(V) T_t in Range 3 and T in Range 2

(1) $c_p = 0.2318 + 0.1040 \times 10^{-4}T + 0.7166 \times 10^{-8}T^2$

(2) $\gamma = c_p/[c_p - (R/J)]$

(3) $M^2 = (2J)\ [(0.2214T_t + 0.1760 \times 10^{-4}T_t^2 - 0.1258 \times 10^{-8}T_t^3) - (0.2318T + 0.0520 \times 10^{-4}T^2 + 0.2388 \times 10^{-8}T^3)]/\gamma RT$

(4) $p/p_t = (T/1700)^{3.389} \times (1700/T_t)^{3.229} \times \exp[(2.753 \times 10^{-8})T_t^2 - (5.134 \times 10^{-4})T_t + 0.7932] \times \exp[(1.516 \times 10^{-4})T + (5.224 \times 10^{-8})T^2 - 0.4087]$

(5) $\rho/\rho_t = (T/1700)^{2.389} \times (1700/T_t)^{2.229} \times \exp[(2.753 \times 10^{-8})T_t^2 - (5.134 \times 10^{-4})T_t + 0.7932] \times \exp[(1.516 \times 10^{-4})T + (5.224 \times 10^{-8})T^2 - 0.4087]$

(VI) T_t in Range 3 and T in Range 1

(1) $c_p = 0.2393$

(2) $\gamma = c_p/[c_p - (R/J)]$

(3) $M^2 = (2J)\ [(0.2214T_t + 0.1760 \times 10^{-4}T_t^2 - 0.1258 \times 10^{-8}T_t^3) - 0.2393T]/\gamma RT$

(4) $p/p_t = (1700/T_t)^{3.229} \times (T/400)^{3.491} \times (0.00535) \times \exp[(2.753 \times 10^{-8})T_t^2 - (5.134 \times 10^{-4})T_t + 0.7932]$

(5) $\rho/\rho_t = (1700/T_t)^{2.229} \times (T/400)^{2.491} \times (0.0228) \times \exp[(2.753 \times 10^{-8})T_t^2 - (5.134 \times 10^{-4})T_t + 0.7932]$

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On Flat Plates and Detached Shock Waves

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RECENT UNDERGRADUATE THESIS work by the author at the University of Detroit leads to the possible modification of Serbin's Eq. (5) of reference 2. This modification would make possible the prediction of the shock detachment distance for any flat-plate configuration. As stated in reference 2, the shock detachment distance for a disc can be determined by the following formula:

$$\Delta/R = 1.03/(K - 1)^{1/2} \qquad \text{(disc)}$$

where Δ is the detachment distance, R is the radius of the disc, and K is the density ratio across a normal shock at the free-stream Mach Number.

The modified formula is:

$$\Delta = 0.581\ [A/(K - 1)]^{1/2} \qquad \text{(flat plate)}$$

where A is the area of the flat plate. This modified formula was verified by the test results obtained on flat plates of a circular, square, and rectangular configuration. The tests were limited to a Mach Number of 2.82, and the aspect ratios of the flat plates were less than 1.5.

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On the Flow of a Hydromagnetic Fluid Near an Oscillating Flat Plate

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THE IMPULSIVE MOTION of an infinite flat plate in a viscous, incompressible magnetic fluid in the presence of an external magnetic field has been discussed by Rossow.[1] In this note, his method is extended to cover the case of the flow near an infinite flat wall which executes linear harmonic oscillations parallel to itself. The velocity profile will be found for the two cases: (1) the magnetic lines of force fixed relative to the fluid, (2) the magnetic lines of force fixed relative to the plate.

MAGNETIC FIELD FIXED RELATIVE TO THE FLUID

Let x denote the coordinate parallel to the direction of motion and y the coordinate perpendicular to the wall. At time $t < 0$, the fluid, plate, and magnetic field are assumed to be stationary everywhere. The plate starts to move at time $t = 0$.

The flow may be approximately described by the following differential equation:[1]

$$(\partial u/\partial t) + (\sigma B_0^2/\rho)\ u = \nu(\partial^2 u/\partial y^2) \qquad (1)$$

where: $u = x$ − component of fluid velocity; B_0 = external magnetic field directed perpendicular to the plate; σ = conductivity of the fluid; ν = kinematic viscosity of the fluid. B_0, σ, and ν are assumed to be constant. The boundary conditions are: $u = u_0 \cos nt$ at $y = 0, t \geq 0$; and u = finite at $y = \infty$, $t \geq 0$.

