# ON THE GEOMETRIC LANGLANDS CONJECTURE

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Let $X$ be a (smooth, projective) curve and $G$ be a reductive group over a finite field $\mathbb{F}_q$.

The field $K_X$ of rational functions on $X$ is what number theorists call a global field and we can do the theory of automorphic functions over it. Namely, we consider the quotient $G(K_X)\backslash G(\mathbb{A})$ (here $\mathbb{A}$ is the ring of adeles corresponding to $K_X$) and study the space of functions on it, as a representation of the adele group $G(\mathbb{A})$.

It was essentially an observation of A.Weil that the quotient $G(K_X)\backslash G(\mathbb{A})$ is closely related to the set of isomorphism classes of principal $G$-bundles on our curve $X$. However, $G$-bundles on $X$ possess a richer structure: one can view the above set as a set of $\mathbb{F}_q$-points of an algebraic variety (or, rather, a *stack*), denoted $\mathrm{Bun}_\mathrm{G}$. Moreover, instead of studying the vector space of functions on the "discrete" set of points of $\mathrm{Bun}_\mathrm{G}$, we will consider the *category* of $\ell$-adic sheaves on it.

Basically, what people call the geometric Langlands program is an attempt to understand a spectral decomposition of the above category under the action of the so-called *Hecke functors* (the latter will be defined in the talk).

The answer predicted by the Geometric Langlands Conjecture links this spectral decomposition to the moduli stack of local systems on $X$ with respect to the *Langlands dual* group $\check{G}$.

## References


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