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On the indentation of brittle solids which experience fragmentation

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Abstract

The paper examines the problem of the interaction between a rigid indentor and a brittle solid which can experience fragmentation during the indentation process. The analysis is achieved through the modification of an existing distinct element code, which incorporates viscoplastic effects in the medium and relatively simple criteria for fragmentation. The procedures are applied to investigate the scale effects in such indentation processes.

1. Introduction

A characteristic feature of brittle solids is that they are susceptible to fragmentation during the application of loads. This transition from an initially continuum region to a discontinuum can be most conveniently handled through the application of discrete element techniques. This paper examines the process of fragmentation associated with the two dimensional problem of the interaction between a rigid indentor and a brittle elastic-viscoplastic solid. The modelling is assumed to be representative of the indentation of polycrystalline materials such as ice. The initial elastic behaviour of the ice is assumed to be isotropic and linear and the rate effects are accommodated through a viscoplastic model with a Mohr-Coulomb failure criterion. In the particular instance when the failure model is incorporated with post peak softening effects, the efficiency of the computational scheme is influenced by the tendency for the development of progressive fragmentation in highly localized zones. Thus uncontrolled continued fragmentation of the medium is, however, an unrealistic model since, in reality, the strength characteristics of the fragments can be influenced by their size. This paper examines the application of a discrete element method to the two dimensional plane strain indentation of a brittle elastic viscoplastic layer by a smooth rigid indentor with a flat surface. The development of a discrete element based computational procedure which accommodates the transition from a continuum to a fragmented discontinuum is an essential requirement in order to address the problem of indentation of brittle solids during the application of loads. The primary advantage of the discrete element method is the facility to achieve a transition from a continuum to a discontinuum by consideration of relatively simplified fragmentation criteria. This investigation modifies the discrete element code DECICE (INTERA Technologies[2]) to examine the process of fragmentation associated with the two dimensional plane strain problems related to the interaction between a rigid indentor and a brittle elastic-viscoplastic solid such as polycrystalline ice. The code is capable of examining the mechanics of multiple interacting deformable bodies undergoing finite displacements. These fragments can undergo progressive fragmentation which can result in the generation of progressively smaller distinct material fragments. The fragmentation is based on a simple tensile strength criterion and the interaction between contacting fragments is modelled by appeal to contact stiffness criteria. The constitutive modelling of the brittle solid accounts for viscoplastic behaviour which can be accompanied by softening phenomena. The continued fragmentation is assumed to be influenced by the fragment size. Such size-dependency has been identified in connection with rock fragmentation. The progress of the fragmentation of the elastic-viscoplastic continuum is thus governed by a size dependent strength criterion. The modified discrete element model is used to establish the time history of force development at the indentor for different indentor sizes, resulting in various contact areas between the indentor and the brittle solid during the constant rate of penetration of the indentor. The paper documents the time-dependent development of the average contact stress at the indentor-solid interface during the interaction process. The corresponding development of fragmentation within the brittle solid due to fragment interaction is also documented. Numerical results indicate a significantly larger collapse contact pressure for smaller contact ares, thus confirming the ability of the numerical model to duplicate size-dependent strength characteristics of the polycrystalline materials.

2. Constitutive models

The DECICE discrete element code has several constitutive processes for which appropriate constitutive parameters should be specified. These include basic constitutive responses for the intact ice and constitutive responses which characterize both the generation of ice fragments from an existing continuum and criteria which characterize the interaction of fragments.

2.1. Viscoplastic brittle material behaviour

Viscoplastic elements model material behaviour that is governed by yield criteria in which time-dependent plastic deformation takes place if these yield criteria are attained. The plastic flow can continue for all stress states which satisfy the yield criteria. Viscoplasticity theories have been successfully employed to investigate rate dependent failure phenomena associated with geomaterials (see e.g. Perzyna[5]; Zienkiewicz and Cormeau[9]; Owen and Hinton[4]; Sepehr and Stimpson[8]; Selvadurai and Sepehr[7]). All classical failure or yield criteria can be extended to include effects of strain softening. Admittedly, the choice of a particular form of a yield criterion and softening relationship must largely be governed by results of experimental observations. In the ensuing sections a modified Mohr-Coulomb yield criterion which incorporates softening effects is employed.

