On-the-Job Search and Precautionary Savings

JEREMY LISE

University College London and Institute for Fiscal Studies

In this paper I develop and estimate a model of on-the-job search in which risk averse workers choose search effort and can borrow or save using a single risk free asset. I derive the implications for optimal savings behavior in this environment and relate this to the frictions that characterize the endogenous earnings process implied by on-the-job search. Savings behavior depends in a very intuitive way on the rate at which offers are received, the rate at which jobs are destroyed, and a worker's current rank in the wage distribution. The implication is that workers, who are identical in terms of preferences and opportunities, have substantially different savings behavior depending on their history and current position in the wage distribution. The mechanism that generates the substantial differences in savings behavior in the model is the dynamic of the "wage ladder" resulting from the search process. There is an important asymmetry between the incremental wage increases generated by on-the-job search (climbing the ladder) and the drop in income associated with job loss (falling off the ladder). The behavior of workers in low paying jobs is primarily governed by the expectation of wage growth, while the behavior of workers near the top of the distribution is driven by the possibility of job loss. The distributions of earnings, wealth and consumption implied by the model (suitably aggregated) align reasonably well with the data, with the notable exception of implying substantially less concentration of wealth among the richest one percent of the population.

1. INTRODUCTION

In this paper I develop and estimate a model of on-the-job search in which risk averse workers choose search effort and can borrow or save using a single risk free asset. I derive the implications for optimal savings behavior in this environment and relate this to the frictions that characterize the endogenous earnings process implied by on-the-job search. Savings behavior depends in a very intuitive way on the rate at which offers are received, the rate at which jobs are destroyed, and a worker's current rank in the wage distribution. The implication is that workers at different point in the wage distribution have substantially different savings behavior, resulting in wealth dispersion that is much greater than earnings dispersion.

The mechanism that generates the high degree of wealth dispersion in the model is the dynamic of the "wage ladder" resulting from the search process. There is an important asymmetry between the incremental wage increases generated by on-the-job search (climbing the ladder) and the drop in income associated with job loss (falling off the ladder). This feature of the model generates differential savings behavior at different points in the earnings distribution. The behavior of workers in low paying jobs is primarily governed by the expectation of wage growth, while the behavior of workers near the top of the distribution is primarily driven by the possibility of job loss. The wage growth expected by low wage workers, combined with the fact that their earnings are not much higher than unemployment benefits, causes them to dis-save. As a worker's wage increases, the incentive to save increases: the potential for wage growth declines and it becomes increasingly important to insure against the large income reduction associated with job loss. The fact that high wage and low wage workers have such different savings behavior leads to a wealth distribution that is much more unequal

than the wage distribution. The literature to date that has considered savings in a labor search model has been primarily concerned with the effect of wealth on an unemployed worker's search effort or reservation wage.[1] In this paper I fully develop a theory for optimal savings with on-the-job search, and relate the parameters characterizing the labor market frictions directly to the workers' optimal savings decision.

Qualitatively, the model I develop can be viewed as a micro-foundation for the exogenous stochastic discount factor of Krusell and Smith (1998) and the particular exogenous income process of Castañeda, Díaz-Giménez, and Ríos-Rull (2003). Both of these papers aim to account for the distributions of wealth within a general equilibrium model of *ex ante* identical workers who face uninsured idiosyncratic shocks. Krusell and Smith (1998) find that the fit to wealth inequality can be improved dramatically if heterogeneity in the rate of time preference is used. Small differences in the rate of time preference across individuals result in large differences in savings behavior over time. Castañeda, Díaz-Giménez, and Ríos-Rull (2003) adopt an alternative approach. Instead of using an income process estimated from the data, they target the Lorenz coordinates for income and wealth inequality, and let the income dynamics be whatever is necessary to generate the observed inequality. As a result, the model can replicate the cross sectional income and wealth distributions found in the data, but the dynamics of the model's income process do not have a direct empirical counterpart.

As a quantitative exercise, I estimate the dynamics of the income process within the partial equilibrium labor search model, and then aggregate up earnings and wealth to check whether the implied inequality in earnings and wealth from the model replicates that observed in the data. This exercise asks whether the model, which is fit to the dynamics of individual labor market histories and the asset accumulation of a cohort of worker form the NLSY 1979, can aggregate to replicate the cross-sectional implications for the distribution of earnings and assets in the economy. The model performs well on many dimensions, although there is a tension when fitting employment dynamics and wage dynamics simultaneously, and the implied concentration of wealth among the richest one percent is substantially below what is observed in the US data.

