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ON THE HIERARCHY OF PETRI NET LANGUAGES (*)

by Matthias JANTZEN [1]

Communicated by W. BRAUER

Abstract. – *We prove* $\mathscr{M}_\cap(D_1'^*) \subsetneq \hat{\mathscr{M}}(D_1'^*)$, *where* $D_1'^*$ *is the one-sided Dyck language, and discuss some old and new results concerning Petri net languages. The above result shows that Petri nets without* λ*-labeled transitions are less powerful than general nets as regards their firing sequences since the class* $\mathscr{L}_0^\lambda$ *of general Petri net languages (Hack [13]) is identical with* $\hat{\mathscr{M}}_\cap(D_1'^*)$, *and the class* $\mathscr{C}\mathscr{S}\mathscr{S}$ *of computation sequence sets (Peterson [21]) equals* $\mathscr{M}_\cap(D_1'^*)$.

INTRODUCTION

The reader is supposed to be familiar with the notion of Petri nets and with formal language theory. For exact definitions of Petri net languages, *see* Hack [13] and Peterson [21]. AFL theory, *see* Ginsburg [8], is used extensively.

For readers who like to read this note without going too much into details some informal explanation of abbreviations follows:

$\mathscr{L}_0^\lambda$ denotes the family of languages each of which is a set of firing sequences leading some arbitrary labeled Petri net from a start marking to a final marking;

$\mathscr{L}_0$ denotes the family of languages each of which is a set of firing sequences leading some arbitrary but λ-free labeled Petri net from a start marking to a different final marking;

$\mathscr{C}\mathscr{S}\mathscr{S}$ is defined like $\mathscr{L}_0$ but without the restriction that the final marking is different from the start marking;

$\mathscr{L}^\lambda$ denotes the family of languages each of which is a set of firing sequences leading some arbitrary labeled Petri net from a start marking to some other marking;

$\mathscr{L}$ is defined like $\mathscr{L}^\lambda$ without using λ-labels.

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$\mathcal{S}_z$ denotes the family of Szilard languages (Salomaa [24]) which are also known as derivation languages of context-free grammars (Penttonen [22]) or associate languages (Moriya [19]).

Note: Szilard languages do not contain the empty word λ ! $\mathcal{M}(\mathcal{L})$ [$\hat{\mathcal{M}}(\mathcal{L})$, $\mathcal{U}(\mathcal{L})$, $\hat{\mathcal{U}}(\mathcal{L})$ resp.] denotes the least trio (least full trio, least semi-AFL, least full semi-AFL resp.) containing $\mathcal{L}$.

For $\mathcal{O}$ being $\mathcal{M}$ ($\hat{\mathcal{M}}$, $\mathcal{U}$, $\hat{\mathcal{U}}$ resp.) $\mathcal{O}_\cap(\mathcal{L})$ denotes the least intersection-closed family containing $\mathcal{L}$ and closed under the operations which define $\mathcal{O}$.

$\mathcal{R}$ (resp. $\mathcal{RE}$) denotes the family of regular (resp. recursively enumerable) sets.

The shuffle operation on languages L_1 and L_2 is defined by:

$$\text{Shuf}(L_1, L_2) := \{ w = x_1 y_1 \ldots x_n y_n \mid x_1 x_2 \ldots x_n \in L_1, y_1 y_2 \ldots y_n \in L_2 \}.$$

The operation perm (L) denotes the commutative closure of the language L.

For families of languages $\mathcal{L}_1$, $\mathcal{L}_2$ we use the following notations

$$\mathcal{L}_1 \vee \mathcal{L}_2 := \{ L \mid L = L_1 \cup L_2 \text{ for some } L_1 \in \mathcal{L}_1, L_2 \in \mathcal{L}_2 \},$$

$$\mathcal{L}_1 \wedge \mathcal{L}_2 := \{ L \mid L = L_1 \cap L_2 \text{ for some } L_1 \in \mathcal{L}_1, L_2 \in \mathcal{L}_2 \},$$

$$\text{Shuf}(\mathcal{L}_1, \mathcal{L}_2) := \{ L \mid L = \text{Shuf}(L_1, L_2), L_1 \in \mathcal{L}_1, L_2 \in \mathcal{L}_2 \},$$

$$\bigwedge \mathcal{L} := \{ L \mid \text{there exists } n \geqq 1, L_1, \ldots, L_n \in \mathcal{L} \text{ such that } L = L_1 \cap L_2 \cap \ldots \cap L_n \}.$$

