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On the hydrodynamics of 'slip–stick' spheres

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The breakdown of the no-slip condition at fluid–solid interfaces generates a host of interesting fluid-dynamical phenomena. In this paper, we consider such a scenario by investigating the low-Reynolds-number hydrodynamics of a novel 'slip–stick' spherical particle whose surface is partitioned into slip and no-slip regions. In the limit where the slip length is small compared to the size of the particle, we first compute the translational velocity of such a particle due to the force density on its surface. Subsequently, we compute the rotational velocity and the response to an ambient straining field of a slip–stick particle. These three Faxén-type formulae are rich in detail about the dynamics of the particles: chiefly, we find that the translational velocity of a slip–stick sphere is coupled to all of the moments of the force density on its surface; furthermore, such a particle can migrate parallel to the velocity gradient in a shear flow. Perhaps most important is the coupling we predict between torque and translation (and force and rotation), which is uncharacteristic of spherical particles in unbounded Stokes flow and originates purely from the slip–stick asymmetry.

1. Introduction

The quintessential boundary constraint in fluid dynamics is the 'no-slip' condition which states that a fluid element 'sticks' when in contact with a solid surface (see e.g. Lamb 1993 and Batchelor 2000). However, the physical origins of this condition are a point of controversy and measurements of its breakdown have been reported by many, including Thompson & Robbins (1990*a*) and Zhu & Granick (2002). The Navier slip condition, a classic albeit limited model of this breakdown, was explored in detail by Thompson & Troian (1997). This model introduces the notion of a scalar slip length, λ, which relates linearly the fluid velocity along the interface to the normal component of the shear stress at the interface. Clearly, the no-slip condition is a special case of this model where $\lambda = 0$. Although the Navier slip condition is rather simple, experimental and theoretical measurements of slip lengths for various fluid–solid interfaces are common (see e.g. Hocking 1976; Thompson & Robbins 1990*b*; Einzel *et al.* 1990).

In this article, we consider a novel spherical particle divided by a plane into two 'faces' such that the slip length is zero on one face of the particle while there is a finite (but small relative to the particle size) slip length on the other. Naturally, such particles will stick to the fluid more on one part of their surface than the other. Such a particle could be manufactured by coating or roughening an initially uniform spherical particle asymmetrically or by bonding two hemispheres of materials with different slip lengths. Since the construction of 'Janus' or two-faced particles at colloidal scales is already possible (Cayre, Paunov & Velev 2003; Nie *et al.* 2006; and Perro *et al.* 2005), we anticipate that the fabrication of particles with the aforementioned slip–stick quality is indeed feasible.

Particles possessing non-uniform surface, or interfacial, properties have been studied extensively in the context of phoretic motion (see Anderson 1989 for a review). In such situations the gradient in an imposed potential — e.g. voltage for electrophoresis, temperature for thermophoresis, or solute concentration for diffusiophoresis — drives a fluid flow in a region adjacent to the fluid–particle interface. When the thickness of this region is small compared to the particle size, the flow may be interpreted as an effective 'slip' velocity, which is equal to the product of the local tangential gradient of the potential and a slip coefficient, or mobility, that plays the role of a slip length. (In electrophoresis, for instance, the slip coefficient is proportional to the zeta potential of the particle surface.) The slip flow causes the particle to move such that there is no net force or torque on the particle plus interfacial (slip) layer. For example, a spherical particle with uniform slip coefficient translates along the gradient of the imposed potential, but does not rotate. However, if the surface symmetry is broken by a non-uniform slip coefficient, the particle may translate perpendicular to the gradient of the potential and also rotate (Anderson 1985). In § 2 we find analogous results for our 'slip–stick' particle; namely, in steady translation the fluid exerts a non-zero torque on the particle, and the force on the particle is not solely along the direction of motion. However, it is important to note that for our slip–stick sphere a gradient in an imposed potential is not required to drive the particle motion.

The design of particles and surfaces with discontinuous changes in slip coefficient has received considerable attention recently. For example, Yariv (2004) has considered electro-osmotic flow past a planar wall with a jump in zeta potential. Moreover, as shown by Golestanian, Liverpool & Ajdari (2007), particles that generate their own concentration gradients (via e.g. a surface catalyzed chemical reaction) may be designed to move autonomously, or 'swim', via appropriate patterning of their slip coefficient. On a rather different note, You & Moin (2007) investigated a circular cylinder whose surface is partitioned into alternating stick and slip regions, as a means of drag and lift reduction in high Reynolds number ($\geqslant 300$) flows.

The hydrodynamics of 'slip–stick' particles promises to be interesting; for instance, as the slip length increases, the fluid's resistance to the motion of the particle decreases. Undoubtedly, the drag coefficient is bounded from above by the drag on a solid spherical particle and from below by the drag on a spherical bubble. Furthermore, the two-facedness of the particle breaks the fore–aft symmetry typically associated with spherical particles in Stokes flow. This introduces a coupling between rotation and translation – a feature often associated with chiral bodies in low Reynolds number flows (e.g. a corkscrew). In fact, slip–stick spheres begin to have more in common with ellipsoids and other axisymmetric bodies than (uniform) spheres, with the slip length, λ, playing the role of an eccentricity. As the slip length grows, the symmetry is further broken and the particle appears increasingly eccentric from a hydrodynamic viewpoint. We formalize these phenomenological ideas in a Faxén-type formula relating the translational velocity of the particle to the various moments of the force density on its surface, as well as the effect of an ambient flow field (i.e. another flow field which exists when the particle is not present and satisfies the Stokes equations). We calculate two additional Faxén formulae coupling the rate of rotation and the effect of a straining field to the moments of the force density on the particle and an ambient flow field.

The rest of the article is organized as follows. In § 2, we define the Navier slip condition explicitly and show that when the slip length is small compared to the size of the particle, the condition reduces to proportionality between the slip velocity and the velocity gradient normal to the surface. In this section, we also determine the appropriate boundary conditions for the velocity field on the surface of a slip–stick sphere. In § 3, we study the problem of a slip–stick sphere translating at low Reynolds number. We determine an analogous Faxén's first law for the slip–stick sphere which couples the translational velocity of the particle to the moments of the force density on its surface as well as the effects of an additional, ambient Stokes flow velocity field. In § 4, we generate the analogous Faxén's second law by studying a slip–stick sphere in a linear flow. These expressions couple the rate of rotation of the particle and the rate of strain in the fluid to the force density on the particle's surface and the effects of an additional ambient Stokes field. We conclude with some thoughts on the experimental realization of these results and a brief discussion of anisotropic slip lengths with other particle shapes as well as possible extensions to the present work.

2. Boundary conditions for the flow around a slip–stick sphere

The Navier slip condition may be written down explicitly as

$$\boldsymbol{t}^{(i)} \cdot \boldsymbol{u}(\boldsymbol{x}) = \frac{\lambda}{\eta} \boldsymbol{t}^{(i)} \boldsymbol{n} : \boldsymbol{\tau}(\boldsymbol{x}), \tag{2.1}$$

where $\boldsymbol{u}(\boldsymbol{x})$ is the velocity of the fluid in the frame of reference of the particle, $\boldsymbol{x}$ is a point on the surface, η is the viscosity of the fluid, $\boldsymbol{n}$ and $\boldsymbol{t}^{(i)}$ are unit vectors normal and tangential to the surface respectively and $\boldsymbol{\tau}$ is the shear stress in the fluid (note that the double-dot-product used here is defined so that the indices are contracted in the 'inside-out' fashion such that in index notation $\boldsymbol{A} : \boldsymbol{B} = A_{ij} B_{ji}$). One can show quite simply that for any curvilinear surface whose smallest radius of curvature, R, is much larger than λ (i.e. $\lambda/R \ll 1$), the Navier slip condition reduces to a linear relationship between the velocity at the surface and the normal velocity gradient to $O(\lambda/R)$:

$$\boldsymbol{t}^{(i)} \cdot \boldsymbol{u}(\boldsymbol{x}) = \lambda \boldsymbol{t}^{(i)} \boldsymbol{n} : \nabla_x \boldsymbol{u}(\boldsymbol{x}) + O\left(\frac{\lambda}{R}\right)^2. \tag{2.2}$$

We illustrate this explicitly for the case of a spherical surface with tangential vectors in the polar and azimuthal directions, $\boldsymbol{t}^{(1)}$ and $\boldsymbol{t}^{(2)}$ (sometimes denoted $\boldsymbol{e}_\theta$ and $\boldsymbol{e}_\phi$). By substituting into equation (2.1) the shear stress of a Newtonian fluid and asserting that the particle is impenetrable, we rewrite that equation as

$$\begin{aligned} \boldsymbol{t}^{(i)} \cdot \boldsymbol{u} &= \lambda \left(\boldsymbol{t}^{(i)} \boldsymbol{n} + \boldsymbol{n} \boldsymbol{t}^{(i)} \right) : \nabla \boldsymbol{u} \\ &= \lambda \left[\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right) \delta_{i1} + \left(\frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r} \right) \delta_{i2} \right], \end{aligned} \tag{2.3}$$

where u_θ and u_ϕ are the polar and azimuthal components of the velocity field on the surface of the particle and r is the radial coordinate. The above expression leads to one particularly useful conclusion: that both u_θ and u_ϕ are quantities which scale as $O(\lambda/R)$, where R is the radius of the particle. The derivatives of these two velocities with respect to r on the surface of the particle are still $O(1)$ quantities however, so we can write the Navier slip condition on the surface of a sphere as

$$\boldsymbol{t}^{(i)} \cdot \boldsymbol{u}(\boldsymbol{x}) = \lambda \left[\frac{\partial u_\theta}{\partial r} \delta_{i1} + \frac{\partial u_\phi}{\partial r} \delta_{i2} \right] + O\left(\frac{\lambda}{R}\right)^2, \tag{2.4}$$

which is identical to equation (2.2).

