

# On-the-job search and moral hazard.\*

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## Abstract

We analyze on-the-job search when moral hazard among employees calls for incentive schemes that include deferred compensation. While deferred compensation improves the workers' incentives to exert effort, it distorts the workers' on-the-job search decisions. We show that deferred compensation is less attractive when overall turnover in the market is high. Moreover, the interplay between search frictions and wage contracts creates feedback effects. If firms in equilibrium use contracts with deferred compensation, entry of new firms into the on-the-job search market becomes less profitable. We find that multiple equilibria may exist: a low-turnover equilibrium where firms use deferred compensation to motivate workers, and a high-turnover equilibrium where they do not.

**Key Words:** On-the-job search, Moral Hazard, Deferred Compensation, Multiple Equilibria .

**JEL Codes:** J41, J63.

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# 1 Introduction

The high worker turnover rates in the economy has spurred a significant literature on on-the-job search. In this literature, the focus is on the role of search. In particular, the models in this literature abstract from any agency problems that may exist between workers and firms along other dimensions than on-the-job search. In the present paper we argue that optimal incentive schemes that motivate workers to provide effort may include an inter temporal element, and this element may interfere with on-the-job search decisions.

Our starting point is that reallocation of workers on firms is necessary in order to obtain an efficient allocation of resources, as experienced workers may have comparative advantage at different tasks and in different firms than inexperienced workers. To capture this we set up an on-the-job search model where experienced workers search for new jobs. Efficient on-the-job search then requires that the experienced workers' wage in the original firm should equal their future production value in that firm (like in Moen and Rosén 2004).

The new feature of our model is that we include moral hazard caused by imperfect monitoring. As in Shapiro and Stiglitz (1984), firms can only imperfectly monitor worker effort. We follow Lazear (1979, 1981) and allow firms to use deferred compensation to provide incentives for workers to provide effort. With deferred compensation, an experienced worker's wage exceeds her productivity. As a result we get a tension: A moral hazard problem which calls for deferred compensation and optimal on-the-job search which calls for wages equal to marginal productivity.

We first show that incentives systems based on deferred compensation become less attractive when turnover is more important for economic efficiency. More interestingly there are feedback effects between the wage contracts used by firms and the number of firms searching for employed workers. These feedback effects may lead to multiple equilibria: A high-effort /low-turnover equilibrium in which firms use deferred compensation to motivate workers, and a low effort - high-turnover equilibrium in which they do not. Furthermore, the larger are the search frictions in the market, the more likely is it that the high-effort/low turnover equilibrium emerges.

As an extension we show that firms in a low-turnover equilibrium with deferred compen-

sation are more reluctant to use piece rate payments to motivate workers, and more inclined to invest in firm-specific human capital, than are firms in a high-turnover equilibrium.

Our paper offers a new explanation for the large variations in turnover rates across countries and regions. For instance, in 1999 the median tenure among employees in 1991 was 3.0 years in the US and 4.4 years in UK, while it was 7.5 years in Germany and 8.2 years in Japan. The percentage of workers with a tenure of less than one year was 28.8 percent in the U.S. and 18.6 % in the U.K., 12.8 % in Germany and only 9.8% in Japan (OECD 1993). Large differences in turnover rates also exist between regions of the same country. For instance, turnover rates are extremely high in Silicon Valley, but much lower along "Route 128" in Massachusetts, another prosperous area with well developed high-technology industry (Saxenian, 1994).

Our model predicts that firms in countries (or regions) with lower turnover rates rely more on long-term wage contracts with deferred compensation (seniority-based wages, promotions etc.) and less on short-term performance-based systems than do firms in countries (regions) with higher turnover rates. This implication is in accordance with popular conceptions of the differences between the US and Japan and between Silicon Valley and Route 128.<sup>1</sup> The prediction of a negative relationship between deferred compensation and short-term performance pay is supported by Bayo-Moriones et.al. (2004). They document that firms which use deferred compensation less than other firms tend to use short-term performance pay as an incentive mechanism.

**Related literature** Our paper proposes an explanation for differences in turnover between countries and regions. In a recent paper, Pries and Rogerson (2005) argues that the differences in worker turnover between the US and Europe may be explained by institutional factors. There also exist papers that analyze multiple equilibrium turnover rates. Acemoglu and Pischke (1998) develop a model where adverse selection may lead to multiplicity in quit rates. Related arguments are made in Chang and Wang (1995), Owan (2004), Saint-Paul (1995), and Moene and Wallerstein (1997). Morita (2001) shows how multiple turnover

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<sup>1</sup>We are not aware of any systematic evidence on the relationship between overall turnover and deferred compensation.

rates may arise as a result of firms' choice of production technology and learning-by-doing. Our paper differs from this literature in several ways. First, multiplicity in our model is caused by incentive contracts and worker moral hazard. Second, our paper is the only one that explicitly model on-the-job search as an equilibrium outcome in the presence of search frictions.

The second contribution of our paper is that we introduce private information into a model of on-the-job search. There is currently a small, but thriving literature on private information in search models. Moen and Rosen (2009) introduce moral hazard and Guerrieri (2008) asymmetric information in competitive search equilibrium. Guerrieri, Shimer and Wright (2009) analyze self-selection of heterogeneous workers in a search environment, and Rudanko (2009) and Menzio and Moen (2009) analyze optimal insurance with limited commitment in a search context. We contribute to this literature by analyzing the relationship between (intertemporal) wage contracts and on-the-job search.

Also related are extensions of the Burdett -Mortensen model (Burdett and Mortensen 1998) which allow for back-loading of wages, see Burdett and Coles (2003) and Stevens (2004). We want to point out that the mechanism at play in these papers is very different from the one in our paper. In their models, search is inefficient from the point of view incumbent firm and the employee, as it reduces their joint income. The employer discourages job quits by back-loading wages (but never to the extent that the wage is higher than output). In our model, by contrast, on-the-job search is efficient, as it increases the joint value of the incumbent firm and the employee. Back-loading is used to motivate workers to exert effort, and implies that wages for senior workers exceed output. Reduced on-the-job search then comes as a costly and unintended by-product of this back-loading.

Finally, as deferred compensation plays an important role in our paper, it is interesting to note that several empirical studies do suggest that deferred compensation is important. Medoff and Abraham (1980) find that pay increases with seniority, although supervisors' rating of performance do not. Lazear and Moore (1984) compare age-income profiles for tenured workers and for self-employed workers, for whom there exists no agency problems. They find that the returns to seniority are higher for tenured workers, and attribute this to deferred compensation. Katlikof and Gokhale (1992) compare wages and productivity

of more than 300,000 workers in a Fortune 1000 firm. They find a substantial degree of deferred compensation for all categories of workers. In particular, managers' productivity exceeds compensation by a factor of more than two at the age of 35, while the opposite is true at the age of 57. Barth (1997) documents that workers on piece-rate compensation schemes have negligible returns to seniority, while workers who are not paid by piece-rates earn significant returns to seniority.

The paper is organized as follows. Section 2 describes our model. Section 3 defines equilibrium and section 4 characterizes equilibrium. Section 5 analyzes multiple equilibria. In section 6 we study implications for contractible effort, firm-specific human capital and entrepreneurship. Section 7 discusses our main assumptions and section 8 concludes. Proofs are relegated to the appendix.

## 2 The Model

We study an overlapping generations model where workers live for two periods. The economy is inhabited by two types of firms, ordinary firms and specialized firms. All workers start their career in ordinary firms. After the first period they qualify for a job in a specialized firm, where their productivity is higher. However, finding a specialized job is hard due to search frictions. All agents are risk neutral with zero discount rate. As there is no interaction between the generations, each generation can be studied in isolation.

Ordinary firms may employ both young and old workers. The productivity of a young worker in an ordinary firm is  $y_1 + e$ , where  $e \in \{0, \bar{e}\}$  is her effort level. The cost of effort is  $ec$ ,  $c \in (0, 1)$ . We introduce a moral hazard problem which may call for deferred compensation, and do this in the simplest possible way by assuming that the effort level of a worker first can be observed in the following period. This may reflect that effort is hard and time-consuming to observe, for instance because it takes time to complete the project the worker is participating in and the effort level cannot be observed before the project is completed. Alternative model specifications that give rise to deferred compensation is presented in the discussion section. Old workers make no effort choice, and produce  $y_2$  units in ordinary firms

and  $y_p$  units in specialized firms,  $y_p > y_2$ .<sup>2</sup>

For simplicity, we assume that the labor market for jobs in ordinary firms is Walrathian. In the labor market for jobs in specialized firms, search frictions are non-negligible. There are unmatched agents on both side of the market, and wages are determined by bargaining. Free entry of both firm types implies zero profits in equilibrium.

Ordinary firms go into a period with a set of existing (old) employees. Specialized firms only hire old workers, and therefore have new workers each period. Each period is divided into four stages, the hiring stage, the production stage, the remuneration stage and the search stage.

### *Ordinary firms*

At the hiring stage, ordinary firms hire young workers. The new employees are offered a wage schedule  $\omega = \{w_1, w_2(e)\}$ , where  $w_1 \in R$  denotes the wage in the current period and  $w_2(e) : \{0, \bar{e}\} \rightarrow R$  is the wage in the next period, given that the worker is still employed in that firm.<sup>3</sup>

At the production stage, junior workers choose effort level  $e$  and produce  $y_1 + e$  units of output. At the remuneration stage, the workers are paid their wages according to the contract.

At the search stage, junior workers search for specialized jobs. This is costly. A search intensity of  $s$  implies an effort cost of  $\gamma s^2/2$ .

If on-the-job search is unsuccessful, the worker may chose to switch to another ordinary

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<sup>2</sup>Several arguments support that turnover can be efficient. Workers may try out several jobs to determine their comparative advantage (Johnson, 1978) or because of match-specific productivity differences (Jovanovic, 1979). A worker's relative productivity in different firms may also change over time as she gains experience and expertise (Moen and Rosén, 2004). Furthermore, sectorial shocks to the economy may warrant a re-allocation of workers. Finally, with technological progress, efficient dissemination of knowledge may require turnover as workers may learn from each other (Saxenian, 1994). The main results of this paper also hold under the less restrictive assumption that only some rather than all workers have a higher productivity in search firms and that only those workers engage in search.

