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On the ideal structure of algebras of LMC-algebra valued functions

by

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Abstract. Let X be a completely regular topological space and A a commutative locally m-convex algebra. We give a description of all closed and in particular closed maximal ideals of the algebra $C(X,A)$ (= all continuous A-valued functions defined on X). The topology on $C(X,A)$ is defined by a certain family of seminorms. The compact-open topology of $C(X,A)$ is a special case of this topology.

Introduction. Let A be a commutative locally m-convex algebra with identity e over the field $\mathbf{C}$ of the complex numbers. Let $\mathcal{P} = \{p_\lambda \mid \lambda \in \Lambda\}$ be a family of seminorms which defines the topology in A denoted by $T(\mathcal{P})$. It is assumed that $T(\mathcal{P})$ is a Hausdorff topology, in other words $p_\lambda(x) = 0$ for all $\lambda \in \Lambda$ only if $x = 0$. Furthermore, we assume that the family $\mathcal{P}$ is directed. For $\lambda \in \Lambda$ we set $N_\lambda = \{x \in A \mid p_\lambda(x) = 0\}$. For each $\lambda \in \Lambda$ the quotient algebra $A/N_\lambda = A_\lambda$ is a normed algebra with the norm $\dot{p}_\lambda(x + N_\lambda) = p_\lambda(x)$, $x + N_\lambda \in A/N_\lambda$. We shall denote the completion of A_λ by $\widetilde{A}_\lambda$. General properties of locally m-convex algebras can be found for example in [4], [12] and [13]. Let $\Delta(A)$ be the set of all nontrivial continuous $\mathbf{C}$-homomorphisms on A. The set $\Delta(A)$ will be equipped with the relative $\sigma(A', A)$-topology called the *Gelfand topology*. With this topology $\Delta(A)$ is called the *carrier space* of $(A, T(\mathcal{P}))$.

Let $x \in A$ be given. The $\mathbf{C}$-valued function $\widehat{x}$ on the carrier space $\Delta(A)$ defined by $\widehat{x}(\tau) = \tau(x), \tau \in \Delta(A)$, is continuous, whence $\widehat{x} \in C(\Delta(A))$.

Let I be an ideal of A. The *hull* of I, denoted by $h(I)$, is then defined as $h(I) = \{\tau \in \Delta(A) \mid \widehat{x}(\tau) = 0,\ x \in I\}$. The *kernel* $k(E)$ of a subset E of $\Delta(A)$ is defined by $k(E) = \{x \in A \mid \widehat{x}(\tau) = 0,\ \tau \in E\}$ and for the empty set $\emptyset$ we define $k(\emptyset) = A$. Obviously $h(I)$ is a closed subset of $\Delta(A)$ and $k(E)$ is a closed ideal of $(A, T(\mathcal{P}))$.

For a completely regular topological space X denote by $C(X, A)$ the set

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of all continuous A-valued functions defined on X. Algebraic operations in $C(X,A)$ are defined pointwise. For a given element x in A we denote by f_x the constant function $f_x(t)=x$, $t\in X$. Thus f_e is the unit element and f_0 is the zero element of $C(X,A)$.

Let $\mathcal{K}$ be a compact cover of X which is closed under finite unions, in other words, $\mathcal{K}$ is a family of compact subsets of X for which

$$\bigcup\{K \mid K\in\mathcal{K}\}=X, \tag{1}$$

$$\text{if } K_1,K_2\in\mathcal{K}, \text{ then } K_1\cup K_2\in\mathcal{K}. \tag{2}$$

For $K\in\mathcal{K}$ and $\lambda\in\Lambda$, we define a seminorm $p_{(K,\lambda)}$ on $C(X,A)$ by

$$p_{(K,\lambda)}(f)=\sup_{t\in K} p_\lambda(f(t)), \quad f\in C(X,A).$$

We write $\mathcal{P}(\mathcal{K},\Lambda)=\{p_{(K,\lambda)} \mid K\in\mathcal{K},\ \lambda\in\Lambda\}$. The family $\mathcal{P}(\mathcal{K},\Lambda)$ defines a locally m-convex Hausdorff topology on $C(X,A)$ denoted from now on by $T(\mathcal{K},\Lambda)$. Obviously $\mathcal{P}(\mathcal{K},\Lambda)$ is a directed family of seminorms. If $\mathcal{K}=\mathcal{K}(X)=$ the set of all compact subsets of X, then $T(\mathcal{K},\Lambda)$ is the compact-open topology of $C(X,A)$. Given $K\in\mathcal{K}$ and $\lambda\in\Lambda$, define $N_{(K,\lambda)}=\{f\in C(X,A) \mid p_{(K,\lambda)}(f)=0\}$.

