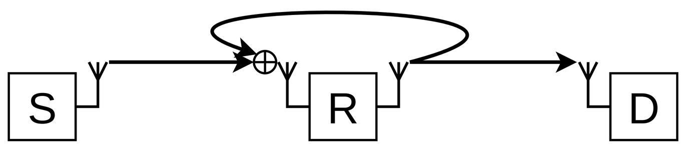
HELSINKI UNIVERSITY OF TECHNOLOGY
SMARAD Centre of Excellence

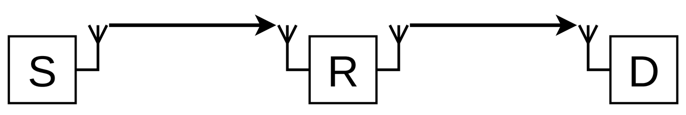
On the Feasibility of Full-Duplex Relaying in the Presence of Loop Interference

T. Riihonen, S. Werner, R. Wichman, and E. Zacarias B.

Introduction

- Fundamental classifications:
  - Amplify-and-forward (AF) vs. decode-and-forward (DF)
  - Relaying *modes*:





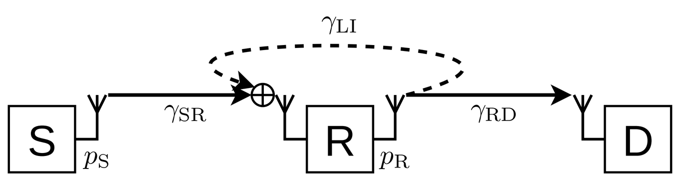
* Full Duplex (FD)
  - Loop interference
  - Fixed infrastructure relays
  - Separate rx and tx antennas
  - Loop cancellation algorithms

* Half Duplex (HD)
  - Pre-log 1/2 in capacity
  - Mobile relays and cooperative communication
  - Single antenna is enough

> What is the benefit of choosing the proper mode?
> When is the full-duplex mode feasible?
> How does power allocation affect the performance?

End-to-end capacities

- The system model:



  - Full Duplex:

$$\mathcal{C}_{\mathrm{FD}}^{\mathrm{AF}} = \log_2\left(1+\frac{\frac{p_{\mathrm{S}}\gamma_{\mathrm{SR}}}{p_{\mathrm{R}}\gamma_{\mathrm{LI}}+1}p_{\mathrm{R}}\gamma_{\mathrm{RD}}}{\frac{p_{\mathrm{S}}\gamma_{\mathrm{SR}}}{p_{\mathrm{R}}\gamma_{\mathrm{LI}}+1}+p_{\mathrm{R}}\gamma_{\mathrm{RD}}+1}\right)$$

$$\mathcal{C}_{\mathrm{FD}}^{\mathrm{DF}} = \log_2\left(1+\min\left\{\frac{p_{\mathrm{S}}\gamma_{\mathrm{SR}}}{p_{\mathrm{R}}\gamma_{\mathrm{LI}}+1}, p_{\mathrm{R}}\gamma_{\mathrm{RD}}\right\}\right)$$

  - Half Duplex:

$$\mathcal{C}_{\mathrm{HD}}^{\mathrm{AF}} = \frac{1}{2}\log_2\left(1+\frac{p_{\mathrm{S}}\gamma_{\mathrm{SR}}p_{\mathrm{R}}\gamma_{\mathrm{RD}}}{p_{\mathrm{S}}\gamma_{\mathrm{SR}}+p_{\mathrm{R}}\gamma_{\mathrm{RD}}+1}\right)$$

$$\mathcal{C}_{\mathrm{HD}}^{\mathrm{DF}} = \frac{1}{2}\log_2\left(1+\min\left\{p_{\mathrm{S}}\gamma_{\mathrm{SR}}, p_{\mathrm{R}}\gamma_{\mathrm{RD}}\right\}\right)$$

