Inventory performance of multi-criteria classification methods: An empirical investigation

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INVENTORY PERFORMANCE OF MULTI-CRITERIA CLASSIFICATION METHODS: AN EMPIRICAL INVESTIGATION

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ABSTRACT: *A number of Multi Criteria Inventory Classification (MCIC) methods have been proposed in the academic literature. However, most of this literature focuses on the development of ranking methods of stock keeping units (SKUs) in an inventory system without any interest in the original and most important goal of this exercise which is the inventory performance. Moreover, these MCIC methods have never been compared in an empirical study. Such an investigation is carried out in this paper by means of both theoretical and empirical data. The theoretical dataset is a common example used in the relevant literature and consists of 47 SKUs. The empirical dataset consists of more than 9,086 SKUs and comes from a retailer in the Netherlands that sells do-it-yourself products.*

KEYWORDS: *ABC classification, multi-criteria, inventory, performance, empirical investigation.*

1 INTRODUCTION

The ABC classification is widely used by companies to deal with inventories consisting of very large numbers of distinct stock keeping units (SKUs). Single-Criterion Inventory Classification (SCIC) methods are often used in practice with traditional criteria such as the demand value or the demand volume (Syntetos *et al*., 2009). These methods have also been analyzed for many decades by inventory researchers (Teunter *et al*., 2010). Furthermore more recently, Multi-Criteria Inventory Classification methods (MCIC) have been developed where a combination of criteria such as the lead time, the criticality, the price, etc. are proposed (Flores and Whybark, 1987; Partovi and Burton, 1992).

The MCIC methods that have attracted most attention from the academic community are: i) the R-method (Ramanathan, 2006); ii) the ZF-method (Zhou and Fan, 2007); iii) the Ng-method (Ng, 2007) and iv) the H-method (Hadi-Vencheh, 2010). However, it should be noted that most of these methods focus on the classification process, i.e. they aim to develop ranking methods of SKUs in an inventory system without any interest in the original and most important goal of this exercise which is the inventory performance. To the best of our knowledge, only two research works have recently tackled this issue by proposing new classification methods that aim to improve the inventory performance whilst classifying the SKUs.

(Teunter *et al*., 2010) have proposed a new ABC classification method based on a cost criterion for ranking the SKUs. The classification criterion is based on the demand rate, the inventory holding cost and the order quantity of each SKU. The inventory performance superiority (lower safety stock cost and higher service levels) of this criterion as compared to the traditional demand value and demand volume criteria and to another criterion proposed by (Zhang *et al*., 2001) has been empirically shown through the use of three real life datasets composed of more than 50,000 SKUs.

(Mohammaditabar *et al*., 2011) have proposed an integrated method based on mathematical programming to simultaneously classify the SKUs and find the inventory control policy. The objective of the mathematical model is to minimize both the total inventory holding cost and the dissimilarity of items classified in the same class. Simulated annealing was used to solve the proposed nonlinear model. The results of the proposed model are compared to those of the annual dollar usage, Analytic Hierarchy Process (AHP) weighted score and optimal inventory score and show that it outperforms all of them in both dissimilarity and total inventory value. No service level objective has been considered in this investigation.

Furthermore, the literature review on inventory classification methods reveals clearly that empirical investigations are lacking. To the best of our knowledge, (Teunter *et al*., 2010) have conducted the only extensive empirical comparative study on the inventory performance of ABC classification methods. However, only mono-criterion methods have been considered in this investigation without any look at MCIC methods. Hence, the aim of this paper is to conduct an analysis of the inventory performance of MCIC methods by means of an investigation based on both theoretical and empirical data. The theoretical dataset is a common example used in the relevant literature and consists of 47 SKUs. The empirical dataset consists of more than 9,086 SKUs and comes from a retailer in the Netherlands that sells do-it-yourself products.

The remainder of our paper is structured as follows. In the next Section the research background focusing on the MCIC methods is presented, and in Section 3 we provide details of the inventory performance evaluation method considered in this paper. The data available for our empirical investigation and the empirical results are discussed in Section 4 followed by our conclusions and suggestions for further research in Section 5. Details related to the implementation of the MCIC methods discussed in this paper are presented in *Appendix A*.

