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On the injection of electrons in oblique shocks

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ABSTRACT

The mechanism by which superthermal electrons generate turbulence inside and behind an oblique shock is studied, and the implications for electron injection are considered. It is shown that, whereas in quasi-parallel shocks the streaming instability dominates, in superluminal shocks the waves are driven unstable by compressional anisotropies produced as a result of betatron acceleration of electrons traversing the shock. The injection process, in the diffusion approximation, is controlled by the structure of the shock and the corresponding diffusion coefficient, and appears to be markedly different for quasi-parallel than for superluminal shocks, owing to the differences between the field-aligned and cross-field transport properties. The requirements for efficient injection are examined and found to be less stringent in quasi-parallel shocks than in superluminal shocks. A naïve estimate suggests that efficient injection by this process may take place in superluminal shocks with Mach numbers in excess of $\sim$100, provided that the shocked electron plasma undergoes very effective collisionless heating. The implications of cross-field diffusion by field-line wandering for electron injection in perpendicular shocks are also discussed.

Key words: diffusion – radiation mechanisms: nonthermal – shock waves.

1 INTRODUCTION

The injection and acceleration of electrons by collisionless shocks is a problem of considerable interest in astrophysics. Although the electrons provide our main *in situ* probes of shock acceleration in most astrophysical sites, the physics of electron injection and acceleration is poorly understood. Much work has been concerned with the acceleration of relativistic electrons (for recent reviews see Eichler 1992, 1994; Blandford 1992, 1994; Biermann 1995). Ellison & Reynolds (1991) have studied the effects of shock non-linearity on the spectrum of accelerated electrons, using Monte Carlo simulations. Reynolds & Ellison (1992) have subsequently shown that non-linear effects are reflected in the radio spectra of several young SNRs, and illustrated how this may enable one to estimate the magnetic field strength in those systems. The overall normalization of the electron spectrum is treated, in their analysis, as a free parameter. (For further discussions on non-linear effects, see Ellison 1994.) Achterberg, Blandford & Reynolds (1994), and Reynolds (1994) have argued that the presence of sharp (unresolved) synchrotron edges in high-resolution radio maps of young, bright SNRs is indicative of enhanced turbulence ahead of the shock, which is one of the characteristic features of diffusion shock acceleration models. Eichler & Usov (1992) considered electron acceleration in colliding wind shocks of Wolf–Rayet binaries, and the implications for radio and γ-ray emission. Motivated by young SNRs where the shock is expanding into the progenitor's stellar wind, Achterberg & Ball (1994) have carefully examined the conditions required for efficient electron acceleration in perpendicular shocks, and have applied their analysis to SN1987A and SN1978K.

However, in most of those discussions, the injection of electrons from the thermal pool to mildly relativistic energies has been neglected. Of fundamental importance is the determination of injection efficiency as a function of shock velocity and obliquity (a related issue is the extent of electron heating in the shock). The synthetic radio images generated by Reynolds (1994) demonstrate that the morphology of young SNRs reflects, essentially, this dependency of injection efficiency on shock speed and obliquity. Biermann & Cassinelli (1993) have argued that a

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comparison of Wolf-Rayet stars and radio SNRs suggests that the injection efficiency of electrons in perpendicular shocks appears to be very sensitive to shock speed; the efficiency drops by several orders of magnitude when the shock speed becomes smaller than some critical value, which they estimated to be $\sim 0.03c$.

Electron injection at non-linear, parallel shocks has been addressed recently by Levinson (1992, 1994), and Bykov & Uvarov (1993). Both authors concluded (although on somewhat different grounds) that efficient injection requires relatively high Mach number shocks (but as we shall argue below, not as high as those required in the case of perpendicular shocks). In Section 2.1 we generalize the injection mechanism mentioned above to quasi-parallel shocks. The rest of the paper is devoted to studying some aspects of diffusive accelerations (as opposed to drift acceleration) in perpendicular shocks. Specifically, we shall concentrate on the plasma physics associated with injection by self-generated turbulence from thermal to mildly relativistic energies, and examine the conditions under which efficient injection may take place, by solving the corresponding transport equation. The appropriate transport equation is derived in the Appendix.

There are several competitive processes that may give rise to electron energization in quasi-perpendicular shocks. Galeev (1984) proposed a mechanism whereby electrons were accelerated in the ion precursor through the interaction with ions reflected from the shock. The acceleration efficiency in this process depends on the distribution and the relaxation of reflected ions upstream, which is still an open issue, in particular in very high Mach number shocks. Non-resonant scattering in the shock by non-linear, small-scale structure in the ramps may also be important. In this paper we shall not consider such processes, and will focus merely on injection into the first-order Fermi process.