- Power allocation (PA)
  - Uniform power allocation: $\boxed{p_{\mathrm{S}} = p_{\mathrm{R}} = 1}$
  - Individual constraints:

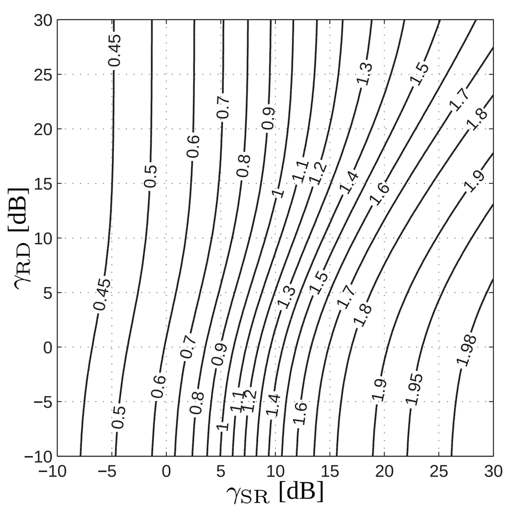
$$\boxed{(p_{\mathrm{S}}^*, p_{\mathrm{R}}^*) = \arg\max_{(p_{\mathrm{S}},p_{\mathrm{R}})} \mathcal{C}_{\mu}^{\pi} \text{ subject to } p_{\mathrm{S}} \leq 1 \text{ and } p_{\mathrm{R}} \leq 1}$$

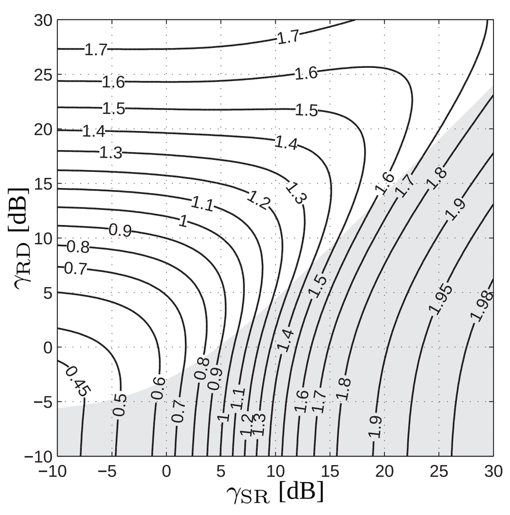
  - A sum constraint:

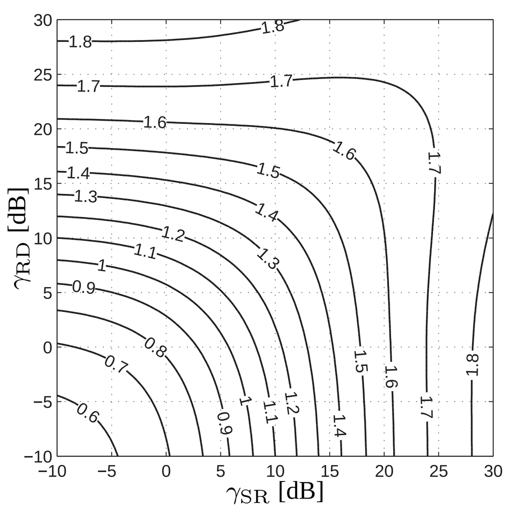
$$\boxed{(p_{\mathrm{S}}^*, p_{\mathrm{R}}^*) = \arg\max_{(p_{\mathrm{S}},p_{\mathrm{R}})} \mathcal{C}_{\mu}^{\pi} \text{ subject to } p_{\mathrm{S}} + p_{\mathrm{R}} \leq 2}$$