Let $t\in X$ and let I be an ideal of A. We define an ideal $J_{(t,I)}$ of $C(X,A)$ by $J_{(t,I)}=\{f\in C(X,A) \mid f(t)\in I\}$ and furthermore $C(X,I)=\{f\in C(X,A) \mid f(t)\in I,\ t\in X\}$. Obviously $C(X,I)=\bigcap_{t\in X} J_{(t,I)}$. If I is a closed ideal of $(A,T(\mathcal{P}))$ and $t\in X$, then it is easy to see that both the ideals $J_{(t,I)}$ and $C(X,I)$ are closed in $(C(X,A),T(\mathcal{K},\Lambda))$.

Let $K\in\mathcal{K}$, $\lambda\in\Lambda$ and $\varepsilon>0$. We denote by $V_{(K,\lambda)}(\varepsilon)$ the set $\{f\in C(X,A) \mid p_{(K,\lambda)}(f)<\varepsilon\}$. Obviously the sets $V_{(K,\lambda)}(\varepsilon)$, $K\in\mathcal{K}$, $\lambda\in\Lambda$, $\varepsilon>0$, form a subbase of neighbourhoods of f_0 (see [14], p. 8).

On closed ideals of $(C(X,A),T(\mathcal{K},\Lambda))$. We give a description of the closed ideals of the algebra $(C(X,A),T(\mathcal{K},\Lambda))$. We also prove that the carrier space $\Delta(C(X,A))$ is homeomorphic to $X\times\Delta(A)$ if $\Delta(A)$ is locally equicontinuous.

The following lemmas are easy to verify.

LEMMA 1. *$N_{(K,\lambda)}=\bigcap_{t\in K} J_{(t,N_\lambda)}$ for all $K\in\mathcal{K}$ and $\lambda\in\Lambda$.*

LEMMA 2. *Let $\{I_\alpha \mid \alpha\in\Gamma\}$ be a family of closed ideals in $(A,T(\mathcal{P}))$. Then $\bigcap_{\alpha\in\Gamma} C(X,I_\alpha)=C(X,\bigcap_{\alpha\in\Gamma} I_\alpha)$.*

Let J be a closed ideal of $(C(X,A),T(\mathcal{K},\Lambda))$ and let $t\in X$ be given. We define $I_t=\{f(t) \mid f\in J\}$ and $I(t)=\mathrm{cl}(I_t)=$ the closure of I_t in $(A,T(\mathcal{P}))$. It is easy to see that either $I(t)$ is a closed proper ideal of $(A,T(\mathcal{P}))$ or $I(t)=A$.

Next we shall prove a useful result.

THEOREM 3. *If J is a closed proper ideal of $(C(X,A),T(\mathcal{K},\Lambda))$, then there is t in X such that $I(t)$ is a closed proper ideal of $(A,T(\mathcal{P}))$.*

Proof. Suppose that $I(t)=A$ for all $t\in X$. Let $K\in\mathcal{K}$, $\lambda\in\Lambda$ and $\varepsilon>0$ be arbitrary. Then for each $t_0\in K$ there is $f_{t_0}\in J$ such that $p_\lambda(f_{t_0}(t_0)-e)=p_\lambda(f_{t_0}(t_0)-f_e(t_0))<\varepsilon$. By continuity, there is a neighbourhood $U(t_0)$ of t_0 such that

$$p_\lambda(f_{t_0}(t)-f_e(t))<\varepsilon \quad \text{for all } t\in U(t_0). \tag{3}$$

