

On the fate of gas accreting at a low rate on to a black hole

Roger D. Blandford¹★ and Mitchell C. Begelman^{2,3}★

¹*Theoretical Astrophysics, Caltech 130-33, Pasadena, CA 91125, USA*

²*JILA, University of Colorado, Boulder, CO 80309-0440, USA*

³*Department of Astrophysical and Planetary Sciences, University of Colorado, Boulder, CO 80309-0440, USA*

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ABSTRACT

Gas supplied conservatively to a black hole at rates well below the Eddington rate may not be able to radiate effectively and the net energy flux, including the energy transported by the viscous torque, is likely to be close to zero at all radii. This has the consequence that the gas accretes with positive energy so that it may escape. Accordingly, we propose that only a small fraction of the gas supplied actually falls on to the black hole, and that the binding energy it releases is transported radially outward by the torque so as to drive away the remainder in the form of a wind. This is a generalization of and an alternative to an ‘ADAF’ solution. Some observational implications and possible ways to distinguish these two types of flow are briefly discussed.

Key words: accretion, accretion discs – black hole physics – hydrodynamics.

1 INTRODUCTION

It has often been supposed that gas that accretes on to a black hole radiates liberated binding energy with an efficiency of $\sim 0.1c^2 \sim 10^{20} \text{ erg g}^{-1}$. This is not always so, as exemplified by observations of the black hole in our Galactic Centre, where it appears that gas is supplied at a rate that has been estimated to lie in the range $\sim 10^{20-22} \text{ g s}^{-1}$ (Falcke & Melia 1997), while the bolometric luminosity is found to be $\sim 10^{36-37} \text{ erg s}^{-1}$ (Mahadevan 1998). Consequently, the radiative efficiency could be as low as $\sim 10^{14} \text{ erg g}^{-1} \sim 10^{-7} c^2$ and is unlikely to be more than $\sim 10^{-4} c^2$.

If gas falls on to a black hole from a few gravitational radii via a thick disc, with shear stress per unit pressure α , then the fraction of the energy of an individual hot ion that is transferred by Coulomb scattering to the electrons (which are almost solely responsible for the radiation) is $f_e \sim (\dot{M}/\dot{M}_E)\alpha^{-2}$, where $\dot{M}_E = L_E/c^2$ is the Eddington accretion rate. Therefore, if (i) viscous dissipation heats only the ions, (ii) the ions couple to the electrons only through Coulomb scattering, and (iii) $\alpha \gtrsim 0.1$, then the radiative efficiency is plausibly low enough to account for the observations of the Galactic Centre. The plausibility of condition (i) has been argued recently by Gruzinov (1998) and Quataert (1998) [but see also Bisnovatyi-Kogan & Lovelace (1997) and Blackman (1999)], (ii) seems reasonable in the absence of a specific proposal for non-Coulombic heating, and (iii) can be a feature of an advection-dominated accretion flow, or ADAF, in which gas accretes quasi-spherically on to a black hole carrying a large amount of internal energy across the horizon (Narayan & Yi 1994; Kato, Fukue, & Mineshige 1998, and references therein).

However, as we discuss below, the gas in ADAF solutions appears to be generically unbound. We therefore propose in this

Letter that ADAF solutions be modified to include a powerful wind that carries away mass, angular momentum and energy from the accreting gas. We describe a family of solutions where the rate at which gas is swallowed by the black hole is only a tiny fraction of the rate at which it is supplied, and where, in the limiting case, the binding energy of a gram of gas at a few gravitational radii drives off 100 kg of gas from $\sim 10^5$ gravitational radii. Disc wind solutions based on quite different principles have also been recently proposed by Xu & Chen (1997) and Das (1998).

2 FUNDAMENTALS OF ACCRETION THEORY

First, we review some principles. Consider thin disc accretion with angular velocity Ω , inflow speed $v \ll \Omega r$, mass per unit radius μ and specific angular momentum ℓ . In assuming that the disc is thin, we are implicitly supposing that the gas can remain cold by radiating away its internal energy. Let the torque that the disc interior to radius r exerts upon the exterior disc be $G(r)$. The equations of mass and angular momentum conservation are then

$$\frac{\partial \mu}{\partial t} = \frac{\partial \mu v}{\partial r}; \quad \frac{\partial \mu \ell}{\partial t} = \frac{\partial \mu v \ell}{\partial r} - \frac{\partial G}{\partial r} \quad (1)$$

(e.g. Kato et al. 1998). These equations immediately imply

$$\frac{\partial G}{\partial r} = \frac{\mu v \ell}{2r}; \quad \frac{\partial \mu}{\partial t} = 2 \frac{\partial}{\partial r} r^{1/2} \frac{\partial G}{\partial r}, \quad (2)$$

where we have assumed the Keplerian relation $\ell = r^{1/2}$ and set $GM = c = 1$ (Lynden-Bell & Pringle 1974).