<sup>3</sup>We assume that firms don't pay workers who have left the firm. This may be because: a) It is hard to verify whether movers had high effort in the first period. b) A firm's reputation may suffer more from breaking the contract if the worker in question is still employed than if she has quitted. c) Deferred compensation may reflect the (expected) gain from promotions. As argued in Carmichael (1983), it may be easier for a firm to commit to promotions than to cash payments not associated with particular positions, e) It may be easier for a worker to retaliate in informal ways after a breach of contract if she is still employed than if she works in another firm.

firm and gets a one-period wage equal to her productivity  $y_2$ . Thus, if the period 2 wage prescribed by the initial contract is less than  $y_2$ , the worker will certainly leave.

### *Specialized firms*

At the search stage the specialist firms enter the search market at a cost  $K$ . The search cost  $K$  has to be repeated in each period the firm searches for workers. They bargain with their workers over the wage. The resulting bargained wage is denoted by  $w_p$ . At the production stage the workers produce without any moral hazard problems, and in the remuneration phase the workers receive a wage according to the contract.

### *Matching*

Matching takes place between the periods. The number of matches between searching workers and specialized firms is determined by a constant return to scale matching function  $x(su, v)$ , where  $u$  is the measure of searching workers,  $s$  their average search intensity, and  $v$  the measure of vacancies posted by specialized firms. We assume that the matching function is Cobb-Douglas, i.e.,  $x(su, v) = A(su)^\beta v^{1-\beta}$ . Let  $p$  denote the probability of finding a job per unit of search intensity and  $q$  the expected number of applicants to a firm. It follows that

$$p(\theta) = A\theta^{1-\beta}, \tag{1}$$

$$q(\theta) = A\theta^{-\beta}, \tag{2}$$

where  $\theta = v/su$ .<sup>4</sup> The probability of finding a job for a worker with search intensity  $s$  is  $sp$ . In equilibrium we require that  $sp \leq 1$ .<sup>5</sup> For technical reasons we allow  $q$  to be greater than one (hence a firm can attract and hire more than one worker).<sup>6</sup>

### *Bargaining and search equilibrium*

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<sup>4</sup>We may think of our matching process as a reduced form of a matching process set in continuous time. The probability of finding a job may then be interpreted more broadly as the fraction of the available time the worker is in the specialized firm.

<sup>5</sup>Albrecht, Gautier and Vroman (2006) have pointed out that coordination externalities associated with multiple applications may arise in a discrete setting if workers can obtain more than one offer and has to choose between them. However, the coordination externality disappears if the matching processes is set in continuous time or if firms may give a job offer to more applicants if the first applicant(s) turn down the offer (Kircher, 2008).

<sup>6</sup>The alternative is to impose the constraint that  $x(u, v) \leq v$ . While clearly doable, this is inconvenient for the existence and efficiency proofs.

When bargaining, the outside option of the worker is the contracted wage  $w_2$  in the ordinary firm. In order to avoid uninteresting technicalities we assume that  $w_2$  is unobservable to the specialized firm, which only knows the distribution of wages in the economy.<sup>7</sup> As we only consider pure strategy equilibria, all workers in equilibrium have the same fallback wage  $w_2$ , and this equilibrium wage is thus the outside option of the workers. The outside option of the firms are zero. Wages are determined by the Hosios condition (Hosios, 1990), i.e., the Nash sharing rule with the worker's bargaining power equal to  $\beta$ . In Appendix 7 we show that the search equilibrium maximizes the income of searching workers subject to the zero profit condition of firms.

It follows from the bargaining game that the wage in a specialized firm is given by

$$w_p = \beta y_p + (1 - \beta)w_2. \quad (3)$$

The expected income to a specialized firm from entering the market is

$$\begin{aligned} V &= q(y_p - w_p) \\ &= K, \end{aligned} \quad (4)$$

where the last equation follows from entry. From (1) and (2) it follows that  $q = A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}}$ . Substituting  $q = A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}}$  and (3) into the zero profit condition (4) gives

$$p = A^{\frac{1}{\beta}} \left[ \frac{(1 - \beta)(y_p - w_2)}{K} \right]^{\frac{1-\beta}{\beta}}. \quad (5)$$

which uniquely pins down  $p$  as a function of  $w_2$ .

### *Parameter assumptions*

In order to ensure that the market for specialized firms is operating, we have to make assumptions on the productivity differential between specialized firms and ordinary firms relative to the cost of opening a specialized vacancy, also in the case with deferred compensation. More specifically, we make the following assumption:

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<sup>7</sup>If  $w_2$  was observable, this would imply that the current employer could jack up the wage and thereby the value of search for her employees. As shown by Shimer (2006), this may lead to an untractable equilibrium distribution of wages.



**Assumption 1**

$$y_p - y_2 - c\bar{e} > 0.$$

In addition we have to make parameter assumptions to ensure that the probability of finding a job,  $ps$ , is strictly lower than 1 in equilibrium.

**Assumption 2**

$$\gamma > A^{\frac{2}{\beta}} \left[ \frac{(1-\beta)(y_p - y_2)}{K} \right]^{\frac{2(1-\beta)}{\beta}} \beta (y_p - y_2).$$

Why the assumptions on the parameters take exactly these forms will be clear in lemma 3 and 4.

### 3 Equilibrium

Before characterizing the equilibrium of the model we make observation: In order to retain an old worker that has not obtained a job offer in a specialized firm, the wage contract has to specify a wage  $w_2(e) \geq y_2$ . We refer to this as the worker's interim participation constraint. As we will see shortly, the interim participation constraint does not bind if the worker exerts effort. If the worker does not exert effort, the firm is indifferent between retaining the worker at wage  $y_2$  and letting the worker go. We assume without loss of generality that the wage schedule satisfies the worker's interim participation constraint. Thus, the expected utility of a worker is,

$$u(\omega, e, s) = w_1 - ec - \gamma s^2/2 + spw_p + (1-sp)w_2(e) \quad (6)$$

The profit of an ordinary firm reads

$$\pi(\omega, e, s) = y_1 + e - w_1 + (1-sp)(y_2 - w_2(e)). \quad (7)$$

Let  $\bar{u}$  denote the expected utility of a young worker that enters the market. The optimal contract can be defined as follows:

**Definition 1** *The optimal contract  $(\tilde{\omega}, \tilde{e}, \tilde{s})$  is a wage schedule  $\tilde{\omega} = \{\tilde{w}_1, \tilde{w}_2(e)\}$ , an effort level  $\tilde{e}$ , and a search intensity  $\tilde{s}$  that solves*

$$\max \pi(\omega, e, s) \quad \text{subject to}$$

1. *Incentive compatibility:*

$$u(\tilde{\omega}, \tilde{e}, \tilde{s}) = \max_{e,s} u(\tilde{\omega}, e).$$

2. *Interim participation:*

$$\tilde{w}_2(e) \geq y_2, \quad e \in \{0, \bar{e}\}.$$

3. *Participation:*

$$u(\tilde{\omega}, \tilde{e}, \tilde{s}) \geq \bar{u}.$$

We are now ready to define the equilibrium.

**Definition 2** *The equilibrium is a contract  $(\omega^*, e^*, s^*)$ , a job finding rate  $p^*$ , a wage  $w_p^*$  and a utility  $\bar{u}^*$  such that*

1. *The contract  $(\omega^*, e^*, s^*)$  is an optimal contract.*
2. *Equilibrium in the search market:  $w_p^*$  and  $p^*$  solve (3) and (5).*
3. *Zero profit of ordinary firms:  $\pi(\omega^*, e^*, s^*) = 0$ .*

## 4 Characterizing equilibrium

In this section we characterize equilibrium. Inserting the participation constraint  $u \geq \bar{u}$ , where  $u$  is given by (6), into the expression for the firm's profit (7) gives (with the participation constraint binding)

$$\begin{aligned} \pi &= y_1 + y_2 - \gamma s^2/2 + e(1-c) + sp(w_p - y_2) - \bar{u} \\ &= y_1 + y_2 + e(1-c) + \Omega(s) - \bar{u}, \end{aligned} \tag{8}$$

where

$$\Omega(s) = sp(w_p - y_2) - \gamma s^2/2, \tag{9}$$

is the *value of search* (the functional dependence on  $p$  and  $w_p$  is suppressed). Define  $\Omega^{\max} = \max_s \Omega(s)$  and let  $s^{\max}$  denote the corresponding value of  $s$ . Then

$$s^{\max} = \frac{p(w_p - y_2)}{\gamma}, \quad (10)$$

$$\Omega^{\max} = \frac{p^2(w_p - y_2)^2}{2\gamma}. \quad (11)$$

Then consider a worker's search behaviour. Incentive compatibility requires that  $\tilde{s}$  maximizes  $u(\tilde{\omega}, \tilde{e}, s)$ , and from (6) the first order condition of this maximization problem reads

$$\tilde{s} = \frac{p(w_p - w_2(\tilde{e}))}{\gamma}. \quad (12)$$

By comparing (12) and (10) it follows that the worker maximizes the value of search  $\Omega$  if and only if  $w_2(\tilde{e}) = y_2$ , in which case there is no externality on the firm from the worker's search behavior. Define

$$L = \Omega^{\max} - \Omega(\tilde{s}). \quad (13)$$

We refer to  $L$  as the (deadweight) loss associated with inefficient search intensity when  $w_2(\tilde{e}) \neq y_2$ . The profit function (8) can thus be written as

$$\pi = y_1 + y_2 + e(1 - c) + \Omega^{\max} - L - \bar{u}, \quad (14)$$

Let  $D \equiv w_2(\tilde{e}) - y_2$  denote the *amount of deferred compensation* the worker receives. If the firm implements effort,  $D > 0$ .