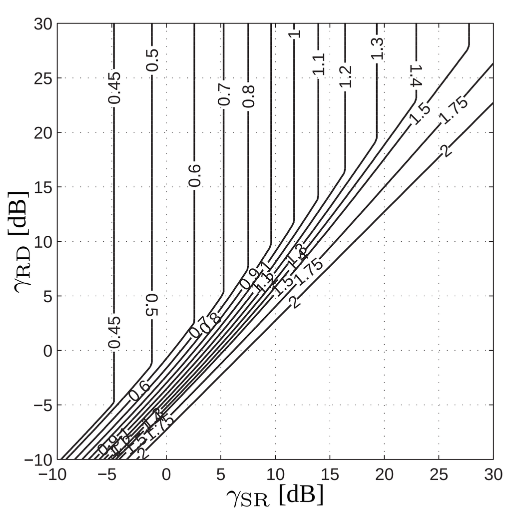
  → Closed-form expressions for p_{S} and p_{R} in the paper
- Comparison of the relaying modes:

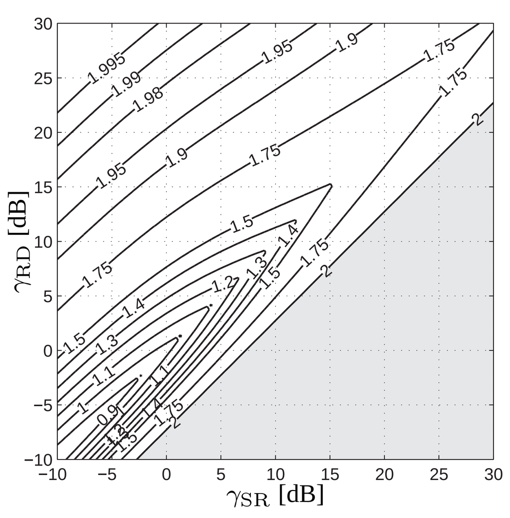
Contour plots for the capacity ratio $\frac{\mathcal{C}_{\mathrm{FD}}}{\mathcal{C}_{\mathrm{HD}}}$ when $\gamma_{\mathrm{LI}} = 6$dB in the full-duplex mode. $\frac{\mathcal{C}_{\mathrm{FD}}}{\mathcal{C}_{\mathrm{HD}}} \leq 2$ for all γ_{SR} and γ_{RD}.

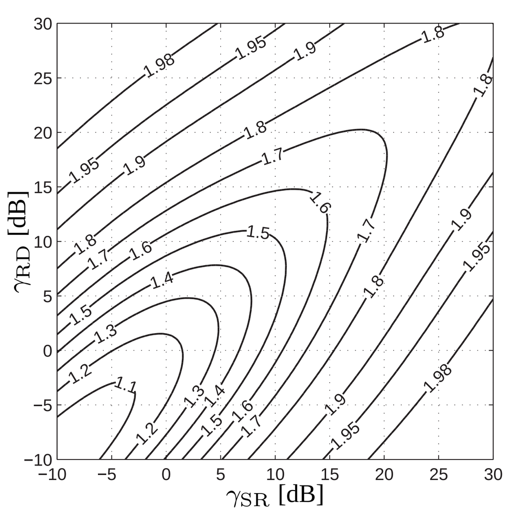

(a) AF and uniform PA.


(b) AF and PA with individual constraints. Maximum power is used ($p_{\mathrm{R}}^* = 1$) in the shaded region.


(c) AF and PA with a sum constraint.


(d) DF and uniform PA.


(e) DF and PA with individual constraints. Maximum power is used ($p_{\mathrm{R}}^* = 1$) in the shaded region.


(f) DF and PA with a sum constraint.

