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On the Integral Representation and Applications of the Generalized Function of Two Variables

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Abstract. In this paper, the author defines the Aleph function of two variables, which is a generalization of the I-function of two variables due to Sharma et al.[9]. In this regard the integral representation and applications of new function has been discussed. Similar results obtained by other authors follows as special cases of our findings.

Keywords: Double Barnes integral, H-function of two variables, I-function of two variables.

2010 Mathematics Subject Classifications: 33C99, 33C60, 44A20.

1 Introduction and Preliminaries

Recently, I-function of two variables has been introduced and studied by Sharma et al.[9], which is a generalization of the H-function of two variables due to Gupta et al.[6]. Singh et al.[10] has been investigated the certain double integrals involving $\overline{H}$ -function of two variables due to Buschman et al.[2]. These double integrals are of most general character and can be suitably specialized to yield a number of known or new integral formulae of much interest to mathematical analysis which are likely to prove quite useful to solve some typical boundary value problems. Srivastava et al.[13] has been obtained the results involving I -function of two variables due to Sharma et al.[9].

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The double Barnes integral occurring in the paper will be referred to as the Aleph function of two variables throughout our present study and will be defined and represented as follows:

$$\aleph[x,y] = \aleph^{0,n1:m2,n2:m3,n3}_{P_i,Q_i,\tau_i:r;P_{i'},Q_{i'},\tau_{i'}:r';P_{i''},Q_{i''},\tau_{i''}:r''}[x,y]$$

$$= \aleph^{0,n_1:m_2,n_2:m_3,n_3}_{P_i,Q_i,\tau_i:r;P_{i'},Q_{i'},\tau_{i'}:r';P_{i''},Q_{i''},\tau_{i''}:r''}\left[x,y\left|\begin{matrix}(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}\\ [\tau_i(b_{ji},\beta_{ji},B_{ji})]_{1,Q_i}\end{matrix}\right.\right.$$

$$\left.\begin{matrix}:(c_j,\gamma_j)_{1,n_2},[\tau_{i'}(c_{ji'},\gamma_{ji'})]_{n_2+1,P_{i'}}:(e_j,E_j)_{1,n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1,P_{i''}}\\ :(d_j,\delta_j)_{1,m_2},[\tau_{i'}(d_{ji'},\delta_{ji'})]_{m_2+1,Q_{i'}}:(f_j,F_j)_{1,m_3},[\tau_{i''}(f_{ji''},F_{ji''})]_{m_3+1,Q_{i''}}\end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\Phi(s,t)\theta_1(s)\theta_2(t)x^s y^t\,ds\,dt, \tag{1}$$

where

$$\Phi(s,t) = \frac{\prod_{j=1}^{n_1}\Gamma(1-a_j+\alpha_j s+A_j t)}{\sum_{i=1}^{r}\tau_i\left\{\prod_{j=1}^{Q_i}\Gamma(1-b_{ji}+\beta_{ji}s+B_{ji}t)\prod_{j=n_1+1}^{P_i}\Gamma(a_{ji}-\alpha_{ji}s-A_{ji}t)\right\}}, \tag{2}$$

$$\theta_1(s) = \frac{\prod_{j=1}^{m_2}\Gamma(d_j-\delta_j s)\prod_{j=1}^{n_2}\Gamma(1-c_j+\gamma_j s)}{\sum_{i'=1}^{r'}\tau_{i'}\left\{\prod_{j=1}^{Q_{i'}}\Gamma(1-d_{ji'}+\delta_{ji'}s)\prod_{j=n_2+1}^{P_{i'}}\Gamma(c_{ji'}-\gamma_{ji'}s)\right\}}, \tag{3}$$

$$\theta_2(t)=\frac{\prod_{j=1}^{m_3}\Gamma(f_j-F_jt)\prod_{j=1}^{n_3}\Gamma(1-e_j+E_jt)}{\sum_{i''=1}^{r''}\tau_{i''}\left\{\prod_{j=m_3+1}^{Q_{i''}}\Gamma(1-f_{ji''}+F_{ji''}t)\prod_{j=n_3+1}^{P_{i''}}\Gamma(e_{ji''}-E_{ji''}t)\right\}},$$