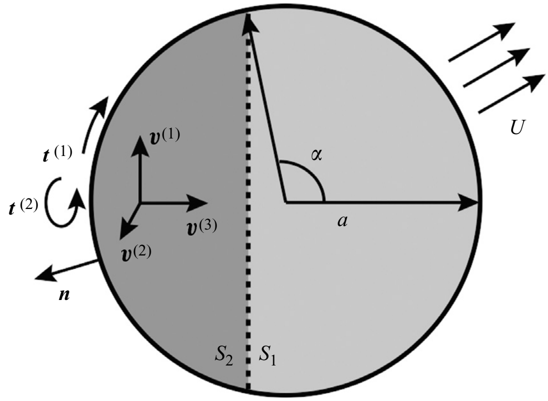

FIGURE 1. Definition sketch of a slip–stick sphere characterized by radius a, dividing angle α, translational velocity $\boldsymbol{U}$ and directional vectors $\boldsymbol{v}^{(i)}$. Fluid slips over surface S_1 and sticks over surface S_2. The vectors $\boldsymbol{n}$, $\boldsymbol{t}^{(1)}$ and $\boldsymbol{t}^{(2)}$ are the spherical unit vectors (often called $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$ and $\boldsymbol{e}_\phi$ respectively) of the right-handed coordinate system formed by the unit vectors $\boldsymbol{v}^{(i)}$.

One simple physical interpretation of this result is that to $O(\lambda/R)$ and on the scale of the slip length, a surface with relatively large curvature is essentially flat. In the case of colloidal spheres, the radius of curvature is characterized by the radius of the colloid, a. Taking the limit that the slip length is small relative to the radius of the particle is reasonable since measured slip lengths on the order of tens of nanometres are typical (Zhu & Granick 2002) while colloidal particles are usually microns in size. Since we have made this approximation, all subsequent expressions are regular perturbation expansions in slip length which we have truncated at the $O(\lambda/a)^2$ level.

We are now in a position to define an explicit set of boundary conditions for the velocity field surrounding a slip–stick sphere, in the frame of reference of the particle. We divide the surface of the particle into two faces (slippery and sticky, see figure 1) and write down the conditions on the fluid in contact with each face separately such that

$$\left.\begin{aligned} &\boldsymbol{n}\cdot\boldsymbol{u}(\boldsymbol{x}) = 0,\\ &\boldsymbol{t}^{(i)}\cdot\boldsymbol{u}(\boldsymbol{x}) = \lambda\boldsymbol{t}^{(i)}\boldsymbol{n}:\nabla_x\boldsymbol{u}(\boldsymbol{x}), \quad i = 1, 2, \end{aligned}\right\} \tag{2.5}$$

on the slippery face of the particle denoted S_1, and

$$\boldsymbol{u}(\boldsymbol{x}) = 0, \tag{2.6}$$

on the sticky face of the particle, denoted S_2. The vectors $\boldsymbol{n}$ and $\boldsymbol{t}^{(i)}$ are the unit normal to the sphere and the ith unit tangential vector to the sphere respectively. In the next section, we proceed by describing the flow around a translating slip–stick particle using the boundary-integral formulation for Stokes flow. This leads quite naturally to an extension of Faxén's first law to $O(\lambda/a)$ that relates the translation of the particle to various moments of the force density.

3. Faxén's first law for slip–stick spheres

A slip–stick sphere of radius a translating with velocity $\boldsymbol{U}$ at low Reynolds number generates a fluid velocity field in the frame of the moving sphere denoted $\boldsymbol{u}(\boldsymbol{x})$, where $\boldsymbol{x}$ is the position vector and $\boldsymbol{x}_0$ denotes the centre of the sphere. Such a sphere is illustrated in figure 1, with the division between slip and stick faces being characterized by the polar angle α, so that when $\alpha = 0$ the entire sphere is no-slip and when $\alpha = \pi/2$ the sphere is evenly divided into slip and stick halves. Associated with this sphere are three unit vectors: $\boldsymbol{v}^{(3)}$, which is normal to the plane dividing the sphere and pointing into the slippery face (S_1) and $\boldsymbol{v}^{(1)}$ and $\boldsymbol{v}^{(2)}$ which together with $\boldsymbol{v}^{(3)}$ describe a mutually orthonormal right-handed system of coordinate axes.

Without loss of generality, the velocity field due to the translating sphere can be written in three parts: a contribution due to the translational frame, $-\boldsymbol{U}$; a contribution due to some other ambient field, $\boldsymbol{u}^{\infty}(\boldsymbol{x})$, perhaps due to the presense of another particle, which itself satisfies the Stokes equations ($\eta\nabla^2\boldsymbol{u}^{\infty} = \nabla p^{\infty}$, $\nabla\cdot\boldsymbol{u}^{\infty} = 0$); and a contribution due to the presence of the particle which we call the disturbance velocity, $\boldsymbol{u}'(\boldsymbol{x})$, namely

$$\left.\begin{aligned} \boldsymbol{u}(\boldsymbol{x}) &= -\boldsymbol{U} + \boldsymbol{u}^{\infty}(\boldsymbol{x}) + \boldsymbol{u}'(\boldsymbol{x}), \\ \boldsymbol{u}'(\boldsymbol{x}) &= \int_{S_1+S_2} [\boldsymbol{J}(\boldsymbol{x}-\boldsymbol{y})\cdot(\boldsymbol{f}(\boldsymbol{y}) + P\boldsymbol{n}) + \boldsymbol{u}(\boldsymbol{y})\cdot\boldsymbol{K}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{n}]\,\mathrm{d}S_y, \end{aligned}\right\} \tag{3.1}$$

where $\boldsymbol{J}(\boldsymbol{r})$ is the stokeslet or Oseen tensor:

$$J_{ij}(\boldsymbol{r}) = \frac{1}{8\pi\eta}\left(\frac{\delta_{ij}}{r} + \frac{r_i r_j}{r^3}\right), \tag{3.2}$$

$\boldsymbol{K}(\boldsymbol{r})$ is the couplet:

$$K_{ijk}(\boldsymbol{r}) = \frac{3}{4\pi}\frac{r_i r_j r_k}{r^5}, \tag{3.3}$$

$\boldsymbol{f}(\boldsymbol{y})$ is the force density on the surface of the particle at point $\boldsymbol{y}$, and P is a static pressure due to the rigid-body motion of the particle in the ambient field. This is just a statement of the boundary-integral solution to the Stokes flow equations (see Ladyzhenskaya 1963). The disturbance velocity $\boldsymbol{u}'(\boldsymbol{x})$ describes the manner in which the force density on the surface of the particle changes in order to satisfy the boundary conditions on its surface. One can show quite easily that the static pressure, as in all Stokes flows, makes no contribution to the dynamical behaviour of a rigid particle. We include it in equation (3.1) for completeness, but the integral of the dot product between the normal and the stokeslet over the surface of a sphere is identically zero, so P does not affect the flow. The contribution to $\boldsymbol{u}'(\boldsymbol{x})$ due to $\boldsymbol{K}(\boldsymbol{r})$ comes from the double-layer (or force dipole) distribution on the particle's surface, a quantity which is zero for all rigid no-slip particles, but makes a finite contribution in this case and in some less exotic circumstances such as flow around a viscous drop.

We proceed by integrating $\boldsymbol{u}(\boldsymbol{x})$ over the surface of the particle such that

$$\begin{aligned} &\int_{S_1+S_2} \boldsymbol{u}(\boldsymbol{x})\,\mathrm{d}S_x \\ &\quad = \int_{S_1+S_2}\left(-\boldsymbol{U} + \boldsymbol{u}^{\infty}(\boldsymbol{x}) + \int_{S_1+S_2} [\boldsymbol{J}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{f}(\boldsymbol{y}) + \boldsymbol{u}(\boldsymbol{y})\cdot\boldsymbol{K}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{n}]\,\mathrm{d}S_y\right)\mathrm{d}S_x. \end{aligned} \tag{3.4}$$

To simplify the left-hand side, we substitute the boundary conditions on the velocity field into the integral over $\boldsymbol{u}(\boldsymbol{x})$. That is, we substitute the tangential components of the slipping field, $\lambda(\boldsymbol{t}^{(1)}\boldsymbol{t}^{(1)} + \boldsymbol{t}^{(2)}\boldsymbol{t}^{(2)})\boldsymbol{n}:\nabla_x\boldsymbol{u}(\boldsymbol{x})$, for $\boldsymbol{u}(\boldsymbol{x})$, and reduce the limits of integration to include only the slipping face since the velocity field is exactly zero on the sticking face. Now, we know $\boldsymbol{u}(\boldsymbol{x})$ explicitly in terms of equation (3.1), so

$$\nabla_x\boldsymbol{u}(\boldsymbol{x}) = \nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x}) + \nabla_x\boldsymbol{u}'(\boldsymbol{x}). \tag{3.5}$$