2 RESEARCH BACKGROUND

Many MCIC methods have been developed recently in the academic literature. In this paper, we only focus on four methods that constitute the objective of our comparative performance investigation. Note that the principle of all these methods is that they aggregate the performance of an inventory item in terms of different criteria to a single score which is used for the ranking purpose.

(Ramanathan, 2006) proposed a weighted linear optimization model to address the MCIC problem. The proposed model, called the R-model, aims to offset the impact of subjectivity of the AHP method, developed by (Saaty, 1980) and studied in the context of inventory classification by (Gajpal *et al*., 1994) and (Partovi and Burton, 1993) among others. In the R-model, a weighted additive function is used to compute the score of each item in terms of different criteria, called the optimal inventory score. The R-model was based on four criteria, namely, the average unit cost, the annual dollar usage, the critical factor, and the lead-time. The weights are chosen using optimization subject to the constraints that the weighted sum, computed using the same set of weights, for all the items must be less than or equal to one. The model uses a maximization objective function as all the criteria are assumed to be positively related to the importance level of an item. The model, when solved, gives the optimal inventory score for the i^{th} item. To get the optimal scores of other inventory items, the R-model should be solved repeatedly by changing the objective function. These scores can then be used to classify the inventory items. The R- model was applied for classifying inventory using the data provided in (Flores *et al*., 1992) which consists in 47 inventory items. More details of the R-model are described in *Appendix A*1.

Although the R-model avoids the subjectivity in determining weights and provides an objective way for multi-criteria ABC inventory classification, it hides some defects:

- If an item has a value dominating other items in terms of a certain criterion, this item would always obtain an aggregated performance score of 1 even if it has severely bad values with respect to other criteria. This may lead to the situation where an item with a high value in an unimportant criterion but with low values in other important criteria is inappropriately classified as class A, which may not reflect the real position of this inventory item.
- The R-model may lead to the situation that a large number of items have an aggregated performance score of 1 and further classification among them becomes impossible.

To address the shortcomings of the R-method, (Zhou and Fan, 2007) proposed an extended version of the R-model called the ZF-model that proposes a similar weighted linear optimization model. The ZF-model provides a new index since it uses two sets of weights that are most favorable and least favorable for each item. The authors assume that the R-model provides the good index for each item denoted as gI_i and propose a similar weighted linear optimization model for the bad index denoted as bI_i. The model uses a minimization objective function and when solved, it gives the worst inventory score for the i^{th} item. To compute these scores of all inventory items, bI should be solved repeatedly by changing the objective function. Consequently, the bad index bI_i provides a way for further classification among those incomparable items based on the good index gI_i. More details on the ZF-model are given in *Appendix A*2.

The ZF model was applied on the same set of inventory items using the data provided in (Flores *et al*., 1992). However, ZF-model was based only on three criteria, namely, the average unit cost, the annual dollar usage, and the lead time.

As previously mentioned, the R-model has been proposed to avoid the subjectivity in the weight assignments. However, it requires a linear optimization for each item. Therefore, the processing time can be very long when the number of inventory items is large. Therefore, (Ng, 2007) proposed an alternative weight linear optimization model for the MCIC problem called the Ng-model. They have first built a weighted linear optimization model for each item i with the same objective function of the R-model but where the criteria are ranked in a descending order. Then, the first Ng-model has undergone a multiple transformations to give rise to a simpler Ng-model which can be solved without a linear optimizer. To do so, the author proposed transformed variable as detailed in *Appendix A3*.

The Ng- model was applied for classifying inventory on the same set of item as the previous models. It was also based on three criteria, namely, average unit cost, annual dollar usage, and lead time. When compared with the R-model classification output, *Appendix B* shows that 37 out of 47 items coincide. That is explained by the newly introduction of ranking in criteria and the schemes of weights generation in scoring.

Despite the advantages, the Ng-model leads to a situation where the score of each item is independent of the weights obtained from the model. That is, the weights do not have any role for determining the total score of each item. This may lead to a situation where an item is inappropriately classified. To address this issue, (Hadi, 2010) presented an extended version of the Ng-model called the H-model in which the weights values for multi-criteria ABC inventory classification were considered. The details of the H-model are presented in *Appendix A*4.