$$\mathcal{H}(\mathcal{L}) := \{ L \mid L = h(L') \text{ for some nonerasing homomorphism } h \text{ and some } L' \in \mathcal{L} \},$$

$$\hat{\mathcal{H}}(\mathcal{L}) := \{ L \mid L = h(L') \text{ for some arbitrary homomorphism } h \text{ and some } L' \in \mathcal{L} \}.$$

$$\mathcal{H}^{-1}(\mathcal{L}) := \{ L \mid L = h^{-1}(L') \text{ for some homomorphism } h \text{ and some } L' \in \mathcal{L} \}.$$

$$\text{perm}(\mathcal{L}) := \{ L \mid L = \text{perm}(L') \text{ for some } L' \in \mathcal{L}. \}.$$

SOME SIMPLE FACTS ON PETRI NETS

A number of proofs have been published to exhibit several closure properties for Petri net languages. The proofs can be found in Höpner [14], Hack [13] and Peterson [21]. We summarize the results in proposition 1:

PROPOSITION 1: *$\mathcal{CSS}$ and $\mathcal{L}_0^\lambda$ are closed with respect to union, concatenation, intersection, shuffle, substitution by λ-free regular sets, inverse homomorphism and* *limited erasing. $\mathscr{C}\mathscr{S}\mathscr{S}$ and $\mathscr{L}_0^\lambda$ contain all the regular sets, whereas $\mathscr{L}_0$ contains only the λ-free regular sets.*

Of course these operations are not independent from each other.

The characterization $\mathscr{L}_0^\lambda = \mathscr{H}(\mathscr{S}_z \wedge \mathscr{R})$ is more or less folklore because of the obvious connections between Petri net languages and derivation languages of matrix grammars. *See* Nash [20], van Leeuwen [18], Crespi-Reghizzi and Mandrioli [4, 6], Höpner [14], Salomaa [24], and many others cited there.

The equality $\mathscr{L}_0 = \mathscr{H}(\mathscr{S}_z \wedge \mathscr{R})$ has been proven by Crespi-Reghizzi and Mandrioli [6] though it is not explicitly stated there.

Using the equations above, proposition 1 and AFL theory we can characterize the Petri net languages in the following way:

PROPOSITION 2:

$$\mathscr{L}_0 = \mathscr{M}(\mathscr{S}_z) = \mathscr{U}(\mathscr{S}_z) = \mathscr{H}(\mathscr{H}^{-1}(\mathscr{S}_z) \wedge \mathscr{R}),$$
$$\mathscr{L}_0^\lambda = \hat{\mathscr{M}}(\mathscr{S}_z) = \hat{\mathscr{U}}(\mathscr{S}_z) = \hat{\mathscr{H}}(\mathscr{H}^{-1}(\mathscr{S}_z) \wedge \mathscr{R}),$$
$$\mathscr{C}\mathscr{S}\mathscr{S} = \mathscr{M}(\mathscr{S}_z \vee \{\{\lambda\}\}).$$

This characterization, as we whall see, is not optimal, since the family $\mathscr{S}\mathscr{Z}$ which generates $\mathscr{L}_0$, $\mathscr{L}_0^\lambda$ and $\mathscr{C}\mathscr{S}\mathscr{S}$ via *a-transductions can be replaced by a smaller family.*

It is easy to see that each Szilard language $L \in \mathscr{S}_z$ is a finite intersection of one-counter languages. A first hint in this direction has been given by Brauer [3], and in [6] it has been shown that certain Petri net languages can be written as finite intersections of deterministic context-free languages. We state this as:

PROPOSITION 3: *If $L \in \mathscr{S}_z$, then there exist $n \geq 1$ and deterministic one-conter languages $K_1, \ldots, K_n \in \mathscr{M}(D_1'^*)$ such that $L = K_1 \cap \ldots \cap K_n$ holds.*

Proof: The proof is obvious: each K_1 is a language accepted by an automaton which counts the number of occurences of the nonterminal A_i in the sentential form of the derivation in progress.

If the context-free grammar has m nonterminals then at most m one-counter languages are needed. Moreover, if the number of occurences of the nonterminal A_i within each sentential form of a terminating derivation is bounded by some constant, then the corresponding language K_i is a regular set. This shows that the integer n in proposition 3 can be chosen equal to the number of unbounded nonterminals of the grammar generating L.