We can combine equations (1) to obtain an energy equation

$$\frac{\partial \mu e}{\partial t} + \frac{\partial(\Omega G - \mu v e)}{\partial r} = G \frac{\partial \Omega}{\partial r}, \quad (3)$$

where $e = -\Omega \ell / 2$ is the Keplerian binding energy, the sum of the kinetic and potential energies per unit mass. (Note the presence of a

★E-mail: rdb@tapir.caltech.edu (RDB); mitch@jila.colorado.edu (MCB)

contribution to the energy flux from the rate at which the torque G does work on the exterior disc.) The right-hand side represents a radiative loss of energy. Evaluating it, we recover the standard result that the local radiative flux, in a stationary disc, is three times the rate of local loss of binding energy (D. Lynden-Bell and K. Thorne, quoted in Pringle & Rees 1972).

Next consider the opposite limiting case when the gas cannot cool and there is no extraneous source or sink of energy. Adding thermodynamic terms to the energy equation, we obtain

$$\frac{\partial \mu(e + u)}{\partial t} + \frac{\partial [\Omega G - \mu v(e + h)]}{\partial r} = G \frac{\partial \Omega}{\partial r} + \mu T \frac{ds}{dt}, \quad (4)$$

where u is the vertically averaged internal energy density, h is the enthalpy density, and s is the entropy density (Landau & Lifshitz 1959). As there are no sources or sinks of energy, the right-hand side must vanish:

$$\mu T \frac{ds}{dt} = T \left[\frac{\partial \mu s}{\partial t} - \frac{\partial \mu v s}{\partial r} \right] = -G \frac{\partial \Omega}{\partial r}. \quad (5)$$

As the gas has pressure, we must also satisfy the radial equation of motion:

$$\frac{\partial v}{\partial t} - v \frac{\partial v}{\partial r} + \Omega^2 r = \frac{1}{r^2} + \frac{1}{\rho} \frac{\partial P}{\partial r}. \quad (6)$$

3 ADVECTION-DOMINATED ACCRETION FLOWS

The basic idea and assumptions are set out most transparently in Narayan & Yi (1994; cf. also Ichimaru 1977, Abramowicz et al. 1995 and Narayan & Yi 1995). In the simplest, limiting case, it is assumed that there is a stationary, one-dimensional, self-similar flow of gas with $\mu \propto r^{1/2}$, $\Omega \propto r^{-3/2}$, and $v, a \propto r^{-1/2}$, where $a = [(\gamma - 1) h / \gamma]^{1/2}$ is the isothermal sound speed and the radial velocity $v \ll \Omega r$. The requirement that $P \propto r^{-5/2}$ transforms the radial equation of motion into

$$\Omega^2 r^2 - \frac{1}{r} + \frac{5a^2}{2} = 0. \quad (7)$$

Conservation of mass, momentum and energy gives

$$\mu v \equiv \dot{m} = \text{constant}, \quad (8)$$

$$\dot{m} r^2 \Omega - G = F_\ell, \quad (9)$$

$$G \Omega - \dot{m} \left[\frac{1}{2} \Omega^2 r^2 - \frac{1}{r} + \frac{\gamma a^2}{\gamma - 1} \right] = F_E, \quad (10)$$

where the inwardly directed angular momentum flux, F_ℓ , and the outwardly directed energy flux, F_E , are constant if there are no sources and sinks of angular momentum or energy. Now, the terms on the left-hand side of equation (9) scale $\propto r^{1/2}$ and those of equation (10) scale $\propto r^{-1}$. Therefore, if we require the flow to be self-similar over several decades of radius, both constants must nearly vanish. In the limit, $F_\ell = F_E = 0$.