Now the neighbourhoods $\{U(t_0) \mid t_0\in K\}$ form an open cover of the compact set K. Take its finite subcovering $U_1,\ldots,U_n$ and denote by $f_1,\ldots,f_n$ those functions of J for which (3) is valid. By Lemma 2.1.1 of [6], there are $\alpha_i\in C(X)$, $i=1,\ldots,n$, such that $0\le\alpha_i(t)\le1$ for all $t\in X$ and $i=1,\ldots,n$, $\operatorname{supp}\alpha_i\subset U_i$ for all $i=1,\ldots,n$ and $\sum_{i=1}^n\alpha_i(t)=1$ for all $t\in K$. Now, if we define a function $F_{(K,\lambda)}$ by

$$F_{(K,\lambda)}(t)=\sum_{i=1}^n(\alpha_i f_i)(t), \quad t\in X,$$

we see that $F_{(K,\lambda)}\in J$ and

$$p_{(K,\lambda)}(F_{(K,\lambda)}-f_e)<\varepsilon.$$

Thus, $F_{(K,\lambda)}\in f_e+V_{(K,\lambda)}(\varepsilon)$. Since the $V_{(K,\lambda)}(\varepsilon)$ form a subbasis of the zero neighbourhoods, the unit element f_e is in $\mathrm{cl}(J)=J$, which is impossible.

Next we shall give a description of the carrier space $\Delta(C(X,A))$. The structure of $\Delta(C(X,A))$ has been considered in many papers under various topological assumptions on X and A (see [1], [3], [5], [6], [8]–[11], [16] and [17]). For example in [6] Dietrich assumed that X is a k-space and A is a complete locally convex algebra for which $\Delta(A)$ is locally equicontinuous. He used the tensor product representation of $C(X,A)$. Tensor product techniques have also been used in [11]. Abel has proved a corresponding result for a more general case in [1]. In this paper we consider the carrier space of the algebra $(C(X,A),T(\mathcal{K},\Lambda))$.

Let $t\in X$ and $\tau\in\Delta(A)$ be given. We define a mapping $\phi_{(t,\tau)}: C(X,A)\to\mathbb{C}$ by

$$\phi_{(t,\tau)}(f)=\tau(f(t)), \quad f\in C(X,A).$$

Obviously $\phi_{(t,\tau)}\in\Delta(C(X,A))$ and $\ker\phi_{(t,\tau)}=J_{(t,\ker\tau)}$.

LEMMA 4. *If N is a closed maximal ideal of $(C(X,A),T(\mathcal{K},\Lambda))$, then there are unique points $t\in X$ and $\tau\in\Delta(A)$ such that $N=\ker\phi_{(t,\tau)}$.*

Proof. By Theorem 3, there is $t\in X$ such that $I(t)=\mathrm{cl}(\{f(t) \mid f\in N\})$ is a closed proper ideal of $(A,T(\mathcal{P}))$. Now there is $\tau\in\Delta(A)$ such that

$I(t) \subset \ker\tau$. If $f \in N$ is arbitrary, then $\phi_{(t,\tau)}(f) = \tau(f(t)) = 0$. Thus $N \subset \ker\phi_{(t,\tau)}$, and so $N = \ker\phi_{(t,\tau)} = J_{(t,\ker\tau)}$. It is easy to see that the points $t \in X$ and $\tau \in \Delta(A)$ are unique.

We now define a mapping $\varphi : X \times \Delta(A) \to \Delta(C(X,A))$ by

$$\varphi(t,\tau) = \phi_{(t,\tau)}, \quad (t,\tau) \in X \times \Delta(A). \tag{4}$$

THEOREM 5. *The mapping φ defined in* (4) *is a bijection from $X \times \Delta(A)$ onto $\Delta(C(X,A))$. The inverse mapping φ^{-1} is continuous, and φ is continuous if $\Delta(A)$ is locally equicontinuous.*

Proof. Obviously $\varphi(t,\tau) \in \Delta(C(X,A))$ if $(t,\tau) \in X \times \Delta(A)$. It is also easy to see that φ is an injection. Now, if $\phi \in \Delta(C(X,A))$, then $\ker\phi$ is a closed maximal ideal of $(C(X,A), T(\mathcal{K},A))$. By Lemma 4 there are unique $t \in X$ and $\tau \in \Delta(A)$ such that $\ker\phi = \ker\phi_{(t,\tau)}$ and thus $\phi = \phi_{(t,\tau)}$. So φ is also a surjection.