Note: This does not mean that n equals the number of simultaneously unbounded nonterminals of that grammar There are examples where no nonterminal is bounded but only one at a time may occur arbitrarily often.

THE HIERARCHY

To obtain a simple and obvious characterization for Petri net languages we define a special kind of k-counter language which is the k-fold shuffle of the one-counter Dyck language.

DEFINITION: Let C_1^i denote the semi-Dyck language over the pair of brackets $\{a_i, \overline{a}_i\}$.

Then C_k is recursively defined by:

$$C_1 := C_1^1,$$
$$C_k := \mathrm{Shuf}(C_{k-1}, C_1^k).$$

Using AFL theory we easily show:

THEOREM 1:
$$\mathscr{L}_0^\lambda = \hat{\mathscr{M}}(\{C_i \mid i \geqq 1\}) = \hat{\mathscr{M}}_\cap(D_1'^*) = \hat{\mathscr{U}}_\cap(D_1'^*),$$
$$\mathscr{C}\mathscr{S}\mathscr{S} = \mathscr{M}(\{C_i \mid i \geqq 1\}) = \mathscr{M}_\cap(D_1'^*) = \mathscr{U}_\cap(D_1'^*).$$

Proof: Since $\mathscr{L}_0^\lambda = \hat{\mathscr{H}}(\mathscr{C}\mathscr{S}\mathscr{S}) = \hat{\mathscr{H}}(\mathscr{L}_0)$ (*see* proposition 2 and the definitions) we only have to show

$$\mathscr{C}\mathscr{S}\mathscr{S} = \mathscr{M}(\{C_i \mid i \geqq 1\}) = \mathscr{M}_\cap(D_1'^*).$$

The equality $\mathscr{M}_\cap(D_1'^*) = \mathscr{U}_\cap(D_1'^*)$ [resp. $\hat{\mathscr{M}}_\cap(D_1'^*) = \hat{\mathscr{U}}_\cap(D_1'^*)$] follows from proposition 1 and AFL theory.

Since

$$\mathscr{M}_\cap(\mathscr{M}(D_1'^*)) = \mathscr{M}(\wedge\mathscr{M}(D_1'^*)) = \mathscr{M}(\wedge\mathscr{M}(D_1'^*))$$

(*see* Ginsburg [8], prop. 3.6.1) and $\mathscr{S}_z \subseteq \wedge\mathscr{M}(D_1'^*)$ (by prop. 3) we get

$$\mathscr{M}(\mathscr{S}_z) \subseteq \mathscr{M}(\wedge\mathscr{M}(D_1'^*)) = \mathscr{M}_\Psi(\mathscr{M}(D_1'^*)) = \mathscr{M}(D_1'^*)$$

thus by proposition 2:

$$\mathscr{L}_0 \subseteq \mathscr{M}_\cap(D_1'^*) \quad \text{and} \quad \mathscr{C}\mathscr{S}\mathscr{S} \subseteq \mathscr{M}_\cap(D_1'^*).$$

Since $\mathscr{C}\mathscr{S}\mathscr{S}$ contains the language $D_1'^*$ (*see* [13, 17]) and is closed with respect to λ-free a-transductions (*see* prop. 1 and 2) we get:

$$\mathscr{C}\mathscr{S}\mathscr{S} = \mathscr{M}_\cap(D_1'^*).$$

To verify $\mathscr{M}(\{C_i \mid i \geqq 1\}) = \mathscr{M}_\cap(D_1'^*)$ we first observe that for each $k \geqq 1$ the language C_k is a member of $\mathscr{M}_\cap(D_1'^*)$ since this family contains $C_1 = D_1'^*$ and is closed with respect to shuffle.

Note: A trio is intersection-closed if and only if it is closed with respect to shuffle (exercice 5.5.6 in [8] or corollary 3 in [7]).

Thus we have $\mathcal{M}(\{C_i \mid i \geqq 1\}) \subseteq \mathcal{M}_\cap(D_1'^*)$.