**Lemma 1** *The loss  $L$  is a function of  $p$  and  $D$ , and reads*

$$L(D, p) = \frac{D^2 p^2}{2\gamma}. \quad (15)$$

**Proof.** See Appendix 1 ■

The loss is increasing in the amount of deferred payment  $D$  and tightness  $p$  in the search market. The higher  $D$  is, the further away is the worker's search intensity from the search intensity that maximizes the value of search. The same is true for  $p$ . In addition, the higher is  $p$ , the more it matters that the search intensity is too low. Hence deferred compensation is less attractive when the job finding rate in the market is high. Note that the loss is independent of  $w_p$ .

If the firm wants to implement high effort,  $\tilde{e} = \bar{e}$ , it must satisfy the incentive compatibility constraint  $u(\omega, \bar{e}, \tilde{s}) \geq \max_s u(\omega, 0, s)$ . We define a shirker as a worker that deviates and sets  $e = 0$  when the contract prescribes  $e = \bar{e}$ . The contract punishes shirkers as hard as possible, hence the interim participation constraint binds;  $w_2(0) = y_2$ . Let  $\bar{D} = w_2(\bar{e}) - y_2$  denote the lowest amount of deferred compensation consistent with the incentive compatibility constraint. The following then holds:

**Lemma 2**  $\bar{D} = \bar{D}(p, w_p; c)$  is implicitly defined by the expression

$$\bar{D}p^2[\bar{D} - 2(w_p - y_2)] + 2\gamma\bar{D} = 2\gamma c\bar{e}. \quad (16)$$

Furthermore,  $\bar{D}$  is strictly increasing in  $p$ ,  $w_p$  and  $c$ .

**Proof.** See Appendix 2. ■

The value of  $\bar{D}$  has to be higher when the probability that the worker is there to pick it up is low, that is, if  $p$  is high or  $w_p$  is high (since this means that  $\tilde{s}$  is high). If the cost of effort is high, the wage increase if exerting effort must be high.

The structure of the equilibrium depends on whether the firms implement effort or not. It is convenient to define two two equilibrium candidates, a *no-effort equilibrium candidate*  $(\omega^n, 0, s^n, p^n, w_p^n, \bar{u}^n)$  and an *effort equilibrium candidate*  $(\omega^e, \bar{e}, s^e, p^e, w_p^e, \bar{u}^e)$ . The no-effort (effort) candidate is defined in the same way as the equilibrium, with the restriction that the firms are forced to implement  $e = 0$  ( $e = \bar{e}$ ).

Consider first a no-effort equilibrium candidate. In this case firms set  $D = 0$ , and  $w_2^n(\bar{e}) = w_2^n(0) = y_2$ . The zero profit constraint and (7) implies that  $w_1^n = y_1$ . From (3) and (5) it follows that

$$w_p^n = y_2 + \beta(y_p - y_2), \quad (17)$$

$$p^n = A^{\frac{1}{\beta}} \left[ \frac{(1 - \beta)(y_p - y_2)}{K} \right]^{\frac{1 - \beta}{\beta}}. \quad (18)$$

Since the right-hand sides only contain exogenous variables, existence and uniqueness of  $p^n$  and  $w_p^n$  follows directly from equations (17) and (18). Given  $w_p^n$  and  $p^n$ , equation (10) and (11) uniquely determines  $s^n = s^{\max}(w_p^n, p^n)$  and  $\Omega = \Omega^{\max}(w_p^n, p^n)$ . The loss is thus zero,  $L = 0$ .

From (14) and the zero profit condition we have that

$$\bar{u}^n = y_1 + y_2 + \Omega^{\max}(p^n, w_p^n). \quad (19)$$

which uniquely determines  $\bar{u}^n$ .

**Lemma 3** *The no-effort equilibrium candidate exists and is unique. Furthermore,  $p^n > 0$  and  $p^n s^n < 1$ .*

The first part of the lemma (existence and uniqueness) was proved above. The last part is proved in appendix 3, and utilizes assumption 1 and 2.

In order for the no-effort equilibrium candidate to constitute an equilibrium of the full model, the firms cannot find it profitable to implement effort, i.e.,  $\pi(\omega^n, 0, s^n) \geq \max_{\omega, s} \pi(\omega, \bar{e}, s)$  given the market parameters  $(p^n, w_p^n, \bar{u}^n)$ . If the firm implements effort, recall that it sets  $w_2(0) = y_2$  and  $w_2(\bar{e}) = y_2 + \bar{D}(p^n, w_p^n)$ . A deviating firm thus obtains a profit given by (from 14)

$$\pi = y_1 + y_2 + \bar{e}(1 - c) + \Omega^{\max}(p^n, w_p^n) - L(\bar{D}(p^n, w_p^n), p^n) - \bar{u}^n, \quad (20)$$

For the deviation to be strictly profitable,  $\pi$  has to be strictly positive. By inserting (19) into (20) it follows that the no-effort equilibrium is an equilibrium of the full model if and only if

$$L(\bar{D}(p^n, w_p^n), p^n) \geq \bar{e}(1 - c). \quad (21)$$

i.e, if the loss of value of search due to deferred compensation exceeds the gain from effort. If (21) is satisfied, we say that a *no-effort equilibrium* exists.

Consider then an effort-equilibrium candidate. In order to implement effort, firms set  $w_2(0) = y_2$  and  $w_2^e(\bar{e}) = y_2 + \bar{D}(p^e, w_p^e)$ . From (3) and (5) it then follows that

$$w_p^e = \beta y_p + (1 - \beta)(y_2 + \bar{D}^e), \quad (22)$$

$$p^e = A^{\frac{1}{\beta}} \left[ \frac{(1 - \beta)(y_p - y_2 - \bar{D}^e)}{K} \right]^{\frac{1 - \beta}{\beta}}, \quad (23)$$

$$\bar{D}^e = D(p^e, w_p^e). \quad (24)$$

In the appendix we show that (22)- (24) have a unique solution. It follows that  $w_2^e(\bar{e}) = y_0 + \bar{D}^e$  is uniquely determined. From (10) and (11) uniquely determines  $s^e = s^{\max}(w_p^e, p^e)$ .

From the profit equation (8) and the definition of the loss function (13) it follows that

$$\pi^e(\bar{e}) = y_1 + y_2 + \bar{e}(1 - c) + \Omega^{\max}(p^e, w_p^e) - L(\bar{D}(p^e, w_p^e), p^e) - \bar{u}^e. \quad (25)$$

Zero profits gives that

$$\bar{u}^e = y_1 + y_2 + \bar{e}(1 - c) + \Omega^{\max}(p^e, w_p^e) - L(\bar{D}(p^e, w_p^e), p^e) \quad (26)$$

which uniquely defines  $\bar{u}^e$ . Equation (6) then defines  $w_1^e$  uniquely.

**Lemma 4** *The effort equilibrium candidate exists and is unique. The solution is such that  $p^e > 0$  and  $p^e s^e < 1$ , where  $s^e$  is given by (12).*

The proof is given in appendix 4.

In order for the effort equilibrium candidate to constitute an equilibrium of the full model, the firms cannot find it profitable to deviate and not implement effort. A necessary and sufficient condition for this is that  $\pi(\omega^e, \bar{e}, s^e) \geq \max_{\omega, s} \pi(\omega, 0, s)$ , given the market parameters  $(p^e, w_p^e, \bar{u}^e)$ . A deviating firm that implements zero effort and sets  $D = 0$ , obtains a loss of zero and a profit given by (from (14))

$$\pi = y_1 + y_2 + \Omega^{\max}(p^e, w_p^e) - \bar{u}^e. \quad (27)$$

Deviation is only strictly profitable if  $\pi$  is strictly positive. By substituting out  $\bar{u}^e$  in (27) by the virtue of (26), it follows that the effort equilibrium candidate is an equilibrium of the full model if and only if

$$L(\bar{D}(p^e, w_p^e), p^e) \leq (1 - c)\bar{e}. \quad (28)$$

The equation states that the loss of value of search due to deferred compensation is outweighed by the gain from effort. If (28) is satisfied, we say that an *effort equilibrium* exists

**Proposition 1** *a) There exists a threshold value  $c^n$  such that the no-effort equilibrium exists if and only if  $c \geq c^n$ .*

*b) There exists a threshold value  $c^e$  such that the effort equilibrium exists if  $c \leq c^e$ .*

**Proof.** See Appendix 5. ■

Note the difference between a) and b). The threshold  $c^n$  is the unique solution to (21) with equality, and the no-effort equilibrium exists if and only if  $c \geq c^n$ . However, (28) with equality may have more than one solution. We have therefore defined  $c^e$  as the smallest solution to (28) with equality. Due to continuity such a smallest value always exists. It follows that the effort equilibrium also may exist for values above  $c^e$  (hence there is no "only if"). The reason why (28) with equality does not necessarily have a unique solution is that a higher  $c$  implies a higher  $\bar{D}$  (which increases  $L$ ) but also a lower  $p^e$ , and this tends to reduce  $L$ . In Appendix 6 we show that a sufficient condition for (28) with equality having a unique solution (unfortunately in terms of endogenous variables) is that  $\bar{D} \leq \frac{\beta}{1-\beta}(y_p - y_2)$ .

We want to illustrate the different equilibria in a figure. In appendix 4 we show that the generic expressions (22) and (24) defines  $\bar{D}$  as a function of  $p$  only,  $\bar{D} = \tilde{D}(p)$ , and that

$$\frac{d\tilde{D}(p)}{dp} > 0 \quad (29)$$

The curve is upward sloping, reflecting that the higher is the job finding rate, the larger amount of deferred compensation is necessary to induce effort.