2 ELECTRON INJECTION BY SELF-GENERATED WAVES IN OBLIQUE SHOCKS

2.1 Electron injection in quasi-parallel shocks

We now generalize the injection mechanism proposed by Levinson (1992, 1994) to quasi-parallel shocks. We suppose that the anisotropy of the electron distribution function is small. To second order in U_-/v, where U_- is the upstream fluid velocity and v is the velocity of a resonant electron, we obtain from equation (A3), after averaging over pitch angle, an equation for the isotropic part of the distribution, f_0,

$$U\frac{\partial f_0}{\partial z}-\frac{\partial}{\partial z}(\kappa_\parallel \cos^2\psi+\kappa_\perp \sin^2\psi)\frac{\partial f_0}{\partial z}-\frac{1}{3}\frac{\mathrm{d}U}{\mathrm{d}z}p\frac{\partial f_0}{\partial p}=0, \qquad (1)$$

where ψ is the angle between the shock normal and the magnetic field, $\kappa_\perp$ is the cross-field diffusion coefficient given by equation (A4), and $\kappa_\parallel$ is the field-aligned diffusion coefficient, and is given by equation (4) of Levinson (1992). We also have an equation relating the anisotropic part of the electron distribution function, f_1, to f_0,

$$\frac{\partial f_1}{\partial \mu}=-\frac{v\cos\psi}{2\nu_s}\frac{\partial f_0}{\partial z}. \qquad (2)$$

Here ν_s is the pitch angle scattering rate (see equation A6). The above equations must be supplemented by an equation for the growth rate of the whistler instability and an equation describing the evolution of the wave spectral intensity.

The linear growth rate for whistlers can be written as (Melrose 1986)

$$\gamma_w=\frac{\pi^2 k}{2n_e}\int_{-1}^{1}\mathrm{d}\mu(1-\mu^2) \times\left[p^3 v\left(\chi\frac{\partial f}{\partial\mu}+\frac{v_w^2}{v^2|\mu|}p\frac{\partial f}{\partial p}\right)\left(1-\frac{\mu}{|\mu|}\chi\right)^2\right]_{p=p_R}, \qquad (3)$$

where $v_w=(m_p/m_e)^{1/2}v_A$, v_A is the Alfvén velocity, p_R is a resonant momentum, given by $p_R=m_e\Omega_e/k|\mu\chi|$, where Ω_e is electron gyrofrequency, χ is the cosine of the angle between the wavevector and the direction of the magnetic field, and n_e is the electron density of the background plasma. Substituting equation (2) into equation (3), and assuming that the electron distribution is a power law, we find, using the analysis of Levinson (1992), that growth occurs when the upstream velocity satisfies

$$U_->\frac{\cos^2\psi+\Delta^2\sin^2\psi}{|\chi|\cos\psi}v_\phi \qquad (4)$$

omitting factors of order unity (see Levinson 1992 for details). In the last equation $\Delta=\kappa_\perp/\kappa_\parallel\simeq r_g\nu_s/v$, where r_g is the gyroradius of a resonant electron. Typically, $\Delta \ll 1$. At angles for which $\tan\psi\ll\Delta$, the last term on the right-hand side of equation (4) can be neglected. It then follows from equation (4) that in this regime the condition for wave growth is given, to a good approximation, by equation (9) of Levinson (1992) with v_ϕ replaced by $v_\phi\cos\psi$. Moreover, in this limit, cross-field diffusion can be neglected. The assumption that f_1 is small holds when $U_-/v>\cos\psi$. The foregoing analysis then indicates that when the angle between the shock normal and the magnetic field $\psi>\cos^{-1}(U_-/v)$, the anisotropy of the electron distribution function inside the shock is predominantly due to the streaming of electrons along field lines. When ψ exceeds this value, the anisotropy along field lines is too small to derive instability. The upstream turbulence in this case should be due to other processes. We can now use the equation for the spectral intensity (see equation 12) together with equation (1) to determine the field-aligned diffusion coefficient. The result is

$$\kappa_\parallel\simeq\frac{2U_-n_e r_g}{\pi n_c\cos\psi}, \qquad (5)$$

where $n_c=4\pi p^3 f_0(p)$ is the total number density of cosmic-ray electrons with momentum greater than p, and n_e is the electron density of the bulk plasma. We see that the diffusion coefficient is inversely proportional to the density of injected electrons, which leads to a negative feedback (Levinson 1994). Equation (5) generalizes the result obtained earlier for strictly parallel shocks. In view of this negative feedback, we anticipate that the injection efficiency will be also insensitive to obliquity.