The remainder of the article is organized as follows. Section 2 presents the model and characterizes the optimal search and savings decisions of workers. I discuss estimation challenges and a strategy for identification of the model parameters using simulation-based estimation in Section 3. Estimation results and the quantitative implications of the model are presented in Section 4. Section 5 concludes and provides directions for further research. All proofs are collected in the Appendix.[2]

2. A MODEL OF ON-THE-JOB SEARCH AND SAVINGS

Time is continuous and there is no aggregate uncertainty. Within a well-defined labor market workers are homogeneous in terms of productivity. Workers are *ex ante* identical,

---

1. Direct empirical support for a positive effect of wealth levels on unemployment durations is provided by Card, Chetty, and Weber (2007). The theoretical literature includes the original contribution on risk aversion and reservations wages by Danforth (1979), and the recent contributions of Acemoglu and Shimer (1999), Costain (1999), Lentz and Tranaes (2005), Rendón (2006), Browning, Crossley, and Smith (2007), and Lentz (2009). One of the innovations of the current paper is the incorporation of on-the-job search, which I show to be an important mechanism for delivering a very dispersed wealth distribution. Recent work incorporating savings into a Mortensen-Pissarides type model with aggregate fluctuations includes Bils et al. (2011) and Krusell et al. (2010).

2. Extended derivations, some numerical details, and robustness exercises are collected in the companion Web Appendix.

but will differ *ex post* due to differing labor market histories. Workers are risk averse and derive utility from consumption and disutility from the effort of searching for a new job. Markets are incomplete in the sense that workers cannot trade a complete set of contingent claims for consumption. Workers are restricted to self-insure against income loss by saving and borrowing at a constant risk-free interest rate.

Let the workers' planning horizon be infinite and let streams of consumption and search effort be ordered according to

$$\mathbb{E}_0 \int_0^\infty \mathrm{e}^{-\rho t}[u(c_t) - e(s_t)]dt, \tag{2.1}$$

where ρ is the subjective rate of time preference, c_t is the instantaneous consumption flow at time t, and s_t is the search effort at time t. Period utility has the Constant Relative Risk Aversion (CRRA) form,

$$u(c_t) = \begin{cases} \dfrac{c_t^{1-\gamma} - 1}{1 - \gamma}, & \gamma > 0 \\ \log(c_t), & \gamma = 1, \end{cases}$$

where γ is the coefficient of relative risk aversion. Search costs have the power form,

$$e(s_t) = \frac{\mu s_t^\eta}{\eta},$$

where $\eta > 1$ is the elasticity of search costs with respect to effort, and $\mu > 0$ is a scaling parameter. Workers are impatient in that the subjective rate of time preference exceeds the risk free rate, $\rho > r$.

At any time t the worker may be unemployed or employed. Workers search for jobs and make consumption decisions both when unemployed and when employed. The probability of finding a job is described by a Poisson arrival process, where the arrival rate depends positively on the intensity of the worker's search effort: λs. Upon contacting a job, the workers face a known stationary offer distribution $F(w)$, $w \in [\underline{w}, \overline{w}]$, where the offer is a constant wage for the duration of the job. Jobs end when either a worker finds a higher paying job or is exogenously separated at exponential rate δ. $W(a, w)$ denotes the expected present value of being employed with assets a and a wage w, and $U(a)$ denotes the expected present value of being unemployed with asset level a.