The Laplace transform of the velocity u is defined as

$$\bar{u} \equiv \int_0^\infty e^{-st}\ u dt$$

Applying the Laplace transform to Eq. (1), gives

$$\nu(d^2\bar{u}/dy^2) - (s + m)\bar{u} = 0 \qquad (2)$$

where $m \equiv \sigma B_0^2/\rho$.

The solution of Eq. (2) is given by

$$\bar{u} = C_1(s) \exp\left[-y\{(s + m)/\nu\}^{1/2}\right] + C_2(s) \exp\left[y\{(s + m)/\nu\}^{1/2}\right]$$

Since u = finite at $y = \infty$, $C_2(s)$ is chosen to be zero. The integration constant $C_1(s)$ is found from the boundary condition at $y = 0$—i.e., on the plate—giving

$$C_1(s) = u_0 s/(s^2 + n^2)$$

Hence,

$$\bar{u}(y, s) = \{u_0 s/(s^2 + n^2)\} \exp\left[-y\{(s + m)/\nu\}^{1/2}\right] \qquad (3)$$

Eq. (3) may now be inverted by means of the inversion theorem:[2]

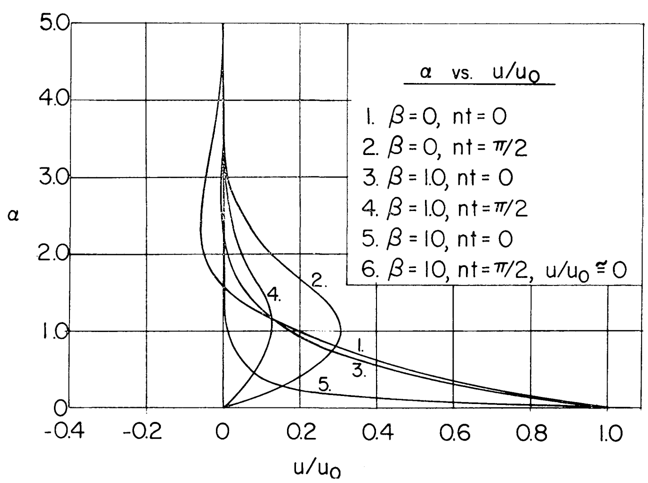

FIG. 1. Velocity profile with magnetic field fixed relative to the fluid.

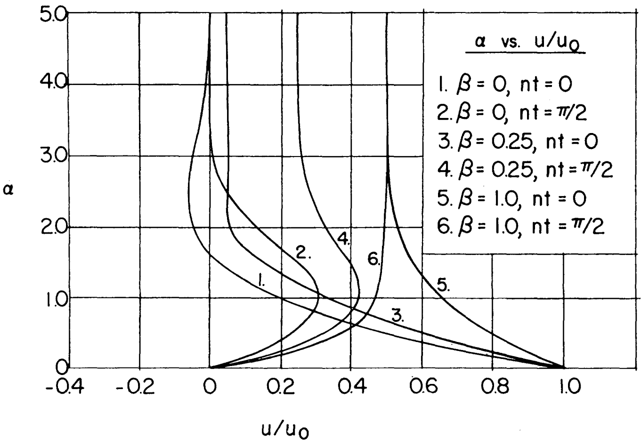

FIG. 2. Velocity profile with magnetic field fixed relative to the plate.

$$u(t) = \{1/2\pi i\} \lim_{\beta \to \infty} \int_{\gamma - i\beta}^{\gamma + i\beta} e^{st}\bar{u}(s)\, ds \qquad (4)$$

After a straightforward but lengthy computation, the solution to Eq. (1) may be expressed by

$$u(y, t) = u_0 \exp\{-y[(1/2\nu)((m^2 + n^2)^{1/2} + m)]^{1/2}\} \times \cos\{nt - y[(1/2\nu)((m^2 + n^2)^{1/2} - m)]^{1/2}\} \qquad (5)$$

It is to be noted that if $m = 0$—i.e., with no external magnetic field—the above expression reduces to the well-known solution for the ordinary viscous fluid flow problem.[3] Fig. 1 represents qualitatively this motion for several instants of time and for various strengths of the external magnetic field where $\alpha \equiv y(n/2\nu)^{1/2}$ and $\beta \equiv m/n$.

