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On the influence of galaxy magnetic fields on the rotation curves in the outer discs of galaxies

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Summary. The dynamical effect of magnetic stress on the tenuous outer gaseous discs of galaxies is discussed using a simplified cylindrical model. It is shown that the effects can be very significant, yielding rotation velocities well above the gravitational orbital velocity based on the visible matter. If this conclusion persists in fully three-dimensional galaxy models then the interpretation of flat rotation curves in the outer discs as being due to hidden matter may be erroneous.

1 Introduction

One of the strongest pieces of evidence for the possibility of hidden matter existing in the universe is the flat rotation curves seen in H I in the outer parts of many galaxies (Bosma 1978). For these galaxies the H I rotation curves remain flat well outside the radius of the optical stellar disc, and if we interpret the rotation as being due to circular orbits in the galactic potential then the flatness of the curve must imply large amounts of hidden matter, since the orbital velocity should be roughly Keplerian outside the optical disc if most of the matter is in the luminous stars. My purpose in this paper is to suggest that such an interpretation may be wrong. The crucial point here is that where the rotation curves are anomolously high, the density of the H I gas drops to very low values and this may significantly alter the energy balance between magnetic energy and rotational energy. Consequently in the outer parts of galaxies the galactic magnetic field may play a primary role in large-scale gas dynamics, and the gas streamlines are not simple gravitational orbits.

H I observations show that the surface H I density drops suddenly by a factor of 10–50 from the edge of the optical disc to about twice this radius (Sancisi 1983), and taking into account the increase in the thickness of the gas disc of perhaps a factor of 10 we may expect a decrease in the volume density of H I by a factor of 100–500. It could be argued that the total density stays constant, with the ionized hydrogen component increasing as the H I decreases. However, this seems unlikely in that outside the optical disc the sources of ionizing radiation drop off rapidly. In addition recent observations of Ca II and Mg II absorption lines (Morton, York & Jenkins 1986) indicate that these ions have a sharp cut-off at between 8–18 Kpc for a sample of 12 galaxies, indicating low heavy ion densities at large radii. It therefore seems likely that the total gas density drops off rapidly outside the Holmberg radius, and we can therefore divide the gas disc of galaxies into two parts, a dense inner disc and a tenuous outer disc which I shall call the disc corona. Now in the dense disc with a field of say $3\,\mu$G, a density of 1H I atom cm^{-3} and a rotational velocity of $200\,km\,s^{-1}$ the rotational energy density is 400 times the magnetic energy density, therefore the field here has little dynamical influence; but in the disc corona, if the field remains as high as $3\,\mu$G, and the total density drops by 500 then the field energy would become comparable to the rotational energy density, and magnetic stresses would significantly affect the dynamics of the corona. The neutral H I gas, which exhibits the flat rotation curve, is not of course directly affected by the magnetic field; however, the gas is at least partially ionized, and mutual friction ensures that the ionized gas (inductively coupled to the magnetic field) moves very closely with the neutral gas. It is thus a good approximation to treat the whole gas as a perfect conductor [under assumption (ii) below] subject to the magnetic force [see equations (5) and (2) below].

2 Magnetic field model and equations

To quantify this idea a simple model of the field in the outer corona has been developed, allowing the motion of the disc to be calculated. The assumptions of the model are as follows:

(i) The magnetic field **B** is generated by dynamo processes in the dense disc and is spiral in form (Sofue, Fujimoto & Wielebinski 1986) – see Fig. 1.

(ii) Turbulence in the corona is small, therefore there is no magnetic dynamo action or diffusion here, simply a frozen-in field.

(iii) Thermal pressure is assumed to be negligible in the corona in comparison to the magnetic stress.

(iv) The field pattern rotates with fixed angular velocity Ω, and in the rest frame of the field the field lines have the equation, in polar coordinates,

$$r_B = r_0 \exp(\theta/\alpha) \qquad \text{with } \tan i = \frac{1}{|\alpha[1+(r\theta/\alpha^2)(d\alpha/dr)]|}$$

and α is a function of radius (it turns out that the pitch angle i increases outwards).

(v) $\mathbf{B}=(B_r, B_\theta, 0)$, $\mathbf{v}=(v_r, v_\theta, 0)$, i.e. **B** and **v** are wholly in the plane.