2.2 Electron injection in perpendicular shocks

In superluminal shocks, particles, once they have been swept downstream by the shock, cannot recross the shock into the upstream region by sliding (or diffusing) along magnetic field lines. Shock recrossing may only be accomplished by cross-field diffusion, which requires sufficiently strong scattering behind the shock. In the absence of cross-field transport, particles traversing the shock can gain energy through the shock-drift mechanism (for reviews see Toptyghin 1980; Drury 1983), but the maximum energy gain is limited to $\Delta E/E$ of order a few. In order to produce power-law distribution behind the shock, particles must scatter (diffuse) back and forth across the shock many times. Diffusive acceleration of relativistic particles in perpendicular shocks has been considered by Jokipii (1982, 1987), who was particularly concerned with the dependence of the acceleration rate on shock obliquity. More recently, Achterberg & Ball (1994) have reconsidered this problem in the context of young supernova blast wave expanding into a stellar wind. They have discussed, in some detail, cross-field transport of electrons induced by both resonant scattering and field line wandering, and the applications to the radio emission from SN1978K and SN1987A. However, they have addressed neither the physics of injection, which is crucial for the determination of the overall normalization of the electron spectrum, nor the generation of turbulence ahead and behind the shock.

In this section, we focus on electron injection from thermal to mildly relativistic energies by self-generated waves in perpendicular shocks. The idea behind the mechanism proposed below is fairly simple: first, it should be noted that the thickness of the shock is typically of order a few ion gyroradii, which is much larger than the gyroradius of thermal electrons (by roughly a factor of $[m_pT_{+p}/m_eT_{+e}]^{1/2}$, where $T_{+\alpha}$ is the downstream temperature of a species α). Therefore, thermal electrons will see the shock as a smooth transition rather than a plane discontinuity, and the adiabatic approximation is applicable. Now, as a result of compression of the magnetic field in the shock, the perpendicular momentum of an electron traversing the shock will increase such that the first adiabatic invariant $p_\perp^2/2B$ is preserved (the betatron effect). Consequently, an isotropic electron distribution transmitted through the shock will develop compressional anisotropy (depicted in Fig. 1), resulting in generation of whistler waves when $p<p_A\equiv m_p v_A$ or Alfvén waves when $p>p_A$ (e.g. Melrose 1980). The excited waves, in turn, would give rise to pitch-angle scattering, thereby maintaining the anisotropy at a level at which wave growth is balanced by energy-loss processes. The resonant scattering will also induce cross-field transport, causing electrons inside the shock to diffuse in the upstream direction. This injection mechanism is essentially similar to the one proposed for quasi-parallel shocks (see also Section 2.1), but with one important difference; whereas in parallel shocks the injection efficiency increases with decreasing v_s, by virtue of the shock non-linearity and the fact that the field-aligned diffusion coefficient is inversely proportional to v_s, in perpendicular shocks the efficiency increases with increasing v_s, because the cross-field diffusion is proportional to v_s. This leads to a positive feedback which may suppress the injection efficiency considerably, except, perhaps, for very high Mach number shocks. Consequently, the conditions required for effective electron injection are expected to be more stringent in perpendicular shocks.

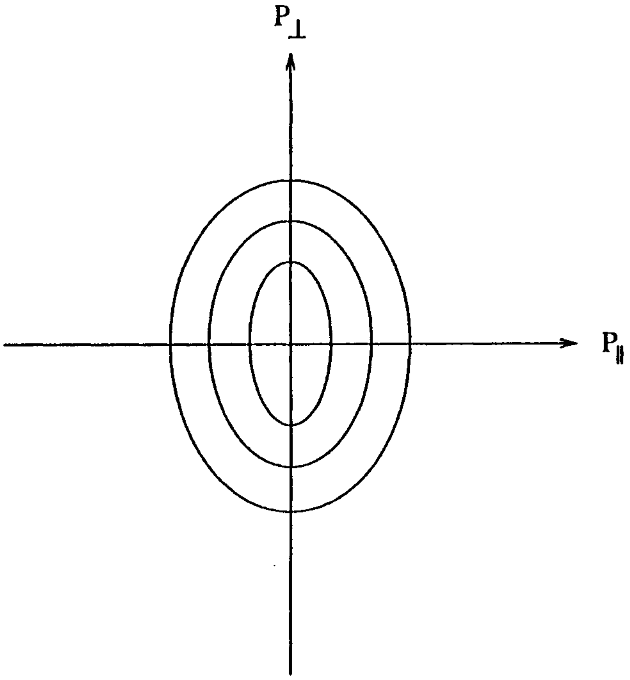

Figure 1. Schematic contour plot of a compressional distortion distribution.