Additionally, we directly evaluate the well-known integrals of $\boldsymbol{U}$, $\boldsymbol{u}^{\infty}(\boldsymbol{x})$ and the stokeslet over the surface of the particle such that

$$\int_{S_1+S_2} \boldsymbol{U}\,\mathrm{d}S_x = 4\pi a^2 \boldsymbol{U}, \tag{3.6}$$

$$\int_{S_1+S_2} \boldsymbol{u}^{\infty}(\boldsymbol{x})\,\mathrm{d}S_x = 4\pi a^2 \left(1 + \frac{a^2}{6}\nabla_x^2\right) \boldsymbol{u}^{\infty}(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0}, \tag{3.7}$$

$$\iint_{S_1+S_2} \boldsymbol{J}(\boldsymbol{x}-\boldsymbol{y})\boldsymbol{\cdot}\boldsymbol{f}(\boldsymbol{y})\,\mathrm{d}S_x\,\mathrm{d}S_y = \frac{2a}{3\eta}\int_{S_1+S_2} \boldsymbol{f}(\boldsymbol{y})\,\mathrm{d}S_y, \tag{3.8}$$

where the integral of the force density over the surface of the particle is the total force on the particle – see e.g. Kim & Karrila (1991, 2005 pp. 73–77) for a detailed explanation. We also use a Taylor expansion of $\boldsymbol{K}(\boldsymbol{x}-\boldsymbol{y})$ with respect to $\boldsymbol{x}$ about $\boldsymbol{x}_0$ to make the double-layer contribution more tractable, as the integral with respect to $\boldsymbol{x}$ over the double-layer contribution becomes

$$\int_{S_1+S_2} \boldsymbol{u}(\boldsymbol{y})\boldsymbol{\cdot} \left(1 + \frac{a^2}{6}\nabla_x^2\right) \boldsymbol{K}(\boldsymbol{x}-\boldsymbol{y})\bigg|_{\boldsymbol{x}=\boldsymbol{x}_0}\boldsymbol{\cdot}\boldsymbol{n}\,\mathrm{d}S_y. \tag{3.9}$$

Because $\boldsymbol{K}(\boldsymbol{r})$ is a solution of the Stokes equations and therefore is biharmonic, this truncated Taylor expansion is in fact exact. The tensor $\boldsymbol{K}(\boldsymbol{x}_0-\boldsymbol{y})\boldsymbol{\cdot}\boldsymbol{n}$ and its Laplacian can be computed directly in terms of tensorial products between spherical normal and tangential vectors. We contract this with the surface velocity by referring back to the boundary conditions in equations (2.5) and (2.6), so that the integral is reduced to one over the slipping surface of the particle alone, and only tangential components of the velocity field, $\boldsymbol{u}(\boldsymbol{y})$, remain, such that when $\boldsymbol{y}$ is a point on the surface of the particle

$$\boldsymbol{u}(\boldsymbol{y})\boldsymbol{\cdot}\boldsymbol{K}(\boldsymbol{x}_0-\boldsymbol{y})\boldsymbol{\cdot}\boldsymbol{n} = 0, \tag{3.10}$$

and

$$\boldsymbol{u}(\boldsymbol{y})\boldsymbol{\cdot}\nabla_{x_0}^2\boldsymbol{K}(\boldsymbol{x}_0-\boldsymbol{y})\boldsymbol{\cdot}\boldsymbol{n} = \frac{3}{2\pi a^4}\boldsymbol{u}(\boldsymbol{y}). \tag{3.11}$$

Now we simply substitute our original definition of the velocity field $\boldsymbol{u}(\boldsymbol{y})$ (3.1) into the above to complete the first part of our derivation. These manipulations result in an expression relating the translational velocity of the particle to four quantities: (i) the translational velocity of a no-slip particle due to a force $\boldsymbol{F}$ (Stokes' law), (ii) a correction to Stokes' law due to the slip condition, (iii) the effect of the ambient field $\boldsymbol{u}^{\infty}(\boldsymbol{x})$ on a no-slip particle, and (iv) a correction to that effect arising from the slipping face, namely

$$\begin{aligned}\boldsymbol{U} = \frac{\boldsymbol{F}}{6\pi\eta a} - \frac{\lambda}{2\pi a^2}\int_{S_1} \boldsymbol{t}^{(\beta)}\boldsymbol{t}^{(\beta)}\boldsymbol{n} : \nabla_x \boldsymbol{u}'(\boldsymbol{x})\,\mathrm{d}S_x + \left(1 + \frac{a^2}{6}\nabla_x^2\right)\boldsymbol{u}^{\infty}(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0} \\ - \frac{\lambda}{2\pi a^2}\int_{S_1} \boldsymbol{t}^{(\beta)}\boldsymbol{t}^{(\beta)}\boldsymbol{n} : \nabla_x \boldsymbol{u}^{\infty}(\boldsymbol{x})\,\mathrm{d}S_x.\end{aligned} \tag{3.12}$$

Note that the repeated Greek index β means summation over the index values $(1, 2)$ only such that the integrals amount to weighted averages over both of the diagonal angular dyads, $\boldsymbol{t}^{(1)}\boldsymbol{t}^{(1)}$ and $\boldsymbol{t}^{(2)}\boldsymbol{t}^{(2)}$. It is clear that when either $\alpha = 0$ or $\lambda = 0$, the classic Faxén formula for the translation of a no-slip sphere is recovered.

At this point we have reached an impasse. To move forward, we need to refer back to the boundary conditions over the slipping face and recognize that gradients in

$\boldsymbol{u}'(\boldsymbol{x})$ associated with the double-layer contribution to the force density on the particle are in fact $O(\lambda/a)$. In order to integrate the contribution due to the double layer (the second term in (3.12)) and close the expression, we again consider the case where the slip length is small relative to the radius of the particle ($\lambda \ll a$) and discard all terms of $O(\lambda/a)^2$. To move ahead we need to compute the surface integrals over the ambient, $\boldsymbol{u}^\infty(\boldsymbol{x})$, and the disturbance fields, $\boldsymbol{u}'(\boldsymbol{x})$. The computation of these integrals is straightforward and is explained fully below.

To compute surface integrals of the disturbance field over the slippery face of the sphere, we refer back to the integral-stokeslet formulation for $\boldsymbol{u}'(\boldsymbol{x})$. We substitute equation (3.1) into the correction to Stokes' law (the second term in (3.12)) and discard the double-layer term $\boldsymbol{K}(\boldsymbol{r})$ as this represents an $O(\lambda/a)^2$ contribution to the result. We then reverse the order of integration, re-expressing the correction as

$$\int_{S_1} \boldsymbol{t}^{(\beta)}\boldsymbol{t}^{(\beta)}\boldsymbol{n} : \nabla_x \boldsymbol{u}'(\boldsymbol{x})\,\mathrm{d}S_x = \int_{S_1+S_2} \boldsymbol{A}(\boldsymbol{y})\cdot \boldsymbol{f}(\boldsymbol{y})\,\mathrm{d}S_y, \tag{3.13}$$

where

$$\boldsymbol{A}(\boldsymbol{y}) = \int_{S_1} \boldsymbol{t}^{(\beta)}\boldsymbol{t}^{(\beta)}\boldsymbol{n} : \nabla_x \boldsymbol{J}(\boldsymbol{x}-\boldsymbol{y})\,\mathrm{d}S_x. \tag{3.14}$$

In general, $\boldsymbol{A}(\boldsymbol{y})$ is difficult to compute since the normal and tangential vectors are radial functions centred on $\boldsymbol{x}_0$, while the stokeslet is centred on $\boldsymbol{y}$. To avoid such complications, we perform a multipole moment expansion of equation (3.13) about the centre of the particle, $\boldsymbol{x}_0$,

$$\begin{aligned}\int_{S_1+S_2} \boldsymbol{A}(\boldsymbol{y})\cdot \boldsymbol{f}(\boldsymbol{y})\,\mathrm{d}S_y &= \boldsymbol{A}(\boldsymbol{y})|_{\boldsymbol{x}_0}\cdot \boldsymbol{F} + \frac{1}{2}\boldsymbol{T}\cdot\nabla_y\times\boldsymbol{A}(\boldsymbol{y})\Big|_{\boldsymbol{x}_0} \\ &\quad + \frac{1}{2}\boldsymbol{S} : \left(\nabla_y\boldsymbol{A}(\boldsymbol{y}) + \left[\nabla_y\boldsymbol{A}(\boldsymbol{y})\right]^T\right)\Big|_{\boldsymbol{x}_0} + \cdots,\end{aligned} \tag{3.15}$$

where $\boldsymbol{F}$ is the total force, $\boldsymbol{T}$ is the torque and $\boldsymbol{S}$ is the stresslet on the particle. We define force moments these as follows:

$$\boldsymbol{F} = \int_{S_1+S_2} \boldsymbol{f}(\boldsymbol{y})\,\mathrm{d}S_y, \tag{3.16}$$

$$\boldsymbol{T} = \int_{S_1+S_2} (\boldsymbol{y}-\boldsymbol{x}_0)\times\boldsymbol{f}(\boldsymbol{y})\,\mathrm{d}S_y, \tag{3.17}$$

$$\boldsymbol{S} = \frac{1}{2}\int_{S_1+S_2} (\boldsymbol{y}-\boldsymbol{x}_0)\,\boldsymbol{f}(\boldsymbol{y}) + \boldsymbol{f}(\boldsymbol{y})\,(\boldsymbol{y}-\boldsymbol{x}_0)\;\mathrm{d}S_y. \tag{3.18}$$

Not only does this approach simplify matters, but it also provides physical insight into the effects of different force moments on the particle. For example, it is clear that the translational velocity of a slip–stick particle is coupled to all of the moments of the force density on its surface. This stands in contrast with the translation of a no-slip sphere which is coupled only to the total force (the same is true of a bubble). It is the symmetry-breaking aspect of slip–stick particles that leads to couples with higher-order force moments. With this expansion, the calculation of $\boldsymbol{A}(\boldsymbol{y})$ and its higher-order derivatives at the centre of the slip–stick particle is all that is needed to describe the translation in an otherwise quiescent fluid. The integrals involved in computing these terms are simply averages of the normal and tangential vectors over the slippery face of the particle weighted by derivatives of the stokeslet. For completeness, we illustrate the computation of $\boldsymbol{A}(\boldsymbol{x}_0)$ for an arbitrary division angle α:

$$\begin{aligned}\boldsymbol{A}(\boldsymbol{x}_0) &= -\int_0^{2\pi}\int_0^{\alpha} \boldsymbol{t}^{\beta}\boldsymbol{t}^{\beta}\sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\phi \\ &= \frac{1}{24\eta}\left[\left(\cos^3(\alpha)+3\cos(\alpha)-4\right)\delta_{ij}+3\cos(\alpha)\sin^2(\alpha)\,\delta_{i3}\delta_{j3}\right]\boldsymbol{v}^{(i)}\boldsymbol{v}^{(j)}. \end{aligned} \tag{3.19}$$

Similar calculations may be performed to compute the derivatives of $\boldsymbol{A}(\boldsymbol{y})$ as well.