(vi) All variables are independent of θ and time in the rest frame of **B**. Strictly speaking the bisymmetric spiral fields observed are not independent of θ, with the field going alternately in and out in different parts of the galaxy (Sofue *et al.* 1986). However, this assumption greatly simplifies

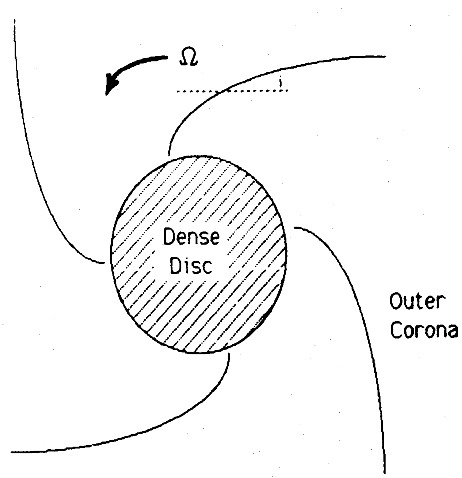

Figure 1. Geometry of the spiral field lines in the outer disc corona.

the equations, and the effects of the magnetic stress are independent of whether the field is going in or out.

(vii) All variables are independent of z, i.e. we consider an idealized cylindrical galaxy only. This avoids complications connected with the vertical equilibrium of the gas disc, and yields a particularly simple solution for the horizontal dynamics. The implications of this idealization are discussed later.

(viii) Gravitational potential $\propto 1/r$ (i.e. no massive halo).

The equations that have to be solved are the continuity and momentum equations for the gas, Maxwell's equations for **B**, and the infinite conductivity Ohm's Law. With the above assumptions these can be reduced to a single ordinary differential equation for the function α, which can easily be solved numerically. The set of equations in the rotating rest frame of **B** is:

$$\nabla \cdot (\varrho \mathbf{v}) = 0, \tag{1}$$

$$\varrho(\mathbf{v} \cdot \nabla)\mathbf{v} + 2\varrho \mathbf{\Omega} \times \mathbf{v} + \varrho \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) = -\varrho \nabla \phi + \mathbf{j} \times \mathbf{B}, \tag{2}$$

$$\mathbf{j} = \frac{\nabla \times \mathbf{B}}{\mu_0}, \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{4}$$

and

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = 0. \tag{5}$$

Gauss's Law (4) in cylindrical polar coordinates using assumptions (v) and (vi) is

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) = 0,$$

i.e.

$$B_r = \frac{\beta}{r}$$

where β is a constant, while from assumption (iv) we have

$$\frac{B_r}{B_\theta} = \frac{1}{r_B}\frac{dr_B}{d\theta} = \frac{1}{\alpha},$$

i.e.

$$B_\theta = \frac{\alpha(r)\beta}{r}.$$

It is then easy to show from equation (3) that

$$(\mathbf{j} \times \mathbf{B})_r = -\frac{\alpha\beta^2}{\mu_0 r^2}\frac{d\alpha}{dr}$$

and

$$(\mathbf{j} \times \mathbf{B})_\theta = \frac{\beta^2}{\mu_0 r^2}\frac{d\alpha}{dr}.$$

The r and θ components of the momentum equation (2) are

$$v_r \frac{dv_r}{dr} - \frac{v_\theta^2}{r} - 2\Omega v_\theta - \Omega^2 r = -\frac{\partial \phi}{\partial r} + \frac{(\mathbf{j}\times\mathbf{B})_r}{\varrho}$$

and

$$v_r \frac{dv_\theta}{dr} + \frac{v_r v_\theta}{r} + 2\Omega v_r = \frac{(\mathbf{j}\times\mathbf{B})_\theta}{\varrho}.$$

Here v_θ=the azimuthal velocity in the rotating frame$=v_{\rm rot}-r\Omega$, where $v_{\rm rot}$ is the rotational velocity in the inertial frame. From the continuity equation (1) in cylindrical polars we have, using assumptions (v) and (vi),

$$\frac{1}{r}\frac{\partial}{\partial r}(r\varrho v_r)=0$$

therefore

$$\varrho v_r = \frac{m}{r} \qquad (m=\text{const}).$$

Then substituting for ϱ in the θ component of the momentum equation and dividing by v_r we obtain

$$\frac{dv_\theta}{dr} + \frac{v_\theta}{r} + 2\Omega = \frac{\beta^2}{m\mu_0 r}\frac{d\alpha}{dr} = \frac{\gamma}{r}\frac{d\alpha}{dr} \qquad (\gamma=\text{const.}),$$

i.e.