Second, from the zero profit condition (5) we can write  $p$  as a function of  $\bar{D}$ ,

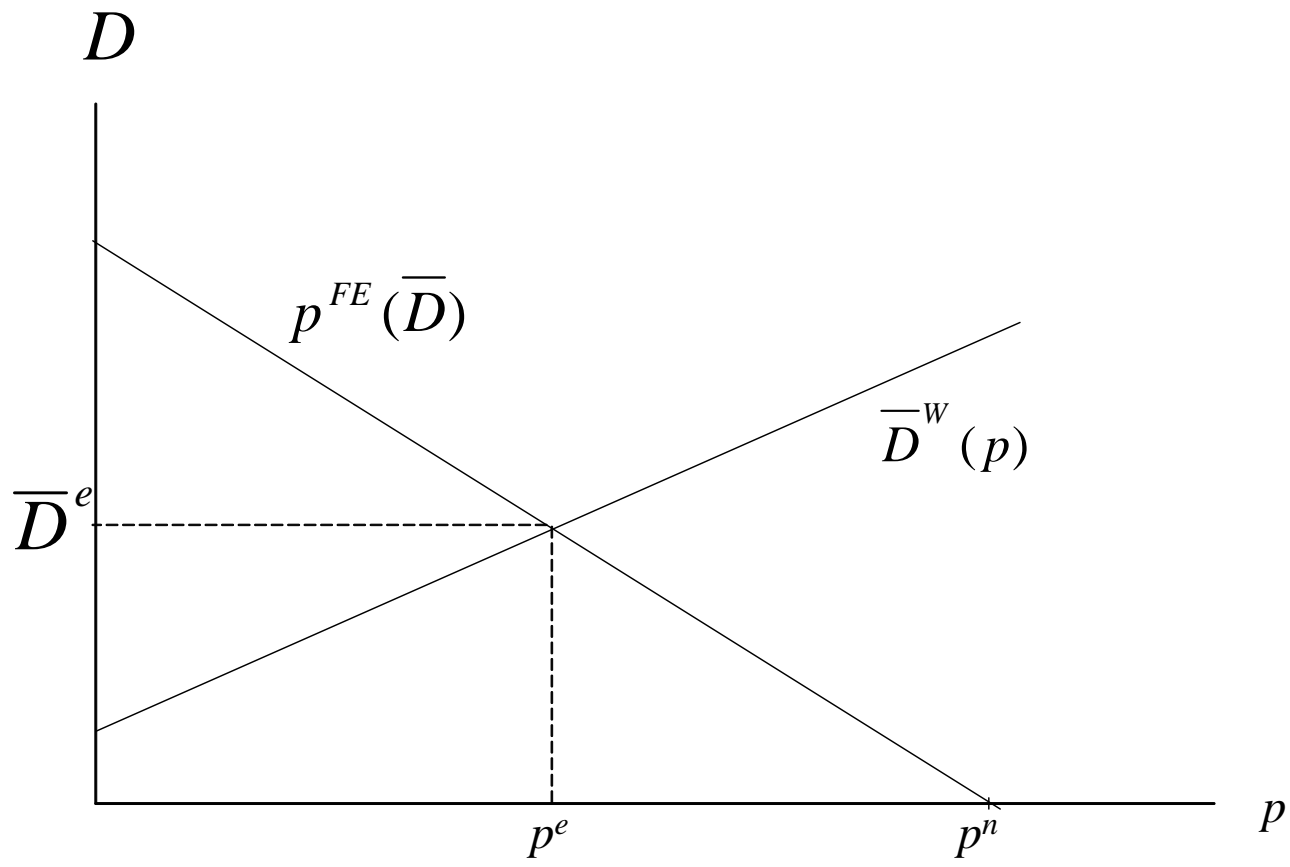
$$p = p^{FE}(\bar{D}), \quad \frac{dp^{FE}(\bar{D})}{d\bar{D}} < 0. \quad (30)$$

The curve shows the job finding rate  $p$  that is consistent with the zero profit condition for specialized firms (or free entry condition) as a function of  $\bar{D}$ . The higher is the deferred compensation, the higher is  $w_p$ , and hence the lower is the number of firms that enter the market, and hence  $p^{FE}$  is lower.

In the  $p - \bar{D}$  space the effort equilibrium is obtained at the intersection of the two curves. The no-effort equilibrium is defined by  $\bar{D} = 0$  and  $p^n = p^{FE}(0)$ . The two equilibria are shown in figure 1.

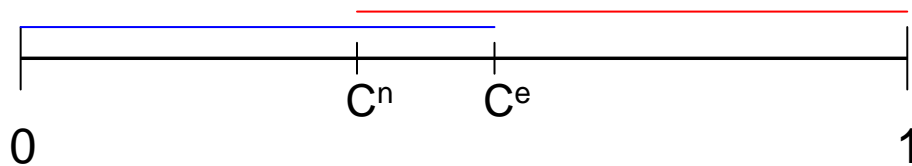
## 5 Multiple equilibria

In the previous section we derived conditions under which the effort and the no effort equilibrium may exist. An interesting issue is whether they may exist simultaneously. As the



Effort and no-effort equilibrium





next proposition shows, the answer is confirmatory:

**Proposition 2** *The threshold values are such that  $c^n < c^e$ . The model exhibits multiple equilibria whenever  $c \in [c^n, c^e]$ . One equilibrium is characterized by high effort, low  $p$  and deferred compensation, while the other is characterized by no effort, high  $p$  and no deferred compensation.*

**Proof.** See Appendix 8. ■

The proposition is illustrated in the figure. The no-effort equilibrium exists for  $c \geq c^e$ . The effort equilibrium exists for  $c \leq c^e$ . Multiple equilibria exists if  $c \in [c^n, c^e]$ .

The intuition for multiplicity is as follows. Suppose we are in the no-effort equilibrium. Then  $w_2$  is relatively low, as no firms defer compensation. Therefore many specialized firms enter the market and on-the-job search is valuable. If a firm deviates and defers compensation in order to implements high effort, it has to defer compensation and distort workers' search effort this comes at a high cost ( $L$  is high). By contrast, in the effort equilibrium all firms defer wages, hence few specialized firms enter the market, and the return from the workers' on-the-job search is lower. A deviating firms that does not implement a high effort and thus does not defer compensation only obtains a modest increases the value of search, since there are relatively few specialized firms to search for anyway.

More generally, when all the other firms use deferred compensation, the search market is "designed" for workers with a high period-two wage, in the sense that the equilibrium maximizes the value of search for such workers. That means few specialized firms paying high wages. The gain for a worker-firm pair of improving the incentives for the worker to do on-the-job search is lower in this situation than in the situation where the equilibrium of the search market is designed for workers with a low period two wage, with more specialized firms paying lower wages.

Put differently, the outcome in the search market depends on the behavior of the agents on the other side of the market, and that will again depend on the agents on the same side of the market. Thus, there exists a feedback effect from the search behavior of the average worker in the market to the gain from search for any individual worker. Since the search behavior depends on the wage contract in question, it follows that the gain from implementing high effort and defer payment depends on the extent to which the other firms in the market defer compensation.

We want to analyze how the equilibrium configurations depend on the parameters of the model. For any parameter  $z$ , let  $\mathcal{M}_{z=\bar{z}}^e$  denote the set of permissible parameters for which the effort equilibrium exists given that  $z = \bar{z}$ . We say that an increase in  $z$  makes the effort equilibrium more likely if  $\mathcal{M}_{z=z_1}^e \subset \mathcal{M}_{z=z_2}^e$  for any  $z_1 < z_2$ . Analogously, let  $\mathcal{M}_{z=\bar{z}}^n$  denote the set of parameters for which the no-effort equilibrium exists given that  $z = \bar{z}$  and define more likely accordingly.

**Proposition 3** *1) An increase in the search frictions (reduced  $A$ , increased  $K$  or increased  $\gamma$ ) makes the effort equilibrium more likely and the no-effort equilibrium less likely.*

*2) An increase in  $y_p - y_2$  makes the no-effort equilibrium more likely and the effort equilibrium less likely.*

**Proof.** See Appendix 9. ■

A reduction in  $A$ , as well as an increase in  $K$  or  $\gamma$ , reflects that it becomes more costly to find a trading partner in the specialized-firms submarket, and hence can be interpreted as search frictions being more severe. The proposition states that such a change will tend to favour the effort equilibrium. Increased search frictions implies that the losses associated with distorting the on-the-job search margins becomes less important, and this tends to favour the effort equilibrium.

We also want to analyse under what conditions multiple equilibria is "likely" to occur. This is difficult, as the model is highly nonlinear. However, we can derive a limit result. First assume that the value of search in the absence of deferred compensation is greater than the gain from effort,  $\Omega^{\max} > (1 - c)\bar{e}$ , or (from 11, 3 and 5)

$$A^{\frac{2}{\beta}} \left[ \frac{(1 - \beta)(y_p - y_2)}{K} \right]^{\frac{2(1-\beta)}{\beta}} \beta(y_p - y_2) \geq 2\gamma(1 - c)\bar{e}$$

We refer to this as assumption 3. By comparing with assumption 2, we see that the two requirements are consistent for all  $c \in [0, 1]$  if  $\bar{e} \leq 1/2$ . Now suppose the cost of effort  $c\bar{e}$  converges to  $y_2 - y_1$ , i.e., towards the border defined by assumption 1. We can then show the following result

**Proposition 4** *Suppose assumption 1-3 is satisfied. Then if  $y_p - y_2$  is sufficiently close to  $c\bar{e}$ , the model exhibits multiple equilibria*

The proof is given in Appendix 10. More generally, when  $c\bar{e}$  is relatively large compared with  $y_p - y_2$ , there is a large difference between  $p^e$  and  $p^n$ . This implies that the difference in deadweight loss of implementing effort in the no-effort equilibrium candidate and the effort equilibrium candidate is large, which broadens the scope for multiple equilibria.

**Welfare** As the workers receive the entire economic surplus, the relevant welfare measure is the utility of workers entering the market,  $\bar{u}$ . From (19) and (26) it follows that the utility in the no-effort and effort equilibrium can be written as.

$$\begin{aligned}\bar{u}^n &= y_1 + y_2 + \Omega^{\max}(p^n, w_p), \\ \bar{u}^e &= y_1 + y_2 + \Omega^{\max}(p^e, w_p) + \bar{e}(1 - c) - L(p^e, \bar{D}^e).\end{aligned}\tag{31}$$

We show in appendix 7 that the search market maximizes the income of searching workers given the zero profit condition of firms. Efficiency in the on-the-job search market is therefore obtained when  $w_2 = y_2$ , as in the no-effort equilibrium. The unconstrained efficient allocation thus requires that  $e = \bar{e}$ ,  $p = p^n$ , and  $s = s^n$ . It follows that both equilibria are inefficient. The no-effort equilibrium because there is no effort. The effort equilibrium is inefficient because there is too little turnover.