Now suppose

$$L \in \mathcal{M}_\cap(D_1'^*) = \mathcal{M}(\bigwedge \mathcal{M}(D_1'^*)),$$

then by definition of $\bigwedge \mathcal{L}$ there exists $k \geqq 1$ such that

$$L \in \mathcal{M}(\mathcal{M}(C_1^1) \wedge \ldots \wedge \mathcal{M}(C_1^k)).$$

Using proposition 5.1.1 and theorem 5.5.1(*d*) in [8] we get

$$\mathcal{H}(\mathcal{M}(C_{k-1}) \wedge \mathcal{M}(C_1^k)) = \mathcal{H}(\mathcal{U}(C_{k-1}) \wedge \mathcal{U}(C_1^k))$$
$$= \mathcal{U}(\mathrm{Shuf}(C_{k-1}, C_1^k)) = \mathcal{U}(C_k) = \mathcal{M}(C_k).$$

By induction we obtain

$$\mathcal{H}(\mathcal{M}(C_1) \wedge \ldots \wedge \mathcal{M}(C_1^k)) =$$

$$\mathcal{H}(\mathcal{H}(\mathcal{M}(C_1^1) \wedge \ldots \wedge \mathcal{M}(C_1^{k-1})) \wedge \mathcal{M}(C_1^k)) = \mathcal{H}(\mathcal{M}(C_{k-1}) \wedge \mathcal{M}(C_1^k)) = \mathcal{M}(C_k).$$

Thus we have shown $L \in \mathcal{M}(C_k)$ which proves $\mathcal{M}(\{C_i \mid i \geqq 1\}) = \mathcal{M}_\cap(D_1'^*)$, and the proof of theorem 1 is finished.

Theorem 1 gives a similar characterization for $\mathcal{L}_0^\lambda$ as theorem 5.6 in [13]. Whereas Hack uses $D_1'^*$ and the regular sets as basis and the operations homomorphism, shuffle and intersection, we use $D_1'^*$ as basis and the following operations: homomorphism, inverse homomorphism, intersection with regular sets and either shuffle or intersection.

Using ideas of Greibach [10] one can show that for each $k \geqq 1$ the language

$$L_k := \{a_1^{n_1} \ldots a_k^{n_k} b a_k^{n_k} \ldots a_1^{n_1} \mid n_i \geqq 0\}$$

is not a member of the family $\mathcal{M}(C_{k-1})$ (*see* example 4.5.2 in [8]).

But obviously $L_k \in \mathcal{M}(C_k)$, thus there exists an infinite hierarchy of families of Petri net languages

$$\mathcal{M}(C_1) \subsetneqq \mathcal{M}(C_2) \subsetneqq \ldots \subsetneqq \mathcal{M}(C_k) \subsetneqq \mathcal{M}(C_{k+1}) \subsetneqq \ldots$$

Since $\mathcal{M}_\cap(D_1'^*) = \bigcup_{i \geqq 1} \mathcal{M}(C_i)$ (by the definition of $\bigwedge \mathcal{L}$ and previous results) we apply theorem 5.1.2 in Ginsburg [8] which shows that $\mathcal{M}(D_1'^*) = \mathcal{CSS}$ is not a principal semi-AFL.

REMARK: With the method of counting the number of reachable configurations Peterson [21] proved that PAL: $=\{ww^R \mid w \in \{0, 1\}^*\}$ is not a member of $\mathscr{C}\mathscr{S}\mathscr{S}$.

Now if the reachability problem for Petri nets is decidable as announced by Tenney and Sacerdote [23]:

(i) PAL is not a member of $\mathscr{L}_0^\lambda$;

(ii) $\mathscr{C}\mathscr{S}\mathscr{S}$ is not closed with respect to Kleene star.

Proof: Suppose PAL $\in \mathscr{L}_0^\lambda$ then $\mathscr{L}_0^\lambda = \mathscr{R}\mathscr{E}$, since $\mathscr{R}\mathscr{E}$ is the least intersection-closed full semi-AFL containing PAL (*see* [1]).

But this would contradict the result of Tenney and Sacerdote.

Suppose $\mathscr{C}\mathscr{S}\mathscr{S}$ to be star-closed, then $\mathscr{L}_0^\lambda$ would be star-closed too and thus a full AFL. But then again $\mathscr{L}_0^\lambda = \mathscr{R}\mathscr{E}$ would yield the contradiction since $\mathscr{R}\mathscr{E}$ is the least intersection closed full AFL containing the language $\{a^n b^n \mid n \geqq 0\}$ which is in $\mathscr{L}_0^\lambda$ (*see* [1]).

Unfortunately there is no direct proof of (i) or (ii) which does not use the result of Tenney and Sacerdote.