In treating the correction to the response of a no-slip particle to the ambient field $\boldsymbol{u}^{\infty}(\boldsymbol{x})$, we perform a Taylor expansion about the centre of the particle such that

$$\int_{S_1}\boldsymbol{t}^{(\beta)}\boldsymbol{t}^{(\beta)}\boldsymbol{n}:\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x})\,\mathrm{d}S_x=\sum_{k=0}^{\infty}\frac{a^k}{k!}\int_{S_1}\boldsymbol{t}^{(\beta)}\boldsymbol{t}^{(\beta)}(\boldsymbol{n})^{k+1}\,\mathrm{d}S_x\odot^{k+1}(\nabla_x)^k\boldsymbol{u}^{\infty}(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0}, \tag{3.20}$$

where $\odot^k$ is the kth-order dot product and $(\nabla_x)^k$ is the kth-order gradient. This Taylor expansion does not truncate for an arbitrary $\boldsymbol{u}^{\infty}(\boldsymbol{x})$, but since any velocity field which satisfies the Stokes equations is biharmonic, physically practical disturbance fields like those generated by the presence of other particles must decay. This means that the higher-order derivatives of $\boldsymbol{u}^{\infty}(\boldsymbol{x})$ will be smaller than the lower-order ones. Therefore, one may reliably approximate this series simply by truncating it. Similar to the correction for the force moments, the integrals involved in this expansion amount to averages of the normal and tangential vectors over the slippery face of the particle. For example computing the $k=0$ term in (3.20) is done quite simply by considering the following integral where S_1 corresponds to division angle α:

$$\begin{aligned}\int_0^{2\pi}\int_0^{\alpha}\boldsymbol{t}^{\beta}\boldsymbol{t}^{\beta}\boldsymbol{n}a^2\sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\phi &= -\frac{\pi a^2}{8}\left[2\sin^4(\alpha)(\delta_{ij}\delta_{k3}+\delta_{ik}\delta_{j3})\right.\\ &\left.+(7+\cos(2\alpha))\sin^2(\alpha)\delta_{jk}\delta_{i3}+(3+5\cos(2\alpha))\sin^2(\alpha)\delta_{i3}\delta_{j3}\delta_{k3}\right]\boldsymbol{v}^{(k)}\boldsymbol{v}^{(j)}\boldsymbol{v}^{(i)}.\end{aligned} \tag{3.21}$$

We proceed by using these simplifications to assemble a Faxén formula describing the translation of a slip–stick sphere.

Consider a slip–stick particle with a specific division angle α. The integrals described above are readily computed for any given α bounded by zero and π. We select a few representative division angles and discuss the resulting expressions for the translational velocity. First, consider a symmetrically divided, or 'half and half,' slip–stick sphere ($\alpha=\pi/2$). The Faxén formula for such a particle is the expanded form of equation (3.12),

$$\begin{aligned}\boldsymbol{U} &= \frac{1}{6\pi\eta a}\left(1+\frac{\lambda}{2a}\right)\boldsymbol{F}+\frac{\lambda}{8\pi\eta a^3}\epsilon_{3ij}\boldsymbol{v}^{(i)}\boldsymbol{v}^{(j)}\boldsymbol{\cdot}\boldsymbol{T}+\cdots+\left(1+\frac{a^2}{6}\nabla_x^2\right)\boldsymbol{u}^{\infty}(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0}\\ &-\frac{\lambda}{8}\left(3\boldsymbol{I}\boldsymbol{v}^{(3)}-\boldsymbol{v}^{(i)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(i)}+\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}:\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x})\right|_{\boldsymbol{x}_0}-\frac{\lambda a}{15}\nabla_x^2\boldsymbol{u}^{\infty}(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0}+\cdots,\end{aligned} \tag{3.22}$$

where $\boldsymbol{F}$ and $\boldsymbol{T}$ are the force and torque on the particle, $\boldsymbol{I}$ is the identity tensor, ϵ_{3ij} is the permutation tensor and the repeated Roman indices (i, j) mean summation over the index values $(1, 2, 3)$. Note that the ellipses here represent the contributions from higher-order force moments and higher-order derivatives of the ambient field respectively. From this expression it is clear that the translation of an evenly divided slip–stick sphere is coupled to a torque in any direction other than $\boldsymbol{v}^{(3)}$, the axis about which the particle is symmetric. It turns out that an evenly divided slip–stick sphere is a special case and has particularly 'nice' symmetry; i.e. the translational velocity due to either a force $\boldsymbol{F}$ or an applied mean pressure gradient,

$$\nabla_x^2 \boldsymbol{u}^\infty(\boldsymbol{x})\big|_{\boldsymbol{x}_0} = \frac{1}{\eta}\,\nabla_x p^\infty(\boldsymbol{x})\big|_{\boldsymbol{x}_0} \tag{3.23}$$

points soley in the direction of these forcings. Note that although the Navier slip condition involves only tangential strain of the fluid at the surface of the particle, it results in a correction to the effects of the mean pressure gradient (the last term on the right-hand side of (3.22)), which is an applied normal stress. Now consider the Faxén formula for a slip–stick particle with an asymmetric division ($\alpha = \pi/3$):

$$\begin{aligned}\boldsymbol{U} = {} & \frac{1}{6\pi\eta a}\left[\boldsymbol{I} + \frac{\lambda}{64a}\left(19\boldsymbol{I} - 9\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\right)\right]\cdot\boldsymbol{F} + \frac{3\lambda}{64\pi\eta a^3}\epsilon_{3ij}\boldsymbol{v}^{(i)}\boldsymbol{v}^{(j)}\cdot\boldsymbol{T} + \cdots \\ & + \left(1 + \frac{a^2}{6}\nabla_x^2\right)\boldsymbol{u}^\infty(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0} - \frac{3\lambda}{128}\left(13\boldsymbol{I}\boldsymbol{v}^{(3)} - 3\boldsymbol{v}^{(i)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(i)} - \boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\right) : \nabla_x\boldsymbol{u}^\infty(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0} \\ & - \frac{\lambda a}{3840}\left(347\boldsymbol{I} - 135\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\right)\cdot\nabla_x^2\boldsymbol{u}^\infty(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0} - \frac{3\lambda a}{256}\left[13\boldsymbol{I}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)} + 5\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\right. \\ & \left. -3\left(\boldsymbol{v}^{(i)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(i)}\boldsymbol{v}^{(3)} + \boldsymbol{v}^{(i)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(i)}\right)\right] \vdots \nabla_x\nabla_x\boldsymbol{u}^\infty(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0} + \cdots \end{aligned} \tag{3.24}$$

The responses to the total force and the mean pressure gradient no longer point in the same direction as the applied forcings. Instead, there is a correction due to the asymmetry of the division such that a force/mean pressure gradient along $\boldsymbol{v}^{(3)}$ propels the particle more slowly/quickly than if the same strength force/mean pressure gradient were pointing in any other direction. The increase or decrease of the applied force and mean pressure gradient along the $\boldsymbol{v}^{(3)}$ axis is characteristic of all asymmetrically divided slip–stick particles and more generally of axisymmetric particles (such as ellipsoids) in Stokes flow.

In Appendix A, we take a slightly less robust approach to this problem by considering the uniform streaming flow past a slip–stick sphere (equivalent to translation strictly along the axis $\boldsymbol{v}_3$) via the stream function ψ and direct solution of the Stokes equations to $O(\lambda/a)$. We explicitly compute the first term in (3.22) using this approach and find that it is the same. From this approach, we can produce a plot of the streamlines surrounding a slip–stick sphere with any division angle, α. In figure 2, we illustrate two such plots of the streamlines around slip–stick spheres with division angles of $\pi/2$ and $2\pi/3$. In this figure, the break in fore–aft symmetry is clear for each division angle and is made obvious by noting where streamlines enter and exit the dotted half-circle surrounding the particle. Even though the scale of the perturbation to the no-slip boundary condition is set by λ/a, which is reasonably small in this figure, it is plain to see that the perturbation to the flow field is significant over a least two particle radii. This suggests that even a small asymmetry in the slip length should have measurable consequences for the hydrodynamic interactions between a slip–stick particle and another body. However, this problem is considerably more difficult to study analytically, and we shall not pursue it here.