Suppose the parameter constellation is such that multiple equilibria exist. We want to explore whether the equilibria can be welfare ranked. The conclusion is negative, one cannot generally show that one of the equilibria welfare dominates the other. The exception is if the effort cost,  $c$ , is close to the upper boundary  $c^n$  for when the high-effort equilibrium exists. In this case, we have that (from 26 and 19 )

$$\bar{u}^e \approx y_1 + y_2 + \Omega^{\max}(p^e, w_p^e) < y_1 + y_2 + \Omega^{\max}(p^n, w_p) = \bar{u}^n.$$

Hence the low-effort equilibrium welfare dominates the high-effort equilibrium in this case. To understand why, note that when  $c = c^e$ , the net gain from effort in effort equilibrium exactly balances the loss associated with distortions in the workers' search intensity. However, since wages are above the workers' productivity, the search market do not maximize the joint gain from search, as too few firms enter the market. Suppose instead that no firms implement effort. Then the wages in specialized firms fall, and more specialized firms enter the market. Now the search market does maximize the joint gain from search, hence this increases the gain from search and thus also welfare.<sup>8</sup>

For values of  $c$  in the interior of  $(c^n, c^e)$ , the effort equilibrium may welfare dominate the no-effort equilibrium. To understand, note the following. Given that a firm implements effort, the joint income from on-the-job search  $\Omega$  may be higher in the effort equilibrium than in the no effort equilibrium. The reason is that a higher specialized firm wage will induce more on-the-job search by workers in firms that defer compensation. This effect may be sufficiently strong so that joint value of search for firms implementing effort may possibly be higher in the effort equilibrium than in the no-effort equilibrium. Hence it is possible that the economy is locked into the no-effort equilibrium although the effort equilibrium is more efficient. With exogenous search intensity of workers, one can show that the no-effort equilibrium always welfare dominates the effort equilibrium.

## 6 Implications

In this section we derive predictions from our model, and compare them with existing empirical findings where such findings exist.

**The slope of the wage-tenure curve and turnover** An interesting issue is the relationship between the slope of the wage-tenure profile in firms and the turnover rate in the economy. In our model, firms are identical. Hence all firms in the market offer the same wage contracts. However, since the model exhibits multiple equilibria, different markets

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<sup>8</sup>VARA LITE MER EXPANSIVA? TEX KOMMENTERA PÅ VAD SOM HÄNDER UTANFÖR MULTIPLICITET.

DISKUTERA HÄR ELLER SENARE OM INTE SÖK-FRIKTIONER- VAD HÄNDER DÅ?

may experience different outcomes even if the parameter constelations are identical. In the markets were firms implement effort, wage profiles are steep and the turnover rate is low, while the opposite is the case the equilibrium with no effort. In this sense the model predicts a negative relationship between the steepness of the wage-tenure profile and turnover. If the parameters differ between the regions we may get similar results. Suppose the cost to workers of implementing effort is higher in one industry than another. If firms implement effort in the low-cost industry and does not in the high-cost economy, there will be a negative relationship between the slope of the wage-tenure profile and the turnover rate across the industries. If effort is implemented in both industries, there will be less deferred compensation and more turnover in the industry with high effort costs, since effort here is not implemented. In both situation the model predicts a negative relationship between the steepness of the wage-tenure profile and the turnover rate in the economy.<sup>9</sup>

In order to analyse within-industry differences (which is relevant case for the empirical studies below) the model may be extended to allow for differences in the cost of implementing effort.<sup>10</sup> Suppose firms in the industry are hetrogeneous, however, their workers undertake their on-the-job search in the same search market and face the same value of  $p$ . For instance, let the cost of effort is low in some firms and high in other firms. If both firm types implement effort, the firms with high effort costs will have to defer compensation more (have a higher  $D$ ) than firms with low effort cost. Turnover rates will be lower in firms with more deferred compensation, as their workers search less intensively. If firms where effort costs are high choose not to implement effort, they will not defer compensation at all. If the firms with low effort costs implement effort, they will still have to defer compensation. Now it is the firms where effort costs are high that will face the higher turnover rates. In both cases there is a negative relationship between the steepness of the wage-tenure profile and the turnover rate in the economy. Similar effects can be obtained if the gain from effort varies between firms. Also in this case the firms that defer compensation most will have the lowest turnover rates.

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<sup>9</sup>The conclusions are not so clear for all parameter differences. Suppse two industries differ in terms of matching efficiency  $A$ . Suppose both industries implement effort. The industry with the higher  $A$  will have more deferred compensation. The turnover rate in this industry may or may not be higher than in the low- $A$  industry.

<sup>10</sup>A formal presentaton of the extension of the model is available upon request.

Several papers have investigated empirically the relationship between the wage profile offered by firms and the turnover rate of their employees. Galizzi and Lang study turnover and wage growth within firms for a set of firms in Turin, Italy. They argue that the tenure-dependent wage growth within a firm can be proxied by the average wage in that firm. Conditional on own wage they find that the turnover rate is negatively related to average wages, and conclude from this that a steep wage profile reduces turnover. Leonard and Audenrode study wage policy in Belgian manufacturing firms. They find that a one standard deviation increase in return for tenure reduces blue collar quits by 39 percent and white-collar quits by 47 percent. Fairris find that quits are lower when the job ladders within firms are long, pay growth from the bottom to the top of the ladder is high, and seniority is used as a criterion for promotion. Finally, Barth and Dale-Olsen find that firms with a steep tenure-wage profile obtains reduced turnover.

The finding seems to fit well with the predictions of our theory. That being said, other theories may also explain a negative relationship between the slope of the wage-tenure contract and the turnover rate, for instance investments in firm-specific human capital.

**When are the different equilibrium more likely?** Our model also have implications for when we can expect to see deferred compensation. First, proposition 3 state that the less frictions there are in the on-the-job search market, the more likely is the no-effort equilibrium and the less likely is the effort equilibrium. This may indicate that in dense areas, with a large number of potential firms, deferred compensation is less attractive. Similarly, 3 also states that if the productivity differences between the specialized firms and the ordinary firms is small, the effort equilibrium is more likely and the no-effort equilibrium is less likely. This may indicate that deferred compensation is more likely when the productivity differences between firms are relatively small. If firms contains several jobs of different types, the probability that a good worker-job match can be found internally increases, and the gain from changing jobs decreases. Again this may call for deferred compensation.

Finally, proposition 4 indicates that multiple equilibria are more likely to occur if the cost of effort is relatively close to the output gap  $y_2 - y_1$ . Hence one can expect large differences between contract forms and turnover rates between countries in sectors were the gain from

turnover is relatively modest compared to the effort cost.

**Piece rate payment or deferred compensation?** So far we have solely focused on effort that only can be observed with a time lag - below referred to as long-term effort. Clearly effort can be multi-dimensional. Some dimensions of effort may be observable, or alternatively the resulting output may be observable with noise. We refer to this as observable effort, and include it in the model as a continuous variable  $d$ . The desired value of  $d$  can be implemented by piece-rate payment. Both effort types are undertaken by young workers only. The driving assumption is that the effort cost  $c^{tot}(e, d)$  is strictly convex. Hence the cost of long-term effort is an increasing function of  $d$ .

If the firm does not implement long-term effort, firms maximize profits given by

$$\pi^n = y_1 + y_2 + d - c^{tot}(0, d) + \Omega^{\max} - \bar{u}.$$

It follows trivially that the firm wants to implement the first best level of  $d$ , given by  $\frac{\partial \kappa(0, d)}{\partial d} = 1$ . With a linear incentive scheme  $w_1 = a + b(y_1 + d)$  this can be implemented by setting  $b = 1$ . Since  $w_2 = y_2$  the zero profit condition then implies that  $a = y_1$ .

Consider then a firm that does implement effort ( $e = \bar{e}$ ). Let  $c(d) = c^{tot}(\bar{e}, d) - c^{tot}(\bar{e}, 0)$ . Since  $c^{tot}$  is strictly convex, increasing in  $d$  From lemma 2 we know that  $\bar{D}$  is increasing in  $c$ . The profit in this case reads

$$\pi = y_1 + y_2 + \bar{e} + d - c^{tot}(\bar{e}, d) + \Omega^{\max} - L(p, \bar{D}) - \bar{u}^e.$$

The first order condition for  $d$  reads

$$\frac{\partial \pi^e}{\partial d} = 1 - \frac{\partial c^{tot}(\bar{e}, d)}{\partial d} - L_{\bar{D}} \frac{d\bar{D}}{dc} c'(d) = 0,$$

or

$$\frac{\partial c^{tot}(\bar{e}, d)}{\partial d} = 1 - L_{\bar{D}} \frac{d\bar{D}}{dc} c'(d) < 1.$$

Thus, the marginal effort cost of  $d$  is less than one. One may implement this by setting  $b = 1 - L_{\bar{D}} \frac{d\bar{D}}{dc} c'(d^e)$  and have a cap at the bonus obtained at  $d^e$ , where  $d^e$  is the effort level

the firm wants to implement. Note that without a cap, a shirking worker will increase  $d$  above  $d^e$  and increase her utility. This makes it even more tempting to shirk.

Our conclusion is thus that firms in effort equilibrium will implement less observable effort, and be more restrictive in its use of short-term bonuses, than firms in the no-effort equilibrium. Analogously, if firms in an industry are exogenous, say regarding the value of long-term effort, the firms that chose to implement long-term effort will use less incentive-powered short term bonus systems and cut back on short-term effort.

There is some evidence that firms which use deferred compensation to a lesser extent than other firms are likely to use short-term bonuses. Bayo-Moriones et.al. (2004) document that firms which use deferred compensation less than other firms tend to use short-term performance pay as an incentive mechanism. MORE AT THIS POINT.

**Firm-specific human capital and entrepreneurs** Clearly, the choice of contracts will influence the incentives of firms to invest in firm-specific human capital. If a firm implements effort, the turnover rate is reduced, and the firm has a higher probability of retaining the worker in the second period. Hence the gain from the firm-specific human capital investments increases.