$$rv_\theta + \Omega r^2 = \gamma\alpha + C \qquad (C=\text{const.}),$$

therefore

$$v_{\rm rot} = v_\theta + r\Omega = \frac{\gamma\alpha(r)+C}{r}.$$

while if

$$\frac{\partial\phi}{\partial r} = \frac{v_{\rm orb}^2}{r} \qquad \left(v_{\rm orb} \propto \frac{1}{r^{1/2}}\right)$$

then the r-component of the momentum equation yields

$$v_r \frac{dv_r}{dr} + \frac{v_{\rm orb}^2 - v_{\rm rot}^2}{r} = -\gamma\alpha \frac{v_r}{r}\frac{d\alpha}{dr},$$

i.e.

$$\frac{dv_r}{dr} + \frac{v_{\rm orb}^2 - v_{\rm rot}^2}{rv_r} = -\frac{\gamma\alpha}{r}\frac{d\alpha}{dr}. \qquad (6)$$

But equation (5) implies

$$\frac{\partial}{\partial r}(v_r B_\theta - v_\theta B_r)=0,$$

i.e.

$$v_r B_\theta - v_\theta B_r = \text{const}.$$

We choose solutions such that the constant here is zero, i.e.

$$\frac{v_r}{v_\theta}=\frac{B_r}{B_\theta},$$

therefore

$$v_r=\frac{1}{\alpha}(v_{\rm rot}-r\Omega)=\frac{1}{\alpha}\left(\frac{\gamma\alpha+C}{r}-r\Omega\right).$$

Substituting this for v_r into equation (6) we obtain the following equation for α:

$$\frac{d\alpha}{dr}\left[\frac{\gamma}{r}\left(\frac{1}{\alpha^2}+1\right)-\frac{1}{\alpha^3}(v_{\rm rot}-r\Omega)\right]=\frac{(v_{\rm rot}^2-v_{\rm orb}^2)}{r(v_{\rm rot}-r\Omega)}+\frac{1}{\alpha^2}\left(\frac{\gamma\alpha+C}{r^2}+\Omega\right). \quad (7)$$

3 Results and discussion

Equation (7) can be easily solved by the predictor–corrector method and the results are shown in Figs 2 and 3. These calculations start at an initial corona radius of 10 kpc, and extend out to 50 kpc. The initial values of the Alfven velocity $[=\sqrt{(B^2/\varrho\mu_0)}]$, $v_{\rm orb}$, and $v_{\rm rot}$ at 10 kpc are 50, 200 and 199 km s^{-1} respectively, and the four curves in Fig. 2 correspond to initial pitch angles of 22°, 24°, 27° and 34°. As can be seen, the rotational velocities are maintained significantly higher than the gravitational orbital velocity, although the shapes of the curves are not flat. It may be that further refinement of the model (e.g. finite magnetic diffusion, vertical structure with varying thickness of the disc and field) might produce a flatter curve. At any rate it can be concluded from this calculation that it may be very unwise to ignore the effects of **B** when considering the dynamics of gas in the disc corona, particularly when one notes that the values of the Alfven and rotational velocities used at the initial radius correspond to a magnetic energy density 16 times lower than the rotational energy density. Further out this rapidly reverses as the density of the gas drops off, with the magnetic field dropping much less rapidly so that the Alfven velocity increases outwards (see Fig. 3). In addition the radial velocity increases outwards from an initial value of approximately 1 km s^{-1} at 10 kpc to about 400 km s^{-1} at 50 kpc, so that an essential feature of this model is a magnetically driven wind along the galactic plane.