Now, φ is continuous provided so is the mapping $\widehat{f} : X \times \Delta(A) \to \mathbb{C}$ defined by

$$\widehat{f}(t,\tau) = \widehat{f}(\phi_{(t,\tau)}) = \tau(f(t)), \quad (t,\tau) \in X \times \Delta(A).$$

But it is well-known that $\widehat{f}$ is continuous if $\Delta(A)$ is locally equicontinuous (see e.g. [10], Lemma 3). So φ is continuous if $\Delta(A)$ is locally equicontinuous. The continuity of φ^{-1} can be shown by a similar method to that used in [17] for the Nachbin algebras.

COROLLARY 6. *The carrier space $\Delta(C(X,A))$ of the algebra $(C(X,A), T(\mathcal{K},A))$ is homeomorphic to $X \times \Delta(A)$ if $\Delta(A)$ is locally equicontinuous.*

COROLLARY 7. *If $\phi \in \Delta(C(X,A))$, then there is exactly one $t_0 \in X$ such that $I(t_0) = \mathrm{cl}(\{f(t_0) \mid f \in \ker\phi\})$ is a closed proper ideal of $(A, T(\mathcal{P}))$, and furthermore $I(t_0) = \ker\tau$ for some $\tau \in \Delta(A)$.*

Next we give a description of closed (proper) ideals J of $(C(X,A), T(\mathcal{K},A))$. It has been proved in [2] (Theorem 2) that in the case where X is a compact Hausdorff space and A is a locally convex topological algebra there is $E \subset X$ and a family $\{I(t) \mid t \in E\}$ of closed proper ideals of A such that $J = \bigcap_{t\in E} J_{(t,I(t))}$. We now prove this for $(C(X,A), T(\mathcal{K},A))$.

THEOREM 8. *If J is a closed proper ideal of $(C(X,A), T(\mathcal{K},A))$, then there is $E \subset X$ and a family $\{I(t) \mid t \in E\}$ of closed ideals of $(A, T(\mathcal{P}))$ such that $J = \bigcap_{t\in E} J_{(t,I(t))}$.*

Proof. For $t \in X$ set $I(t) = \mathrm{cl}(\{f(t) \mid f \in J\})$. As noted earlier, either $I(t)$ is a proper closed ideal of $(A, T(\mathcal{P}))$ or $I(t) = A$. Define $E = \{t \in X \mid I(t)$ is a proper subset of $A\}$. By Theorem 3, E is nonempty.

It is easy to see that $J \subset \bigcap_{t\in E} J_{(t,I(t))}$. Now, if $t \in X \sim E$ = the complement of E in X, then $J_{(t,I(t))} = C(X,A)$ and therefore $\bigcap_{t\in X} J_{(t,I(t))} = \bigcap_{t\in E} J_{(t,I(t))}$. Let $f \in \bigcap_{t\in X} J_{(t,I(t))}$ and fix $K \in \mathcal{K}$, $\lambda \in \Lambda$, $t_0 \in K$ and $\varepsilon > 0$. From the definition of $I(t_0)$ it follows that there is $f_{t_0} \in J$ such that $p_\lambda(f_{t_0}(t_0) - f(t_0)) < \varepsilon$. By continuity, there is a neighbourhood $U(t_0) \subset X$ of t_0 such that

$$p_\lambda(f_{t_0}(t) - f(t)) < \varepsilon, \quad t \in U(t_0). \tag{5}$$

Now $\{U(t_0) \mid t_0 \in K\}$ is an open cover of K. Taking a finite subcovering as in the proof of Theorem 3, we obtain a function $F_{(K,\lambda)} \in J$ such that

$$p_{(K,\lambda)}(F_{(K,\lambda)} - f) < \varepsilon.$$

Hence $f \in \mathrm{cl}(J) = J$, and so $\bigcap_{t\in E} J_{(t,I(t))} \subset J$, which completes the proof.

The set E above is not necessarily closed as the following example shows.