Magnetic Field Fixed Relative to the Plate

In this case, the magnetic field is moving and the fluid is initially at rest, so the relative motion must be accounted for. Hence, the relation analogous to Eq. (1) is

$$(\partial u/\partial t) + m(u - u_0 \cos nt) = \nu(\partial^2 u/\partial y^2) \qquad (6)$$

The boundary conditions are: $u = u_0 \cos nt$ at $y = 0$, $t \geq 0$; and $u =$ finite at $y = \infty$, $t \geq 0$. Application of the Laplace transform to Eq. (6) gives

$$\nu d^2\bar{u}/dy^2 = (m + s)\bar{u} - mu_0\{s/(s^2 + n^2)\} \qquad (7)$$

The solution of Eq. (7) is given by

$$\bar{u}(s, y) = \frac{mu_0}{(s + m)} \frac{s}{s^2 + n^2} + \frac{mu_0}{(s + m)} \frac{s}{(s^2 + n^2)} \left[C_1(s) \exp\left\{-y\left(\frac{s + m}{\nu}\right)^{1/2}\right\} + C_2(s) \exp\left\{y\left(\frac{s + m}{\nu}\right)^{1/2}\right\}\right] \qquad (8)$$

From the boundary conditions: $C_2(s) = 0$, and $C_1(s) = s/m$. Hence,

$$\bar{u}(y, s) = \frac{mu_0}{(s + m)} \frac{s}{(s^2 + n^2)} + \frac{u_0}{(s + m)} \frac{s^2}{(s^2 + n^2)} \exp\left\{-y\left(\frac{s + m}{\nu}\right)^{1/2}\right\} \qquad (9)$$

The first term on the right-hand side may be inverted by elementary methods to give

$$\mathcal{L}^{-1}\left\{\frac{mu_0 s}{(s + m)(s^2 + n^2)}\right\} = \frac{mu_0}{m^2 + n^2}\left[m \cos nt + n \sin nt - m \exp(-mt)\right]$$

The second term is inverted by means of the inversion formula, Eq. (4), and the final solution to Eq. (6) is expressed by

$$u(y, t) = \frac{m\, u_0}{m^2 + n^2}[m \cos nt + n \sin nt] + \frac{u_0 n}{m^2 + n^2} \exp\left\{-y\left[\frac{1}{2\nu}\left((m^2 + n^2)^{1/2} + m\right)\right]^{1/2}\right\} \times \left[n \cos\left\{nt - y\left[\frac{1}{2\nu}\left((m^2 + n^2)^{1/2} - m\right)\right]^{1/2}\right\} - m \sin\left\{nt - y\left[\frac{1}{2\nu}\left((m^2 + n^2)^{1/2} - m\right)\right]^{1/2}\right\}\right] \qquad (10)$$

Fig. 2 illustrates this motion for various strengths of the external magnetic field and for several instants of time. Again $\alpha \equiv y(n/2\nu)^{1/2}$ and $\beta \equiv m/n$.

In both figures, the curves for $nt = \pi$ and $3\pi/2$ are not included due to the complete symmetry (Fig. 1) and almost complete symmetry (Fig. 2) with respect to the ordinate.

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Generalized-Newtonian Theory

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Lees[1] has shown that in application to blunt-nose bodies an improvement upon Newtonian theory is realized by use of the modified form

$$C_p/C_{p_{max}} = \sin^2 \delta \qquad (1)$$

together with the value of $C_{p_{max}}$ that is derived from the normal shock relations. (The angle δ is the local inclination of the body surface.) If the left-hand and right-hand terms of Eq. (1) are divided, respectively, by $C_{p_{max}}$ and by its equivalent for blunt noses $C_{p_{max}} \sin^2 \delta_{max}$, there results

$$C_p/C_{p_{max}} = \sin^2 \delta / \sin^2 \delta_{max} \qquad (2)$$

This equation will be referred to as the generalized-Newtonian theory.

For $\delta_{max} = 90°$ the utility of the generalized-Newtonian theory is obviously well established since in this case it reverts to Lees[1] blunt-nose modification. For $\delta_{max} < 90°$ (pointed-nose bodies) the generalized form is equally useful, and in addition to exhibiting advantages over the Newtonian theory ($C_p = 2 \sin^2 \delta$), it tends to unify the results for body shapes in general at hypersonic speeds. To illustrate, Fig. 1 compares exact solutions (including rotational effects) for several ogives with the generalized-Newtonian theory and the Newtonian theory. The exact