Combining equations, we derive expressions for the sound speed a and the Bernoulli constant Be :

$$a^2 = \left[\frac{3(\gamma - 1)}{5 - 3\gamma} \right] \Omega^2 r^2 = \frac{6(\gamma - 1)}{(9\gamma - 5)r}, \quad (11)$$

$$Be \equiv \frac{1}{2} \Omega^2 r^2 - \frac{1}{r} + \frac{\gamma a^2}{\gamma - 1} = \Omega^2 r^2. \quad (12)$$

The elementary ADAF solution is then completed by defining an α -viscosity parameter through e.g. $G = \dot{m} r^2 \Omega = \alpha_{\text{SS}} \mu r a^2$, which then

implies $v = \alpha_{\text{SS}} a^2 / \Omega r$, assuming that $\alpha_{\text{SS}} \ll (5/3 - \gamma)^{1/2}$. [Note that this conventional definition of α , following Shakura & Sunyaev (1973), differs slightly from α_{NY} , the Newtonian prescription used by Narayan & Yi (1994).]

This solution has some features (as noted by Narayan & Yi 1994) that make it somewhat problematic. The first is a technical, although somewhat subtle, point. As can be seen from equation (11), $\gamma = 5/3$ is a singular case and, if imposed strictly, requires the flow to be non-rotating. This is familiar from the Bondi (1952) analysis which found a self-similar non-rotating inflow only when $\gamma = 5/3$. Narayan & Yi (1994) avoid this problem by supposing that the magnetic energy density is comparable to the ion energy density so that the effective specific heat ratio has a value intermediate between $\gamma = 5/3$ and the value $\gamma = 4/3$, which is appropriate when the magnetic energy density is dominant. However, if the magnetic energy density is maintained below equipartition values, as numerical simulations of shearing flows suggest is the case (e.g. Balbus & Hawley 1998), then the internal energy must be dominated by the non-relativistic ions, γ is very close to $5/3$, and $\Omega^2 \approx (5 - 3\gamma)/5r^3$, well below the Keplerian value. To match on to the ADAF solutions, weakly magnetized flows would have to lose most of their angular momentum at large radii in a manner likely to unbind much of the gas. Note also that for slowly rotating weakly magnetized flows the α -prescription is inappropriate, and the small differential rotation makes the generation of field less likely.

The second concern is more fundamental. The Bernoulli constant, Be (equation 12), is necessarily positive. This implies that any exposed gas can escape to infinity with positive energy. Furthermore, the value of Be increases as γ decreases from $5/3$, hence addressing the first problem by including an equipartition magnetic energy density would exacerbate this difficulty. These problems are not simply a consequence of assuming self-similarity, but stem from the fact that torque transports energy as well as angular momentum and that, in a steady state, the angular momentum and energy fluxes are conserved. Provided that the mechanical and torque contributions to the angular momentum (energy) flux are separately increasing (decreasing) functions of r , while their sums assume the constant values $F_\ell = O(r_{\text{in}}^{1/2})$ [$F_E = O(r_{\text{tr}}^{-1})$] in terms of the inner (outer) radius r_{in} (r_{tr}), we deduce that F_ℓ (F_E) must be relatively close to zero at intermediate radii, r , where $r_{\text{in}} \ll r \ll r_{\text{tr}}$. Equation (12) then follows without using equation (7) which is where self-similarity is introduced. Therefore $Be \sim \Omega^2 r^2$ at intermediate radii because as much energy has to be transported outward by the torque as inward by the mass. (By contrast, there is a net radial inflow of entropy.)

Thirdly, in an elaboration upon this model, it is supposed that a conical velocity field extends to the polar axis, at least for large α_γ (Narayan & Yi 1995; Narayan, Kato & Honma 1997). However, in this solution, the gas is in hydrostatic equilibrium along the axis but is unsupported at its base, and it seems hard to avoid the formation of a funnel from which gas can escape.

These considerations motivate us to investigate flows in which winds carry off enough of the mass, angular momentum and energy to bind the gas to the hole and to allow accretion to proceed.

