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On the Flow Induced by a Maxwell–Chartoff Rheometer

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The Maxwell-Chartoff rheometer does not supply the surface tractions necessary to maintain the flow customarily assumed. This can be seen from considerations of the energy balance.

Key words: Normal stresses; rheology; rheometer; viscoelasticity.

Most steady-state rheometers in use today utilize some form of viscometric flow, i.e., a flow which is dynamically equivalent to a plane steady simple shearing motion [1].[1] Recently, there has been a great deal of interest in extending rheological studies to nonviscometric flows, such as the flow in a Maxwell-Chartoff rheometer [2–5]. This apparatus is basically a pair of parallel plates rotating steadily and in the same sense about noncoincident axes perpendicular to the plates. The test sample is placed between the plates and rotates with them and must consequently undergo some shearing.

A flow has been proposed [4] which is supposed to take place in the Maxwell-Chartoff rheometer, and which has been studied in considerable detail along with its rheological consequences [5, 6]. In this proposed flow the material particles travel within planes parallel to the confining planes and describe circles about a center on the straight line connecting the centers of rotation of the confining planes. Thus, the material forms a cylinder whose cross section perpendicular to its generators is an ellipse. This flow can be described by the following deformation

$$\begin{aligned} x(t-s) &= X\cos\Omega s+(Y-\Psi Z)\sin\Omega s \\ y(t-s) &= -X\sin\Omega s+(Y-\Psi Z)\cos\Omega s+\Psi Z \\ z(t-s) &= Z \end{aligned}$$

where X, Y, Z are the coordinates of a material particle at time t and x, y, z are the coordinates of the same particle at any previous time, $t-s$. The angular velocity of the confining planes is Ω and Ψ is the tangent of the angle between the axis of rotation and the line connecting the centers of rotation.

Because this deformation is a linear mapping, the relative deformation gradient history is the same throughout the sample. For an incompressible isotropic simple fluid (in which the extra stress is an isotropic function of the history of the relative deformation gradient) the stress must also be a constant throughout the sample. Thus, the divergence of the stress is zero within the sample (ignoring any inertial forces), and the flow is dynamically possible if suitable surface tractions are supplied at the boundaries. This flow is a member of the class called controllable,[2] for which the necessary surface tractions can be supplied by geometrical constraints on the boundaries *without reference to the properties of the material*. On the plane boundaries, these tractions are supplied by the reaction forces of the rotating planes. A consideration of the symmetries of the situation is enough to establish that these forces do no work since there are no net displacements and no net torques. It is clear, however, that whatever power is consumed by the assumed flow must be supplied by the surface tractions on the cylindrical boundary, since they are the only external forces which do work. The power input from the surface tractions on the cylindrical boundary is proportional to the volume of the sample, and, of course, it cannot be made ignorable by decreasing the thickness of the sample.

Although the rotating plates of the Maxwell-Chartoff rheometer are physically equivalent to the confining planes of this flow, this rheometer does not control the sample surfaces between the plates, and one must expect the resulting flow to depend on the properties of the material being sheared. Crude experiments in our laboratory showed several different flows for different materials, none of which resembled the flow of the above equations. It is clear that the flow induced by the Maxwell-Chartoff rheometer can approximate that assumed above only to the extent that the test material dissipates a negligible amount of energy. It is interesting that the Maxwell-

[1] Figures in brackets indicate the literature references at the end of this paper.

[2] More exactly, this flow is controllable [6] (ignoring inertia) for simple incompressible isotropic fluids. Controllable flows are of particular rheological interest because they offer the possibility of producing a known flow in a material with unknown properties. Then, if stress measurements are taken simultaneously, rheological information may be developed.

Chartoff rheometer seems to have been a direct outgrowth of a device used by Mooney [7] to study the stored energy in vulcanized rubber cylinders.

Modifications of the Maxwell-Chartoff rheometer to supply the necessary constraints, such as the addition of an elastic cylindrical confining wall around the sample, are conceivable, but not without considerable design problems.

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