As compared with the Ng-model, *Appendix B* shows that 2 out of the 47 items do not have the same classification. The difference in classification of the two approaches is attributed to the fact that in the Ng-model the score of each item is independent of the weights. As we see item 14 dominates item 6 in terms of the first and third criterion. But the Ng-model ignores this fact and since item 6 dominates item 14 in terms of the second criterion, according to the Ng-model the score of item 6 is higher than item 14. This yields item 6 classified as class A and item 14 as class B in the Ng-model. On the other hand, H- model uses the weights of each criterion for determining score of each item. Hence, item 6 is classified as class B and item 14 as class A in our model. When compared with the ZF-model, it can be seen that 9 out of the 47 items do not have the same classification. The difference in classification of the two approaches is because of the new introduction of ranking in criteria and the schemes of weights generation in scoring.

3 INVENTORY PERFORMANCE STUDY

In this section, we first introduce the method that we use to evaluate the inventory performance of each MCIC method and then we present the results based on the theoretical dataset.

3.1 Inventory performance evaluation method

We consider a multi-SKU inventory system where the objective is to classify the SKUs and to evaluate the total holding inventory cost (safety stock cost) for all SKUs and the achieved overall service level of the system, when a specified service level is fixed for each class. The inventory performance evaluation method is similar to the one used in (Teunter *et al*., 2010). We assume that the system is controlled with a reorder point, reorder quantity (*s*,*Q*) policy but as shown in (Teunter *et al*., 2010), the analysis can be applied as well to reorder point, order-up-to-level (*s*,*S*) policy without any fundamental change in the results (Silver *et al*. 1998; Axsäter, 2006). For simplification purposes in our analysis, we will only consider Normal distributed demand, but it should be noted that the analysis can also be used for any type of demand distribution by just modifying the probability distribution functions. In order to measure the service for the inventory system, the fill rate measure, i.e. the fraction of demands that are satisfied directly from stock on hand, is used. The main advantage of using the fill rate is that it directly reflects the service as experienced by the customers.

The objective of the numerical investigation is to determine, for each MCIC classification method, the total holding inventory cost (safety stock cost) and the achieved overall fill rate of the system, when a fixed service level is specified for each class. This would enable us to compare the inventory performance of the considered MCIC classification methods.

We introduce the following notation:

N : Number of SKUs in the inventory system
n : Number of classes
D_i : Mean demand (per time unit) of item *i*
σ_i : Standard deviation of the demand of item *i*
L_i : Lead-time of item *i*
Q_i : Order quantity of item *i*
FR_i : Fill rate of item *i*
FR_T : Overall fill rate of the inventory system
C_T : Total safety stock inventory cost
CSL_i : Cycle service level of item *i*
k_i : Safety factor of item *i*
h_i : Inventory holding cost of item *i*
$\Phi(.)$: Standard normal probability distribution function
$G(x)$: Loss function of the standard normal distribution

The overall fill rate of the inventory system is given by

$$FR_T = \frac{\sum_{i=1}^{N} FR_i D_i}{\sum_{i=1}^{N} D_i} \tag{1}$$

The total safety stock inventory cost is given by

$$C_T = \sum_{i=1}^{N} h_i k_i \sigma_i \sqrt{L_i} \tag{2}$$

where the safety factor k_i of each item *i* is given by

$$k_i = \Phi^{-1}(CSL_i) \tag{3}$$

The fill rate of each item *i* can be approximated by

$$FR_i = 1 - \frac{\sigma_i \sqrt{L_i}}{Q_i} G(k_i) \tag{4}$$

where

$$G(k_i) = \frac{1}{\sqrt{2\pi}} e^{-\frac{k_i^2}{2}} - k_i \left[1 - \Phi(k_i)\right] \tag{5}$$

In order to evaluate the inventory performance of the ABC classification method, we assume that we have n classes, where all the SKUs in each class u have the same target cycle service level (CSL_u). The objective of our work is to evaluate the total safety stock inventory cost and the overall fill rate for a fixed *CSL* per class. Obviously, as a classification method, we assume that the n classes have decreasing optimal safety factors (i.e. $k_u \geq k_{u+1}$) in order to ensure decreasing *CSL*s and fill rates from a class u to a less important class $u+1$.