The viscoplastic brittle solid material is assumed to be homogeneous isotropic, and the constitutive parameters are assumed to remain constant throughout the interaction process. The constitutive processes include isotropic elastic behaviour, viscoplastic behaviour characterized by a Mohr-Coulomb failure criterion and an associative flow rule, consideration of the fluidity of the medium and a criterion for brittle fragmentation due to tension.

Elastic properties: Elastic modulus(E) = 3.5×10^9 Pa, Poisson's ratio (ν) = 0.35

Viscoplastic properties: Cohesion (c) = 1.5×10^6 Pa, Friction Angle (ϕ) = 30°
Tensile Strength (T) = 0.5×10^6 Pa

In this study the brittle solid is assumed to represent polycrystalline ice at low temperature. We note here that the value of fluidity parameter (γ) for ice is not reported in the literature. However, a fluidity parameter value of 1.0×10^{-3} /sec. is reported in the literature for concrete and hard rock (Owen and Hinton[4]). Intact ice, however, is expected to be less viscous than these materials, thus a higher fluidity parameter for ice is considered to be realistic. Therefore for the purposes of this study, an approximate fluidity parameter of 1.0×10^{-2} /sec is assumed for intact ice.

Post failure behaviour

Ice can assume residual strength values representative of its post failure behaviour (strain softening). The residual strength parameters are assigned the following values:

Residual Cohesion (c_r) = 2.0 Pa, Residual Friction Angle (ϕ_r) = 3°, Residual Tensile Strength = 1.0 Pa
Residual Fluidity Parameter = 1.0×10^{-2} /sec

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2.2 Brittle fragmentation behaviour

The elements composing the ice sheet can experience fragmentation either in tension or compression depending on the state of stress within each element and its material properties. Upon fragmentation, new elements are generated and interaction takes place along surfaces of the elements. This interaction is governed by the stiffness and frictional properties of the separate fragments. The fragmentation process can be such that the initially intact ice sheet can be fragmented into various sizes and shapes consistent with the stress state and the strength criteria. The ability to achieve such a fragmentation process is a distinct advantage of the discrete element method over the conventional continuum computational models including finite element techniques. In this study the brittle failure criterion for a Mohr-Coulomb material with a tension cut-off is adopted. Fragmentation which involves, failure through element centroids, occurs as described in the ensuing sections.

Mohr-Coulomb/tension cut-off brittle failure criteria

Two kinds of failure are allowed to occur:

a) Failure along mesh-lines
The failure criterion applied along mesh lines is given by

$$\tau > c + \mu\sigma \qquad (1)$$

where σ and τ are the normal and shear stresses across the plane; c is the cohesion of material; μ is the coefficient of internal friction of the material (i.e. $\mu = \tan\phi$)

b) Failure through element centroids
In order to apply this failure criterion to check whether an element will fragment through its centroid, it is necessary to express the criterion in terms of the maximum and minimum principal stresses denoted by σ_1 and σ_3, respectively, as follows:

$$\sigma_1 > S + \sigma_3 \tan^2(\pi/4 + \phi/2) \qquad (2)$$

where ϕ is the internal angle of friction defined by $\mu = \tan\phi$, and S is the unconfined compressive strength. Internal cohesion of the material, c is related to the unconfined compressive strength, S, by

$$c = S\{(\mu^2 + 1)^{1/2} - \mu\}/2 \qquad (3)$$

The post yield viscoplastic behaviour of the solid is also assumed to be described by a Mohr-Coulomb failure criterion with reduced values of the compressive strength parameters ϕ_r, c_r and S_r. The strength loss between the peak and residual states is assumed to be instantaneous.