In addition, the potential to invest in firm-specific human capital makes the effort equilibrium more likely and the no-effort equilibrium less likely. To see this, denote the firm-specific human capital level of a worker by  $h$ , and her period-2 productivity in that firm by  $y_2 + h$ . The wage that implements optimal on-the-job search is then  $w_2 = y_2 + h$ . A shirking worker is still payed a wage  $y_2$  if remaining in the firm in period 2, hence the  $w_2(\bar{e})$  is unchanged. Hence the amount of deferred compensation is reduced to  $\bar{D} - h$ , and the loss function (15) can thus be written as

$$L(\bar{D}, p, h) = \frac{(\bar{D} - h)p^2}{2\gamma}.$$

Firm-specific human capital thus reduces the loss of implementing effort, and therefore makes the effort equilibrium more likely.

Entrepreneurs are often former employees of firms in the same industry. Furthermore, entrepreneurs often need access to particular kinds of funding, (e.g., venture capital), for



which the market may be thin. The matching process between venture capitalists and entrepreneurs may be similar to the search market described above.

For a potential entrepreneur, the shadow price of becoming an entrepreneur is continued employment. This shadow price is higher with deferred compensation than without. Furthermore, when bargaining over terms of trade with a venture capitalist, the economic compensation of continued employment is likely to influence a potential entrepreneur's bargaining position. Thus, in low-turnover equilibrium with deferred compensation, entrepreneurship is less attractive, because the shadow price in terms of foregone wages is high. Just as with specialized firms, this may also reduce the number of entrepreneurs entering the market. The mechanism creating multiple equilibria may then again be at work: in low-turnover equilibrium, few venture capitalists enter the market, hence the loss of deferred compensation caused by reduced entrepreneurship is small. In high-turnover equilibrium, by contrast, a large number of venture capitalists enter the market, and distortions associated with low entrepreneurial activity are large.

## 7 Discussion

In this section we will discuss some of the assumptions of our model in some detail

**The Hosios condition** We have assumed that the Hosios condition is satisfied, so that the search equilibrium maximizes the income of searching workers given the free entry constraint of the firms. Hence the search market in itself does not create inefficiencies. We will now relax this assumption. The critical assumption in our proof of multiplicity is that  $L(\bar{D}(p^n, w_p^n), p^n) > L(\bar{D}(p^e, w_p^e), p^e)$ ; it is more costly to implement effort in the no-effort equilibrium than in the effort equilibrium. The proof of the inequality builds on the fact that  $p^e(w_p^e - y_2) < p^n(w_p^n - y_2)$ , which in turn builds on the Hosios condition being satisfied. First note that due to continuity, the inequality is satisfied for small deviations from the Hosios condition.

Suppose that the Hosios condition is not satisfied. Suppose first that the workers' bargaining power is higher than the Hosios condition prescribes. In this case wages are too

high and  $p$  too low so that the search market does not maximize the income of the searching workers. We conjecture that this is more detrimental to the joint value of search in the effort than in the no-effort equilibrium, since the wages in the effort equilibrium is too high even under the Hosios condition. As a result, the cost of implementing effort is still higher in the no effort equilibrium than in the effort equilibrium so that multiplicity still exists.

Suppose then that the bargaining power of the workers is lower than what the Hosios condition prescribes, so that wages are too low and  $p$  too high compared with the values that maximizes the value of the searching worker. This will tend to increase the value of search in the effort equilibrium and reduce the value of search in the no effort equilibrium. If the deviation from the Hosios condition is sufficiently large, we conjecture that the loss of implementing effort is lower in the no effort equilibrium than in the effort equilibrium, in which case multiple equilibria cease to exist.

Our conjecture regarding welfare is analogous. Suppose the workers' bargaining power is higher than the Hosios condition prescribes. From a welfare point of view we conjecture that this will tend to make the no-effort equilibrium more attractive relative to the effort equilibrium. If the workers' bargaining power is lower, we conjecture that the opposite holds. Note that there may exist a value of the bargaining power which is such that the efficient value of  $p$  is realized in the effort equilibrium. Note though that this does not imply that the effort equilibrium is efficient, as workers still search too little.

<sup>11</sup>In this paper we assume that effort is observable with a time lag, and this forces firms to use deferred compensation in order to motivate the worker. However, this is only one reason why deferred compensation may be warranted. Another reason may be that effort is observed in the same period but with noise. We want to demonstrate the need for deferred compensation under this alternative assumption more precisely, and show that also in this case the trade-off between effort provision and efficient turnover arise. We make the following additional assumptions:

1. A worker exerts effort in both periods, so that output in period  $i$  is  $y_i = y + e_i$ , where  $e_i$  is effort in period  $i$ ,  $i = 1, 2$

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<sup>11</sup>SÄGA ATT STARTAR PÅ NÅGOT NYTT

2. There is a lower bound on the wage a worker can offer in any period. To simplify the exposition we set the lower bound equal to  $y$ .

The rest of the model is as before. Let  $\delta$  denote the probability that the firm observes that the worker provides no effort,  $e = 0$ . Suppose the contract specifies that  $e = \bar{e}$ . Suppose this is to be implemented in a period by period basis, without deferred compensation. The highest wage the firm can profitably pay is  $y + \bar{e}$ . The cost of effort is  $\bar{e}c$ . The non-shirking condition reads  $y + \bar{e}(1 - c) \geq y + \bar{e}(1 - \delta)$ . High effort can thus only be implemented if

$$c/\delta \leq 1$$

We assume that this is not the case.

Consider deferred compensation. If a worker that is detected "shirking" in period 1, she cannot profitably be incentivized in period 2 as  $c/\delta \leq 1$ . She thus obtains  $y$  in period 2. Consider a contract with deferred compensation, where a worker who is not detected shirking in any period gets  $w_1 = y$  in period 1 and  $w_2 = y + 2\bar{e}$  in period 2. The worker will not shirk in period 2 if  $c/\delta \leq 2$ . The period 2 utility of a worker is thus  $2\bar{e} + y - c\bar{e}$ , and independent of whether the worker provided effort in period 1 or not.

In period 1, the lifetime utility of a shirker is  $2y\delta + (1 - \delta)(2y + 2e - c\bar{e})$ . The lifetime utility of a non-shirker is  $2y + 2\bar{e} - 2c\bar{e}$ . The non-shirking condition in period 1 thus reads

$$2y + 2\bar{e} - 2c\bar{e} \geq 2y\delta + (1 - \delta)(2y + 2\bar{e} - c\bar{e}),$$

or

$$c/\delta \leq 2 - c.$$

Thus, if the parameters satisfy

$$1 < c/\delta \leq 2 - c,$$

high effort can be implemented if and only if the firm uses deferred compensation.

The point is that even if the period-by period bonus available is insufficient to motivate the worker, the aggregate surplus over the workers' career is. Deferring the compensation to the end of the second period allows the firm to use the bonuses in both periods to motivate the worker. This doubles the incentives to exert effort in the second period. Furthermore,

since the effort cost in period 2 is less than the bonus available in that period ( $c < 1$ ), it also increases the incentives to exert effort in period 1. Put differently, with deferred compensation the firm makes the decision based on two observations instead of one. The increased information increases the scope for implementing a high effort level.

## 8 Conclusion

This paper analyses moral hazard in a model of on-the-job search. As worker effort is observed with a time lag, the optimal incentive contract includes deferred compensation. However, deferred compensation distorts the workers' on-the-job search decisions, as it gives the workers too weak incentives to search on the job. Due to feedback effects between firms' choice of wage contracts and entry in the on-the-job search market, multiple equilibria may emerge. In one equilibrium, firms offer incentive contracts with deferred compensation, which lead to high effort and low turnover rates. In the other equilibrium firms do not offer deferred compensation, and this lead to low effort and high turnover. Our model contributes to a growing literature that incorporates private information into matching models of the labor market. Our model also sheds light on the observed differences in turnover rates between countries (U.S. and Europe/Japan) and regions (Silicon valley and Massachusetts' route 128).

Our model has several empirical implications. The equilibrium with deferred compensation is more likely to prevail in markets with large search frictions, inclined to give weaker incentives to contractible performance (less use of short-term bonuses) and lead to higher investments in firm-specific human capital than in the equilibrium without deferred compensation. Furthermore, entrepreneurship and venture capital may be more frequent in high-turnover equilibrium than in low-turnover equilibrium. These implications are in line with popular perceptions of the differences between e.g. the US and Japan or between Silicon Valley and Massachusetts.

## Appendix

## 8.1 Appendix 1. Proof of Lemma 1

We have that

$$\begin{aligned}
L(D, p) &= \Omega^{\max}(p) - \Omega(p, s^w(p, D)) \\
&= \frac{p^2(w_p - y_2)^2}{2\gamma} - s^w p(p, D)(w_p - y_2) + \frac{\gamma s^{w^2}(p, D)}{2} \\
&= \frac{p^2(w_p - y_2)^2}{2\gamma} - \frac{p^2(w_p - w_2)(w_p - y_2)}{\gamma} + \frac{p^2(w_p - w_2)^2}{2\gamma} \\
&= \frac{p^2}{2\gamma} ((w_p - y_2)^2 - 2(w_p - w_2)(w_p - y_2) + (w_p - w_2)^2) \\
&= \frac{p^2}{2\gamma} ((w_p - y_2) - (w_p - w_2))^2 \\
&= \frac{p^2}{2\gamma} (w_2 - y_2)^2 \\
&= \frac{p^2 D^2}{2\gamma}
\end{aligned}$$

which completes the proof.