4 ADVECTION-DOMINATED INFLOW – OUTFLOW SOLUTIONS

We continue to assume that the accreting gas cannot cool, that $v \ll \Omega r$ and that the ions dominate the equation of state so that $\gamma = 5/3$. (As with ADAFs, generalization to relax each of these

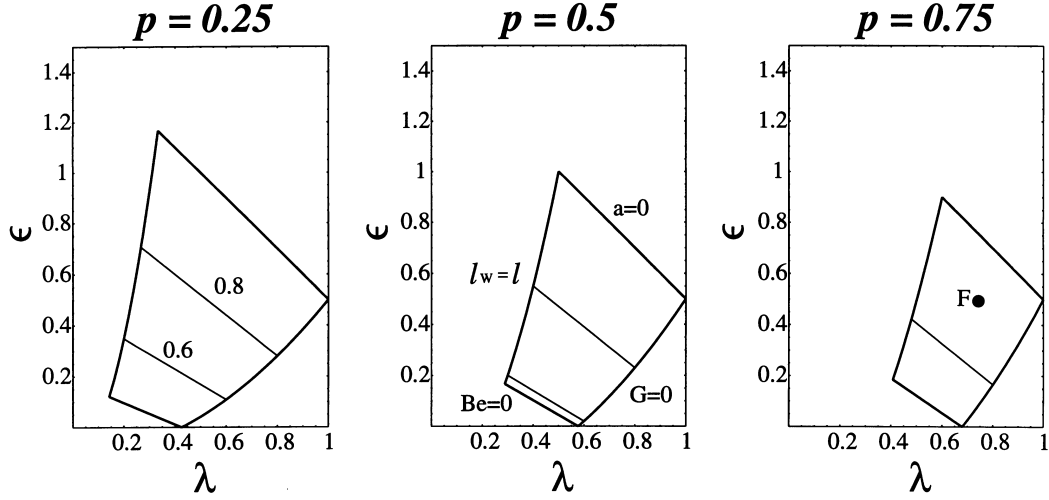


Figure 1. Allowed regions and limiting cases in the λ - ϵ plane for three values of the mass-loss exponent p . The allowed region, shown with bold lines, is defined by the following four constraints: (i) $a, H > 0$, (ii) $G > 0$, (iii) $\ell_w > \ell$, (iv) $Be < 0$. (See text.) The light lines correspond to angular velocities equal to 0.6 and 0.8 times the Keplerian angular velocity. The point F for $p = 0.75$ corresponds to the fiducial solution.

assumptions is straightforward.) Let the mass inflow rate satisfy

$$\dot{m} \propto r^p; \quad 0 \leq p < 1. \quad (13)$$

(The restriction on the exponent p allows the accreting mass to decrease with decreasing radius, while the energy released can still increase.)

The inward flow of angular momentum satisfies

$$F_\ell = (\dot{m}r^2\Omega - G) = \lambda\dot{m}r^{1/2}; \quad \lambda > 0. \quad (14)$$

Similarly, the outward flow of energy is

$$F_E = G\Omega - \dot{m}\left(\frac{1}{2}\Omega^2r^2 - \frac{1}{r} + \frac{5a^2}{2}\right) = \frac{\epsilon\dot{m}}{r}; \quad \epsilon > 0. \quad (15)$$

Equivalently, for the specific angular momentum and energy carried off by the wind, we have

$$\frac{dF_\ell}{d\dot{m}} = \frac{\lambda(p+1/2)r^{1/2}}{p}; \quad \frac{dF_E}{d\dot{m}} = \frac{\epsilon(p-1)}{pr}. \quad (16)$$

We use the ADAF self-similar scalings except that we must allow for mass loss. The radial equation of motion now gives

$$\Omega^2r^2 - \frac{1}{r} + (5/2 - p)a^2 = 0. \quad (17)$$

Similarly, the Bernoulli constant becomes

$$Be = \frac{\Omega^2r^2}{2} - \frac{1}{r} + \frac{5a^2}{2} = pa^2 - \frac{1}{2}\Omega^2r^2 \quad (18)$$

and it can have either sign. Combining these equations, we obtain

$$\Omega r^{3/2} = \frac{(5-2p)\lambda}{15-2p} + \frac{[(5-2p)^2\lambda^2 + (15-2p)(10\epsilon+4p-4\epsilon p)]^{1/2}}{15-2p}; \quad (19)$$

$$\left(\frac{H}{r}\right)^2 = ra^2 = \frac{1 - \Omega^2r^3}{5/2 - p}, \quad (20)$$

where H is the isothermal scaleheight in the disc and a measure of the disc thickness. As a^2 must be positive, we conclude that $\epsilon < 3/2 - \lambda$. The torque is now given by $G = \dot{m}(r^2\Omega - \lambda r^{1/2}) = \alpha_{SS}\mu ra^2$. As $G > 0$, we find that

$$\epsilon > \frac{(5+2p)\lambda^2 - 4p}{10-4p}. \quad (21)$$

Finally, solving for the inflow speed, we obtain $v = \alpha_{SS}r^{1/2}a^2/(\Omega r^{3/2} - \lambda)$.

