reliab.*, R–32 (1983), 475–478.

[24] S. Yamada, S. Osaki, Software reliability growth modeling: Models and Applications, *IEEE Transaction on Software Engineering*, SE–11 (1985), 1431–1437.

[25] N. Pavlov, G. Spasov, A. Rahnev, N. Kyurkchiev, A new class of Gompertz–type software reliability models, [*International Electronic Journal of Pure and Applied Mathematics*, 12 (2018), 43–57.

[26] N. Pavlov, G. Spasov, A. Rahnev, N. Kyurkchiev, Some deterministic reliability growth curves for software error detection: Approximation and modeling aspects, *International Journal of Pure and Applied Mathematics*, 118 (2018), 599–611.

[27] N. Pavlov, A. Golev, A. Rahnev, N. Kyurkchiev, A note on the Yamada–exponential software reliability model, *International Journal of Pure and Applied Mathematics*, 118 (2018), 871–882.

[28] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, A Note on The "Mean Value" Software Reliability Model, *International Journal of Pure and Applied Mathematics*, 118 (2018), 949–956.

[29] N. Pavlov, A. Golev, A. Rahnev, N. Kyurkchiev, A note on the generalized inverted exponential software reliability model, *International Journal of Advanced Research in Computer and Communication Engineering*, 7 (2018), 484–487.

[30] A. L. Goel, Software reliability models: Assumptions, limitations and applicability, *IEEE Trans. Software Eng.* SE–11 (1985), 1411–1423.

[31] J. D. Musa, *Software Reliability Data*, DACS, RADC, New York (1980).

[32] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Transmuted inverse exponential software reliability model, *Int. J. of Latest Research in Engineering and Technology*, 4 (2018) (accepted).

[33] N. Kyurkchiev, A. Iliev and S. Markov, *Some techniques for recurrence generating of activation functions*, LAP LAMBERT Academic Publishing (2017), ISBN 978-3-330-33143-3.

[34] V. Kyurkchiev, N. Kyurkchiev, A family of recurrence generated functions based on Half-hyperbolic tangent activation functions, *Biomedical Statistics and Informatics*, 2 (2017), 87–94.

[35] N. Guliyev, V. Ismailov, A single hidden layer feedforward network with only one neuron in the hidden layer san approximate any univariate function, *Neural Computation*, 28 (2016), 1289-1304.

[36] D. Costarelli, R. Spigler, Approximation results for neural network operators activated by sigmoidal functions, *Neural Networks*, 44 (2013), 101-106.

[37] D. Costarelli, G. Vinti, Pointwise and uniform approximation by multivariate neural network operators of the max-product type, *Neural Networks*, (2016), doi:10.1016/j.neunet.2016.06.002

[38] D. Costarelli, R. Spigler, Solving numerically nonlinear systems of balance laws by multivariate sigmoidal functions approximation, *Computational and Applied Mathematics*, (2016), doi:10.1007/s40314-016-0334-8

[39] D. Costarelli, G. Vinti, Convergence for a family of neural network operators in Orlicz spaces, *Mathematische Nachrichten*, (2016), doi: 10.1002/mana.20160006

[40] N. Kyurkchiev, A. Andreev, *Approximation and antenna and filter synthesis: Some moduli in programming environment Mathematica*, LAP LAMBERT Academic Publishing, Saarbrucken (2014), ISBN 978-3-659-53322-8.

[41] N. Kyurkchiev, Bl. Sendov, Approximation of a class of functions by algebraic polynomials with respect to Hausdorff distance, *Ann. Univ. Sofia, Fac. Math.*, 67 (1975), 573–579. (in Bulgarian)

[42] N. Kyurkchiev, S. Markov, On the Hausdorff distance between the Heaviside step function and Verhulst logistic function, *J. Math. Chem.*, 54 (2016), 109–119.

[43] A. Andreev, N. Kyurkchiev, Approximation of some impulse functions - implementation in programming environment MATHEMATICA, *Proceedings of the 43 Spring Conference of the Union of Bulgarian Mathematicians*, Borovetz, April 2-6, (2014), 111–117.