In the tensile fragmentation criterion it is assumed that the material will fragment when the maximum tensile stress in an element reaches the tensile strength of the material, T. The fragmentation criterion in two dimensions is

$$F = \sigma_3 - T \qquad (4)$$

Upon failure in tension, a residual value of tensile strength T_r is assumed. Thus the residual failure criterion becomes

$$F_r = \sigma_3 - T_r \quad (5)$$

2.3 Size dependency of strength

Polycrystalline solids such as ice and solids such as rock, are composed of crystals and grains in a fabric that includes flaws such as cracks and fissures. When the size of a specimen is small such that only relatively few defects are present, failure is forced to involve new crack growth; whereas a material loaded through a larger volume will encounter pre-existing flaws at critical locations. Thus, the material strength can be size-dependent. In particular, materials such as coal, granitic rocks and shale exhibit the greatest degree of size dependency; the ratio of laboratory to field strengths sometimes attaining values of 10 or more. A few definitive studies have been made to examine the influence of size effect on the strength over a broad range of specimen sizes. Bieniawski[1] reported tests on prismatic in-situ coal specimens up to 1.6m x 1.6m x 1.0m as shown in Figure 1. Jahns[3] reported results of similar tests on cubical specimens of calcareous iron ore also shown in Figure 1. Available data from these investigations are insufficient to reach a conclusive recommendation valid for a variety of polycrystalline materials; however, it does appear that there is generally a limiting size such that larger specimens suffer no further decrease in strength. Figure 1 also shows test results obtained by Pratt, Black, Brown and Brace[6] for fissured quartz diorite where results confirm the size dependency of strength. The polycrystalline brittle solid investigated in this paper is assumed to behave in a similar manner. Tensile and compressive strengths are assumed to undergo strain softening for specimen edge lengths of 0.32m or higher, however, material strength is simulated to increase exponentially as the specimen size falls below an edge length of 0.32m. Figure 2 illustrates the assumed tensile and compressive strengths of the polycrystalline solid versus the specimen size as implemented in the discrete element model.

2.4 Interactive responses

The interactive behaviour between ice fragments is characterized by normal and shear interaction stiffnesses at the contact surface. Such interaction responses can have a variety of features including elastic effects and frictional phenomena. The elastic stiffnesses are calculated by appeal to the theory of Hertzian-type contact between interacting elements and the frictional responses are dependent on the material characteristics of the contacting regions. Normal interaction stiffness (K_n) = 1.0 x 10^{10} Pa, Shear interaction stiffness (K_s) = 1.0 x 10^{10} Pa, Interaction friction(μ) = 30°, Interaction cohesion(c) = 2.0 x 10^6 Pa

During seperation, the interaction cohesion and friction parameters will assume near-zero values.

10. Numerical results

In this section we examine the problem of the indentation of a layer of finite thickness which is underlain by a rigid base. The contact between the rigid base and the layer is assumed to be fixed. The indentor is modelled as relatively rigid body which interacts with the layer at a uniform velocity of 0.1 m/s. The elastic properties for the interacting structure assumed in the analyses are as follows:

Elastic modulus (E) = 2.0×10^{11} Pa, Poisson's ratio (ν) = 0.35

Example 1: Solid Block Indentation - Large Size Indentor (B/H = 10/600)

In this example, we examine the problem of the indentation of a brittle elasto-viscoplastic solid by a relatively wide rigid indentor with a total contact area of 1.0 m^2 (assuming a unit thickness for the block and the indentor as shown in Figure 3) which interacts with the block at a uniform velocity of 0.1 m/s. Figure 4 shows the discrete element mesh at the solid-indentor contact zone. Only half of the solid and indentor is modelled because of the symmetrical nature of the problem. The brittle solid is represented by a discrete element region with viscoplastic material behaviour. Figures 5 to 8 illustrate the process of fragment development of the solid in the vicinity of the indentor. As can be observed, a relatively large area of the solid block has experienced fragmentation. The computational process, however, ceases as a result of extensive fragmentation leading to elements with near-zero area signalling global failure within the solid block. Figure 9 illustrates the average contact pressure versus time history for the elements in contact with the indentor. As can be observed from Figure 9, a maximum contact pressure of 42.5 MPa is predicted.