We now consider a quantitative demonstration of this mechanism. For simplicity, we consider the idealized case of a planar shock propagating in the $-z$ direction. Because, as mentioned above, the shock thickness largely exceeds the gyroradii of injected electrons, the electron distribution function is effectively independent of gyrational phase, allowing one to average the transport equation over this coordinate (e.g. Blandford & Eichler 1987; Schlickeiser 1989). In the case of relativistic electrons for which the gyroradius exceeds the shock thickness, finite gyro-orbit effects may become important, unless, perhaps, the effective scattering rate ahead and behind the shock is extremely large, as often assumed. Even though the adiabatic theory is not applicable in this case, the magnetic moment is still preserved to a good approximation when a particle crosses the shock (Toptyghin 1980), and self-generation of Alfvén waves by the compressional distortion distributions behind the shock will still take place.

For a strictly perpendicular shock, the steady-state transport equation for the electron distribution, averaged over gyrational phase, reduces to (see Appendix)

$$U\frac{\partial f}{\partial z}-\frac{\partial}{\partial z}\kappa_\perp\frac{\partial f}{\partial z}-\frac{(1-\mu^2)}{2}\frac{\mathrm{d}U}{\mathrm{d}z}\left(p\frac{\partial f}{\partial p}-\mu\frac{\partial f}{\partial\mu}\right)=\frac{\partial}{\partial\mu}\left[\frac{(1-\mu^2)}{2}v_s\frac{\partial f}{\partial\mu}\right], \tag{6}$$

where $U(z)$ is the local flow velocity, $\kappa_\perp$ is the cross-field diffusion coefficient and is given by equation (A4), μ is the cosine of the particle's pitch angle, and v_s is the pitch-angle scattering rate given by equation (A6). We have neglected terms involving gradients along the shock front (i.e. in the X and Y directions). Note that the last term on the LHS can be written as

$$\frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}p_\perp\frac{\mathrm{d}p_\perp}{\mathrm{d}t}f,$$

with $\mathrm{d}p_\perp/\mathrm{d}t=(p_\perp/2B)\mathrm{d}B/\mathrm{d}t$ (which follows from the continuity of the electric field across the shock, i.e. UB=constant), representing the betratron effect. In the absence of cross-field transport, i.e. $v_s=\kappa_\perp=0$, equation (6) can be readily solved analytically. The distribution is constant along the characteristic $p_\perp(z)$, with $p_\perp(z=0)$ determined by the distribution upstream. If the upstream distribution is isotropic then the transmitted distribution is a compressional distortion, and is unstable. This would lead to rapid generation of turbulence inside the shock which would scatter the electrons, and would limit the anisotropy. In this case one can solve equation (6) using standard perturbation theory, whereby the parameter $v_s^{-1}(\mathrm{d}U/\mathrm{d}z)$ serves as the smallness parameter. Averaging over pitch angle gives, to the lowest order, the standard convection–diffusion equation,

$$U\frac{\partial f_0}{\partial z}-\frac{\partial}{\partial z}\kappa_\perp\frac{\partial f_0}{\partial z}-\frac{1}{3}\frac{\mathrm{d}U}{\mathrm{d}z}p\frac{\partial f_0}{\partial p}=0. \tag{7}$$

Subtracting the latter equation from equation (6), we obtain, to the next order, an equation for f_1,

$$\frac{\partial f_1}{\partial \mu}=-\frac{\mu}{3v_s}\frac{\mathrm{d}U}{\mathrm{d}z}p\frac{\partial f_0}{\partial p}. \tag{8}$$

Note that if $v_s(\mu)$ is symmetric (which is the last case here) f_1 is an even function of μ such that (to first order) the electron distribution f_0+f_1 is a compressional distortion. We can employ now the expression obtained for f_1 in order to compute the linear growth rate for whistlers inside the shock.

The linear growth rate is given by equation (3). By employing equation (8), we can now express the growth rate in terms of the isotropic part of the distribution function

$$\gamma_w=-\frac{\pi^2 k}{2n_e}\int_{-1}^{1}\mathrm{d}\mu(1-\mu^2)p^4 v\frac{\partial f_0}{\partial p}\times\left[\left(\frac{\chi\mu}{3v_s}\frac{\mathrm{d}U}{\mathrm{d}z}-\frac{v_w^2}{v^2|\mu|}\right)\left(1-\frac{\mu}{\mu|}\chi\right)^2\right]_{p=p_R}. \tag{9}$$

For a given wavevector, the dominant contribution to the integral comes from electrons propagating opposite to its direction, namely, those for which the pitch angle satisfies $\chi\mu<0$. For a power-law distribution, $f_0\propto p^{-\alpha}$; ($\alpha\geq 4$), and assuming that v_s is a slowly varying function of μ (except maybe for small μ), the integral on the RHS of equation (9) can be performed analytically. To a good approximation, we find that the condition for wave growth (i.e. $\gamma_w>0$) is

$$\frac{\chi}{3v_s}\left|\frac{\mathrm{d}U(z)}{\mathrm{d}z}\right|>\frac{v_w^2}{v^2}=2\beta_e^{-1}\left(\frac{V_e}{v}\right)^2, \tag{10}$$