Even given our simplifying assumptions, there are three independent, adjustable parameters, p , λ and ϵ , that depend upon the details of the wind (Fig. 1). Let us consider some special cases.

(i) $p = \lambda = \epsilon = 0$. There is no wind and the system reduces to the non-rotating Bondi solution.

(ii) $p = \lambda = 0$, $\epsilon = 3(1-f)/2$. This corresponds to flow with no wind but with radiative loss, which carries away energy but not angular momentum. The parameter f , introduced by Narayan & Yi (1994), is defined by the relation $\dot{m}Tds/dr = fGd\Omega/dr$.

(iii) $p = 0$, $\lambda = 1$, $\epsilon = 1/2$. This describes a magnetically dominated wind with mass flow conserved in the disc. All of the angular momentum and energy are carried off by a wind with $dF_E/dF_\ell = \Omega$ (cf. Blandford & Payne 1982; Königl 1991). There is no dissipation in the disc, which is cold and thin.

(iv) $\lambda = 2p[(10\epsilon+4p-4\epsilon p)/(2p+1)(4p^2+8p+15)]^{1/2}$. This corresponds to a gasdynamical wind where $dF_\ell/d\dot{m} = \ell_w = r^2\Omega = \ell$. The wind carries off its own angular momentum at the point of launching and does not exert any reaction torque on the remaining gas in the disc. Any magnetic coupling to the disc implies $\ell_w > \ell$.

(v) $ra^2 = r^3\Omega^2/2p = 1/(p+5/2)$. This corresponds to a marginally bound flow with vanishing Bernoulli constant (Fig. 2). In practice, it is expected that $Be < 0$.

(vi) $p = 0.75$, $\lambda = 0.75$, $\epsilon = 0.5$. This is an intermediate solution, with $Be = -0.35/r$, which carries off the specific angular momentum of the disc and has a velocity at infinity of 0.41 times the escape velocity from the point of origin. The angular velocity is 90 per cent of the Keplerian value, the disc thickness is $H \sim 0.3r$ and the inflow speed is $v = 0.56\alpha_{SS}$ times the Keplerian velocity. Only a fraction $(r_{in}/r_{tr})^{3/4}$ of the mass supplied will reach the hole.

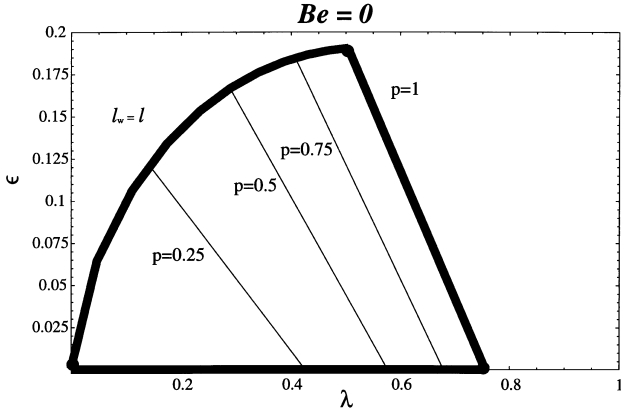


Figure 2. Accretion flows with vanishing Bernoulli constant. The allowed region in the λ – ϵ plane is as defined in Fig. 1. The contours are of constant mass-loss exponent p . Flows with $Be < 0$ carry away more energy.