Note: Theorem 9.8 in [13], stating that the language $Q_0 = (D_1'^* \cdot \{0\})^* \cdot D_1'^*$ is not a member of $\mathscr{L}_0^\lambda$, is based on an incorrect proof as observed by Valk [26]!

THE NONCLOSURE OF $\mathscr{C}\mathscr{S}\mathscr{S}$ UNDER ERASING

There are two problems which are to be solved:

PROBLEM 1: Does or does not hold

$$\hat{\mathscr{M}}_\cap(D_1'^*) = \mathscr{M}_\cap(D_1'^*)\ ?$$

PROBLEM 2: Does or does not hold

$$\hat{\mathscr{M}}(C_k) = \hat{\mathscr{M}}(C_{k+1})\ ?$$

Before we solve the first one, let us shortly discuss the second one.

Of course $\hat{\mathscr{M}}(C_1) \nsubseteq \hat{\mathscr{M}}(C_2)$ since C_2 is not context-free and $\hat{\mathscr{M}}(C_1)$ contains only context-free languages. We will even see that $\hat{\mathscr{M}}(C_2)$ contains a language BIN such that ψ(BIN) is not a semilinear set (ψ denotes the usual Parikh mapping). It can be shown that $\hat{\mathscr{M}}(C_k) = \hat{\mathscr{M}}(C_{k+1})$ implies $\hat{\mathscr{M}}_\cap(C_1) = \hat{\mathscr{M}}(C_k)$, thus the family $\mathscr{L}_0^\lambda$ would be a principal semi-AFL which would be surprising. I conjecture that $\hat{\mathscr{M}}(C_k) \nsubseteq \hat{\mathscr{M}}(C_{k+1})$ holds for each $k \geqq 1$.

Compare this conjecture with results by Latteux [17] who has shown that $\hat{\mathscr{M}}_\cap(D_1^*) = \hat{\mathscr{M}}(\{O_n \mid n \geqq 1\})$ is not principal. The language O_n is defined similar to our language C_n by:

$$O_1 := \mathrm{perm}(\{a_1 \bar{a}_1\}^*) = D_1^*,$$

which is the two-sided Dyck language, and

$$O_n := \mathrm{Shuff}(O_{n-1}, \mathrm{perm}(\{a_n \bar{a}_n\}^*)).$$

To solve problem 1 we define the language BIN which will be the counterexample to show the desired inequality:

DEFINITION:

$$\mathrm{BIN} := \{ wa^k \mid w \in \{0, 1\}^*, 0 \leqq k \leqq n(w) \},$$

where $n(w)$ denotes the integer represented by w as a binary number. Convention: $n(\lambda) := 0$.

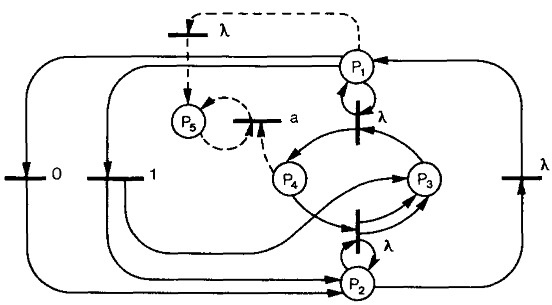
We first prove :

THEOREM 2:

$$\mathrm{BIN} \in \hat{\mathscr{M}}(C_2).$$

Proof: Let N be the following Petri net (*fig.*) including the place p_5, the dotted arcs and the transition labeled with the symbol "a".

Let N′ be the net N without the dotted lines.



We will verify that Petri net N accepts the language BIN, i.e. each firing sequence beginning with the start marking (1, 0, 0, 0, 0) spells out a word from BIN and conversely each element of BIN can be accepted in that way.

Let $|p_i|$ denote the number of tokens at place p_i. By induction we first prove a basic property of the net N':

FACT: After $w \in \{0, 1\}^*$ has been accepted by the net N' starting with the marking (1, 0, 0, 0) then $|p_3| + |p_4| \leqq n(w)$ holds true for the marking which has been reached.

Basic step: For $w \in \{0\}^*$ trivially $|p_3| + |p_4| = 0 = n(w)$.

For $w \in \{0\}^* \cdot \{1\}$ obviously $|p_3| + |p_4| = 1 = n(w)$.