(note that $\mathrm{d}U/\mathrm{d}z<0$), where V_e is the electron thermal speed and β_e is the ratio of thermal electron pressure to magnetic pressure. Since β_e inside the shock is anticipated to exceed unity as a result of heating, condition (10) will be satisfied for electrons having $v>V_e$. Now, in the absence of any energy-loss processes (e.g. non-linear mode coupling, wave convection) other than damping by thermal electrons, the waves will reach a steady state wherein $\gamma_w=0$. The pitch-angle-scattering rate can then be readily found using equation (10). In terms of the shock thickness $L_{sh}=U/(\mathrm{d}U/\mathrm{d}z)$ and the gyroradius of electrons with $v=U$, namely, $r_g(U\_)=U\_/\Omega_e$, it takes the form

$$\frac{v_s}{\Omega_e}\simeq\frac{1}{6}\frac{r_g(U\_)}{L_{sh}}\beta_e\left(\frac{v}{V_e}\right)^2\simeq\frac{\beta_e}{6}\frac{m_e}{m_p}\left(\frac{v}{V_e}\right)^2, \tag{11}$$

where it has been assumed that L_{sh} is of the order of the downstream thermal ion gyroradius. The assumption that non-linear processes can be ignored is likely to break down when the ratio of wave energy density, ε_w, to the energy density of the background magnetic field, $\varepsilon_B=B^2/8\pi$, which for whistlers is given approximately by (Levinson 1992; see also Appendix) $\varepsilon_w/\varepsilon_B\simeq(4/\pi)(v_s/\Omega_e)$, approaches unity. This happens when the electron momentum approaches $p_A=m_p v_A$. Electrons with higher momenta will generate Alfvén waves that will give rise to pitch-angle scattering at a rate $\sim\Omega_e$.

The waves produced in the shock will be convected downstream by the fluid. The associated energy flux is $U\varepsilon_w$. (One way to obain this result is by Lorentz transforming the wave electromagnetic field from the fluid frame into the shock frame and then calculating the corresponding Poynting flux). The resultant energy loss rate must balance the net linear growth rate. This is described by the equation

$$U(z)\frac{\partial\varepsilon_w(k,\chi,z)}{\partial z}=2\gamma_w(k,\chi,z)\varepsilon_w(k,\chi,z). \tag{12}$$

Equations (7), (9) and (12) can be integrated now for a given shock profile in order to yield the distribution of injected electrons inside and downstream of the shock. The structure of the shock is expected to be complicated on these scales because the ions cannot be treated as a fluid. Nevertheless, it may still be possible to define an average flow velocity $U(z)$, and assume that, to some extent, the thermodynamic variables follow the fluid equations. This will be done in a follow-up paper (Levinson 1995, in preparation). Here, we estimate v_s by replacing $\partial/\partial z$ with L_{sh}^{-1} in the last equation, and again assume that $p\partial f/\partial p\sim f$. This yields

$$\frac{v_s}{\Omega_e}\simeq\frac{n_c}{3n_e}\left(1+\frac{n_c v_w^2 L_{sh}\Omega_e}{n_e v^2 U\_}\right)^{-1}, \tag{13}$$

where $n_c=4\pi p^3 f(p)$ is roughly the total number density of injected electrons with momentum $>p$. This result generalizes equation (11). (When v exceeds $[m_p/m_e]v_A$, the term v_w^2/v^2 in the parentheses on the right-hand side of the last equation must be replaced by v_A/v.) Evidently, the scattering rate of injected electrons with sufficiently small velocities is limited by collisionless damping of waves by the background electrons, whereas that of more energetic electrons is limited by wave convection. It is also expected from equation (7) that the injection efficiency will depend quite sensitively on the downstream electron temperature (in contrast to the injection in parallel shocks). In fact, inspection of equation (11) suggests that efficient injection may take place when the ratio of the diffusion length of thermal electrons to shock thickness,

$$\frac{\kappa_\perp}{U_- L_{sh}} \simeq \frac{\beta_{e+}}{6\gamma M_s^2}\left(\frac{T_{e+}}{T_{e-}}\right)\left(\frac{m_p}{m_e}\right), \quad (14)$$