Example 2:Solid Block Indentation - Intermediate Size Indentor (B/H = 6/600)

The geometry of the solid block and the element discretization is similar to Example 1 (Figure 3). However, a relatively smaller indentor is used resulting in a contact area of 0.6 m^2 between the solid block and the indentor. Figure 10 illustrates the discrete element mesh at the solid - indentor contact zone for this case. Figure 11 shows the extensively fragmented area immediately prior to the global failure of the solid block. As can be observed, for this case a relatively smaller volume of the solid block has experienced fragmentation in comparison with Example 1. As a result of the smaller volume which undergoes fragmentation, relatively fewer defects which influence strength are believed to exist in the loaded area. Figure 9 confirms this size dependency as a maximum average contact pressure of 72.0 MPa is predicted to cause

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indentation failure of the block for this example as compared to an average contact pressure of only 42.5 MPa for Example 1, thus confirming the ability of the numerical model to predict the size dependent strength characteristics of the solid block.

Example 3: Solid Block Indentation - Small size Indentor (B/H = 2/600)

The geometry of the solid block and the element discretization is similar to Example 2 (Figure 3). However, unlike in the previous example, a relatively smaller indentor is used resulting in a total contact area of 0.2 m^2. Figure 12 illustrates the initial discrete element mesh at the solid - indentor contact zone for this case. Figure 13 shows the fragmented area immediately before the global failure. As can be observed, relatively smaller volume of the solid block has experienced fragmentation for this case as compared to the previous examples. Figure 9 confirms the size dependency of the problem as a maximum contact pressure of 110 MPa is predicted to cause failure of the block for this case compared to a predicted contact pressures of 42.5 and 72.0 MPa for Examples 1 and 2 respectively.

CONCLUDING REMARKS

The primary objective of this paper was to develop a discrete element based computational procedure which incorporates viscoplastic softening effects and, to establish a procedure whereby the criteria governing fragmentation is related to the fragment size. The modified distinct element code is applied to examine the response of an elastic-viscoplastic layer which is subjected to surface indentation. The problem is of specific interest to the indentation of a polycrystalline rate-dependent materials such as ice. Conventional continuum based computational formulations, even when extended to include large strain phenomena, will not adequately model the brittle fragmentation, crushing, spalling and large scale movements associated with the indentation process. The development of a computational procedure which accommodates the transition from a continuum to a fragmented discontinuum is therefore an essential requirement for the current developments. The discrete element modelling of the interaction process is an alternative to the continuum modelling. Three cases have been examined, involving, the indentation of a brittle solid by a smooth, rigid indentor with different contact areas. The discrete element modelling provides an effective means of examining indentation response of polycrystalline rate-dependent materials. The results of the analyses essentially duplicates the prescribed dependency of the strength on the size of fragments i.e. a significantly larger collapse contact pressures are needed in order to cause global failure of smaller samples of solid block.

ACKNOWLEDGEMENT

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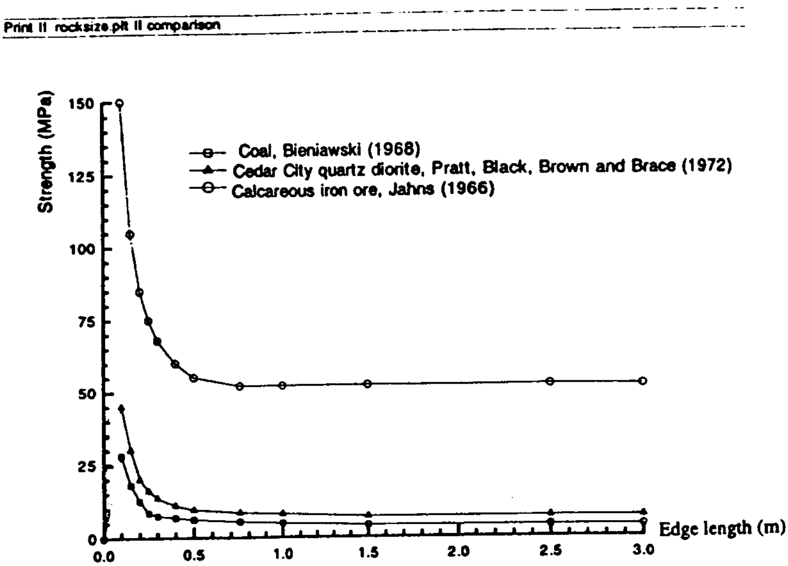