(4)

where x and y (real or complex) are not equal to zero, and an empty product is interpreted as unity and $P_i, P_{i'}, P_{i''}, Q_i, Q_{i'}, Q_{i''}, m_3, m_3, n_1, n_2, n_3$ are non-negative integers such that $0 \le n_1 \le P_i, 0 \le n_2 \le P_{i'}, 0 \le n_3 \le P_{i''}$,
$Q_i > 0, Q_{i'} > 0, Q_{i''} > 0; \tau_i, \tau_{i'}, \tau_{i''} > 0$ $(i = 1,\ldots,r; i' = 1,\ldots,r'; i'' = 1,\ldots,r'')$.

All the $A's, \alpha's, B's, \beta's, \gamma's, \delta's, E's$ and $F's$ are assumed to be positive quantities for standardization purpose; the definition of Aleph function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the s–plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $\Gamma(d_j-\delta_j s); j=1,\ldots,m_2$ lies to the right, and the poles of $\Gamma(1-c_j+\gamma_j s); j=1,\ldots,n_2$, $\Gamma(1-a_j+\alpha_j s+A_j t); j=1,\ldots,n_1$ to the left of the contour.

The contour L_2 is in the t–plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $\Gamma(f_j-F_jt); j=1,\ldots,m_3$ lies to the right, and the poles of $\Gamma(1-e_j+E_jt); j=1,\ldots,n_3, \Gamma(1-a_j+\alpha_j s+A_j t); j=1,\ldots,n_1$ to the left of the contour. The existence conditions of (1) are given below:

$$\Omega=\tau_i\sum_{j=1}^{P_i}\alpha_{ji}-\tau_i\sum_{j=1}^{Q_i}\beta_{ji}+\tau_{i'}\sum_{j=1}^{P_{i''}}\gamma_{ji'}-\tau_{i'}\sum_{j=1}^{Q_{i''}}\delta_{ji'}<0,$$

$$\Delta=\tau_i\sum_{j=1}^{P_i}A_{ji}+\tau_{i'}\sum_{j=1}^{P_{i''}}E_{ji'}-\tau_i\sum_{j=1}^{Q_i}B_{ji}-\tau_{i'}\sum_{j=1}^{Q_{i''}}F_{ji'}<0,$$

$$\Theta=\tau_i\sum_{j=n_1+1}^{P_i}\alpha_{ji}-\tau_i\sum_{j=1}^{Q_i}\beta_{ji}+\tau_{i'}\sum_{j=m_2+1}^{Q_{i'}}\delta_{ji'}+\sum_{j=1}^{n_2}\gamma_j-\tau_{i'}\sum_{j=n_1+1}^{P_{i'}}\gamma_{ji'}>0,$$

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$$\Lambda = -\tau_i \sum_{j=n_1+1}^{P_i} A_{ji} - \tau_i \sum_{j=1}^{Q_i} B_{ji} - \sum_{j=1}^{m_3} F_j - \tau_{i''} \sum_{j=m_3+1}^{Q_{i''}} F_{ji''} + \sum_{j=1}^{n_3} E_j - \tau_{i''} \sum_{j=n_3+1}^{P_{i''}} F_{ji''} > 0,$$

$$|\arg x| < \frac{\pi}{2}\Theta, |\arg y| < \frac{\pi}{2}\Lambda. \tag{5}$$

Remark 1: It may be noted that as Aleph function of two variables defined by (1) in terms of double Barnes integral is most general in nature, which includes a number of special functions which can be deduced by assigning suitable values to the parameters. Some interesting special cases of the main definition are given below:

(i) when all $\tau_i = \tau_{i'} = \tau_{i''} = 1$, (1) yields the I-function of two variables due to Sharma et al.[9].

(ii) when $\tau_i = \tau_{i'} = \tau_{i''} = 1, r = r' = r'' = 1,$ it reduces to the H-function of two variables introduced by Gupta et al.[6].

The following result[5, p.372] will be useful for finding the solution of the problem given in next section:

$$\int_0^L \{\sin(\pi x / L)\}^{\omega-1} \sin(n\pi x / L)dx = \frac{L\{\sin(\pi x)\}^{1/2}\Gamma(\omega)}{2^{\omega-1}\Gamma\{1/2(\omega \pm n+1)\}}, \tag{6}$$

where n is any integer and $\omega > 0$.