where M_s is the sound Mach number, and γ is the adiabatic index, is not much smaller than unity. In deriving the last equation, we have used equation (13) and assumed that $n_c/n_e \sim 1$ for thermal electrons just behind the shock. Condition (14) might be satisfied for strong shocks when the downstream plasma is in rough equipartition; that is, when the downstream electron temperature approaches the Rankine–Hugoniot value, T_+^{eq}. This, in turn, requires very strong coupling via collective processes between ions and electrons in the shock, which might be accessible in high Mach number shocks (Cargil & Papadopoulos 1988), as also indicated by X-ray observations of young SNRs (e.g. Draine & McKee 1993). To elucidate the Mach number required for efficient injection, we employ a rudimentary shock model. We consider a strong shock; that is, high Alfvén and sound Mach numbers, and suppose that the equation of state is $h = (\gamma/\gamma-1)P$, where h is the specific enthalpy and P is the pressure. The adiabatic index is taken to be constant across the shock. The jump conditions then yield (e.g. Draine & McKee 1993), $kT_+^{eq} = (2(\gamma-1)/(\gamma+1)^2) m_p U_-^2$. Let us assume further that the electron temperature downstream is a fraction η of T_+^{eq}. We then find $\beta_{e+} = (2\gamma(\gamma-1)^2/(\gamma+1)^3)\beta_{e-}\eta M_s^2$. Substituting these results into equation (14) gives

$$\frac{\kappa_\perp}{U_- L_{sh}} \simeq \frac{2\gamma(\gamma-1)^3}{3(\gamma+1)^5}\left(\frac{m_e}{m_p}\right)\beta_{e-}\eta^2 M_s^2. \quad (15)$$

Under the physical conditions anticipated, $\beta_{e-} \sim 1$. Taking $\gamma = 5/3$, we find that the Mach number required for efficient injection is

$$M_s \gtrsim 100\eta^{-1}. \quad (16)$$

Thus, even when electrons undergo substantial heating behind the shock (i.e. $\eta \sim 1$), very high Mach numbers are required.

For the sake of completeness, we consider the implications of cross-field transport by field-line braiding for electron injection. A rigorous treatment of cross-field diffusion due to field-line wandering in different scattering regimes is given by Achterberg & Ball (1994). The corresponding diffusion coefficient depends in general on the power spectrum of the magnetic fluctuations. In the case of the narrow-band spectrum, it can be approximated as (Jokipii 1966),

$$\kappa_\perp = vL_c(\delta B/B_0)^2,$$

where L_c is the correlation length and is of the order of the dominant wavelength, B_0 is the mean field, and δB is the corresponding amplitude of the fluctuations. In the case of a broad-band power spectrum $\kappa_\perp$ is reduced by some factor, $R_q \geq 1$, associated with the decorrelation of field lines owing to finite perpendicular correlation distance (Achterberg & Ball 1994). By applying the same analysis as in the preceding example, we obtain

$$\frac{\kappa_\perp}{U_- L_{sh}} \simeq \left(\frac{T_{e+}}{T_{e-}}\right)^{1/2}\left(\frac{L_c}{L_{sh}}\right)\left(\frac{\delta B}{B_0}\right)^2 R_q^{-1} M_s^{-1},$$

which implies again that when T_{e+} approaches T_+^{eq} (in which case $[T_{e+}/T_{e-}]^{1/2} M_s^{-1} \sim 1$),

$$L_c\left(\frac{\delta B}{B_0}\right)^2 \gtrsim R_q L_{sh}$$

is required for efficient injection. Thus, the presence of large-amplitude, long-wavelength modes may help to raise the injection efficiency. Such modes may be an inherent part of the shock structure. A fair estimate of this effect, however, requires extensive numerical simulations, and a better understanding of the shock culture.

3 SUMMARY AND CONCLUSION

We have considered electron injection by self-generated turbulence in oblique shocks, and examined the conditions required for efficient injection. In the following, we summarize the main results of this investigation.

In contrast to quasi-parallel shocks, in superluminal ones shock recrossing must involve cross-field transport, which requires vigorous scattering. Electrons traversing the shock undergo betatron acceleration owing to the compression of the magnetic field in the shock, giving rise to compressional distortion distributions inside and just behind the shock. Shock distributions are unstable and would lead to generation of resonant waves which would, in turn, reduce the anisotropy via pitch-angle scattering, keeping it at a level at which the net growth rate is balanced by convection of waves downstream. The pitch-angle scattering also induces cross-field transport, causing the electrons to diffuse in the upstream direction. The resultant cross-field diffusion coefficient appears to be proportional to the pitch-angle scattering rate, thereby giving rise to a positive feedback which strongly suppresses the injection efficiency, and renders it sensitive to the Mach number and the extent of collisionless electron heating inside and behind the shock. Efficient injection requires the diffusion length of thermal electrons downstream to be not much smaller than the shock length-scale (which, in the absence of shock smoothing owing to back reaction of accelerated protons, is expected to be of the order of thermal ion gyroradius). This condition might be satisfied in very high Mach number shocks. Naïve estimate suggests that the implied sound Mach number should exceed 100. This seems to be compatible with the conclusion of Biermann & Cassinelli (1993), namely that the electron injection efficiency drops by several orders of magnitude in perpendicular shocks when the shock velocity becomes smaller than about 0.03c. Quantitative determination of the spectrum of injected electrons is the subject of numerical simulations. The simplest treatment is to solve equation (7) numerically, using the diffusion coefficient computed in equation (13), for different shock parameters. Such analysis will be presented elsewhere.