EXAMPLE. Let X be the real line $\mathbf{R}$ with the usual topology and consider the open interval $(0,1) \subset \mathbf{R}$. Let $A = C(0,1)$ with the topology $T(\mathbf{N})$ given by the sequence of seminorms

$$p_n(x) = \sup_{t\in[1/(n+1),1-1/(n+1)]} |x(t)|, \quad x \in A,\ n \in \mathbf{N},$$

denoted by $\mathcal{P}(\mathbf{N})$. For a natural number m and a seminorm $p_n \in \mathcal{P}(\mathbf{N})$ let $p_{(m,n)}$ be the seminorm on $C(X,A)$ defined by

$$p_{(m,n)}(f) = \sup_{t\in[-m,m]} p_n(f(t)), \quad f \in C(X,A).$$

Now $\{p_{(m,n)} \mid m,n \in \mathbf{N}\}$ is a directed family of seminorms on $C(X,A)$ which defines a locally m-convex topology on $C(X,A)$ denoted by $T(\mathbf{N},\mathbf{N})$.

Let $k \in \mathbf{N}$ be given. If $t \in X$, we define

$$E_t = \begin{cases} \{s \in (0,1) \mid -t/k \le s < 1\} & \text{if } t < 0, \\ \{s \in (0,1) \mid t/k \le s < 1\} & \text{if } t \ge 0. \end{cases}$$

Obviously E_t is a closed subset of $(0,1)$ for all $t \in \mathbf{R}$ and $E_t = \emptyset$ if $t \in X \sim (-k,k)$. If we define $k(E_t) = \{x \in A \mid x(s) = 0,\ s \in E_t\}$, then obviously $k(E_t)$ is a closed ideal of $(A, T(\mathbf{N}))$ for each $t \in \mathbf{R}$ and $k(E_t) = A$ if $t \in X \sim (-k,k)$.

If we choose $J = \bigcap_{t\in\mathbf{R}} J_{(t,k(E_t))}$, then J is a closed ideal of $(C(X,A), T(\mathbf{N},\mathbf{N}))$ and clearly $J = \bigcap_{t\in(-k,k)} J_{(t,k(E_t))}$. Thus

$$E = \{t \in X \mid I(t) \text{ is a proper subset of } A\} = (-k,k),$$

whence E is an open subset of X.

Next we shall give some conditions which guarantee that the set E in Theorem 8 is closed. First we recall that $(A, T(\mathcal{P}))$ is a *Q-algebra* if the set of regular elements is open.

THEOREM 9. *Let J be a closed ideal of $(C(X,A), T(\mathcal{K},\Lambda))$ and let $E = \{t \in X \mid I(t) \neq A\}$ where $I(t) = \mathrm{cl}(\{f(t) \mid f \in J\})$. Then E is closed in each of the following cases:*

1° *$(A, T(\mathcal{P}))$ is a Q-algebra.*

2° *$\Delta(A)$ is locally equicontinuous and $\{\tau \mid \phi_{(t,\tau)} \in h(J)\} \subset h(N_\lambda)$ for some $\lambda \in \Lambda$.*

3° *For each boundary point t in E there is $f \in J$ such that $f(t) \in \ker \tau$ for some $\tau \in \Delta(A)$.*

Proof. 1° and 3° are obvious. We only prove case 2°. Suppose that $\Delta(A)$ is locally equicontinuous. Now $\varphi^{-1}(h(J)) = \{(t,\tau) \mid \phi_{(t,\tau)} \in h(J)\}$ is a closed subset of $X \times \Delta(A)$. If $\{\tau \in \Delta(A) \mid \phi_{(t,\tau)} \in h(J)$ for all $t \in X\} \subset h(N_\lambda)$ for some $\lambda \in \Lambda$, then $\varphi^{-1}(h(J)) \subset X \times h(N_\lambda)$, and so E is just the projection of the closed set $\varphi^{-1}(h(J)) \subset X \times h(N_\lambda)$ into X. By Theorem 4.1 and Corollary 2.1 of [12], $h(N_\lambda)$ is homeomorphic to $\Delta(\widetilde{A}_\lambda)$. So $h(N_\lambda)$ is compact, since $\Delta(\widetilde{A}_\lambda)$ is. The closedness of E now follows from Theorem 2.5 of [7].

COROLLARY 10. *If $K \in \mathcal{K}$ and $\lambda \in \Lambda$, then*

$$\{(t,\tau) \in X \times \Delta(A) \mid \phi_{(t,\tau)} \in h(N_{(K,\lambda)})\} = K \times h(N_\lambda).$$

In particular, if $\Delta(A)$ is locally equicontinuous, then $h(N_{(K,\lambda)})$ is homeomorphic to $K \times h(N_\lambda)$.