The Bessel polynomial[4] is given as

$$y_n(x,a,b) = \sum_{k=0}^{n} \frac{(-n)_k(a+n-1)_k}{k!}\left(\frac{-x}{b}\right)^k$$

$$= {}_2F_0[-n, a+n-1; -; -x/b]. \tag{7}$$

The orthogonal property of Bessel polynomial is given by [4]

$$\int_0^\infty x^{a-2} e^{-1/x} y_m(x;a,1)dx = \frac{(-1)^n n!(n+a-1)\pi}{\Gamma(a+n)\Gamma(2n+a-1)\sin \pi a} \delta_{mn}, \tag{8}$$

where

$$\delta_{mn} = \begin{cases} 1 \; if \quad m = n \\ 0 \; if \quad m \neq n \end{cases}$$

and $\mathrm{Re}(a) < 1 - m - n.$

Following result[12] is the integral representation of Bessel polynomial

$$\int_0^\infty x^{\sigma-1} e^{-1/x} y_n(x;a,1)dx = \frac{\Gamma(-\sigma-n)(a-\sigma-1+n)}{\Gamma(a-\sigma-1)}, \tag{9}$$

where $\mathrm{Re}(\sigma) < 0, \mathrm{Re}(a-\sigma) < 2, \sigma \neq -1,-2,...$

2 The Integral Representation

The integral representation of the Aleph function of two variables is given by

$$\int_0^\infty x^{\varepsilon-1} e^{-1/x} y_n(x;a,1)\aleph\left[\xi x^\lambda, \eta\right]dx$$

$$= \aleph^{0,n_1:m_{2+2},n_2:m_3,n_3}_{P_i,Q_i,\tau_i:r;P_{i'}+1,,Q_{i'}+2,\tau_{i'}:r';P_{i''},Q_{i''},\tau_{i''}:r''}\left[\xi,\eta \left| \begin{matrix} (a_j,\alpha_j,A_j)_{1,n_1},[\tau_j(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i} \\ \\ [\tau_j(b_{ji},\beta_{ji},B_{ji})]_{1,Q_i} \end{matrix}\right.\right]$$

$$\left.\begin{matrix} :(c_j,\gamma_j)_{1,n_2},[\tau_j(c_{ji'},\gamma_{ji'})]_{n_2+1,P_{i'}}(c_j,\gamma_j),(a-\varepsilon-1,\lambda):(e_j,E_j)_{1,n_3},[\tau_j(c_{ji''},\gamma_{ji''})]_{n_3+1,P_{i''}} \\ :(-\varepsilon-n,\lambda),(a-\varepsilon-1+n,\lambda),(d_j,\delta_j)_{1,m_2},[\tau_j(d_{ji'},\delta_{ji'})]_{m_2+1,Q_{i'}}:(f_j,F_j)_{1,m_3},[\tau_j(f_{ji''},F_{ji''})]_{m_3+1,Q_{i''}} \end{matrix}\right],$$

(10)

where

$$\tau_i,\tau_{i'},\tau_{i''}>0(i=1,\ldots,r;i'=1,\ldots,r;i''=1,\ldots,r''),$$

$$\mathrm{Re}(\varepsilon)<0,\mathrm{Re}(a-\varepsilon)<2,\varepsilon\neq-1,-2,\ldots$$

and $\Theta,\Lambda>0,|\arg\xi|<\frac{\pi}{2}\Theta$, where Θ and Λ are given by(5).

Proof:

To prove (10), expressing the Aleph function of two variables as a Mellin-Barnes type integral (1) and interchanging the order of integrations which is justified due to the absolute convergence of the integrals involved in the process, evaluating the inner integral with the help of (9) and then using the definition (1), we arrive at the result(10).

Remark 2: If we set $\tau_i=\tau_{i'}=\tau_{i''}=1$, (10) yields the result[12] for I-function of two variables due to Sharma et al.[9].