4. Faxén's second law for slip–stick spheres

Now consider a slip–stick sphere in a linear flow denoted $\boldsymbol{\Gamma}\cdot(\boldsymbol{x} - \boldsymbol{x}_0)$, where $\boldsymbol{\Gamma}$ is a constant velocity gradient satisfying continuity (i.e. the tensor is traceless). This linear field consists of two contrasting parts: the symmetric part of $\boldsymbol{\Gamma}$ represents the

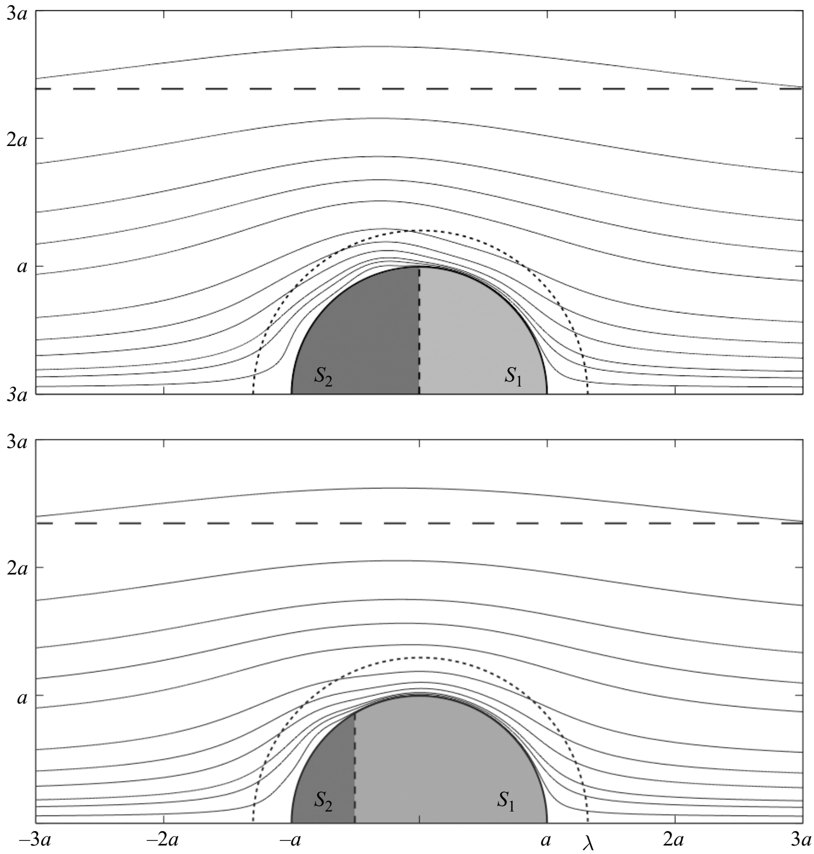

FIGURE 2. The streamlines surrounding two slip–stick spheres with slip length $\lambda = a/3$ indicated by the dotted half-circle and division angles $\pi/2$ and $2\pi/3$. The particles are translating along the axis $\boldsymbol{v}_3$ in a co-moving frame so that the flow field is strictly axisymmetric. The regions S_1 and S_2 correspond to the slippery and sticky faces of the particle respectively. The far-field asymmetry in the streamlines is indicated explicitly by the horizontal dashed lines which intersect streamlines on the right but fail to intersect them again on the left.

rate of strain and relates to fluid deformation, while the antisymmetric part of $\boldsymbol{\Gamma}$ represents the vorticity and reflects rigid rotation of the fluid relative to the particle. In what follows, the rate of strain and vorticity will be decoupled in order to study their effects on a slip–stick particle independently. We can use the same boundary conditions on the surface of the particle as in the previous section, but this case is defined by a slightly different velocity field (see (2.5) and (2.6)),

$$\begin{aligned} \boldsymbol{u}(\boldsymbol{x}) &= \boldsymbol{\Gamma}\cdot(\boldsymbol{x}-\boldsymbol{x}_0) + \boldsymbol{u}^{\infty}(\boldsymbol{x}) + \boldsymbol{u}'(\boldsymbol{x}), \\ \boldsymbol{u}'(\boldsymbol{x}) &= \int_{S_1+S_2} [\boldsymbol{J}(\boldsymbol{x}-\boldsymbol{y})\cdot(\boldsymbol{f}(\boldsymbol{y}) + P\boldsymbol{n}) + \boldsymbol{u}(\boldsymbol{y})\cdot\boldsymbol{K}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{n}]\,\mathrm{d}S_y. \end{aligned} \tag{4.1}$$

Here, there is no need for a comoving frame since there is no uniform translation of the far field. We proceed as in the derivation of the first Faxén law but take the tensorial product between $\boldsymbol{x}$ and $\boldsymbol{u}(\boldsymbol{x})$ before integrating, namely

$$\int_{S_1+S_2} \boldsymbol{x}\boldsymbol{u}(\boldsymbol{x})\,\mathrm{d}S_x, \tag{4.2}$$

in order to study the first moment of the velocity field. We use a Taylor expansion of $\boldsymbol{x}\boldsymbol{K}(\boldsymbol{x}-\boldsymbol{y})$ similar to the one in the equation (3.9) resulting in

$$\int_{S_1+S_2}\left(1+\frac{a^2}{10}\nabla_x^2\right)\nabla_x \boldsymbol{u}(\boldsymbol{y})\cdot\boldsymbol{K}(\boldsymbol{x}-\boldsymbol{y})\Big|_{\boldsymbol{x}=\boldsymbol{x}_0}\cdot\boldsymbol{n}\,\mathrm{d}S_y, \tag{4.3}$$

to simplify the double layer while also disregarding the static pressure, P, as it makes no contribution to the problem after this integration. After substituting the boundary conditions for the velocity field into the integrals as in the previous section, we recover an expression for the shear rate, $\boldsymbol{\Gamma}$:

$$\begin{aligned}-\left(\hat{\boldsymbol{I}}-\frac{\lambda}{a}\boldsymbol{B}\right):\boldsymbol{\Gamma}^T &= \frac{\boldsymbol{\epsilon}\cdot\boldsymbol{T}}{8\pi\eta a^3}+\frac{\boldsymbol{S}}{\frac{20}{3}\pi\eta a^3}\\ &\quad-\frac{3\lambda}{4\pi a^3}\int_{S_1}\boldsymbol{A}\boldsymbol{t}^{(\beta)}\boldsymbol{n}:\nabla_x\boldsymbol{u}'(\boldsymbol{x})\,\mathrm{d}S_x+\left(1+\frac{a^2}{10}\nabla_x^2\right)\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x})\Big|_{\boldsymbol{x}=\boldsymbol{x}_0}\\ &\quad-\frac{3\lambda}{4\pi a^3}\int_{S_1}\boldsymbol{A}\boldsymbol{t}^{(\beta)}\boldsymbol{n}:\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x})\,\mathrm{d}S_x,\end{aligned} \tag{4.4}$$

where the superscript ^{T} indicates transposition,

$$\boldsymbol{A}=\frac{4}{5}\left(\boldsymbol{n}\boldsymbol{t}^{(\beta)}+\boldsymbol{t}^{(\beta)}\boldsymbol{n}\right)+\left(\boldsymbol{n}\boldsymbol{t}^{(\beta)}-\boldsymbol{t}^{(\beta)}\boldsymbol{n}\right), \tag{4.5}$$

$$\boldsymbol{B}=\frac{3}{4\pi a^2}\int_{S_1}\boldsymbol{A}\boldsymbol{t}^{(\beta)}\boldsymbol{n}\,\mathrm{d}S_x, \tag{4.6}$$

the stresslet ($\boldsymbol{S}$) is the symmetric part of the first moment of the force density on the particle's surface, the fourth-order tensor $\hat{\boldsymbol{I}}$ is defined such that $\hat{\boldsymbol{I}}_{ijkl}=\delta_{ik}\delta_{jl}$ and $\boldsymbol{\epsilon}$ is the permutation symbol. Note that we have already separated $\boldsymbol{A}$ into symmetric and antisymmetric parts in anticipation of needing these parts individually. While this expression is complicated, it is analogous to equation (3.12) and contains four parts which relate to Faxén's second law for rigid spheres and corrections to that expression due to the slipping surface just as in equation (3.12). One additional complication in deriving this Faxén formula is the inversion of a fourth-order tensor to isolate the shear rate. To simplify things, we again consider the limit that the slip length is small compared to the particle radius ($\lambda \ll a$) and note that taking an expansion and truncation yields

$$\left(\hat{\boldsymbol{I}}-\frac{\lambda}{a}\boldsymbol{B}\right)^{-1}\approx\hat{\boldsymbol{I}}+\frac{\lambda}{a}\boldsymbol{B}+O\left(\frac{\lambda}{a}\right)^2. \tag{4.7}$$

Making this simplification and discarding terms of order $(\lambda/a)^2$ and higher results in:

$$\begin{aligned}-\boldsymbol{\Gamma}^T &= \left(\hat{\boldsymbol{I}}+\frac{\lambda}{a}\boldsymbol{B}\right):\left(\frac{\epsilon\cdot\boldsymbol{T}}{8\pi\,\eta a^3}+\frac{\boldsymbol{S}}{\frac{20}{3}\pi\eta a^3}\right)-\frac{3\lambda}{4\pi a^3}\int_{S_1}\boldsymbol{A}\boldsymbol{t}^{(\beta)}\boldsymbol{n}:\nabla_x\boldsymbol{u}'(\boldsymbol{x})\,\mathrm{d}S_x\\ &\quad+\left(\hat{\boldsymbol{I}}+\frac{\lambda}{a}\boldsymbol{B}\right):\left(1+\frac{a^2}{10}\nabla_x^2\right)\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x})\Big|_{\boldsymbol{x}=\boldsymbol{x}_0}-\frac{3\lambda}{4\pi a^3}\int_{S_1}\boldsymbol{A}\boldsymbol{t}^{(\beta)}\boldsymbol{n}:\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x})\,\mathrm{d}S_x.\end{aligned} \tag{4.8}$$