Field-line braiding can relax the requirements for efficient injection if intense, narrow-band power-spectrum turbulence with coherent length of the order of the shock thickness, is present. In any case, very effective heating of the shocked electron plasma is necessary.

As already mentioned in Section 2.2, the adiabatic approximation breaks down for energetic particles for which the diffusion length exceeds the shock thickness. Nevertheless, it is still true that the magnetic moment of these particles will be preserved (Toptyghin 1980) when crossing the shock from the upstream to the downstream region. Thus, the same mechanism that governs the generation of downstream turbulence by injected electrons, as discussed above, may be applicable to relativistic electrons. The generation of upstream waves in perpendicular shocks is still an unresolved issue. One plausible mechanism is the generation of waves by the reflected ions (Galeev 1984). The interaction of electrons with those waves may also give rise to electron energization in the ion precursor. However, a fair estimate of the efficiency requires self-consistent simulations. Even in the absence of upstream turbulence, efficient acceleration may still be possible, as suggested by Tzhong (1989). We have performed test-particle simulations to examine the acceleration of electrons in perpendicular shocks, in the presence of and in the absence of upstream turbulence. We find that, although the presence of a turbulence precursor helps to raise the acceleration efficiency significantly, a non-negligible fraction of particles can gain energy, by several orders of magnitude, also in the absence of scattering upstream. Detailed results of these simulations will be reported elsewhere.

Several important processes have been neglected in our analysis, some of which may be of importance in perpendicular shocks. In particular, the interaction of electrons with the turbulence generated by reflected ions, non-resonant interactions in the shock, and second-order Fermi acceleration by the self-excited waves themselves. Such processes may become relatively more important in low and moderate Mach number shocks, as suggested by measurements of the fluxes of high-energy particles in the earth bow shock. However, as suggested by our analysis, first-order Fermi acceleration may dominate in high Mach number shocks such as young supernova blast waves, or shocks in extragalactic radio sources. Again, a fair estimate may require extensive numerical simulations of shocks.

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APPENDIX A: DERIVATION OF THE PITCH-ANGLE SCATTERING EQUATION IN THE PRESENCE OF CROSS-FIELD DIFFUSION

The derivation of the transport equation has been made by several authors. A rigorous derivation in the absence of cross-field diffusion is due to e.g. Skilling (1975), (see also Schlickeiser 1989). Webb (1985) has extended it to the general relativistic case. Cross-field diffusion has been treated, under the assumption of strong scattering, by e.g. Earl, Jokipii & Morfill (1988), Webb (1989), who modelled wave–particle interactions with a Krook scattering operator, which was inappropriate for describing diffusion in momentum space through resonant, small-angle scattering. Below, we follow the derivation of Skilling (1975) (see also Blandford & Eichler 1987 and references therein), but we include cross-field diffusion in a manner discussed by Melrose (1980).

We denote by $\boldsymbol{b}(\boldsymbol{x})$ the unit vector in the direction of the local magnetic field, and choose a coordinate system (X_1, X_2, X_3) such that X_3 is measured along $\boldsymbol{b}$. The momentum and pitch angle of the particle are denoted by p and $\cos^{-1}\mu$, respectively. We assume that the distribution of waves is cylindrically symmetric with respect to the magnetic field, i.e. $k_1=k_2=k_\perp$. It is convenient to transform into a frame in which the scattering is elastic. In the case of scattering by waves, this would be the frame in which the wave electric field vanishes. Because the phase velocities involved are typically much smaller than the fluid velocity, the corresponding frame is essentially the fluid frame. In this frame the Vlasov equation can be written (to order U/v) as

$$\frac{\partial f}{\partial t}+(\boldsymbol{v}+\boldsymbol{U})\nabla-(\boldsymbol{p}\nabla)\boldsymbol{U}\frac{\partial f}{\partial \boldsymbol{p}}$$

$$=\sum_s\int\frac{d^3k}{(2\pi)}\hbar\hat{D}\{[w^\sigma(s,\boldsymbol{p},\boldsymbol{k})=\hbar N(\boldsymbol{k})]\hat{D}\}f \qquad \text{(A1)}$$

where $\boldsymbol{v}$ and $\boldsymbol{p}$ are the particle velocity and momentum, respectively, $\boldsymbol{U}$ is the shock velocity, $w^\sigma(s,\boldsymbol{p},\boldsymbol{k})$ is the probability per unit time for gyromagnetic emission (absorption of a photon in a mode σ in the range $d^3k/(2\pi)^3$ (see Melrose 1980 for explicit expressions for this quantity), $N(\boldsymbol{k})$ is the corresponding occupation number, and