The budget constraint can be described by the asset accumulation equation and the stochastic process governing labor income. A worker accumulates assets according to

$$da = [ra + i - c]dt \quad \text{subject to} \quad a \geq \underline{a}, \tag{2.2}$$

where r is the risk free interest rate, a is the current asset level, i is income from wages or unemployment benefits, c is consumption, and $\underline{a}$ is the lower bound on assets. A worker's wage or benefit income changes stochastically according to

$$di = \begin{cases} dq_{\lambda s}1(W(a, x) \geq U(a))[x - b], & \text{when unemployed,} \\ dq_{\lambda s}1(W(a, x) \geq W(a, w))[x - w] + [b - w]dq_\delta, & \text{when employed,} \end{cases} \tag{2.3}$$

where $1(\cdot)$ is the indicator function that takes a value of one when the argument is true and zero otherwise, x is drawn from the wage offer distribution $F(w)$, $dq_{\lambda s} = 1$ when a job offer arrives and 0 otherwise, and $dq_\delta = 1$ when a job is exogenously destroyed and 0 otherwise.[3]

---

3. A general model with additional stochastic non-labor income is outlined in the Web Appendix.

Consider the problem of a worker who is currently unemployed with assets a. At each instant the worker faces the possibility, which is increasing in search effort s, of a job offer with Poisson probability λs. When an offer arrives, he will accept it if the value of working at the offered wage $W(a,w)$ exceeds the value of remaining unemployed $U(a)$. Since both the time until a job offer arrives and the potential wage once an offer is received are uncertain, the problem facing the unemployed worker is to decide how much to consume at each instant while unemployed, how hard to search for work, and the minimum acceptable wage offer, $\tilde{w}(a)$, that will induce a move from unemployment to employment. In general all of these decisions may depend on current assets a.

When employed, workers engage in on-the-job search, and face an exogenous probability δ of job destruction.

The problem outlined in equations (2.1), (2.2), and (2.3) can be conveniently represented using the continuous time Bellman equations for the value of being unemployed with assets a and the value of being employed with assets a and wage w:

$$\rho U(a) = \max_{0 \leq c \leq a - \underline{a}, 0 \leq s} \left\{ u(c) - e(s) + U_a(a)[ra + b - c] \right.$$
$$\left. + \lambda s \int \max\{W(a,x) - U(a), 0\} dF(x) \right\}, \quad (2.4)$$

$$\rho W(a,w) = \max_{0 \leq c \leq a - \underline{a}, 0 \leq s} \left\{ u(c) - e(s) + W_a(a,w)[ra + w - c] \right.$$
$$\left. + \lambda s \int \max\{W(a,x) - W(a,w), 0\} dF(x) + \delta[U(a) - W(a,w)] \right\}. \quad (2.5)$$

The flow value of being unemployed with assets a is given by the utility flow from consumption $u(c)$ less the disutility of search effort $e(s)$ plus the expected change in the value of unemployment. The latter has two parts. First, the value of unemployment changes because assets change due to accumulation (or decumulation). This is the term $U_a(a)[ra + b - c]$: the marginal value of assets multiplied by the instantaneous change in assets. Second, the value of being unemployed changes in expectation by the product of the arrival rate of job offers and the expected net gain associated with job offers: $\lambda s \int \max\{W(a,x) - U(a), 0\} dF(x)$. When employed, the flow value of employment will additionally change in expectation by the product of the job destruction rate and the net loss associated with losing wage w: $\delta[U(a) - W(a,w)]$.

The lower bound on assets $\underline{a}$ is taken to be the self-imposed borrowing limit $\underline{a} = -b/r$, where r is the risk free rate (Aiyagari, 1994). A worker will never choose to borrow an amount in excess of what he can service and maintain positive consumption, even if unemployed indefinitely.[4]

I now turn to the discussion of optimal search effort and optimal consumption choices for workers. The first order necessary conditions for optimal consumption and search

---

4. Making use of the self imposed borrowing constraint simplifies the derivation of optimal consumption since the marginal utility of consumption can always be equated to the marginal value of wealth.

effort when unemployed and employed are:

$$u'(c) = U_a(a), \tag{2.6}$$

$$e'(s) = \lambda \int \max\{W(a, x) - U(a), 0\} dF(x), \tag{2.7}$$

$$u'(c) = W_a(a, w), \tag{2.8}$$

$$e'(s) = \lambda \int \max\{W(a, x) - W(a, w), 0\} dF(x). \tag{2.9}$$

Conditions (2.6) and (2.8) require the marginal utility flow of consumption to be equal to the marginal value of assets, both when unemployed and unemployed. This is the standard inter-temporal result that expected utility cannot be increased by additional savings or borrowing. Conditions (2.7) and (2.9) require the marginal cost of search effort to be equal to the expected change in value associated with an accepted wage offer. Search effort is chosen such that expected utility cannot be increased by exerting more or less effort.