Break-even loop interference

- Two extremes for the trade-off:
  - $\mathcal{C}_{\mathrm{FD}}^{\pi} = 2\mathcal{C}_{\mathrm{HD}}^{\pi}$ with protocol $\pi \in \{\mathrm{AF}, \mathrm{DF}\}$ if $\gamma_{\mathrm{LI}} = 0$
  - $\mathcal{C}_{\mathrm{FD}}^{\pi}/\mathcal{C}_{\mathrm{HD}}^{\pi}$ is continuous and monotonically decreasing in terms of γ_{LI} and $\lim_{\gamma_{\mathrm{LI}}\to\infty} \mathcal{C}_{\mathrm{FD}}^{\pi}/\mathcal{C}_{\mathrm{HD}}^{\pi} = 0$

⇒ There exists a *break-even loop interference level* $\gamma_{\mathrm{LI}} = \Gamma_{\mathrm{LI}}^{\pi}$ for which $\mathcal{C}_{\mathrm{FD}}^{\pi} = \mathcal{C}_{\mathrm{HD}}^{\pi}$

> Determine $\Gamma_{\mathrm{LI}}^{\pi}$ for protocol $\pi \in \{\mathrm{AF}, \mathrm{DF}\}$
> such that $\mathcal{C}_{\mathrm{FD}}^{\pi} \geq \mathcal{C}_{\mathrm{HD}}^{\pi}$ if and only if $\gamma_{\mathrm{LI}} \leq \Gamma_{\mathrm{LI}}^{\pi}$

- Uniform power allocation ($\Gamma_{\mathrm{LI}}^{\pi} \geq 1 = 0$dB)
  - Amplify-and-forward

$$\Gamma_{\mathrm{LI}}^{\mathrm{AF}} = \sqrt{\frac{\gamma_{\mathrm{SR}}+1}{\gamma_{\mathrm{RD}}+1}(\gamma_{\mathrm{SR}}+\gamma_{\mathrm{RD}}+1)}$$

  - Decode-and-forward:

$$\Gamma_{\mathrm{LI}}^{\mathrm{DF}} = \frac{\gamma_{\mathrm{SR}}\left(\sqrt{\min\{\gamma_{\mathrm{SR}},\gamma_{\mathrm{RD}}\}+1}+1\right)}{\min\{\gamma_{\mathrm{SR}},\gamma_{\mathrm{RD}}\}} - 1$$

- Power allocation with individual constraints
  - Amplify-and-forward ($\Gamma_{\mathrm{LI}}^{\mathrm{AF}} \geq 1 = 0$dB):

$$\Gamma_{\mathrm{LI}}^{\mathrm{AF}} = \gamma_{\mathrm{SR}}\gamma_{\mathrm{RD}}\left(2+\frac{1}{A}+\frac{1}{\gamma_{\mathrm{SR}}}-2\sqrt{(1+\frac{1}{A})(1+\frac{1}{\gamma_{\mathrm{SR}}})}\right),$$

  where $A = \sqrt{1+\gamma_{\mathrm{SR}}\gamma_{\mathrm{RD}}/(\gamma_{\mathrm{SR}}+\gamma_{\mathrm{RD}}+1)} - 1$ if $\sqrt{\frac{(\gamma_{\mathrm{SR}}+1)(\gamma_{\mathrm{SR}}+\gamma_{\mathrm{RD}}+1)}{\gamma_{\mathrm{RD}}+1}} \geq \frac{\gamma_{\mathrm{SR}}+1}{\gamma_{\mathrm{RD}}}$ and otherwise $\Gamma_{\mathrm{LI}}^{\mathrm{AF}}$ as with uniform power allocation
  - Decode-and-forward ($\Gamma_{\mathrm{LI}}^{\mathrm{DF}} \geq 2 = 3$dB):

$$\Gamma_{\mathrm{LI}}^{\mathrm{DF}} = \frac{\left(\gamma_{\mathrm{SR}}-\sqrt{\min\{\gamma_{\mathrm{SR}},\gamma_{\mathrm{RD}}\}+1}+1\right)\gamma_{\mathrm{RD}}}{\left(\sqrt{\min\{\gamma_{\mathrm{SR}},\gamma_{\mathrm{RD}}\}+1}-1\right)^2}$$

- Power allocation with a sum constraint
  - Amplify-and-forward ($\Gamma_{\mathrm{LI}}^{\mathrm{AF}} \geq 2(2-\sqrt{2}) \approx 0.69$dB):