Proof. Just note that $N_{(K,\lambda)} = \bigcap_{t \in K} J_{(t,N_\lambda)}$.

We shall say that $(A, T(\mathcal{P}))$ has the *property of spectral synthesis* if $k(h(I)) = I$ for each closed ideal I of $(A, T(\mathcal{P}))$.

COROLLARY 11. *The algebra $(C(X,A), T(\mathcal{K},\Lambda))$ has the property of spectral synthesis if and only if $(A, T(\mathcal{P}))$ has this property.*

Proof. Suppose $k(h(I)) = I$ for each closed ideal of $(A, T(\mathcal{P}))$. Now, if J is a closed ideal of $(C(X,A), T(\mathcal{K},\Lambda))$, then by Theorem 8, we have $J = \bigcap_{t \in E} J_{(t,I(t))}$ for some $E \subset X$ and a family $\{I(t) \mid t \in E\}$ of closed ideals of $(A, T(\mathcal{P}))$. It is easy to see that $h(J) = \{\phi_{(t,\tau)} \mid t \in E, \tau \in h(I(t))\}$. Thus the following equivalences are valid:

$$\begin{aligned} f \in k(h(J)) &\Leftrightarrow \phi(f) = 0 \text{ for all } \phi \in h(J) \\ &\Leftrightarrow \phi_{(t,\tau)}(f) = 0 \text{ for all } t \in E \text{ and } \tau \in h(I(t)) \\ &\Leftrightarrow \tau(f(t)) = 0 \text{ for all } t \in E \text{ and } \tau \in h(I(t)) \\ &\Leftrightarrow f(t) \in k(h(I(t))) \text{ for all } t \in E \\ &\Leftrightarrow f(t) \in I(t) \text{ for all } t \in E \Leftrightarrow f \in \bigcap_{t \in E} J_{(t,I(t))} = J. \end{aligned}$$

Thus $k(h(J)) = J$.

Conversely, suppose that $k(h(J)) = J$ for each closed ideal J of $C(X,A)$, $T(\mathcal{K},\Lambda))$. Let I be an arbitrary closed ideal of $(A, T(\mathcal{P}))$. Now, if $x \in k(h(I))$, then $f_x \in J_{(t,k(h(I)))}$ for each $t \in X$. It is easy to see that $J_{(t,k(h(I)))} = k(h(J_{(t,I)}))$. But by the hypotheses $k(h(J_{(t,I)})) = J_{(t,I)}$. So $f_x \in J_{(t,I)}$ for any $t \in X$ and therefore $x \in I$. Thus $k(h(I)) \subset I$, whence $k(h(I)) = I$, which completes the proof.

The result of Corollary 11 is a generalization of the corresponding result of [3]. The structure of closed ideals of $C(X,A)$ with the compact-open topology has been considered for example in [15].

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Corrigendum to the paper "On the reproducing kernel for harmonic functions and the space of Bloch harmonic functions on the unit ball in R^n"

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On page 24 we got a formula for the projector P:

$$Pu = c(n)\Delta\left[(|x|^2-1)\int_B \frac{(1-|x|^2|y|^2)u(y)\,dV_y}{[|x-y|^2+(1-|x|^2)(1-|y|^2)]^{n/2}}\right].$$

In the next step we made a mistake in calculating the above laplacian. The next formula should read

$$Pu = c(n)\int_B\left(\frac{2n(1-|x|^2|y|^2)^2}{[|x-y|^2+(1-|x|^2)(1-|y|^2)]^{n/2+1}} - \frac{8|x|^2|y|^2}{[|x-y|^2+(1-|x|^2)(1-|y|^2)]^{n/2}}\right)u(y)\,dV_y\,,$$

and the formula on the top of page 25 should read

$$K(x,y) = c(n)\left(\frac{2n(1-|x|^2|y|^2)^2}{[|x-y|^2+(1-|x|^2)(1-|y|^2)]^{n/2+1}} - \frac{8|x|^2|y|^2}{[|x-y|^2+(1-|x|^2)(1-|y|^2)]^{n/2}}\right).$$

The kernel $K(x,y)$ in its correct form satisfies the estimates of page 26 and therefore the other results of the paper remain valid.

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