## Appendix 2. Proof of Lemma 2

Incentive compatibility requires that  $u(\tilde{\omega}, \bar{e}, \tilde{s}) \geq \max_s u(\tilde{\omega}, 0, s)$ . From (6) and (12) it follows that the incentive compatibility constraint thus reads

$$\begin{aligned}
\frac{p^2(w_p - w_2)^2}{2\gamma} + w_2 - c\bar{e} &\geq \frac{p^2(w_p - y_2)^2}{2\gamma} + y_2 \Leftrightarrow \\
\frac{p^2(w_p - y_2 - D)^2}{2\gamma} + y_2 + D - c\bar{e} &\geq \frac{p^2(w_p - y_2)^2}{2\gamma} + y_2 \Leftrightarrow \\
p^2(w_p - y_2 - D)^2 + 2\gamma D - 2\gamma c\bar{e} &\geq p^2(w_p - y_2)^2 \Leftrightarrow \\
p^2 [(w_p - y_2 - D)^2 - (w_p - y_2)^2] + 2\gamma D &\geq 2\gamma c\bar{e} \Leftrightarrow \\
p^2 [(w_p - y_2)^2 - 2D(w_p - y_2)] + D^2 - (w_p - y_2)^2 &\geq 2\gamma c\bar{e} \Leftrightarrow \\
p^2 D [D - 2(w_p - y_2)] + 2\gamma D &\geq 2\gamma c\bar{e}
\end{aligned} \tag{32}$$

Thus  $\bar{D}$  is defined by

$$p^2 \bar{D} [\bar{D} - 2(w_p - y_2)] + 2\gamma \bar{D} = 2\gamma c \bar{e} \quad (33)$$

(ii) Differentiating (33) w.r.t  $\bar{D}$  and  $p$  gives

$$(2p^2 \bar{D} - 2p^2(w_p - y_2) + 2\gamma) d\bar{D} + 2p \bar{D} [\bar{D} - 2(w_p - y_2)] dp = 0,$$

which gives

$$\frac{d\bar{D}}{dp} = \frac{-2p \bar{D} [\bar{D} - 2(w_p - y_2)]}{2p^2 \bar{D} - 2p^2(w_p - y_2) + 2\gamma}.$$

Using (33) we have that  $-2p \bar{D} [\bar{D} - 2(w_p - y_2)] = \frac{2\gamma(\bar{D} - c\bar{e})}{p} > 0$  and hence the numerator is positive. Since  $2p^2 \bar{D} - 2p^2(w_p - y_2) + 2\gamma > \frac{p^2 \bar{D} [\bar{D} - 2(w_p - y_2)] + 2\gamma \bar{D}}{\bar{D}}$  and (using (33))  $\frac{p^2 \bar{D} [\bar{D} - 2(w_p - y_2)] + 2\gamma \bar{D}}{\bar{D}} = \frac{2\gamma c \bar{e}}{\bar{D}} > 0$  the denominator is also positive, hence  $\frac{d\bar{D}}{dp} > 0$ .

Differentiating (33) w.r.t to  $\bar{D}$  and  $w_p$ .

$$(2p^2 \bar{D} - 2p^2(w_p - y_2) + 2\gamma) d\bar{D} - 2p^2 \bar{D} dw_p = 0,$$

or

$$\frac{d\bar{D}}{dw_p} = \frac{2p^2 \bar{D}}{2p^2 \bar{D} - 2p^2(w_p - y_2) + 2\gamma}.$$

We have already shown that the denominator is positive. Hence  $\frac{d\bar{D}}{dw_p} > 0$ .

The claim that  $\bar{D}$  is increasing in  $c$  follows directly from that the left-hand side of (16) is increasing in  $\bar{D}$  and the right-hand side increasing in  $c$ .

### Appendix 3. Proof of lemma 3

Most of the proof is given in the main text. From equation (18) and assumption 1, we have that  $p^n > 0$ . From equation (10) and (18) it follows that the probability that a worker finds a job in a specialized firm,  $p^n s^n$ , is given by

$$p^n s^n = A^{\frac{2}{\beta}} \left[ \frac{(1 - \beta)(y_p - y_2)}{K} \right]^{\frac{2(1-\beta)}{\beta}} \beta \frac{y_p - y_2}{\gamma} < 1,$$

where the inequality follows directly from assumption 2. This also rationalizes assumption 2.

## Appendix 4. Proof of lemma 4

Much of the proof is given in the main text. We only have left to show that (22)- (24) uniquely determines  $p^e$ ,  $w_p^e$  and  $w_2(\bar{e}) = y_2 + D^e$ . The proof is constructed as follows. First we show that equation (22) and (24) defines  $\bar{D}$  as a function of  $p$ ,  $\bar{D} = \tilde{D}(p)$ . Second we show that  $\tilde{D}(p)$  is strictly increasing in  $p$ , and thirdly that the equations  $\bar{D} = \tilde{D}(p)$  and (23) have a unique solution.

### 1. Equation (22) and (24) uniquely defines $\bar{D} = \tilde{D}(p)$

Rewrite (22) to (on generic form)

$$\bar{D} = \frac{w_p - \beta y_p}{1 - \beta} - y_2 = f(w_p).$$

Hence the two equations (22) and (24) can be condensed to

$$D(p, w_p) = f(w_p) \tag{34}$$

We first want to show that for any  $p > 0$ , (34) has a unique solution. To this end, first note that

$$\begin{aligned} f(\beta y_p + (1 - \beta)y_2) &= 0. \\ f'(w_p) &= \frac{1}{1 - \beta} > 1 \end{aligned}$$

For any given  $p$ , implementing effort requires that  $D \geq c\bar{e}$ , hence in particular

$$D(p, \beta y_p + (1 - \beta)y_2) > 0$$

Hence  $D(p, w_p) > 0 = f(w_p)$  for  $w_p = \beta y_p + (1 - \beta)y_2$  (i.e., the wage in the no-effort equilibrium). Furthermore,  $f(y_p) = y_p - y_0$  while  $D(p, y_p) < y_p - y_0$  for all  $p$ . We have just seen that  $f'(w_p) = \frac{1}{1 - \beta} > 1$ . A sufficient condition for the existence of a unique solution is thus that  $\frac{\partial \bar{D}(p, w_p)}{\partial w_p} < 1$ .

To show this, recall that incentive compatibility requires that  $u(\tilde{\omega}, \bar{e}, s^e) \geq \max_s u(\tilde{\omega}, 0, s)$ , which is satisfied with equality. Then

$$s^s p^e (w_p - y_2) + y_2 = s^e p^e (w_p - y_2 - \bar{D}) + y_2 + \bar{D} - c\bar{e},$$

where  $s^s$  is the search intensity with no effort and  $s^e$  is the search intensity with effort. Differentiating with respect to  $\bar{D}$  and  $w_p$  gives (due to the envelope theorem we can ignore changes in  $s^s$  and  $s^e$ )

$$s^s p^e dw_p = s^e p^e dw_p + (1 - s^e p^e) d\bar{D},$$

or that

$$\frac{d\bar{D}}{dw_p} = \frac{s^s p^e - s^e p^e}{1 - s^e p^e} < 1,$$

provided that  $s^s p^e < 1$ . The final step is thus to show that  $s^s < s^e$ . To this end, recall that the no-effort equilibrium maximizes the value of search 9. But then it follows directly that  $s^s < s^e$ . It follows that for any given  $p_e$ , the equations (22) and (24) have a solution, and hence that we can write  $\bar{D} = D(p, w_p(p)) \equiv \tilde{D}(p)$ .

**2.  $\tilde{D}(p)$  is strictly increasing in  $p$ .**

We want to show that  $\frac{d\tilde{D}(p)}{dp} > 0$ . To this end, first note that

$$\frac{d\tilde{D}(p)}{dp} \equiv \frac{d\bar{D}(p, w_p(p))}{dp} = \frac{\partial \bar{D}(p, w_p)}{\partial p} + \frac{\partial \bar{D}(p, w_p)}{\partial w_p} \frac{dw_p}{dp}$$

Differentiate (22) to get

$$\frac{dw_p}{dp} = (1 - \beta) \frac{d\bar{D}}{dp},$$

or

$$\frac{d\bar{D}(p, w_p(p))}{dp} (1 - (1 - \beta) \frac{\partial \bar{D}(p, w_p)}{\partial w_p}) = \frac{\partial \bar{D}(p, w_p)}{\partial p}.$$

From lemma 2 we know that the right-hand side is strictly positive, ( $\frac{\partial \bar{D}(p, w_p)}{\partial p} > 0$ ). Above we showed that  $\frac{\partial \bar{D}(p, w_p)}{\partial w_p} < 1$ . The claim thus follows.

**3. The equations  $\bar{D} = \tilde{D}(p)$  and (23) have a unique solution.**

From (23) we can write  $\bar{D}$  as a function of  $p$ ,  $\bar{D} = g(p)$ , where

$$g(p) = -p^{\frac{\beta}{1-\beta}} A^{-\frac{1}{\beta}} \frac{K}{1-\beta} + (y_p - y_2),$$

which is strictly decreasing in  $p$ . Note that  $g(0) = y_p - y_2$  (with  $\bar{D} = y_p - y_2$ , it follows that  $w_2 = y_2 + \bar{D} = y_p$ ). By the definition of  $p_n$ , we know that  $g(p_n) = 0$ .



Then consider  $\tilde{D}(p)$ . From (16) it follows that  $\tilde{D}(0) = c\bar{e} < y_2 - y_1$  (assumption 1). It follows that the equation  $g(p) = \tilde{D}^e(p)$  has a unique solution  $p^e \in (0, p^n)$ . Furthermore, from (12) it follows that  $s^n < s^s$ , hence it follows that  $p^e s^e < p^n s^n < 1$  (from lemma 2).

## Appendix 5. Proof of proposition 1

Proof: a) It is sufficient to show that (21) has a unique solution for  $c$ , i.e. that

$$L(\bar{D}(p^n, w_p^n), p^n) - \bar{e}(1 - c) = 0.$$

has a unique solution. For  $c = 0$ ,  $\bar{D} = 0$  and hence  $L = 0$ , and the left-hand side is strictly negative. Furthermore, from lemma 2 we know that  $\bar{D}$  is increasing in  $c$ , hence the left-hand side of the equation is strictly increasing in  $c$ . Finally, for  $c = 1$ , the last term is zero, and hence the left-hand side is strictly positive. Part a) thus follows.

b) We have to show that (28) has a solution. The proof proceeds as in a). Rewrite the equation to

$$L(\bar{D}(p^e, w_p^e), p^e) - (1 - c)\bar{e} = 0. \quad (35)$$

For  $c = 0$ ,  $D = 0$ , and it follows that the lhs is strictly negative. For  $c = 1$ , the last term is zero, and the left-hand side is strictly positive. Due to continuity it follows that 1), the equation has a solution, and b) there is a smallest solution to the equation, which we define as  $c^e$ . The proposition thus follows.