5 DISCUSSION

The application of our adiabatic inflow–outflow solutions (‘ADIOS’) to describe real astrophysical flows depends upon several considerations. First, we have assumed that the viscosity is primarily hydromagnetic and dissipates most of the energy locally into the ions, and that electron heating is ineffective. If there is efficient electron heating, then neither ADAFs nor these ADIOS flows are likely to be of much relevance. Alternatively, it is possible that the rate of local dissipation is not given by $-Gd\Omega/dr$; instead the energy may be transported away by large-scale magnetic field, which can also drive an outflow. Secondly, we have taken the numerical simulations of magnetohydrodynamic shear flows at face value and supposed that $\alpha_{SS} \sim 0.01$. If $\alpha_{SS} > 0.1$, then the radial kinetic energy must be included. Just as with the ADAF solutions, this does not change their character. Presumably simulations designed to mimic ADAFs or ADIOS flows more closely are possible, and might determine the level to which the field energy density can grow. Thirdly, we assume that there is some means for launching an orderly wind from exposed surfaces that drains energy away from the interior of the accretion flow. We expect that the wind will be hydromagnetic and will extract angular momentum as well as energy, just as in the solar wind. However, pure thermal winds are also possible, especially as ADAFs are probably Høiland-unstable (Begelman & Meier 1982; Narayan & Yi 1994). The resulting convection will further increase Be at high latitude.

ADIOS models can be elaborated in much the same way as the ADAF solutions. The influence of boundary conditions on the similarity solutions can be followed by directly integrating the equations of motion (Kato et al. 1998). General relativity has been included at the inner boundary for ADAFs (Abramowicz et al. 1996; Igumenshchev & Beloborodov 1997; Popham & Gammie 1998), and this approach can be applied to ADIOS as well. If one adopts the lower values of α advocated here, the flow around the black hole may look rather similar to the ion torus model of Rees et al. (1982), although the flow may turn out to be non-stationary.

Perhaps the most careful application of an ADAF is to our own Galactic Centre (Mahadevan 1998). The central black hole mass is measured to be $2.6 \times 10^6 M_{\odot}$ (Eckart & Genzel 1997), and the detailed ADAF model has a steady mass inflow of $5 \times 10^{20} \text{ g s}^{-1}$ and extends over five decades of radius. The model spectrum passes through the millimetre and X-ray observational data, complies with upper limits in the infrared, and slightly underestimates the radio

emission. To illustrate the changes caused by substituting our fiducial ADIOS for the ADAF model, suppose that the flow extends from radii $r \sim 3 \times 10^4 m = 10^{16} \text{ cm}$ to $r \sim 3m$ and that the mass supply rate is as high as $\sim 5 \times 10^{21} \text{ g s}^{-1}$. The mass accretion rate on to the hole is then only $\sim 5 \times 10^{18} \text{ g s}^{-1}$, significantly lower than in the ADAF solution despite the fact that we have assumed a mass supply 10 times higher. If $\alpha_{SS} \sim 0.01$, the ions will not cool and the density close to the hole will be similar to that adopted in the ADAF solution, so that a somewhat similar spectrum can be created, while the hole mass increases at a much smaller rate.

The winds themselves may be sources of observable emission, especially when they pass through a terminal shock. Furthermore, the outflows, with or without magnetic field, can be self-collimating and form jets. [We note that the outflow in M87 appears to be partially collimated within 60 gravitational radii: cf. Junor & Biretta (1995).]

The ADAF model has also been applied to black hole X-ray binaries and their various spectral states, most notably the ‘quiescent’, ‘low’ and ‘high’ states, have been interpreted as a sequence of flows with increasing \dot{M} (Esin, McClintock & Narayan 1997). These models can account for the luminosities at which these transitions occur only if the viscosity is high, $\alpha_{SS} \sim 0.3$, and the same would be true for an ADIOS.

One application of the ADIOS model, which may lead to a clean observational test, is to neutron star accretion. Radiatively inefficient flow on to the surface of a neutron star is not possible for a conventional ADAF solution, because there is inevitably a large release of energy with efficiency $\sim 10^{20} \text{ erg g}^{-1}$ at the surface. However, with an ADIOS, the central density of the gas can be greatly reduced relative to an ADAF with the same mass supply at large radius, and in this sense it is possible to have a flow with low radiative efficiency. Numerical computations of the emergent spectrum will be necessary to see if these flows can model neutron star accretion in low states.

Finally, we note that there are many similarities between accretion at high rates and that at low rates. In the former case, the radiative efficiency is low because electron scattering traps the radiation (Begelman & Meier 1982). These flows also have to lose excess energy and angular momentum, and winds, like those observed in broad-absorption-line quasars, provide one way in which this may be accomplished. Radiation-dominated analogues of ADIOS exist (Blandford & Begelman, in preparation), and may be relevant for this case.

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