[44] N. Kyurkchiev, S. Markov, On the numerical approximation of the "cross" set, *Ann. Univ. Sofia, Fac. Math.*, 66 (1974), 19–25. (in Bulgarian)

[45] N. Kyurkchiev, A. Andreev, Hausdorff approximation of functions different from zero at one point - implementation in programming environment MATHEMATICA, *Serdica J. of Computing*, 7 (2013), 135–142.

[46] N. Kyurkchiev, A. Andreev, Synthesis of slot aerial grids with Hausdorff–type directive patterns – implementation in programming environment Mathematica, *C.R. Acad. Bulgare Sci.*, 66 (2013), 1521–1528.

[47] N. Kyurkchiev, *Synthesis of slot aerial grids with Hausdorff type directive patterns*, PhD Thesis, Department of Radio-Electronics, VMEI, Sofia (1979). (in Bulgarian)

[48] Bl. Sendov, H. Schinev, N. Kjurkchiev, Hausdorff-synthesis of aerial grids in scanning the directive diagram, *Electropromishlenost i Priboroostroene*, 16 (1981), 203–205. (in Bulgarian)

[49] H. Schinev, N. Kjurkchiev, M. Gachev, Experimental investigations of slot aerial grids with Hausdorff type directive patterns, *Electropromishlenost i Priboroostroene*, 14 (1979), 223–224. (in Bulgarian)

[50] H. Shinev, N. Kyurkchiev, M. Gachev, S. Markov, Application of a class of polynomials of best approximation to linear antenna array synthesis, *Izv. VMEI*, Sofia, 34 (1975), 1–6. (in Bulgarian)

[51] A. Golev, T. Djamiykov, N. Kyurkchiev, Sigmoidal functions in antenna-feeder technique, *Int. J. of Pure and Appl. Math.*, 116 (2017), 1081–1092.

[52] N. Kyurkchiev, A new class activation functions with application in the theory of impulse technics, *Journal of Mathematical Sciences and Modelling* (2018), ID 421392. (accepted)

[53] Z. Chen, A new two–parameter lifetime distribution with bathtub shape or increasing failure rate function, *Stat. and Prob. Letters*, 49 (2000), 155–161.

[54] M. Xie, Y. Tang, T. Goh, A modified Weibull extension with bathtub–shaped failure rate function, *Reliability Eng. and System Safety*, 76 (2002), 279–285.

[55] M. Khan, A. Sharma, Generalized order statistics from Chen distribution and its characterization, *J. of Stat. Appl. and Prob.*, 5 (2016), 123–128.

[56] S. Dey, D. Kumar, P. Ramos, F. Louzada, Exponentiated Chen distribution: Properties and Estimations, *Comm. in Stat.–Simulation and Computation*, (2017), 1–22.

[57] Y. Chaubey, R. Zhang, An extension of Chen's family of survival distributions with bathtub shape or increasing hazard rate function, *Comm. in Stat.–Theory and Methods*, 44 (2015), 4049–4069.

[58] B. Sendov, *Hausdorff Approximations*, Kluwer, Boston (1990).

[59] A. Pandey, N. Goyal, *Early Software Reliability Prediction. A Fuzzy Logic Approach*, In: Studies in Fuzziness and Soft Computing (J. Kacprzyk, Ed.), vol. 303, Springer, London (2013).

[60] N. D. Singpurwalla, S. P. Wilson, *Statistical Methods in Software Engineering. Reliability and Risk*, In: Springer Series in Statistics (P. Bickel, Adv.), Springer, New York (1999).

[61] M. Bisi, N. Goyal, *Artificial Neural Network for Software Reliability Prediction*, In: Performability Engineering Series (K. Misra and J. Andrews, Eds.), John Wiley & Sons, Inc., New Jersey (2017).

[62] P. K. Kapur, H. Pham, A. Gupta, P. C. Jha, *Software Reliability Assessment with OR Applications*, In: Springer Series in Reliability Engineering, Springer-Verlag, London (2011).

[63] P. Karup, R. Garg, S. Kumar, *Contributions to Hardware and Software Reliability*, World Scientific, London (1999).

[64] M. Lyu (Ed. in Chief), *Handbook of Software Reliability Engineering*, IEEE Computer Society Press, The McGraw-Hill Companies, Los Alamitos (1996).

[65] M. Ohba, Software reliability analysis models, *IBM J. Research and Development*, 21 (1984).