In addition to making consumption and search effort decisions, unemployed workers must decide on the minimum wage offer that will induce a move from unemployment to employment. This reservation wage is the unique solution to $W(a, \tilde{w}(a)) = U(a)$.

Proposition 1. *The reservation wage for unemployed workers is independent of assets and equal to the unemployment benefit:* $\tilde{w}(a) = b$.

Proof. See Appendix Appendix A.1.

The constant reservation wage is a direct consequence of the fact that the job contact rate λ and the disutility of search $e(s)$ do not depend on the worker's employment status. Since there is no option value associated with remaining unemployed, any wage higher than the unemployment benefit is acceptable. Although the reservation wage is constant and independent of assets, the transition rate out of unemployment varies with assets because search effort varies with assets. Optimal search effort is characterized by the equation

$$s = \varphi \left(\lambda \int_{\underline{w}}^{\overline{w}} \frac{u'(c) + [u''(c)c_w][ra + x - c]}{\rho + \delta + \lambda s \overline{F}(x)} \overline{F}(x) dx \right), \tag{2.10}$$

where φ is the inverse function for the marginal cost of search $e'(s)$.

Finally, optimal consumption growth for the periods between job transitions can be characterized by the differential equation for consumption

$$\frac{\dot{c}}{c} = \frac{1}{\gamma} \left(r - \rho - \lambda s \left(\overline{F}(w) - \int_{w}^{\overline{w}} \frac{u'(\hat{c})}{u'(c)} dF(x) \right) + \delta \left(\frac{u'(\underline{c})}{u'(c)} - 1 \right) \right), \tag{2.11}$$

the equation for asset accumulation

$$\dot{a} = ra + w - c,$$

and the present-value budget constraint

$$\lim_{t \to \infty} e^{-rt} a(t) \geq 0, \text{ (a.s.)}, \tag{2.12}$$

where $\dot{x} = dx/dt$, $\gamma = -u''(c)c/u'(c)$ is the coefficient of relative risk aversion, and I make use of the shorthand $\overline{F}(w) = [1 - F(w)]$, $\hat{c} = c(a, x)$, and $\underline{c} = c(a, \underline{w}) = c(a, b)$. Since

all workers will reject wage offers below the common reservation wage b, I normalize the wage offer distribution to have $\underline{w} = b$.[5] Equations (2.10)–(2.12) must hold at all times, meaning that at the instant a new wage offer is accepted, or a job is destroyed, search effort and consumption must change discretely to ensure the worker is on the saddle path implied by the new wage. In other words, equation (2.11) describes consumption growth between jumps, when $dq_{\lambda s}\mathbf{1}\left(W(a, w') \geq W(a, w)\right) = dq_\delta = 0$.

The consumption growth equation (2.11) provides a direct and intuitive link between the labor market frictions λ and δ and the motives for saving or dis-saving of workers at various points in the earnings distribution. To highlight the direct effect that the "job ladder" has on savings behavior, I first consider the case in which search effort is exogenous, $s = 1$.

Proposition 2. *In the case where search effort is exogenous ($s = 1$) the job contact and job destruction rates have opposing influences on the incentive to save or dis-save, and the tension from these opposing forces results in a target level of savings that depends on the current wage.*

Proof. See Appendix Appendix A.1.

Examination of equation (2.11) reveals the effect of the "wage ladder" on the savings behavior of workers at different points of the earnings distribution.

1 As in an environment with perfect certainty, $r - \rho$ represents the importance of the rate of time preference relative to the interest rate in determining savings.
2 λ represents the potential for wage growth and induces additional impatience over and above the pure rate of time preference ρ. The influence of expected wage growth is greatest at the lowest wage $\underline{w}$ and falls monotonically as the wage increases, having no effect at the highest wage $\overline{w}$. As the current wage increases, the probability of receiving a higher wage offer falls. Similarly, as the current wage increases, the expectation of any change in the marginal utility of consumption (due to *ex post* Euler equation errors) also falls.
3 δ represents the risk of job loss, and induces precautionary savings. The effect on savings of unemployment risk is greatest at the highest wage $\overline{w}$ and falls monotonically, having no effect at the lowest wage $\underline{w}$. Intuitively, the cost of having no savings, in terms of the required change in the marginal utility of consumption, is zero for a worker who looses a job which pays $\underline{w} = b$, since his flow budget constraint has not changed. If we consider a worker who initially has zero savings but is lucky enough to receive the highest wage offer $\overline{w}$, the cost of not saving in the event of a job loss is a change in the marginal utility of consumption from $u'(\overline{w})$ to $u'(b)$. Faced with this possibility, a worker lucky enough to obtain the highest wage offer will save at a very high rate in an attempt to smooth out any such change.
4 Given any wage w there is a target asset level $a^*(w)$ that balances the competing influences in 1) 2) and 3). Once $a^*(w)$ is attained, the worker will maintain a constant consumption level, equal to wage plus interest income, until he either switches jobs or becomes unemployed.[6]