$$\hat{D}\equiv\frac{s\Omega_e}{v_\perp}\frac{\partial}{\partial p_\perp}+k_\parallel\frac{\partial}{\partial p_\parallel}-\frac{k_\perp}{m_e\Omega_e}t_{ij}\frac{\partial}{\partial X_j} \qquad \text{(A2)}$$

where $k_\perp$ is the component of the wave vector perpendicular to the magnetic field, and $t_{ij}\equiv(\delta_{ij}-b_ib_j)$. The last term on the RHS of equation (A2) describes the shift of the centre of gyration across the field as a result of the emission of a photon with wavevector $\boldsymbol{k}$ (see Melrose 1980 for detailed discussions). Under the assumption that the distribution function is independent of gyrational phase, we can average equation (A2) over this coordinate. This yields

$$(\mu v\boldsymbol{b}+\boldsymbol{U})\nabla f+t_{ij}\frac{\partial}{\partial X_j}\left[\kappa_\perp t_{ik}\frac{\partial f}{\partial X_k}\right]$$

$$+p\frac{\partial f}{\partial p}\left[\frac{(1-3\mu^2)}{2}\boldsymbol{b}\frac{\partial}{\partial X_3}\boldsymbol{U}-\frac{(1-\mu^2)}{2}\nabla\boldsymbol{U}\right]$$

$$+\frac{(1-\mu^2)}{2}\frac{\partial f}{\partial\mu}\left[v\nabla\boldsymbol{b}+\mu\nabla\boldsymbol{U}-3\boldsymbol{b}\frac{\partial}{\partial X_3}\boldsymbol{U}\right]$$

$$=\frac{\partial}{\partial\mu}\left[\frac{(1-\mu^2)}{2}\nu_s\frac{\partial f}{\partial\mu}\right] \qquad \text{(A3)}$$

where ν_s is the pitch-angle scattering rate, and is given by

$$\nu_s(\boldsymbol{p})=\sum_s\int\frac{d^3k}{(2\pi)^3}(\Delta\alpha)^2w^\sigma(s,\boldsymbol{p},\boldsymbol{k})N(\boldsymbol{k}).$$

The second term on the left-hand side of equation (A3) describes the cross-field diffusion. The corresponding diffusion coefficient is

$$\kappa_\perp=\sum_s\int\frac{d^3k}{(2\pi)^3}\frac{\hbar^2k_\perp^2}{m_e^2\Omega_e^2}w^\sigma(s,\boldsymbol{p},\boldsymbol{k})N(\boldsymbol{k})$$

$$=\sum_s\int\frac{d^3k}{(2\pi)^3}r_g^2\left(\frac{k_\perp}{k_\parallel}\right)^2(\Delta\alpha)^2w^\sigma(s,\boldsymbol{p},\boldsymbol{k})N(k). \qquad \text{(A4)}$$

Consequently, cross-field diffusion requires the presence of off-axis waves. In equations (A3) and (A4) $r_g=p_\perp/(m_e\Omega_e)$ is the particle's gyroradius, and $\Delta\alpha$ being the change in the pitch angle as a result of emission (absorption) of a photon,

$$\Delta\alpha=\hbar\hat{D}(\cos^{-1}\mu)=\frac{\hbar(\omega\mu-k_\parallel v)}{p_\perp v}\simeq\frac{\hbar k_\parallel}{p_\perp}, \qquad \text{(A5)}$$

(we have used the fact that $\omega\ll\Omega_e\sim k_\parallel v$). Thus, unless the excited waves are confined to a narrow cone around the magnetic field, we can approximate the cross-field diffusion as $\kappa_\perp\sim r_g^2\nu_s$, which is a well-known result. For whistlers, the scattering rate is given explicitly by (Melrose 1980)

$$\nu_s(p,\mu)=\frac{\pi^2e^2}{m_ec^2p|\mu|}\int_{-1}^{1}d\chi\,\frac{W(k_R,\chi)}{\chi^2}\left(1-\frac{\mu}{|\mu|}\chi\right)^2, \qquad \text{(A6)}$$

where $k_R=m_e\Omega_e/p|\mu\chi|$, χ being the angle between the wavevector and the magnetic field, and

$$\varepsilon_w=\int_{-1}^{1}d\chi\int_0^\infty dk\,\frac{W(k,\chi)}{2}$$

is the energy density in whistlers.

Equation (A3) is sufficiently general to describe transport in oblique shocks. We have neglected the contribution arising from the potential drop across the shock ramp which is expected to be small for the energetic particles under consideration.