3.2 Performance comparison with theoretical data

For the purpose of the numerical investigation, we consider the theoretical example of 47 inventory items described in (Flores *et al*., 1992). In this dataset, three classes are defined A, B and C that contain 10, 14 and 23 items respectively. The annual dollar usage, the lead time and the unit cost of all items are given.

Due to the lack of information on the ordering quantity, the unit holding cost and the standard deviation of the demand for the 47 inventory items, which is needed to evaluate the inventory performance of the methods under concern, we have considered some values proposed by (Mohammaditabar *et al*., 2011) and we have also extended the range of the values to investigate the sensitivity of the results to the considered values. We have also made an assumption regarding the ordering quantity since this is needed for the inventory policy under concern. In fact, we assume that the inventory system works with the Economic Ordering Quantity model.

The unit ordering cost for any item *i*, denoted by W_i is fixed to $W_i = 1$, the values considered for the standard deviation of the demand per year for an item *i* are $\sigma_i = 0.1 \times D_i$, $\sigma_i = 0.5 \times D_i$, $\sigma_i = 1 \times D_i$ and the unit inventory holding cost is assumed to be h=20% of the unit cost.

The results are reported for target *CSL*s of classes A, B and C equal to 99%, 95% and 90% respectively. The inventory cost and service results are shown in Table 1 and 2 respectively.

σ_i	*R*	*Ng*	*ZF*	*H*
$0.1\times D_i$	185.503	202.201	189.071	199.978
$0.5\times D_i$	927.517	1011.007	945.357	999.892
$1\times D_i$	1855.033	2022.013	1890.714	1999.784

Table 1: Inventory cost results

σ_i	*R*	*Ng*	*ZF*	*H*
$0.1\times D_i$	0.999	0.999	0.999	0.999
$0.5\times D_i$	0.995	0.997	0.995	0.997
$1\times D_i$	0.991	0.994	0.990	0.994

Table 2: Service results

The results show that overall the R and ZF models provide the lowest inventory costs, whereas the H-method and the Ng-method provide the highest costs. However, it should be noted that the latter results also in a high achieved service level, so the increase in the cost is also an increase in the service level. By looking at Tables 1 and 2 in more details, the results show that for low standard deviation of the demand (i.e. $\sigma_i = 0.1 \times D_i$), equal achieved service level are obtained for all the methods, with a lower cost for the R-model and the ZF-model. By increasing the standard deviation of the demand, the inventory cost of these two methods increases, while the achieved service decreases.

In the next section, we use a larger real dataset where all the cost parameters are available, in order to further investigate the performance of the considered MCIC methods.

4 EMPIRICAL INVESTIGATION

4.1 Empirical data and settings

For the purpose of the empirical investigation, we use a dataset that comes from a retailer located in the Netherlands and selling do-it-yourself products. The dataset contains details on the weekly demand, ordering lead-times and quantities and the purchase costs of 9,086 SKUs. Table 3 shows key statistics for demand, lead time, order quantity, standard deviation of the demand and purchase cost.

9086 SKUs	Demand (per day)	Lead-time (days)	Order Quantity	St. Dev of Demand	Cost (Euro)
Min	0.005	14	1	0	0.010
25%ile	0.247	14	30	1.240	0.910
Median	0.934	14	60	5.269	7.020
75%ile	3.374	14	250	21.038	20.500
Max	11376.940	120	1000000	110917.236	339.000

Table 3: Descriptive statistics of the dataset

For the purpose of the inventory performance evaluation, we assume that the inventory system consists of three classes (i.e. $n = 3$) where the class A consists of 1817 item (20% of the total number of items), the class B consists of 2725 item (30% of the total number of items) and the class C consists of 4544 item (50% of the total number of items). Three target CSLs for classes (A, B, C) are assumed, namely: (99%, 95% and 90%), (95%, 90% and 85%) and (90%, 85% and 80%). We also assume that the unit inventory holding cost is equal to 20% of the unit cost which represents well the context of the company under concern.

4.2 Empirical results

For every MCIC method and every target CSLs the following table shows i) the inventory holding cost and ii) the achieved FR service level. The FR service results have been rounded to the third decimal place.