3 Applications

In this section, we shall investigate the solution of the following problems in terms of the Aleph function of two variables stated below:

Problem 3.1(Temperature in the prism): Let all four faces of an infinitely long rectangular prism, formed by the planes $x = 0, x = a, y = 0\ \ y = b$, are kept at temperature zero and the initial temperature be $f(x, y)$, and the expression for the temperature $u(x, y, t)$ in the prism is given by [1, p.131] as follows:

$$u(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{mn}\, e^{\left\{-\pi^2 kt\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\right\}}\sin(n\pi x/a)\sin(m\pi y/b), \tag{11}$$

where

$$B_{mn}=\frac{4}{ab}\int_0^b\int_0^a f(x,y)\ \sin(m\pi x/b)\sin(n\pi y/a)dxdy. \tag{12}$$

Solution:

Let us assume that

$$f(x,y)=\{\sin n\pi x/a\}^{\varepsilon-1}\{\sin m\pi y/b\}^{\sigma-1}$$
$$\times \aleph\left[s(\sin n\pi x/a)^{\lambda}(\sin m\pi x/b)^{\mu}, t\right] \tag{13}$$

where $\aleph[.,.]$ is the Aleph function of two variables defined in (1).

From (12) and (13), and making use of (1) and changing the order of integration and after using the result(6), we obtain

$$B_{mn}= 2^{4-\varepsilon-\sigma}\ \sin(n\pi/2)\sin(m\pi/2)$$

$$\times \aleph^{0,n_1:m_2,n_2+2:m_3,n_3}_{P_i,Q_i,\tau_i:r;P_{i'}+2,Q_{i'}+4,\tau_{i'}:r';P_{i''},Q_{i''},\tau_{i''}:r''}\left[s\,2^{-\lambda-\mu}, y \,\middle|\, \begin{matrix} (a_j,\alpha_j,A_j)_{1,n_1},[\tau_j(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i} \\ [\tau_j(b_{ji},\beta_{ji},B_{ji})]_{1,Q_i} \end{matrix}\right.$$

$$\left.\begin{matrix} :(1-\varepsilon,\lambda),(1-\sigma,\mu),(c_j,\gamma_j)_{1,n_2},[\tau_j(c_{ji'},\gamma_{ji'})]_{n_2+1,P_{i'}}:(e_j,E_j)_{1,n_3},[\tau_j(c_{ji''},\gamma_{ji''})]_{n_3+1,P_{i''}} \\ :(d_j,\delta_j)_{1,m_2},[\tau_j(d_{ji'},\delta_{ji'})]_{m_2+1,Q_{i'}},(1/2-\varepsilon/2\pm n/2,\lambda/2),(1/2-\sigma/2\pm m/2,\mu/2):(f_j,F_j)_{1,m_3},[\tau_j(f_{ji''},F_{ji''})]_{m_3+1,Q_{i''}} \end{matrix}\right], \quad (14)$$

where $\tau_i, \tau_{i'}, \tau_{i''} > 0 (i = 1,\ldots,r; i' = 1,\ldots,r; i'' = 1,\ldots,r'')$,

$\lambda, \mu \geq 0, \mathrm{Re}(\varepsilon), \mathrm{Re}(\sigma) > 0$ and $\Theta, \Lambda \geq 0, |\arg s| < \frac{\pi}{2}\Theta$, where and are given in (5).

Substituting the value of B_{mn} from (14) in (11), we arrive at

$$u(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} 2^{4-\omega-\delta} e^{\left\{-\pi^2 kt\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\right\}} \sin(n\pi x/a)\sin(m\pi y/b),$$

$$\times \aleph^{0,n_1:m_2,n_2+2:m_3,n_3}_{P_i,Q_i,\tau_i:r;P_{i'}+2,Q_{i'}+4,\tau_{i'}:r';P_{i''},Q_{i''},\tau_{i''}:r''}\left[s\,2^{-\lambda-\mu}, y \,\middle|\, \begin{matrix} (a_j,\alpha_j,A_j)_{1,n_1},[\tau_j(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i} \\ [\tau_j(b_{ji},\beta_{ji},B_{ji})]_{1,Q_i} \end{matrix}\right.$$