## Appendix 6. Uniqueness of $c^e$

A sufficient condition for uniqueness is that  $L(\bar{D}(p^e, w_p^e), p^e)$  is increasing in  $c$ , and hence from the loss equation (15) that  $p \times D$  is increasing in  $c$ . We know from lemma 2 that  $D$  is strictly increasing in  $c$ . By using (23) we get that

$$Dp^e = DA^{\frac{1}{\beta}} \left[ \frac{(1 - \beta)(y_p - y_2 - \bar{D}^e)}{K} \right]^{\frac{1 - \beta}{\beta}}.$$

Taking the elasticity of the rhs, we get that  $Dp^e$  is increasing in  $D$  if

$$1 - \frac{1 - \beta}{\beta} \frac{\bar{D}^e}{y_p - y_2 - \bar{D}^e} \geq 0.$$

Hence a sufficient condition for uniqueness is that

$$\bar{D}^e \leq \frac{\beta}{1 - \beta} [y_p - y_2].$$

## Appendix 7. Efficiency

The problem of maximizing worker utility given the zero profit constraint writes

$$\max_{s,p} sp(w_p - w_2) - \frac{\gamma s^2}{2} + w_2 \quad \text{s.t.} \quad A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}} (y_p - w_p) = K. \quad (36)$$

The optimal search effort reads  $s = \frac{p(w_p - w_2)}{\gamma}$ , which inserted gives

$$\max_p \frac{p^2(w_p - w_2)^2}{2\gamma} + w_2 \quad \text{s.t.} \quad A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}} (y_p - w_p) = K. \quad (37)$$

The associated Lagrangian is

$$L = \frac{p^2(w_p - w_2)^2}{2\gamma} + w_2 - \lambda [A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}} (y_p - w_p) - K],$$

with first order conditions

$$\begin{aligned} \frac{\partial L}{\partial w_p} &= 0 \Leftrightarrow \frac{p^2(w_p - w_2)}{\gamma} + \lambda A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}} = 0, \\ \frac{\partial L}{\partial p} &= 0 \Leftrightarrow \frac{p(w_p - w_2)^2}{\gamma} + \lambda A^{\frac{1}{1-\beta}} \frac{\beta}{1-\beta} p^{-\frac{\beta}{1-\beta}-1} (y_p - w_p). \end{aligned}$$

Solving out gives

$$w_p = \beta y_p + (1 - \beta)w_2 = \beta y_p + (1 - \beta)(y_2 + \bar{D}).$$

## Appendix 8. Proof of proposition 2

It is sufficient to show that  $L(\bar{D}(p^e, w_p^e), p^e) < L(\bar{D}(p^n, w_p^n), p^n)$  evaluated at  $c = c^e$ . Then we know that  $c^e$  satisfies (21) with strict inequality, and hence that  $c^n > c^e$ .

From equation (15) it follows that this is true if and only if  $p^e D^e < p^n D^n$ . From (37) it follows that the no-effort equilibrium solves  $\max_{p, w_p} p(w_p - y_2)$  subject to the zero profit constraint, and hence  $p^e(w_p^e - y_2) \leq p^n(w_p^n - y_2)$ . The condition (16) can be written as

$$p[2(w_p - y_2)p - p\bar{D}] = 2\gamma(1 - \frac{c\bar{e}}{D}). \quad (38)$$

Suppose  $p^e \bar{D}^e > p^n \bar{D}^n$ . Since  $p^e(w_p^e - y_2) < p^n(w_p^n - y_2)$  and  $p^e < p^n$  it follows that

$$p^e[2(w_p^e - y_2)p^e - p^e \bar{D}^e] < p^n[2(w_p^n - y_2)p^n - p^n \bar{D}^n].$$

From (38) it thus follows that

$$2\gamma(1 - \frac{c\bar{e}}{D^e}) < 2\gamma(1 - \frac{c\bar{e}}{D^n}),$$

i.e, that  $\bar{D}^e < \bar{D}^n$ . But then it follows that  $p^e \bar{D}^e < p^n \bar{D}^n$ , a contradiction.

## Appendix 9. Proof of proposition 3

1) Consider first a change in  $A$ . From (21) it follows that a sufficient condition for the no-effort equilibrium to be more likely is that an increase in  $A$  implies a strict increase in  $L(\bar{D}(p^n, w_p^n), p^n)$ . From (16) it follows that  $\bar{D}$  only depends on  $A$  through  $p^n$  and  $w_p^n$ . From (18) and (17) it follows that  $\frac{dp^n}{dA} > 0$  and  $\frac{dw_p^n}{dA} = 0$ . From Lemma (2) it then follows that  $\bar{D}^n$  strictly decreases in  $A$ . Hence an increase in  $A$  makes the no-effort equilibrium more likely. The proof that a reduction in  $K$  makes the no-effort equilibrium more likely is analogous.

From (28) it follows that a sufficient condition for an increase in  $A$  to make the effort equilibrium less likely if  $L(\tilde{D}^e(p^e), p^e)$  is strictly increasing in  $A$ . From (16) it again follows that  $\bar{D}$  only depends on  $A$  through  $p^e$  and  $w_p^e$ . From (29) we know that  $\bar{D} = \tilde{D}^e(p)$ , with  $\frac{\tilde{D}^e(p)}{dp} > 0$ . It is thus sufficient to show that  $p^e$  is strictly increasing in  $A$ . Suppose not. Then  $\bar{D}^e = \tilde{D}^e(p^e)$  is strictly decreasing in  $A$ . Recall that  $w_2^e = y_2 + \bar{D}^e$ , which is then strictly decreasing. From (5) it follows that  $p^e$  is increasing, a contradiction. The proof that a reduction in  $K$  makes the no-effort equilibrium more likely is analogous.

Then consider an increase in  $\gamma$ . We claim that this is equivalent to reducing  $A$ . More specifically, we will show that if  $\gamma$  increases from  $\gamma_0$  to  $\gamma_0 + \Delta\gamma$ , there exists a  $\Delta A > 0$  such that if  $A$  simultaneously increases from  $A_0$  to  $A_0 + \Delta A$ , the equilibrium is unchanged. It then follows that an increase in  $\gamma$  from  $\gamma_0$  to  $\gamma_0 + \Delta\gamma$  given  $A = A_0$  is equivalent to reducing  $A$  from  $A_0 + \Delta A$  to  $A_0$  given that  $\gamma = \gamma_0 + \Delta\gamma$ . We have already shown the effects of the latter.

Suppose  $\gamma$  increases from  $\gamma_0$  to  $\gamma_0 + \Delta\gamma$ . Suppose there exists a change in  $A$  so that the equilibrium is unchanged. The search cost of workers have to stay constant, hence

$$\frac{\gamma s^2}{2} = \frac{p^2(w_p - w_2)^2}{2\gamma} = \text{const}$$

$$p \sim \gamma^{1/2}$$

From the free entry condition (5) it follows that  $p \sim A^{\frac{1}{\beta}}$ . Hence the equilibrium stay unchanged if  $\gamma^{1/2} \sim A^{\frac{1}{\beta}}$ , or  $A \sim \gamma^{\frac{\beta}{2}}$ . Since the equilibrium is unique the claim follows.

2) It is sufficient to show that the result holds for an increase in  $y_p$ . It is sufficient to show that the loss of implementing effort increases both for the effort- and no-effort equilibrium

candidate. Consider first the effort equilibrium. Suppose  $dy_p = d\bar{D}$ . From (22) it follows that  $dw_p = dy_p$ , and from (5)  $dp = 0$ . From (32) we have that

$$\frac{p^2(w_p - \bar{D})^2}{2\gamma} + \bar{D} - c\bar{e} - \frac{p^2(w_p - y_2)^2}{2\gamma} \geq 0 \quad (39)$$

Denote the left-hand side of (39) by  $H$ . It follows that

$$\begin{aligned} dH &= dy_p - \frac{p^2(w_p - y_2)}{2\gamma} dy_p \\ &= dy_p - \frac{ps}{2} dy_p \\ &> 0 \end{aligned}$$

since  $ps < 1$ . Hence the incentive compatibility constraint is slack at this point. Let topscript  $e$  indicate equilibrium values before the shift. It follows that at  $p = p^e$ ,  $\tilde{D}(p^e; y_p + dy_p) < \bar{D}^e + dy_p$ . At the same time,  $\bar{D} = p^{FE-1}(p; y_p + dy_p) = \bar{D}^e + dy_p$  (where  $p^{FE-1}$  is the inverse of the function  $p^{FE}(\bar{D})$  defined by (30)). Hence, at  $p^e$ , the  $p^{FE}$  curve is above the  $\tilde{D}(p^e)$  curve after the shift. It follows that both  $p$  and  $\bar{D}$  increases, and hence also the loss of implementing effort. The result then follows.

Consider then the no-effort equilibrium. From (17) and (18) it follows that  $w_p$  and  $p$  increases in  $y_p$ , and hence also the loss  $L(p, w_p)$ . The result thus follows.

## 8.2 Proof of proposition 4

First consider the no-effort equilibrium. Since  $\bar{D} > c\bar{e}$ , it follows that  $w_2(\bar{e}) \geq y_2 + c\bar{e}$ . Note that  $w_2(\bar{e}) \leq y_p$ , otherwise  $p = 0$  and  $w(\bar{e}) = y_2 + c\bar{e} < y_p$ . Hence as  $y_p - y_2$  converges to  $\bar{e}c$  from above,  $y_p - w_2(\bar{e})$  converges to zero. From (5) it then follows that  $p \rightarrow 0$ . From lemma (15) it follows that  $L$  converges to zero. Hence for any  $c < 1$ , deviation is not profitable, and an effort equilibrium exists.

Consider then a no-effort equilibrium. Consider a deviating firm. Again  $w_2(\bar{e}) > y_2 + c\bar{e} > y_2 + \beta(y_p - y_2) = w_p$  for  $y_p - y_2$  sufficiently close to  $c\bar{e}$ . Hence the workers in the deviating firm does not search. But then it follows from Assumption 3 that the deviation is unprofitable

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