Induction step: Assume the fact to be true for all $w \in \{0, 1\}^*$ of length m and suppose the net N' has already accepted such a word w. Then either p_2 or p_1 has one token. In order to accept a word $w' \in \{0, 1\}^*$ of length $m+1$ we have to reach a situation where p_1 has the token. This can be done using the λ-transitions. Suppose the situation reached so far is described by the marking $(1, 0, x, y)$. By our assumption $x + y \leqq n(w)$ holds true.

Now two cases are of interest:

Case 1: We use the transition labeled with "0". This means we accept $w' = w0$. In this case, not using one of the λ-transitions, we directly reach the marking $(0, 1, x, y)$. Still leaving the token on p_2 we can only reach a marking $(0, 1, x', y')$ where

$$0 \leqq y' \leqq y \qquad \text{and} \qquad x' = 2(y - y') + x.$$

Now we can shift the token from p_2 to p_1 and then we may reach some marking $(1, 0, x'', y'')$ where

$$x'' = x' - z \qquad \text{and} \qquad y'' = y' + z$$

for some $0 \leqq z \leqq x'$. Thus

$$x'' + y'' = x' + y' = 2y - 2y' + x + y' = 2y + x - y' \leqq 2y + x.$$

Since $x + y \leqq n(w)$ implies $y \leqq n(w)$ we get $2y + x \leqq 2n(w)$. Thus finally

$$x'' + y'' \leqq 2n(w) = n(w0) = n(w')$$

This proves the induction step restricted to case 1.

Case 2: Suppose we use the transition labeled with "1". This means we accept $w' = w1$. Then $n(w') = 2n(w) + 1$ and the same considerations as in case 1 show that in this case $|p_3| + |p_4| = x'' + y'' + 1$, so that $|p_3| + |p_4| \leqq n(w')$. Therefore we have proved the fact for all works $w \in \{0, 1\}^*$.

Now, looking at the net N we can easily verify that the transition labeled with "a" can be used at most $|p_4|$ times, thus at most $n(w)$ times if w has been accepted and p_5 has got the token from p_1. This shows that each word accepted by the net N is in BIN.

Conversely, we have to show that each word in BIN can be accepted by the net. This is easily seen in the following way: First of all each word $w \in \{0, 1\}^*$ can be accepted by the net. Moreover, if each λ-transition is used as often as possible until w has been accepted and p_1 has one token, then $|p_4| = n(w)$. Of course the transition labeled with "a" may now be used k times, where $0 \leqq k \leqq n(w)$ is arbitrary.

This shows that the net N accepts exactly the language BIN without using final markings. Of course we could add some more λ-transitions to clear all places if we liked.

Since the net has only the two unbounded places p_3 and p_4 we have the result $\mathrm{BIN} \in \hat{\mathcal{M}}(C_2)$.

The language BIN is similar to a language used by Greibach [11] to show that linear-time is more powerful than real-time recognition by multicounter machines. We now show $\mathrm{BIN} \notin \mathcal{M}_\cap(D_1'^*)$. The proof uses Dedekind's idea of distributing more than n pieces into less than n boxes.

THEOREM 3:

$$\mathrm{BIN} \notin \mathcal{M}_\cap(D_1'^*).$$

Proof: Assume $\mathrm{BIN} \in \mathcal{M}_\cap(D_1'^*)$, then there exists a net N with k places which accepts BIN not using λ-transitions. We will derive a contradition.

Let m be the maximal number of tokens which can be added to the net in firing one transition. Let m_0 be the total number of tokens in the net at the beginning. Then after n steps, each step being the firing of one transition, there are at most $m_0 + n \cdot m$ tokens in the net. Distributing up to that many tokens over the k places of the net yields at most

$$\sum_{i=0}^{m_0+n\cdot m} \binom{i+k-1}{k-1} = \binom{m_0+n\cdot m+k}{k} \leqq (m_0+n\cdot m+1)^k,$$

different markings which are reachable within n steps!

Note: $\binom{i+k-1}{k-1}$ equals the number of different possibilities to distribute exactly i indistinguishable objects into k different boxes.

Of course the upper bound obtained above is quite bad, on the other hand it is good enough for our purpose.

Now, there are 2^n different words $w \in \{0, 1\}^*$ of length n. Each word represents an integer $n(w)$, where $0 \leqq n(w) \leqq 2^n - 1$. Let $w_0, w_1, \ldots, w_{2^n-1}$ be the ordering of all words of length n such that $n(w_i)$ equals i for $i = 0, 1, \ldots, 2^n - 1$.