$$:(1-\varepsilon,\lambda),(1-\sigma,\mu),(c_j,\gamma_j)_{1,n_2},[\tau_j(c_{ji'},\gamma_{ji'})]_{n_2+1,P_{i'}}:(e_j,E_j)_{1,n_3},[\tau_j(c_{ji''},\gamma_{ji''})]_{n_3+1,P_{i''}}$$

$$:(d_j,\delta_j)_{1,m_2},[\tau_j(d_{ji'},\delta_{ji'})]_{m_2+1,Q_{i'}},(1/2-\varepsilon/2\pm n/2,\lambda/2),(1/2-\sigma/2\pm m/2,\mu/2):(f_j,F_j)_{1,m_3},[\tau_j(f_{ji''},F_{ji''})]_{m_3+1,Q_{i''}}\Bigg]$$

$$\times \sin(n\pi/2)\sin(m\pi/2). \tag{15}$$

which is the required solution of the problem.

Problem 3.2(The time-domain synthesis problem of signals): Such type problem occurs in electrical network theory which is given[4] below:

Given an electrical signal described by a real valued conventional function $f(t)$ on $0<t<\infty$, construct an electrical network consisting of finite number of components, R, C and I, which are all fixed, linear and positive, such that the output of $f_N(t)$, resulting from a delta function $\delta(t)$ approximates $f(t)$ on $0<t<\infty$ in some sense.

Solution:

To obtain a solution of this problem, the function $f(t)$ into a convergent series is given [12] as

$$f(t)=\sum_{n=0}^{\infty} \psi_n(t) \tag{16}$$

where $\psi_n(t)$ is the real-valued function.

Let

$$f_N(t)=\sum_{n=0}^{N} \psi_n(t),\ N=0,1,2,... \tag{17}$$

possess the two properties:

(i) $f_N(t) = 0$, for $-\infty < t < 0$,

(ii) The Laplace transform $f_N(s)$ of $f_N(t)$ is a rational function having zero at $s = \infty$

and all its poles in the left-half s-plane, except possibly for a simple pole at the origin.

After choosing n in (17) sufficiently large to satisfy whatever approximation criterion is being used, an orthogonal series expansion may be employed. The Bessel polynomial transformation and (7) yields an immediate solution in the following form:

$$f(t) = \sum_{n=0}^{\infty} c_n \; t^{(a-2)/2} e^{-1/2t} y_n(t;a,1), \tag{18}$$

where

$$c_n = (-1)^n \frac{\Gamma(a+n)\Gamma(2n+a-1)\sin \pi a}{n!\,\Gamma(n+a-1)\pi} \int_0^{\infty} f(t)\, t^{a-2} e^{-1/2t} y_n(t;a,1)\,dt, \tag{19}$$

and $\mathrm{Re}(a) < 1 - 2n$.

The particular solution is given by

$$f(t) = \sum_{n=0}^{\infty} (-1)^n \frac{\Gamma(a+n)\Gamma(2n+a-1)\sin \pi a}{n!\,\Gamma(n+a-1)\pi} t^{(a-2)/2} e^{-1/2t} y_n(t;a,1)$$

$$\times \aleph^{0,n_1:m_2+2,n_2:m_3,n_3}_{P_i,Q_i,\tau_i:r;P_{i'}+1,,Q_{i'}+2,\tau_{i'}:r';P_{i''},Q_{i''},\tau_{i''}:r''}\left[\xi,\eta \,\middle|\, \begin{matrix} (a_j,\alpha_j,A_j)_{1,n_1},[\tau_j(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i} \\ [\tau_j(b_{ji},\beta_{ji},B_{ji})]_{1,Q_i} \end{matrix}\right]$$

$$\left.\begin{matrix} :(c_j,\gamma_j)_{1,n_2},[\tau_j(c_{ji'},\gamma_{ji'})]_{n_2+1,P_{i'}}(cj,\gamma j),(a-\varepsilon-1,\lambda):(e_j,E_j)_{1,n_3},[\tau_j(c_{ji''},\gamma_{ji''})]_{n_3+1,P_{i''}} \\ :(-\varepsilon-n,\lambda),(a-\varepsilon-1+n,\lambda),(d_j,\delta_j)_{1,m_2},[\tau_j(d_{ji'},\delta_{ji'})]_{m_2+1,Q_{i'}}:(f_j,F_j)_{1,m_3},[\tau_j(f_{ji''},F_{ji''})]_{m_3+1,Q_{i''}} \end{matrix}\right],$$