Target CSL for classes (A, B, C)	**(99%, 95%, 90%)**		**(95%, 90%, 85%)**		**(90%, 85%, 80%)**	
	Cost	***Service***	***Cost***	***Service***	***Cost***	***Service***
Ng *Model*	676242	0.992	490216	0.984	385653	0.973
H *Model*	679954	0.992	492132	0.983	386909	0.973
ZF *Model*	684785	0.996	494715	0.989	388657	0.979
R *Model*	684870	0.996	494761	0.989	388688	0.979

Table 4: Empirical Results of the MCIC Methods

These results indicate that the Ng-model results in the lowest inventory cost whereas, the R results in the highest cost. The results also show that the achieved service levels are very close for all the methods. It should be noted that the increase in the achieved FRs also results in an increase in the total inventory cost which makes it difficult to draw conclusions about the outperformance of any method based only on the results in Table 4. Therefore it is necessary to perform an additional analysis of the combined Service-Cost performance, which is the objective of an analysis conducted later in this paper. In order to better appreciate the Service-Cost combined performance of each MCIC method, we indicate in Figures 1 their respective *efficiency curves*. These curves show the achieved FR as a function of the inventory cost. Obviously, these efficiency curves can be interpreted as signifying that for a certain inventory cost, the curve that is further from the x-axis implies more efficiency.

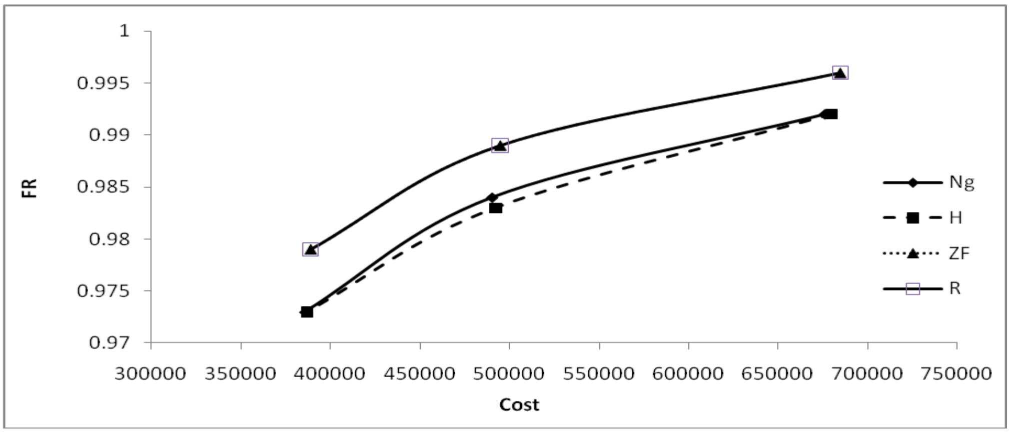

Figure 1: Efficiency curves for the MCIC methods

The efficiency curves show that the R-model and the ZF-model are very close in terms of the combined Service-Cost performance and they outperform the Ng-model and the H-model since for a fixed cost, they provide the highest achieved service level. The H-model provides the lowest efficiency. The outperformance of the R-model and the ZF-model is due to the fact that the H-model and the Ng-model impose an order relation between the criteria where both the annual dollar usage and the unit cost are considered as the most important criteria. This penalizes the inventory cost since the holding cost is proportional to the unit cost and an item with a high unit cost should be classified in lower classes as compared to an item with lower unit cost in order to ensure a minimization of the inventory cost.

5 CONCLUSIONS

A number of MCIC methods have been proposed in the academic literature. The methods that have attracted most of the attention of the researchers and that are considered in this paper are the R-model, the ZF-model, the Ng-model and the H-model. These methods are all based on weighted linear and non linear mathematical programming. The performance of these methods has been empirically investigated in this paper.

The empirical results show that the Ng-model results in the lowest inventory cost whereas, the R results in the highest cost. When the combined Service-Cost performance is considered, the R-model and the ZF-model are very close and they outperform the other methods. This investigation shows that a method that gives a good ranking is not necessarily a method that outperforms the other methods in terms of the inventory performance. This can be true only if the ranking method does not consider the annual dollar usage and the unit cost are considered as the most important criteria. An interesting avenue for further research would be to compare the empirical performance of the MCIC methods to that of single criterion inventory classification methods such as the method proposed by (Teunter *et al*., 2010).