We can separate the symmetric and antisymmetric parts of $\boldsymbol{\Gamma}$ directly now. We substitute equation (4.1) for $\boldsymbol{u}'(\boldsymbol{x})$ and truncate the expression at the $O(\lambda/a)^2$ level by discarding any additional double-layer contributions as in equation (3.13). The symmetric part of $\boldsymbol{\Gamma}$ is the ambient rate of strain of the fluid, $\boldsymbol{E}^{\infty}$, namely

$$
\begin{aligned}
-\boldsymbol{E}^{\infty} = & \frac{\boldsymbol{S}}{\frac{20}{3}\eta\pi a^3} + \frac{\lambda}{2a}\left(\boldsymbol{B}+\boldsymbol{B}^T\right):\left(\frac{\boldsymbol{\epsilon}\cdot\boldsymbol{T}}{8\pi\eta a^3} + \frac{\boldsymbol{S}}{\frac{20}{3}\pi\eta a^3}\right) \\
& -\frac{3\lambda}{5\pi a^3}\int_{S_1}\left(\boldsymbol{n}\boldsymbol{t}^{(\beta)}+\boldsymbol{t}^{(\beta)}\boldsymbol{n}\right)\boldsymbol{t}^{(\beta)}\boldsymbol{n}:\nabla_x\int_{S_1+S_2}\boldsymbol{J}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{f}(\boldsymbol{y})\,\mathrm{d}S_y\,\mathrm{d}S_x \\
& +\frac{1}{2}\left(1+\frac{a^2}{10}\nabla_x^2\right)\left[\left(\nabla_x+\nabla_x^T\right)\boldsymbol{u}^{\infty}(\boldsymbol{x})+\frac{\lambda}{a}\left(\boldsymbol{B}+\boldsymbol{B}^T\right):\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x})\right]\Bigg|_{\boldsymbol{x}=\boldsymbol{x}_0} \\
& -\frac{3\lambda}{5\pi a^3}\int_{S_1}\left(\boldsymbol{n}\boldsymbol{t}^{(\beta)}+\boldsymbol{t}^{(\beta)}\boldsymbol{n}\right)\boldsymbol{t}^{(\beta)}\boldsymbol{n}:\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x})\,\mathrm{d}S_x, \qquad (4.9)
\end{aligned}
$$

where the operator ∇_x^T is defined by $\nabla_x^T\boldsymbol{u}^{\infty}(\boldsymbol{x}) = (\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x}))^T$ and $B_{ijkl}^T = B_{jikl}$. If we change our frame of reference by using the Galilean invariance of Stokes flow, then we can write the antisymmetric part of $-\boldsymbol{\Gamma}$ (in vector form) as the rate of rotation of the particle, $\boldsymbol{\Omega}$, relative to half the ambient vorticity of the fluid, $\boldsymbol{\Omega}^{\infty}$:

$$
\begin{aligned}
\boldsymbol{\Omega}-\boldsymbol{\Omega}^{\infty} = & \frac{\boldsymbol{T}}{8\pi\eta a^3} - \frac{\lambda}{2a}\boldsymbol{\epsilon}:\boldsymbol{B}:\left(\frac{\boldsymbol{\epsilon}\cdot\boldsymbol{T}}{8\pi\eta a^3}+\frac{\boldsymbol{S}}{\frac{20}{3}\pi\eta a^3}\right) \\
& -\frac{3\lambda}{4\pi a^3}\int_{S_1}\left(\boldsymbol{n}\times\boldsymbol{t}^{(\beta)}\right)\boldsymbol{t}^{(\beta)}\boldsymbol{n}:\nabla_x\int_{S_1+S_2}\boldsymbol{J}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{f}(\boldsymbol{y})\,\mathrm{d}S_y\,\mathrm{d}S_x+\frac{1}{2}\nabla_x\times\boldsymbol{u}^{\infty}(\boldsymbol{x})\Bigg|_{\boldsymbol{x}=\boldsymbol{x}_0} \\
& -\frac{\lambda}{2a}\left(1+\frac{a^2}{10}\nabla_x^2\right)\boldsymbol{\epsilon}:\boldsymbol{B}:\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x})\Bigg|_{\boldsymbol{x}=\boldsymbol{x}_0} - \frac{3\lambda}{4\pi a^3}\int_{S_1}\left(\boldsymbol{n}\times\boldsymbol{t}^{(\beta)}\right)\boldsymbol{t}^{(\beta)}\boldsymbol{n}:\nabla_x\boldsymbol{u}^{\infty}(\boldsymbol{x})\,\mathrm{d}S_x. \qquad (4.10)
\end{aligned}
$$

The integrals over the slippery face of the particle are computed in exactly the same way as described in § 3. The explicit computation of these integrals is explained fully in Appendix B.

To illustrate the utility of the preceding analysis, consider a torque-free slip–stick sphere subject to an imposed external force. If the ambient fluid is quiescent, then from equation (4.10) upon expanding the integral over the gradient of the stokeslet about $\boldsymbol{x}_0$ and integrating the tensor denoted $\boldsymbol{n}\times\boldsymbol{t}^{(\beta)}\boldsymbol{t}^{(\beta)}$ over the slippery surface of the sphere, the rotational velocity of the particle is

$$\boldsymbol{\Omega} = -\frac{9}{16}\frac{\lambda}{a}(1-\cos^2\alpha)\epsilon_{3ij}\boldsymbol{v}^{(i)}\boldsymbol{v}^{(j)}\cdot\frac{\boldsymbol{F}}{6\pi\eta a^2}. \qquad (4.11)$$

From this expression, we can draw some conclusions. First, if the force acts along the axis about which the particle is symmetric ($\boldsymbol{F}\times\boldsymbol{v}^{(3)} = 0$), the particle will not rotate. Physically, the fluid exerts a symmetric drag about $\boldsymbol{v}^{(3)}$ on the surface of the particle; thus there is no tendency for the particle to rotate. This situation is akin to a see-saw with equal weights applied at its ends. Note that this is the 'reciprocal' relationship to equations (3.22) and (3.24), which state that a torque applied along $\boldsymbol{v}^{(3)}$ causes no translational motion. Similarly and second, if the particle is entirely slip ($\alpha = \pi$) or stick ($\alpha = 0$), the particle will not rotate as there is no longer a coupling between force and rotation (or torque and translation). Finally, we can see that if the force acts along $\boldsymbol{v}^{(1)}$ or $\boldsymbol{v}^{(2)}$, the particle will rotate about the $\boldsymbol{v}^{(2)}$ or $-\boldsymbol{v}^{(1)}$ axes respectively, regardless of the ratio of slip to no-slip areas (i.e. the value of α). From this we conclude that the particle will continue to rotate until it assumes a terminal orientation with its slip surface pointing in the same direction as the force. That is, orientations of slip–stick particles where $\boldsymbol{F}/|\boldsymbol{F}| = -\boldsymbol{v}^{(3)}$ (stick-side translation) and $\boldsymbol{F}/|\boldsymbol{F}| = \boldsymbol{v}^{(3)}$ (slip-side translation) are respectively the unstable and stable equilibrium modes of translation for a sedimenting slip–stick sphere. Interestingly, the analogy with axisymmetric particles breaks down here in the light of this preferred orientation, since all sedimenting slip–stick spheres have an experimentally distinguishable fore and aft.

One important question concerning a symmetry-breaking particle is whether or not it will migrate along the gradient of a simple shear flow. Such a feature might be particularly useful in microfluidic experiments or as a means of quantifying the degree of slip that a particle possesses. There are two contributions from the linear field. The first effect is direct and arises from the $\boldsymbol{u}^{\infty}(\boldsymbol{x})$ terms in equation (3.12). In the context of that first Faxén formula, a linear field is like another ambient field. Therefore, in addition to the $\boldsymbol{\Gamma}\cdot\boldsymbol{x}$ contribution typical of solid spheres, there is an $O(\lambda/a)$ part that causes the particle to translate along the gradient in the shear field. The exact direction of the translation is a function of the orientation of the particle, but may be calculated directly once the division angle α is specified. The fourth terms in equations (3.22) and (3.24), which are proportional to $\nabla\boldsymbol{u}^{\infty}(\boldsymbol{x}_0)$, explicitly detail this effect for particles with $\alpha = \pi/2$ and $\alpha = \pi/3$ respectively. The second effect of the shear field comes from the change in the force density on the particle surface due to the gradient in the field. To explore this, we need to determine whether the ambient straining field $\boldsymbol{E}^{\infty}$ couples to a force on the particle by performing a multipole expansion of the third term in equation (4.9). From the expansion in Appendix B, we know that

$$\frac{3\lambda}{5\pi a^3}\int_{S_1}\left(\boldsymbol{n}\boldsymbol{t}^{(\beta)}+\boldsymbol{t}^{(\beta)}\boldsymbol{n}\right)\boldsymbol{t}^{(\beta)}\boldsymbol{n}:\nabla_x\boldsymbol{u}'(\boldsymbol{x})\,\mathrm{d}S_x = \frac{3\lambda}{5\pi a^3}\left[\int_{S_1}\left(\boldsymbol{n}\boldsymbol{t}^{(\beta)}+\boldsymbol{t}^{(\beta)}\boldsymbol{n}\right)\boldsymbol{t}^{(\beta)}\,\mathrm{d}S_x\right]\cdot\frac{\boldsymbol{F}}{8\pi\eta a^2}+\dots. \tag{4.12}$$

When the slippery face S_1 is defined by $\alpha \in (0, \pi)$, the coupling to the force is strictly non-zero, and the straining field causes the particle to migrate. The direction of migration depends on the orientation of the particle relative to the straining field. In simple shear flow, the field also causes the particle to rotate through a pair of similar hydrodynamic couplings in equation (4.10) which reorient the slippery face and change the direction of migration due to the flow. The reorientation and migration causes a slip–stick particle to sweep out effective Jeffrey orbits in orientation space like other axisymmetric bodies in shear flows. This is novel behaviour for a body that, geometrically speaking, is radially symmetric.

