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On the interpretation of pulsar braking indices

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Summary. Timing observations of the Crab pulsar rotation frequency of sufficient accuracy and duration to allow a 10 per cent estimate of the third frequency derivative have been reported (Lyne *et al.*). This measurement is consistent with both non-dipolar electromagnetic models and a secular change in the dipole moment. A more accurate determination may discriminate between these two possibilities. Measurements of braking indices in other young pulsars may reveal similar variations.

1 Introduction

Pulsar rotation frequencies are generally assumed to evolve according to the spindown law

$$\dot{\nu}= \propto -\nu^{n}. \tag{1}$$

Measurement of the frequency second derivative $\ddot{\nu}$ provides an estimate of the 'deceleration parameter' $\tilde{n}=\ddot{\nu}\nu/\dot{\nu}^2$ which should then be equal to the 'braking index' n (e.g. Manchester & Taylor 1977). At present three young, fast pulsars have measured values of $\tilde{n}$ not dominated by timing noise (PSR 0531+21, $\tilde{n}=2.509\pm0.001$, Lyne, Pritchard & Smith 1988; PSR 1509−58, $\tilde{n}=2.83\pm0.03$, Manchester, Durdin & Newton 1985; PSR 0540−69, $\tilde{n}=3.6\pm0.8$, Middleditch, Pennypacker & Burns 1987). These measurements imply that the braking index varies from pulsar to pulsar.

Observations of PSR 0531+21 over ~18 yr recently reported by Lyne *et al.* yield a significant measurement of $\dddot{\nu}$, allowing a second deceleration parameter, $\tilde{p}=\dddot{\nu}\nu^2/\dot{\nu}^3$, to be estimated. (There is inevitably a concern, which we shall ignore in the present note, that this measurement is contaminated by 'red' timing noise, e.g. Deeter & Boynton 1982.) Now assuming the spindown law, equation (1), $\tilde{p}$ can be predicted $\tilde{p}=\tilde{n}(2\tilde{n}-1)=10.1$; the measurements are in ~10 per cent agreement with this value, i.e. $\dddot{\nu}\sim-6\times10^{-31}$ and $\tilde{p}=10\pm1$. In this note, we consider what constraints a measurement of this accuracy imposes on the mechanism of pulsar spindown.

2 Theoretical braking indices

In the most basic model of pulsar spindown, the torque is assumed to be mostly electromagnetic and is estimated by $\sim m^2 r_{\mathrm{LC}}^{-3} \propto m^2\nu^3$, where m is the magnetic dipole moment and $r_{\mathrm{LC}}=c/2\nu$ is the radius of the light cylinder. The braking index is therefore $n=3$ (e.g. Goldreich, Pacini & Rees 1971).

Two elaborations of this model can cause the measured braking index to differ from 3. First, the torque may scale with frequency in different way from a pure magnetic dipole radiating *in vacuo*. This can happen if either the out-flowing plasma removes a significant quantity of mechanical angular momentum, or if strong magnetospheric currents produce a non-dipolar magnetic field. (It is extremely unlikely that higher multipoles in the star can account for the range of braking indices observed, as their influence will be very small at the light cylinder, except perhaps in the fastest pulsars.) So, provided the magnetic dipole is constant, we expect the torque to obey a power-law scaling with frequency as given by equation (1), where the braking index n should be constant for a given pulsar but probably depends upon the angle between the magnetic and spin axes and so can vary between pulsars. In this case, the torque does not depend explicitly on age.

The second possibility is that the magnetic moment can vary with time. This may happen through field growth (e.g. Blandford, Applegate & Hernquist 1983), field decay (e.g. Gunn & Ostriker 1970), dipole alignment (e.g. Macy 1974; Candy & Blair 1986) or dipole counter-alignment (e.g. Beskin, Gurevich & Istomin 1984). In this case, the spindown law will depend upon the age of the pulsar and will have the form

$$\dot\nu=-f(t)\nu^n. \tag{2}$$

(There is strong evidence that torque decays in old pulsars and this is usually interpreted as decay or alignment of the magnetic field. It is much less certain that this is happening in pulsars young enough for $\ddot\nu$ to be measurable.) Adopting this law, we can express the first two derivatives of the function $f(t)$ as dimensionless numbers

$$d_1=\frac{\dot f\nu}{f\dot\nu}=\tilde n-n \tag{3}$$

and

$$d_2=\frac{\ddot f}{f}\left(\frac{\nu}{\dot\nu}\right)^2=\tilde p-n[2n-1+3(\tilde n-n)]. \tag{4}$$

If we assume *a priori* a value for n (e.g. 3), then equation (3) gives an estimate for the time-scale of the torque variation and equation (4) allows a check on a given functional form for $f(t)$. Note that any reasonable torque law $\dot\nu=g(\nu,t)$ can be expressed in the form of equation (2) by first solving the differential equation for $\nu(t)$ and then computing $f(t)=-\dot\nu/\nu^n$. So measurements of $\ddot\nu$ and $\dddot\nu$ provide constraints on two terms in the Taylor expansion of f; such measurements can thus exclude the f=constant braking law of equation (1).