For each word w_i there must exist at least one marking M_i of the net which is reachable while accepting w_i and from which it is possible to accept a^i, since the word $w_i a^i$ is in BIN. We shall see that all these markings $M_0, \ldots, M_{2^n-1}$ must be different. But this then is a contradiction, because there are at most $(m_0 + n \cdot m)^k$ different markings reachable within n steps, which for n big enough is strictly less than 2^n.

Now suppose for some $i \neq j$ we would have $M_i = M_j$. Then we could reach this marking accepting the word $w_{\min(i,j)}$, and starting with this marking we could accept the word $a^{\max(i,j)}$, thus we could accept the word $w_{\min(i,j)}\, a^{\max(i,j)}$ which is not a member of BIN. The contradiction is met and we have shown that no Petri net without λ-labeled transitions can accept the language BIN.

COROLLARY 1:

$$\mathcal{M}_{\cap}(D_1'^{*}) \subsetneqq \hat{\mathcal{M}}_{\cap}(D_1'^{*}) \qquad \textit{and} \qquad \mathcal{CSS} \subsetneqq \mathcal{L}_0^{\lambda}.$$

Proof: Trivial, using theorem 2, theorem 3 and the propositions.

COROLLARY 2:

$$\mathcal{L} \subsetneqq \mathcal{L}^{\lambda}.$$

Proof: Since BIN is in $\mathcal{L}^{\lambda}$ and the proof of theorem 3 works for nets with or without final markings.

REMARK: When writing this note, I have been told that Greibach [12] has shown $\mathcal{CSS} = \mathcal{M}_{\cap}(D_1'^{*}) \subsetneqq \hat{\mathcal{M}}_{\cap}(D_1'^{*})$ independently.

Vidal Naquet [27] has proved corollary 2 using a different method which was not applicable for nets with final markings.

Corollary 1 solves the open problem of Hack [13] whether λ-labels can be eliminated in arbitrary Petri nets.

The well known language $L_{St} := \{ a^n b^m \mid 1 \leqq n,\ 1 \leqq m \leqq 2^n \}$, the Parikh image of which is not a semi-linear set (Stotzkij [25]) now simply can be shown to be a member of $\hat{\mathcal{M}}(C_2)$ since

$$L_{St} = h(\mathrm{BIN} \cap \{1\}^{+} \{a\}^{*}) \cdot \{b\},$$

where h is the coding defined by $h(1) := a$ and $h(a) := b$.

Surprisingly enough it can be shown that this language can be accepted by a certain net without λ-labeled transitions. We state this as:

PROPOSITION 4:

$$L_{St} \in \mathcal{M}(C_3).$$

The proof can be found in [16].

Careful inspection of the net for this language L_{St} which in fact is a modified version of the net for BIN shows that the Parikh image of the set of all reachable markings is not a semi-linear set.

Using results of van Leeuwen [18] we see that Petri nets with three unbounded places are strictly more powerful than vector addition systems of dimension 3. This follows since van Leeuwen [18], theorem 6.4, has proved that for each vector addition system of dimension 3 the Parikh image of the set of reachable points is a semi-linear set.

Looking at the proof of theorem 3 one can check that the method used here doesn't work if the language under consideration is bounded, i. e. if $L \subseteq \{w_1\}^* \ldots \{w_m\}^*$ for a fixed collection of words $w_1, \ldots, w_m$. In this case there are at most $D(n; \lg(w_1), \ldots, \lg(w_m))$ different words of length n, where the "denumerant" $D(n; a_1, \ldots, a_m)$ equals the number of different points $x := (x_1, \ldots, x_m)$ for which

$$a_1 \cdot x_1 + a_2 \cdot x_2 + \ldots + a_m \cdot x_m = n \quad \text{holds true.}$$

Using results of Bell [2] it can be shown that for all $n \geqq 1$ $D(n; a_1, \ldots, a_m) \leqq c \, . \, n^{m-1}$ for some appropriate constant c depending only on $a_1, \ldots, a_m$.

Thus the number of words of a certain length n and the number of different markings reachable within n steps both are bounded by some polynomial in n.

These suggestions give rise to the following:

Conjecture: Each bounded language $L \in \hat{\mathscr{M}}_\cap(D_1'^*)$ is in fact a member of $\mathscr{M}_\cap(D_1'^*)$.

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