In addition to the couple between force and rate of strain which drives the body through the fluid, there is also the couple between force and vorticity which we detailed in equation (4.11). This comes from the third term in equation (4.10) which is analogous to the term reflecting the force rate of strain coupling in equation (4.9). While complicated, it is clear that both ambient rate of strain and vorticity can direct the motion of a slip–stick particle by generating a net force on the particle's surface. This does not happen for solid spheres or rigid bubbles in Stokes flow.

We omit the computation of these Faxén formulae for any specific slip–stick sphere since its difficult to extract useful physical information from those expressions alone. Instead we note that the computation of these terms is done in the same way as in § 3. However, it is worth noting that one particular slip–stick particle, the symmetrically divided one ($\alpha = \pi/2$), has especially interesting properties. From the first Faxén formula (3.22), we know that unlike other axisymmetric bodies, the resistance to motion of an evenly divided slip–stick sphere is isotropic (i.e. the force and velocity on the body point in the same direction) just like a sphere and a few other special shapes in Stokes flow such as a cube. In spite of this, an evenly divided slip–stick sphere can still migrate parallel to the velocity gradient in a shear flow since the force on the particle still couples to the straining field, i.e. equation (4.12) for $\alpha = \pi/2$ gives

$$\frac{3\lambda}{5\pi a^3}\left[\int_{S_1,\alpha=\pi/2}\left(\boldsymbol{n}\boldsymbol{t}^{(\beta)}+\boldsymbol{t}^{(\beta)}\boldsymbol{n}\right)\boldsymbol{t}^{(\beta)}\,\mathrm{d}S_x\right]\cdot\frac{\boldsymbol{F}}{8\pi\eta a^2}$$
$$=\frac{3\lambda}{80\pi\eta a^3}\left(\boldsymbol{v}^{(i)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(i)}+\boldsymbol{v}^{(3)}\boldsymbol{v}^{(i)}\boldsymbol{v}^{(i)}-\boldsymbol{v}^{(i)}\boldsymbol{v}^{(i)}\boldsymbol{v}^{(3)}+\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\boldsymbol{v}^{(3)}\right)\cdot\boldsymbol{F}. \quad (4.13)$$

This makes the symmetrically divided slip–stick particle an idiosyncratic sort of axisymmetric particle. When translating alone in an unbounded fluid, it appears to be a sphere. However it traces out orbits as it translates in a shear flow.

5. Conclusions

We want to offer some quantitative incentive for the experimental study of slip–stick particles. Consider the behaviour of a slip–stick sphere with $\alpha = \pi/2$, $a = 1$ μm and $\lambda = 10$ nm in water at room temperature (these are typical quantities for colloidal particles and slip lengths). Equation (3.22) predicts that a torque of magnitude 10^{-3} pN μm about the axes $\boldsymbol{v}^{(1)}$ or $\boldsymbol{v}^{(2)}$ would propel the particle with a translational velocity of approximately 200 μm s^{-1}. The magnitude of this torque is on the order of thermal stresses on the particle (kT). Similarly, equation (4.12) predicts that the same particle in a shear flow, with shear rate 0.1 s^{-1}, may be held fixed by a force of approximately 1 pN (a typical magnitude in the colloidal regime). This same force would propel our slip–stick particle at a little more than 50 μm s^{-1} through the fluid. These are all reasonable values and suggest that our predictions could be observed experimentally via particle tracking and laser tweezer microscopy.

We have computed three Faxén formulae coupling an ambient velocity field and the force density on a slip–stick particle to its translational and rotational velocity and the effects of a straining field. These expressions suggest that breaking the radial symmetry of a spherical particle by altering the slip length along its surface produces an object with interesting hydrodynamic properties. The hydrodynamic behaviour of a slip–stick particle shares features with other axisymmetric bodies. Namely, the resistance to motion of slip–stick particles along and perpendicular to the axis of symmetry is different. While slip–stick particles are axisymmetric, they are not fore–aft symmetric and therefore can migrate parallel to the velocity gradient in shear flows. Perhaps these results can be used to determine if some other Janus, or two-faced, particles slip asymmetrically. If one of these particles migrates in a shear flow, can we infer something about its surface features?

Our slip–stick particle represents a minimal model for a host of particles with asymmetric slip lengths. Only a few modifications to the procedure described in this paper are necessary to study whole classes of slip–stick particles patterned into a multitude of regions or patterned with a continuously varying slip length. However, we believe our model in many ways still captures the essential hydrodynamic features needed to study the more interesting collective behaviour of particles with asymmetrical slip lengths. Might we expect that the many-body hydrodynamic interactions among slip–stick spheres could lead to large-scale ordering or even phase separation? Furthermore, what are the rheological properties of a suspension of these particles? These are problems whose study and eventual solution should be aided by this analysis.

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Appendix A. Uniform streaming flow past a stick–slip sphere

Consider a uniform streaming flow $\boldsymbol{U} = -U\boldsymbol{v}^{(3)}$ past a stick–slip sphere of radius a and polar division angle α (figure 1). The flow is axisymmetric about $\boldsymbol{v}^{(3)}$; hence, we express the velocity field, $\boldsymbol{u}(\boldsymbol{x})$, in terms of the Stokes stream function $\psi(r, \theta)$:

$$u_r = -\frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta}, \quad u_\theta = \frac{1}{r \sin\theta}\frac{\partial \psi}{\partial r}. \tag{A 1}$$

The stream function solves $\mathrm{E}^4\psi = 0$, where

$$\mathrm{E}^2 = \frac{\partial^2}{\partial r^2} + \frac{1-\mu^2}{r^2}\frac{\partial^2}{\partial \mu^2}, \tag{A 2}$$

and $\mu = \cos\theta$. Additionally, ψ must satisfy the condition of uniform flow far from the sphere:

$$\psi \to \tfrac{1}{2}Ur^2(1-\mu^2) \quad \text{as} \quad r \to \infty, \tag{A 3}$$

and the no-flux and stick–slip conditions (see equations (2.1) and (2.2)) on the particle surface ($r = a$):

$$\psi = 0, \tag{A 4}$$

$$\frac{\partial \psi}{\partial r} = \begin{cases} 0 & \text{if } -1 \leqslant \mu \leqslant \cos\alpha \\ \lambda r^2 \dfrac{\partial}{\partial r}\left(\dfrac{1}{r^2}\dfrac{\partial \psi}{\partial r}\right) & \text{if } \cos\alpha < \mu \leqslant 1. \end{cases} \tag{A 5}$$

We non-dimensionalize by scaling lengths with the particle radius a and the stream function by Ua^2 (henceforth all quantities are dimensionless). For small slip length to particle radius ratio, $\lambda/a \ll 1$, the stream function is expanded as $\psi = \psi_0 + (\lambda/a)\psi_1 + O[(\lambda/a)^2]$. The leading–order solution ψ_0 is the well-known result for uniform flow past a no-slip sphere (see e.g. Happel & Brenner 1986)

$$\psi_0 = \tfrac{1}{2}r^2(1-\mu^2)\left(1 - \frac{3}{2r} + \frac{1}{2r^3}\right). \tag{A 6}$$

The $O(\lambda/a)$ stream function, ψ_1, satisfies $\mathrm{E}^4\psi_1 = 0$ subject to the boundary conditions

$$\psi_1/r^2 \to 0 \quad \text{as} \quad r \to \infty, \tag{A 7}$$

$$\psi_1 = 0 \quad \text{at} \quad r = 1 \tag{A 8}$$

$$\frac{\partial \psi_1}{\partial r} = \begin{cases} 0 & \text{on } r = 1 \text{ and } -1 \leqslant \mu \leqslant \cos\alpha \\ r^2 \dfrac{\partial}{\partial r}\left(\dfrac{1}{r^2}\dfrac{\partial \psi_0}{\partial r}\right) = \tfrac{3}{2}(1-\mu^2) & \text{on } r = 1 \text{ and } \cos\alpha < \mu \leqslant 1. \end{cases} \tag{A 9}$$

The general solution for $E^4\psi_1 = 0$ in spherical polar coordinates is (see p. 135 of Happel & Brenner 1986)

$$\psi_1 = \sum_{n=2}^{\infty} \left(A_n r^n + B_n r^{1-n} + C_n r^{2+n} + D_n r^{3-n}\right) G_n(\mu), \tag{A 10}$$

where $G_n(\mu)$ is the Gegenbauer function of the first kind of order n. To satisfy the far-field condition we require $A_n = C_n = 0$ for all n, and the no-flux condition on $r = 1$ gives $B_n = -D_n$. From the stick–slip condition and the orthogonality of the Gegenbauer functions the D_n coefficients are given by

$$D_n = \frac{3}{4} n(n-1)(2n-1) \int_{\cos\alpha}^{1} \frac{G_2(\mu) G_n(\mu)}{1-\mu^2}\, \mathrm{d}\mu, \tag{A 11}$$

where we have used $G_2 = (1-\mu^2)/2$. For $n = 2$ a straightforward integration gives

$$D_2 = \tfrac{9}{8}\left(\tfrac{2}{3} - \cos\alpha + \tfrac{1}{3}\cos^3\alpha\right), \tag{A 12}$$

and for $n > 2$ some lengthy algebra yields

$$D_n = \frac{3}{8} n(n-1)(2n-1) \left[\frac{2[1 - P_{n-3}(\cos\alpha)]}{(2n+1)(2n-3)} - \frac{1 - P_{n-1}(\cos\alpha)}{(2n-1)(2n-3)} - \frac{1 - P_{n+1}(\cos\alpha)}{(2n-1)(2n+1)}\right], \tag{A 13}$$

where $P_n(\mu)$ is the Legendre polynomial of the first kind of order n. Thus, to first order in λ/a the stream function is

$$\begin{aligned}\psi &= \left(r^2 - \frac{3r}{2} + \frac{1}{2r}\right) G_2(\mu) + \frac{\lambda}{a}\frac{9}{8}\left(\frac{2}{3} - \cos\alpha + \frac{1}{3}\cos^3\alpha\right)\left(r - \frac{1}{r}\right) G_2(\mu) \\ &\quad + \frac{\lambda}{a}\sum_{n=3}^{\infty} D_n (r^{3-n} - r^{1-n}) G_n(\mu).\end{aligned} \tag{A 14}$$