Figure 1: Effect of specimen size on unconfined compressive strength for rock

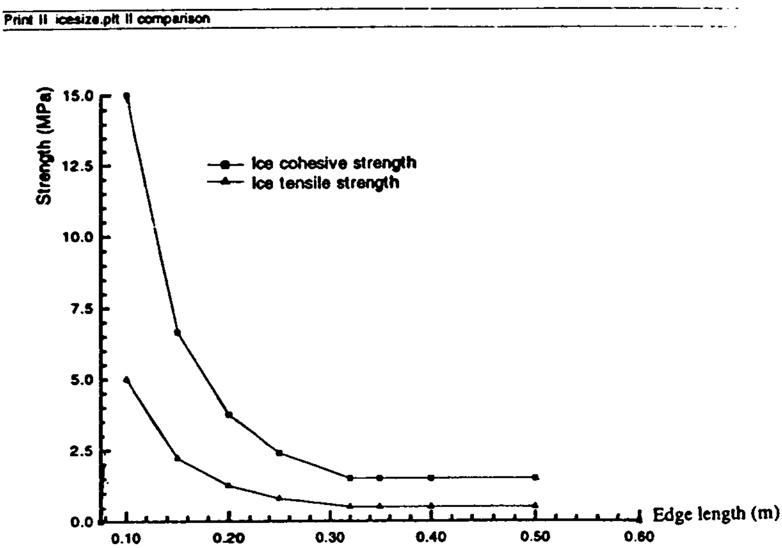

Figure 2: Assumed compressive and tensile strength versus specimen size for ice

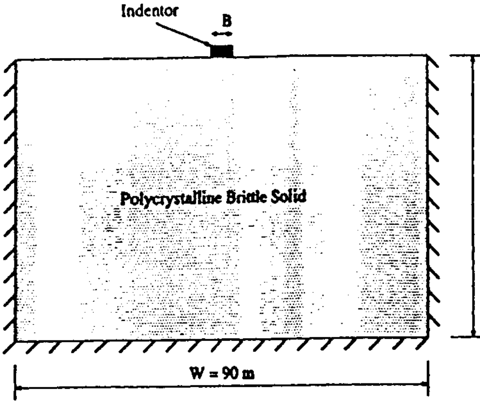

Figure 3: Geometry of the brittle solid and the rigid indentor

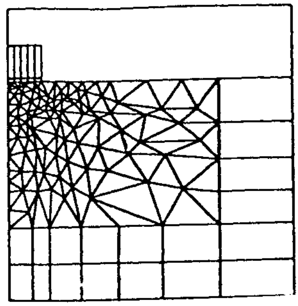

Figure 4: Discrete element mesh at the solid - indentor contact zone (B/H = 10/600)