APPENDIX A. DETAILS OF THE MCIC METHODS

In order to present the details of the MCIC methods, the following assumptions and notations are needed:

- There are N inventory items, and that the items have to be classified as A, B or C based on their performance in terms of J criteria.
- The measurement of the i^{th} item under the j^{th} criteria is denoted as y_{ij}.
- The contribution of performance of the i^{th} item under the j^{th} criteria to the score of the item is denoted as a non-negative weight w_{ij}.

All the criteria are positively related to the importance level of the item: the larger the score of an item in terms of these criteria, the greater is the chance that the item be classified as an A-Class item.

• APPENDIX A1. THE R-MODEL

The R-model is given by the following maximization linear program:

$$\begin{aligned} &\max\ S_i = \sum_{j=1}^{J} w_{ij} y_{ij} \\ &st \quad \sum_{j=1}^{J} w_{ij} y_{nj} \le 1 \qquad n = 1,2,\dots N \\ &\qquad w_{ij} \ge 0 \qquad\qquad\quad j = 1,2,\dots. J \end{aligned}$$

• APPENDIX A2. THE ZF-MODEL

The ZF-model is given by the following two sub-models, namely the gI and the bI models.

$gI_i = \max \sum_{j=1}^{J} w_{ij} y_{ij}$ $st \quad \sum_{j=1}^{J} w_{ij} y_{nj} \le 1 \quad n = 1,2,\dots N$ $w_{ij} \ge 0 \quad j = 1,2,\dots. J$
Good index (gI) model
$bI_i = \min \sum_{j=1}^{J} w_{ij} y_{ij}$ $st \quad \sum_{j=1}^{J} w_{ij} y_{nj} \ge 1 \quad n = 1,2,\dots N$ $w_{ij} \ge 0 \quad j = 1,2,\dots. J$
Bad index (bI) model

Therefore, the ZF-model uses a composite index nI_i for each item i by combining the two extreme cases gI_i and bI_i formulated as follows:

$$nI_i(\lambda) = \lambda \frac{gI_i - gI^-}{gI^* - gI^-} + (1-\lambda)\frac{bI_i - bI^-}{bI^* - bI^-}$$

where $gI^* = \max\{gI_i, i = 1,2,\dots, N\}$, $gI^- = \min\{gI_i, i = 1,2,\dots, N\}$, $bI^* = \max\{bI_i, i = 1,2,\dots, N\}$, $bI^- = \min\{bI_i, i = 1,2,\dots, N\}$, and $0 \le \lambda \le 1$ is a control parameter which may reflect the preference of decision maker on the good and bad indexes.

• APPENDIX A3. THE NG-MODEL

The original Ng-model is given by the following maximization linear program:

$$\begin{aligned} &\max\ S_i = \sum_{j=1}^{J} w_{ij} y_{ij} \\ &st \quad \sum_{j=1}^{J} w_{ij} = 1 \\ &\qquad w_{ij} - w_{i(j+1)} \ge 0 \qquad j = 1,2,\dots(J-1) \\ &\qquad w_{ij} \ge 0 \qquad\qquad\quad j = 1,2,\dots. J \end{aligned}$$

After multiple transformations and setting the transformed variable x_{ij} as $x_{ij} = \sum_{k=1}^{j} y_{ik} = 1$ the final Ng-model is as follows:

$$\begin{aligned} &\min\ z_i \\ &st \quad z_i \ge \frac{1}{j} x_{ij} \qquad j = 1,2,\dots. J \end{aligned}$$

where $\min\ z_i = \max_{j=1,2,\dots.J} (1/j\ x_{ij})$

• APPENDIX A4. THE H-MODEL

The H-model is given by the following nonlinear maximization program:

$$\begin{aligned} &\max\ S_i = \sum_{j=1}^{J} w_{ij} y_{ij} \\ &st \quad \sum_{j=1}^{J} w_{ij}^2 = 1 \\ &\qquad w_{ij} - w_{i(j+1)} \ge 0 \qquad j = 1,2,\dots(J-1) \\ &\qquad w_{ij} \ge 0 \qquad\qquad\quad j = 1,2,\dots. J \end{aligned}$$