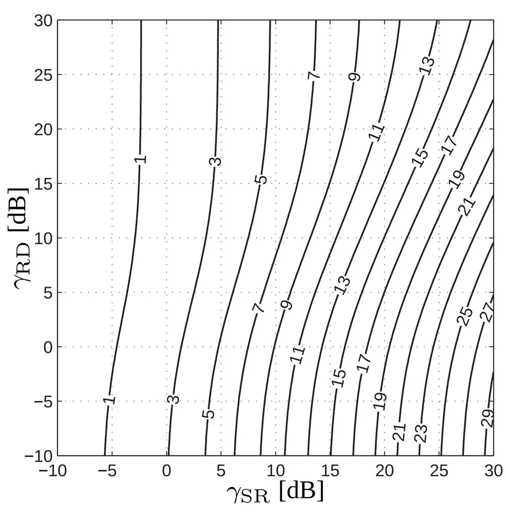
$$\Gamma_{\mathrm{LI}}^{\mathrm{AF}} = \gamma_{\mathrm{SR}}+\gamma_{\mathrm{RD}}+2\gamma_{\mathrm{SR}}\gamma_{\mathrm{RD}}(2+\frac{1}{A}) - 2\gamma_{\mathrm{SR}}\gamma_{\mathrm{RD}}\sqrt{(1+\frac{1}{A})(2+\frac{1}{\gamma_{\mathrm{SR}}})(2+\frac{1}{\gamma_{\mathrm{RD}}})},$$

  where $A = \sqrt{1+2\gamma_{\mathrm{SR}}\gamma_{\mathrm{RD}}/(1+\gamma_{\mathrm{SR}}+\gamma_{\mathrm{RD}}+\sqrt{(2\gamma_{\mathrm{SR}}+1)(2\gamma_{\mathrm{RD}}+1)})} - 1$
  - Decode-and-forward ($\Gamma_{\mathrm{LI}}^{\mathrm{DF}} \geq 4 = 6$dB):

$$\Gamma_{\mathrm{LI}}^{\mathrm{DF}} = \frac{\gamma_{\mathrm{SR}}+\gamma_{\mathrm{RD}}}{1-\sqrt{\frac{\gamma_{\mathrm{SR}}+\gamma_{\mathrm{RD}}}{\gamma_{\mathrm{SR}}+\gamma_{\mathrm{RD}}+2\gamma_{\mathrm{SR}}\gamma_{\mathrm{RD}}}}}$$

- Illustration of above expressions:

Contour plots for the break-even loop interference level Γ_{LI} [dB]: if $\gamma_{\mathrm{LI}} \leq \Gamma_{\mathrm{LI}}$ then $\mathcal{C}_{\mathrm{FD}} \geq \mathcal{C}_{\mathrm{HD}}$.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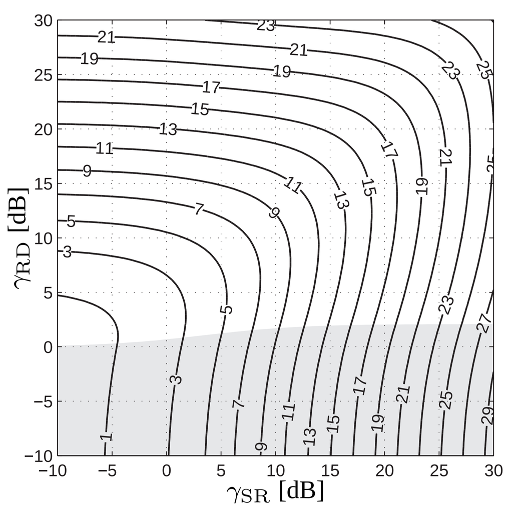
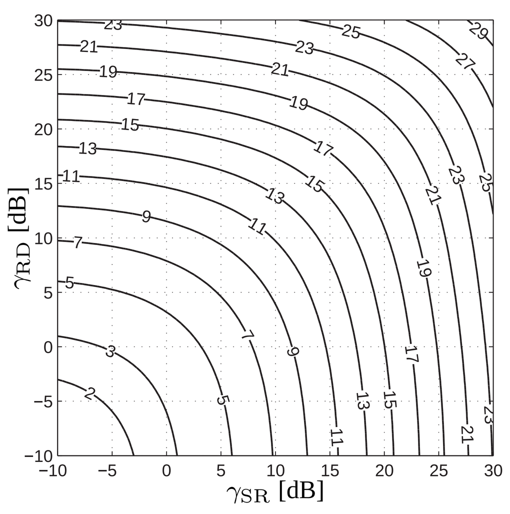
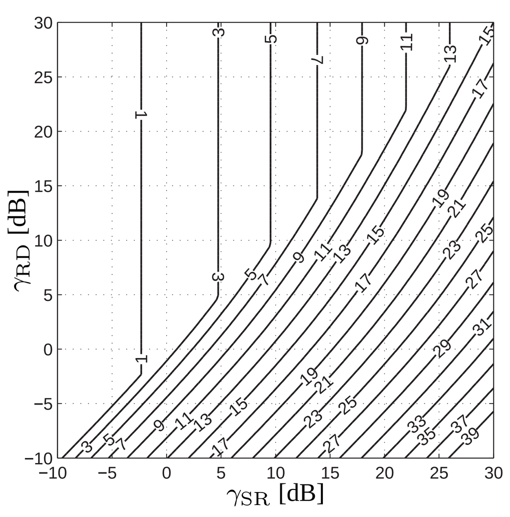
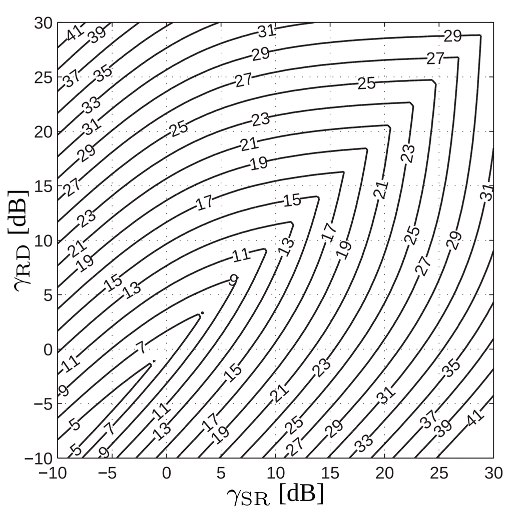
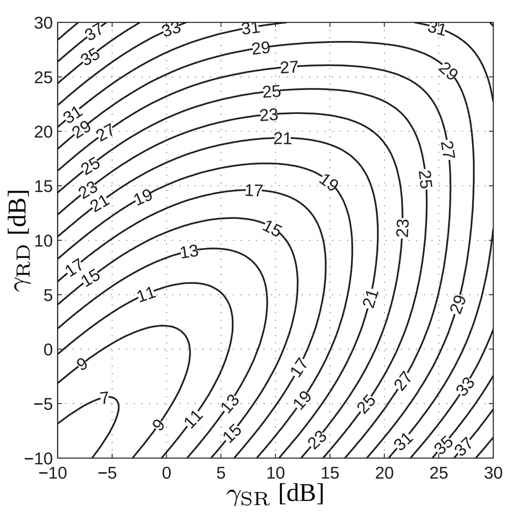

(b) AF and PA with individual constraints. Maximum power is used ($p_{\mathrm{R}}^* = 1$) in the shaded region.


(c) AF and PA with a sum constraint.


(d) DF and uniform PA.


(e) DF and PA with individual constraints.


(f) DF and PA with a sum constraint.

2009 Postal address: P.O. Box 3000 FI-02015 TKK Tel: +358-9-451 2429 Fax: +358-9-452 3614 Email: taneli.riihonen@tkk.fi