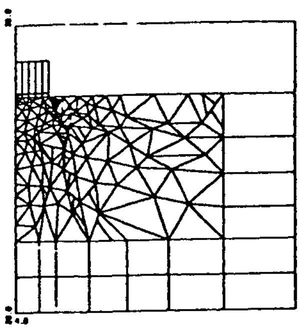

Figure 5: Fragmentation of brittle solid 0.12 seconds after interaction

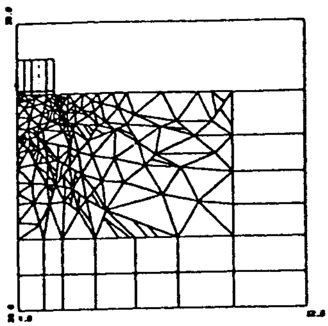

Figure 6: Fragmentation of brittle solid 0.18 seconds after interaction

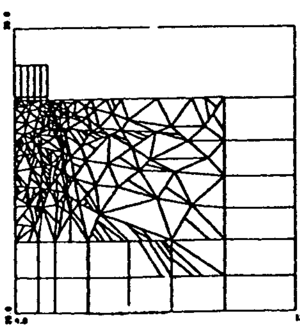

Figure 7: Fragmentation of brittle solid 0.24 seconds after interaction

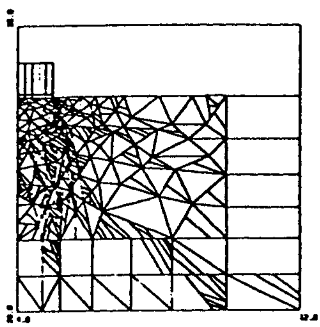

Figure 8: Fragmentation of brittle solid at collapse 0.3 seconds after interaction

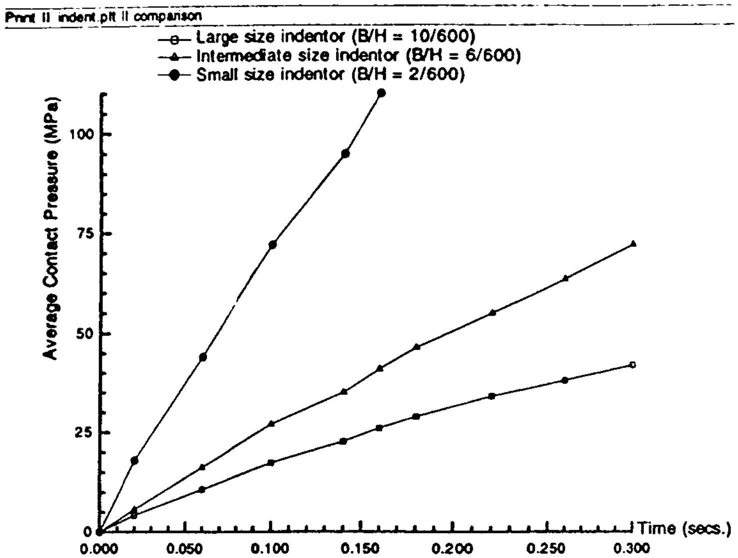

Figure 9: Average contact pressure versus time (indentor velocity = 0.1 m/s)

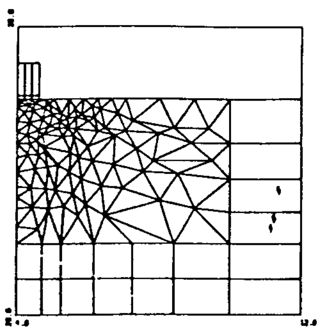

Figure 10: Discrete element mesh at the solid - indentor contact zone (B/H = 6/600)

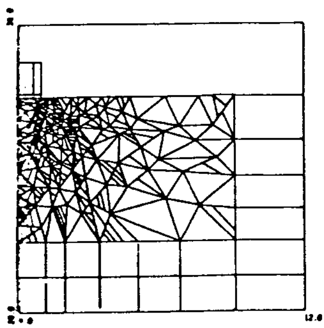

Figure 11: Fragmentation of brittle solid at collapse (intermediate size indentor)

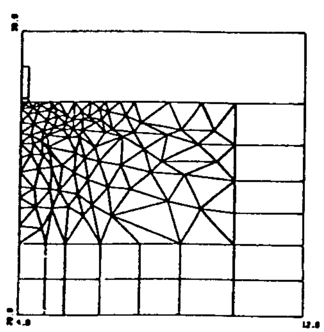

Figure 12: Discrete element mesh at the solid - indentor contact zone (B/H = 2/600)

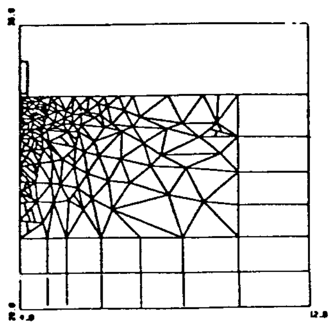

Figure 13: Fragmentation of brittle solid at collapse (small size indentor)