3 Application to specific pulsars

In the case of the Crab pulsar, PSR 0531+21, the two measured deceleration parameters are currently consistent with spin-down law (1) if the braking index is $n=2.5$. As the phase residual associated with $\dddot\nu$ increases $\propto t^4$, inconsistency may be demonstrated in the near future. If, alternatively, the deceleration parameter is $n=3$, but the dipole moment is varying, then $d_1=-0.491$, $d_2=-0.6\pm1$. We can ask if a discrepant third derivative should have already been seen? The answer strictly depends on the assumed functional form for $f(t)$, but is almost surely 'no'. If $f(t)$ is an exponential, then the value for d_1 suggests a magnetic moment growing on a time-scale $\sim 4\tau$ (where $\tau = -\nu/2\dot{\nu}$ is the characteristic age for $n=3$) and the predicted value for $d_2 = d_1^2 = 0.24$, within the current errors but detectable with a fourfold improvement in accuracy. Alternatively, if we assume a power-law variation, $f(t) \propto t^q$, then for $t=\tau$, the measured deceleration parameter gives $q=0.25$, a relatively slow rate of growth in the square of the effective dipole moment. The predicted value of d_2 is then $d_2 = (q-1)\,d_1^2/q = -0.74$, again possibly measurable soon.

The pulsar PSR 1509−58 is associated with the supernova remnant MSH 15−52. However, its timing age is ~1500 yr, much shorter than the estimated age $\sim 10^4$ yr of the remnant (Van den Bergh & Kramper 1984). One possible explanation of this discrepancy is that the magnetic field grew from a small value within the last $\sim 10^3$ yr. The present braking index indicates that field growth has levelled off and now proceeds on a time-scale $\sim (\tilde{n}-n)^{-1}\nu/\dot{\nu} \sim 10\tau$. For exponential field growth, the value of $\tilde{p}$ would be ~2 per cent smaller than that predicted from $\tilde{n}$; with present timing accuracies (and if low frequency 'red' noise does not become dominant in the timing residual) $\gtrsim 10$ years of observations are required to give a measurement sensitive to such small perturbations.

Finally, the 39.5 ms pulsar PSR 1951+32 in the supernova remnant CTB80 (Kulkarni *et al.* 1988) presents an additional puzzle; the small observed $\dot{\nu} \sim 4\times10^{-12}\,\mathrm{s\,s^{-1}}$ corresponds to a magnetic field of $\sim 5\times10^{11}$ G. This is substantially below the typical initial field strength inferred for other pulsars, although with a characteristic age $\tau \sim 10^5$ yr PSR 1951+32 is too young to have experienced substantial field decay. As an alternative, the field may still be growing towards the typical value $\log(B_0) \sim 12.5$. The existence of a ~10′ plerionic trail behind the pulsar, combined with the transverse speed of $\sim 300\,\mathrm{km\,s^{-1}}$ estimated from scintillation observations, suggests that the spindown luminosity has been roughly constant for $\sim 10^4$ yr, so this provides a lower limit to the field growth time-scale. With this estimate for the field growth time we expect $\ddot{\nu} \sim 1.1\times10^{-23}\,\mathrm{s^{-3}}$ and a large *negative* 'braking index' $\tilde{n} \sim -18$, which would be detectable with only a few years of pulse timing.

4 Conclusions

We have shown that measurements of the third frequency derivative can constrain electromagnetic models of pulsars. The measured deceleration parameters suggest that either the magnetic dipole model is inadequate or the dipole moment is changing. A more accurate determination of $\ddot{\nu}$ for the Crab pulsar, possible in the near future, may allow us to choose between these two alternatives. In the case of PSRs 1509−58 and 1951+32, measurements of the higher derivatives of the pulse frequency can put interesting constraints on secular variations in the spindown law, over time-scales $\sim 10^5$ yr.

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