Moreover, note that at large distances from the particle the streamfunction asymptotes to the uniform flow as

$$\psi - G_2(\mu) r^2 = \frac{3}{2}\left[-1 + \frac{\lambda}{a}\frac{3}{4}\left(\frac{2}{3} - \cos\alpha + \frac{1}{3}\cos^3\alpha\right)\right] G_2(\mu) r + O\left(\frac{\lambda}{a} r^0\right); \tag{A 15}$$

therefore, although the slip length is small relative to the particle size, the perturbation to the flow field caused by the stick–slip asymmetry persists far from the particle.

In figure 2 we plot streamlines around the sphere as a function of α for $\lambda = a/3$. It is clearly seen that the heterogeneous stick–slip nature of the particle surface leads to a breakdown in the fore–aft symmetry usually associated with particle motion at low Reynolds number.

Finally, the force, F, exerted by the sphere in the $\boldsymbol{v}_3$ direction is given by (see p. 115 of Happel & Brenner (1986)

$$\begin{aligned}\frac{F}{6\pi\eta a U} &= \frac{4}{3}\lim_{r\to\infty} \frac{\psi - \frac{1}{2}(1-\mu^2) r^2}{1-\mu^2}, \\ &= 1 - \frac{2}{3} D_2 \frac{\lambda}{a}.\end{aligned} \tag{A 16}$$

For $\alpha = 0$, where the fluid sticks over the entire sphere, we recover Stokes' drag law: $F = 6\pi\eta a U$. When $\alpha = \pi/2$ (a 'half-and-half' stick–slip particle) we find $F = 6\pi\eta a U(1 - \lambda/2a)$ as predicted by the first Faxén formula (equation (2.7)). Lastly, for $\alpha = \pi$ we have $F = 6\pi\eta a U(1 - \lambda/a)$ in agreement with equation (4-20.10) of Happel & Brenner (1986) for a fluid that slips over the entire particle surface. Note that D_2 is a monotonically increasing function of α, which simply means that the more surface area available for the fluid to slip over, the less force that must be exerted by the particle for it move with velocity U.

Appendix B. Expansion of spherical integrals for Faxén's second law

As in § 3 we need to compute several integrals over the surface of the particle via multipole expansions. We use the same formulation as in equation (3.13) to compute the integral

$$\int_{S_1+S_2} \boldsymbol{A}(\boldsymbol{y}) \cdot \boldsymbol{f}(\boldsymbol{y})\,\mathrm{d}S_y, \tag{B 1}$$

where for the straining field (4.9):

$$\boldsymbol{A}(\boldsymbol{y}) = \int_{S_1} \left(\boldsymbol{n}\boldsymbol{t}^{(\beta)} + \boldsymbol{t}^{(\beta)}\boldsymbol{n}\right) \boldsymbol{t}^{(\beta)}\boldsymbol{n} : \nabla_x \boldsymbol{J}(\boldsymbol{x} - \boldsymbol{y})\,\mathrm{d}S_x, \tag{B 2}$$

and for the rotational field (4.10):

$$\boldsymbol{A}(\boldsymbol{y}) = \int_{S_1} \left(\boldsymbol{n} \times \boldsymbol{t}^{(\beta)}\right) \boldsymbol{t}^{(\beta)}\boldsymbol{n} : \nabla_x \boldsymbol{J}(\boldsymbol{x} - \boldsymbol{y})\,\mathrm{d}S_x. \tag{B 3}$$

The same multipole expansion of $\boldsymbol{A}(\boldsymbol{y})$ applies to these as to the $\boldsymbol{A}(\boldsymbol{y})$ in equation 3.15.

There are two integrals to compute the straining and rotational response of a slip–stick particle to an ambient field. These are computed in exactly the same way as in § 3 using a Taylor expansion of the ambient field about the centre of the particle. We give the expansions of these integrals explicitly here using the same notation:

$$\int_{S_1} \left(\boldsymbol{n}\boldsymbol{t}^{(\beta)} + \boldsymbol{t}^{(\beta)}\boldsymbol{n}\right) \boldsymbol{t}^{(\beta)}\boldsymbol{n} : \nabla_x \boldsymbol{u}^\infty(\boldsymbol{x})\,\mathrm{d}S_x$$
$$= \sum_{k=0}^{\infty} \frac{a^k}{k!} \int_{S_1} \left(\boldsymbol{n}\boldsymbol{t}^{(\beta)} + \boldsymbol{t}^{(\beta)}\boldsymbol{n}\right) \boldsymbol{t}^{(\beta)}(\boldsymbol{n})^{k+1}\,\mathrm{d}S_x \odot^{k+1} (\nabla_x)^k \boldsymbol{u}^\infty(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0}, \tag{B 4}$$

$$\int_{S_1} \boldsymbol{n} \times \boldsymbol{t}^{(\beta)}\boldsymbol{t}^{(\beta)}\boldsymbol{n} : \nabla_x \boldsymbol{u}^\infty(\boldsymbol{x})\,\mathrm{d}S_x$$
$$= \sum_{k=0}^{\infty} \frac{a^k}{k!} \int_{S_1} \boldsymbol{n} \times \boldsymbol{t}^{(\beta)}\boldsymbol{t}^{(\beta)}(\boldsymbol{n})^{k+1}\,\mathrm{d}S_x \odot^{k+1} (\nabla_x)^k \boldsymbol{u}^\infty(\boldsymbol{x})\bigg|_{\boldsymbol{x}_0}. \tag{B 5}$$

Just as in § 3, because Stokes flow is biharmonic, higher-order derivatives of the ambient field decay faster than the field itself. Because of this, these series can also be truncated and still produce reliable results.

REFERENCES

Anderson, J. L. 1985 Effect of non-uniform zeta potential on particle movement in electric fields. *J. Colloid Interface Sci.* **105**, 45–54.

Anderson, J. L. 1989 Colloid transport by interfacial forces. *Annu. Rev. Fluid. Mech.* **21** 61–99.

Batchelor, G. K. 2000 *An Introduction to Fluid Dynamics*. Cambridge University Press.

Einzel, D., Panzer, P. & Liu, M. 1990 Boundary condition for fluid flow: curved or rough surfaces. *Phys. Rev. Lett.* **64**, 2269–2272.

Golestanian, R., Liverpool, T. B. & Ajdari, A. 2007 Designing phoretic micro- and nano-swimmers. *New J. Phys.* **9**, 126.

Happel, J. & Brenner, H. 1986 *Low Reynolds Number Hydrodynamics*, 2nd edn. Prentice Hall.

Hocking, L. M. 1976 A moving fluid interface on a rough surface. *J. Fluid Mech.* **78**, 801–817.

Kim, S. & Karrila, S. J. 1991, 2005 *Microhydrodynamics*, 2nd edn. Dover.

Ladyzhenskaya, O. A. 1963 *The Mathematical Theory of Viscous Incompressible Flow*. Gordon and Breach.

Lamb, H. 1993 *Hydrodynamics*, 6th edn. Dover.

Nie, Z., Li, W., Seo, M., Xu, S. & Kumacheva, E. 2006 Janus and ternary particles generated by microfluidic synthesis. *J. Am. Chem. Soc.* **128**, 9408–9412.

Cayre O., Paunov V. N. & Velev, O. D. 2003 Fabrication of asymmetrically coated colloid particles by microcontact printing techniques. *J. Mat. Chem.* **13**, 2445–2450.

Perro, A., Reculusa, S., Ravaine, S., Boureat-Lami, E. & Duguet, E. 2005 Design and synthesis of janus micro- and nanoparticles. *J. Mat. Chem.* **15**, 3745–3760.

Thompson, P. A. & Robbins, M. O. 1990*a* Origin of stick-slip motion in boundary lubrication. *Science* **250**, 792–794.

Thompson, P. A. & Robbins, M. O. 1990*b* Shear flow near solids: Epitaxial order and flow boundary conditions. *Phys. Rev.* A **41**, 6830–6837.

Thompson, P. A. & Troian, S. M. 1997 A general boundary condition for liquid flow at solid surfaces. *Nature* **389**, 360–362.

Yariv, E. 2004 Electro-osmotic flow near a surface charge discontinuity. *J. Fluid Mech.* **521**, 181–189.

You, D. & Moin, P. 2007 Effects of hydrophobic surfaces on the drag and lift of a circular cylinder. *Phys. Fluids* **19**, 081701.

Zhu, Y. & Granick, S. 2002 Limits of the hydrodynamics no-slip boundary condition. *Phys. Rev. Lett.* **88**, 106102(4).