APPENDIX B. RESULTS OF THE CLASSIFICATION OF THE 47 ITEMS USING THE MCIC METHODS

Item no	Annual dollar usage ($)	Average unit cost ($)	Lead-time (days)	Classifications			
				ZF-model (λ=0.5)	R-model	Ng-model	H-model
1	5840.64	49.92	2	A	A	A	A
2	5670	210	5	A	A	A	A
3	5037.12	23.76	4	A	A	A	A
9	2423.52	73.44	6	A	A	A	A
10	2407.5	160.5	4	A	B	A	A
13	1038	86.5	7	A	A	A	A
14	883.2	110.4	5	A	B	B	A
18	594	49.5	6	A	A	B	B
28	313.6	78.4	6	A	A	B	B
29	268.68	134.34	7	A	A	A	A
5	3478.8	57.98	3	B	B	A	A
8	2640	55	4	B	B	B	B
12	1043.5	20.87	5	B	B	B	B
19	570	47.5	5	B	B	B	B
20	467.6	58.45	4	B	C	C	C
22	455	65	4	B	C	C	C
23	432.5	86.5	4	B	C	B	B
31	216	72	5	B	B	B	B
33	197.92	49.48	5	B	B	B	B
34	190.89	7.07	7	B	A	B	B
37	150	30	5	B	B	C	C
39	119.2	59.6	5	B	B	B	B
40	103.36	51.68	6	B	B	B	B
45	34.4	34.4	7	B	A	B	B
4	4769.56	27.73	1	C	B	A	A
6	2936.67	31.24	3	C	C	A	B
7	2820	28.2	3	C	C	B	B
11	1075.2	5.12	2	C	C	C	C
15	854.4	71.2	3	C	C	C	C
16	810	45	3	C	C	C	C
17	703.68	14.66	4	C	C	C	C
21	463.6	24.4	4	C	C	C	C
24	398.4	33.2	3	C	C	C	C
25	370.5	37.05	1	C	C	C	C
26	338.4	33.84	3	C	C	C	C
27	336.12	84.03	1	C	C	C	C
30	224	56	1	C	C	C	C
32	212.08	53.02	2	C	C	C	C
35	181.8	60.6	3	C	C	C	C
36	163.28	40.82	3	C	C	C	C
38	134.8	67.4	3	C	C	C	C
41	79.2	19.8	2	C	C	C	C
42	75.4	37.7	2	C	C	C	C
43	59.78	29.89	5	C	B	C	C
44	48.3	48.3	3	C	C	C	C
46	28.8	28.8	3	C	C	C	C
47	25.38	8.46	5	C	B	C	C

APPENDIX C. AN EXAMPLE OF INVENTORY PERFORMANCE EVALUATION USING THE ZF-MODEL (W_i = 1 AND $\sigma_i = 1 \times D_i$)

