On the "Identification and Control of Dynamical Systems Using Neural Networks"

Ernesto Rios-Patron and Richard D. Braatz

***Abstract*—It is noted that [1, p. 15, Example 2] has a third equilibrium state corresponding to the point (0.5, 0.5).**

I. Remarks

In [1], Narendra and Parthasarathy perform an admirable study of the application of neural networks for identification and control. We agree with the statement of the authors of [1, p. 15], that for nonlinear processes, "Some prior information concerning the input–output behavior of the plant is needed before identification can be undertaken. This includes the number of equilibrium states of the unforced system and their stability properties …." The authors then state that the equilibrium states of the unforced system

$$y_p(k+1) = \frac{y_p(k)y_p(k-1)(y_p(k)+2.5)}{1+y_p^2(k)+y_p^2(k-1)} \tag{1}$$

are $(y_p(k), y_p(k-1)) = (0, 0)$ and (2, 2).

We would like to note that this system has a third equilibrium state, which corresponds to the point (0.5, 0.5). This equilibrium state is an unstable saddle point, as can be seen by linearizing (1) around the equilibrium state (0.5,0.5), and writing the system in state-space form

$$\begin{bmatrix} y_p(k+1) \\ y_p(k) \end{bmatrix} = \underbrace{\begin{bmatrix} 5/6 & 2/3 \\ 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} y_p(k) \\ y_p(k-1) \end{bmatrix} + \begin{bmatrix} -1/4 \\ 0 \end{bmatrix}. \tag{2}$$

The eigenvalues of A are $\lambda_1 = 4/3$ and $\lambda_2 = -1/2$, giving $|\lambda_1| > 1$ and $|\lambda_2| < 1$.

Input–output data must be collected around each of these equilibrium states for the neural network model trained on the data to adequately capture the fundamental physics of the process. Although the authors did not describe the third steady state (0.5,0.5), their identification data did adequately sample the region of state space near the third steady state, so that the identification procedure produced an adequate model for the process.

References

[1] K. S. Narendra and K. Parthasarathy, "Identification and control of dynamical systems using neural networks," *IEEE Trans. Neural Networks*, vol. 1, pp. 4–27, 1990.

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Comments on "Stochastic Choice of Basis Functions in Adaptive Function Approximation and the Functional-Link Net"

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***Abstract*—This paper includes some comments and amendments of the above-mentioned paper. Subsequently, Theorem 1 in the above-mentioned paper has been revised. The significant change of the original theorem is the space of the thresholds in the hidden layer. The revised theorem says that the thresholds of hidden units, b_0, should be $-w_0 \cdot y_0 - u_0$, where $w_0 = \alpha \hat{w}_0$; $\hat{w}_0 = (\hat{w}_{01}, \cdots, \hat{w}_{0d})$, $y_0 = (y_{01}, \cdots, y_{0d})$, and u_0 be independent and uniformly distributed in $\mathbf{V}^d = [0; \Omega] \times [-\Omega; \Omega]^{d-1}$, I^d, and $[-2d\Omega, 2d\Omega]$, respectively.**

I. Introduction

The above-mentioned paper[1] has introduced the random vector version of the functional-link (RVFL) net. Igelnik and Pao show the function approximation capability of RVFL by a stochastic approach based on an limit-integral representation of the function to be approximated with subsequent evaluation of the integral by the Monte Carlo method. This stochastic approach is demonstrated to be an efficient approximation method of multivariate functions according to its theoretical justification and simulation results. The most distinctive characteristic of RVFL is that parts of parameters of RVFL, i.e., the weights and thresholds of hidden layer are selected randomly, independently and uniformly in the specific spaces. These parameters of RVFL are fixed and do not require the learning procedure that conventional networks do. Such property of RVFL results in a simple and efficient learning algorithm. In that paper, however, there are several errors in the proof procedure of Theorem 1 such that the selection spaces of random parameters of RVFL are given in a not exact form. Consequently, the thresholds of hidden units are calculated incorrectly. Therefore, Theorem 1 of the above-mentioned paper should be revised. The revised Theorem 1 in this paper gives more exact selection space for the random parameters of RVFL. Subsequently, the establishment and training for RVFL will be subjected to a more appropriate guideline.

II. Revisions

First, we give the revised Theorem 1 of the above-mentioned paper.

The Revised Theorem 1: For any compact K, $K \subset I^d$, $K \neq I^d$ and any absolutely integrable activation function g such that

$$\int_R g^2(x)\,dx < \infty \tag{1}$$

there exist a sequence of RVFL $\{f_{\omega_n}\}$ and a sequence of probability measures $\{\mu_{n\,\Omega\,\alpha}\}$ such that

$$\rho_K(f, f_{\omega_n}) \underset{n\to\infty}{\longrightarrow} 0. \tag{2}$$

The probability measures $\mu_{n\,\Omega\,\alpha}$, can be specified as follows. Let $\hat{w}_0 = (\hat{w}_{01}, \cdots, \hat{w}_{0d})$, $y_0 = (y_{01}, \cdots, y_{0d})$ and u_0 be independent

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[1] B. Igelnik and Y.-H. Pao, *IEEE Trans. Neural Networks*, vol. 6, no. 6, pp. 1320–1329, 1995.