(20)

where $\tau_i,\tau_{i'},\tau_{i''}>0(i=1,\ldots,r;i'=1,\ldots,r;i''=1,\ldots,r'')$,

$\mathrm{Re}(\varepsilon)<0,\mathrm{Re}(a-\varepsilon)<2,\varepsilon\neq-1,-2,\ldots$ and $\Theta,\Lambda>0,|\arg\xi|<\frac{\pi}{2}\Theta$,

Θ and Λ are given by(5).

Proof of (20):

Let us assume that

$$f(t)=t^{\varepsilon-a/2}e^{-1/2t}\aleph[\xi x^{\lambda},\eta]$$

$$=\sum_{n=0}^{\infty}c_n\ t^{(a-2)/2}e^{-1/2t}y_n(t;a,1). \tag{21}$$

where $\aleph[.,.]$ is the Aleph function of two variables defined in (1).

Since $f(t)$ is continuous and of bounded variation in $(0,\infty)$ this implies that (13) is valid.

On multiplying both sides of (21) by $t^{(a-2)/2}e^{-1/2t}y_n(t;a,1)$ and integrating w.r.to t from 0 to ∞, we get

$$\int_0^{\infty}t^{\sigma-1}e^{-1/t}y_n(t;a,1)\ \aleph[\xi x^{\lambda},\eta]\,dt$$

$$= \sum_{n=0}^{\infty} c_n \int_0^{\infty} t^{(a-2)/2} e^{-1/t} y_m(t;a,1)dt$$

From (8) and (10), we have

$$c_m = (-1)^m \frac{\Gamma(a+m)\Gamma(2m+a-1)\sin \pi a}{m!\Gamma(m+a-1)\pi}$$

$$\times \aleph^{0,n_1:m_2+2,n_2:m_3,n_3}_{P_i,Q_i,\tau_i:r;P_{i'}+1,,Q_{i'}+2,\tau_{i'}:r';P_{i''},Q_{i''},\tau_{i''}:r''}\left[\xi,\eta \middle| \begin{matrix} (a_j,\alpha_j,A_j)_{1,n_1},[\tau_j(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i} \\ [\tau_j(b_{ji},\beta_{ji},B_{ji})]_{1,Q_i} \end{matrix}\right.$$

$$\left. \begin{matrix} :(c_j,\gamma_j)_{1,n_2},[\tau_j(c_{ji'},\gamma_{ji'})]_{n_2+1,P_{i'}}(c_j,\gamma_j),(a-\varepsilon-1,\lambda):(e_j,E_j)_{1,n_3},[\tau_j(c_{ji''},\gamma_{ji''})]_{n_3+1,P_{i''}} \\ :(-\varepsilon-m,\lambda),(a-\varepsilon-1+m,\lambda),(d_j,\delta_j)_{1,m_2},[\tau_j(d_{ji'},\delta_{ji'})]_{m_2+1,Q_{i'}}:(f_j,F_j)_{1,m_3},[\tau_j(f_{ji''},F_{ji''})]_{m_3+1,Q_{i''}} \end{matrix}\right], \quad (22)$$

By using (21) and (22), the solution (20) follows immediately.

3 Special Cases

(i) If we set $\tau_i = \tau_{i'} = \tau_{i''} = 1$ in (15), results in terms of the I-function of two variables given earlier by Srivastava et al.[11] are recovered.

(ii) If we put $\tau_i = \tau_{i'} = \tau_{i''} = 1, r = r' = r'' = 1$ in (15), yields the results in terms of the H-function due to Gupta et al.[6].

(iii) If we set $\tau_i = \tau_{i'} = \tau_{i''} = 1$ in (20), results in terms of the I-function of two variables given earlier by Srivastava et al.[12] are recovered.

(iv) If we put $\tau_i = \tau_{i'} = \tau_{i''} = 1, r = r' = r'' = 1$ in (20), we get the results in terms of the H-function due to Gupta et al.[6].

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