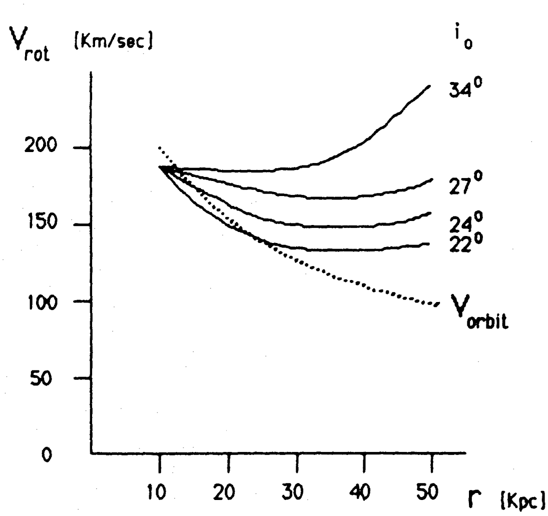

Figure 2. Rotational velocity versus r for various values of i_0. The dotted curve shows the orbital velocity in the gravitational field alone.

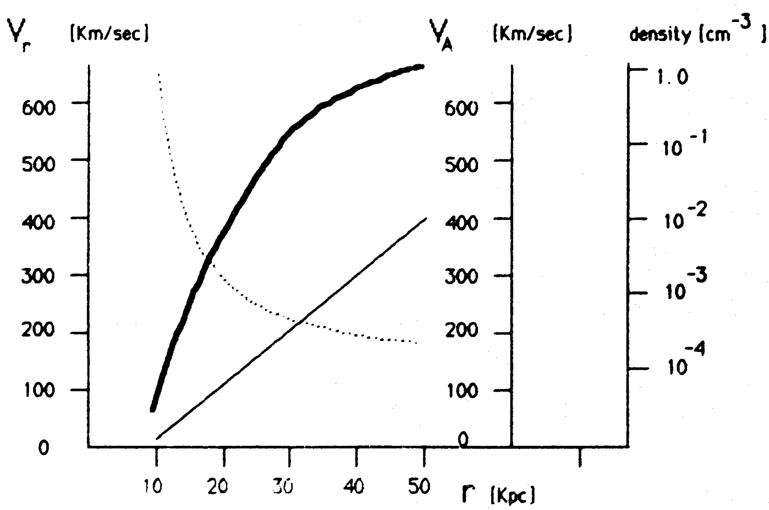

Figure 3. Radial velocity (continuous line), Alfven velocity (bold line), and gas density (broken line) versus r for the case of i_0=24°.

A little more thought reveals why the magnetic field is so influential even when it starts off with a relatively low energy density. In order for the magnetic stress to play an important role in the dynamics of a gas flow with radial velocity v_r we require that

magnetic torque ~ rate of change of angular momentum outwards,

i.e.

$$|\mathbf{r}\times(\mathbf{j}\times\mathbf{B})|\sim v_r\,\frac{\partial}{\partial r}\,(\varrho v_\theta r).$$

Therefore we need only

$$\frac{B^2}{\mu_0}\sim\varrho v_r v_\theta$$

which for small v_r near the edge of the dense disc is a much less stringent condition than

$$\frac{B^2}{\mu_0}\sim\varrho v_\theta^2.$$

Finally we note that the above radial velocity obtained at large radii is rather high in comparison to the observational limits. However, observations of the H I velocity fields of galaxies do not usually extend to more than a few Holmberg radii and often show disturbed velocity in the outer regions. This has usually been interpreted as due to warping alone, but a contribution from radial motion with radial velocities approaching 50 km s^{-1} may not be inconsistent with the observations. If we interpret 20 kpc in the simple model here as 2 Holmberg radii then at that position we have a radial velocity of 100 km s^{-1} from the model, which may not be quite so out of line with the observations. In addition we would expect the effect of finite magnetic diffusion and realistic vertical structure to weaken the magnetic torque effect so that $v_{\rm rot}$ would not rise so rapidly at large radii, and v_r would not attain such large values. The high Alfven velocity at large radii implies that in a disc of finite thickness, in which the galaxy field also has a finite thickness, the field pressure would cause expansion of the disc and field at large radii, to the point at which magnetic tension, with the field lines anchored in the dense disc, balances the pressure; i.e. we would expect the field to be nearly force-free in the vertical direction, but not in the horizontal direction, with the field strength along the plane dropping faster than roughly $1/r$ as in the cylindrical model. To estimate how much this would affect the magnetic torque requires a fully three-dimensional model.

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