Item no	*Class*	*Annual demand* (D_i)	*Ordereing quantity* (Q_i)	*St. dev. LT Dem.* ($\sigma_i \sqrt{L_i}$)	*Holding cost (h)*	*CSL*	*Safety factor k*	*G(k)*	*FR(i)*	*Satisfied demand*	*Inventory cost*
1	A	117	4.841	8.673	9.984	0.99	2.326	0.003	0.994	116.290	201.432
2	A	27	1.134	3.164	42	0.99	2.326	0.003	0.991	26.745	309.188
3	A	212	9.446	22.224	4.752	0.99	2.326	0.003	0.992	210.310	245.678
9	A	33	2.120	4.237	14.688	0.99	2.326	0.003	0.993	32.776	144.769
10	A	15	0.967	1.572	32.1	0.99	2.326	0.003	0.994	14.917	117.422
13	A	12	1.178	1.664	17.3	0.99	2.326	0.003	0.995	11.943	66.973
14	A	8	0.851	0.938	22.08	0.99	2.326	0.003	0.996	7.970	48.161
18	A	12	1.557	1.541	9.9	0.99	2.326	0.003	0.997	11.960	35.483
28	A	4	0.714	0.514	15.68	0.99	2.326	0.003	0.998	3.990	18.733
29	A	2	0.386	0.277	26.868	0.99	2.326	0.003	0.998	1.995	17.336
5	B	60	3.217	5.447	11.596	0.95	1.645	0.021	0.965	57.877	103.895
8	B	48	2.954	5.032	11	0.95	1.645	0.021	0.964	46.292	91.042
12	B	50	4.895	5.860	4.174	0.95	1.645	0.021	0.975	48.749	40.233
19	B	12	1.589	1.406	9.5	0.95	1.645	0.021	0.982	11.778	21.977
20	B	8	1.170	0.839	11.69	0.95	1.645	0.021	0.985	7.880	16.125
22	B	7	1.038	0.734	13	0.95	1.645	0.021	0.985	6.897	15.691
23	B	5	0.760	0.524	17.3	0.95	1.645	0.021	0.986	4.928	14.915
31	B	3	0.645	0.352	14.4	0.95	1.645	0.021	0.989	2.966	8.328
33	B	4	0.899	0.469	9.896	0.95	1.645	0.021	0.989	3.956	7.631
34	B	27	6.180	3.744	1.414	0.95	1.645	0.021	0.987	26.658	8.708
37	B	5	1.291	0.586	6	0.95	1.645	0.021	0.991	4.953	5.783
39	B	2	0.579	0.234	11.92	0.95	1.645	0.021	0.992	1.983	4.596
40	B	2	0.622	0.257	10.336	0.95	1.645	0.021	0.991	1.983	4.366
45	B	1	0.539	0.139	6.88	0.95	1.645	0.021	0.995	0.995	1.569
4	C	172	7.876	9.015	5.546	0.9	1.282	0.047	0.946	162.679	64.076
6	C	94.004	5.486	8.534	6.248	0.9	1.282	0.047	0.926	87.080	68.333
7	C	100	5.955	9.078	5.64	0.9	1.282	0.047	0.928	92.782	65.618
11	C	210	20.252	15.566	1.024	0.9	1.282	0.047	0.964	202.358	20.428
15	C	12	1.298	1.089	14.24	0.9	1.282	0.047	0.960	11.523	19.881
16	C	18	2.000	1.634	9	0.9	1.282	0.047	0.961	17.304	18.848
17	C	48	5.722	5.032	2.932	0.9	1.282	0.047	0.958	46.002	18.907
21	C	19	2.790	1.992	4.88	0.9	1.282	0.047	0.966	18.358	12.456
24	C	12	1.901	1.089	6.64	0.9	1.282	0.047	0.973	11.674	9.270
25	C	10	1.643	0.524	7.41	0.9	1.282	0.047	0.985	9.849	4.977
26	C	10	1.719	0.908	6.768	0.9	1.282	0.047	0.975	9.750	7.874
27	C	4	0.690	0.210	16.806	0.9	1.282	0.047	0.986	3.942	4.516
30	C	4	0.845	0.210	11.2	0.9	1.282	0.047	0.988	3.953	3.009
32	C	4	0.869	0.296	10.604	0.9	1.282	0.047	0.984	3.935	4.029
35	C	3	0.704	0.272	12.12	0.9	1.282	0.047	0.982	2.945	4.230
36	C	4	0.990	0.363	8.164	0.9	1.282	0.047	0.983	3.931	3.799
38	C	2	0.545	0.182	13.48	0.9	1.282	0.047	0.984	1.968	3.137
41	C	4	1.421	0.296	3.96	0.9	1.282	0.047	0.990	3.960	1.505
42	C	2	0.728	0.148	7.54	0.9	1.282	0.047	0.990	1.981	1.433
43	C	2	0.818	0.234	5.978	0.9	1.282	0.047	0.986	1.973	1.796
44	C	1	0.455	0.091	9.66	0.9	1.282	0.047	0.991	0.991	1.124
46	C	1	0.589	0.091	5.76	0.9	1.282	0.047	0.993	0.993	0.670
47	C	3	1.883	0.352	1.692	0.9	1.282	0.047	0.991	2.973	0.762
Sum		**1415.004**								**1369.696**	**1890.714**

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