---

5. Indeed, in an equilibrium version of the model, optimization by firms would ensure this is the case since offers below b would never be accepted.
6. Carroll (2004) proves the existence of a target level of savings in a discrete time framework, a result that had previously been a robust feature of simulations but not proved generally.

The key to generating heterogeneity in savings is that expected gains and losses in income are not symmetric, and differ according to the current wage. Workers in the lowest paying jobs expect to gain much more when offered a new job than they expect to lose if that job is lost; this results in a desire to bring future income forward. Conversely, workers at the highest paying jobs have very little expectation of wage growth, but will lose a lot in the event of job loss, resulting in a strong motive to build up precautionary savings as a means to insure consumption across this transition.

I now return to the case of endogenous search effort. The advantage of modeling search effort is that it allows for the possibility that both unemployment and job durations are increasing in current assets, with job durations also increasing in the current wage, a feature that turns out to be empirically relevant. In order for Proposition 2 to continue to hold with endogenous search, it must be the case that the value function is concave in assets and the wage. This is indeed true at the estimated parameters, presented in Figure 3, and for all simulations that I have carried out, but has not been established to hold in general.

The third proposition examines the target asset level of a worker employed at the highest wage. This level determines the upper bound on assets (and does not depend on whether search effort is endogenous or exogenous).

Proposition 3. *Under the assumption that workers are sufficiently impatient $(\rho > r)$, the upper bound on desired assets is finite, and defined implicitly by the equation*

$$\overline{w} + r\overline{a} = \phi\left(\frac{\delta u'(c(\overline{a}, \underline{w}))}{\rho + \delta - r}\right), \tag{2.13}$$

where ϕ is the inverse function of the marginal utility of consumption.
In the limit as $(\rho - r)/\delta$ tends to zero, this tends to

$$\overline{w} + r\overline{a} = c(\overline{a}, \underline{w}). \tag{2.14}$$

Proof. See Appendix Appendix A.1.

The upper bound on assets is determined endogenously by the desire to smooth the marginal utility of consumption across employment states. When $(\rho - r)/\delta$ is small, workers at the highest wage save up to the point at which they can minimize any discrete change in consumption at the instant of a job loss.

3. IDENTIFICATION AND ESTIMATION

To solve the model requires knowledge of the wage offer distribution $F(w)$, the risk free rate r, the rate of time preference ρ, the coefficient of relative risk aversion γ, the elasticity of search costs with respect to effort η, the scale of the disutility of search μ, the arrival rate of job offers λ, and the job destruction rate δ. I fix the risk free rate at three percent, the rate of time preference at five percent, and the scale of the disutility of search effort at one.[7] This leaves $F(w)$, γ, η, λ, and δ to estimate. It is possible to directly identify $F(w)$

7. The scale of the disutility of search effort is not identified without direct observation of search effort. I choose to fix the rate of time preference since, in the absence of variation in the interest rate, the identification of both the rate of time preference and the coefficient of relative risk aversion leans heavily on the assumption of CRRA preferences (see Crossley and Low (2011) for a discussion on how much CRRA buys in terms of identification). In a general equilibrium model the demand for capital would provide additional structure relating time preference to the interest rate, aiding identification.

using wages accepted out of unemployment. This approach makes use of the fact that, as outlined in Proposition 1, workers only reject wage offers if they fall below their current wage. I normalize the wage offer distribution to have lower support $\underline{w} = b$ so that there are no offers below the common reservation wage of the unemployed b; unemployed workers never reject wage offers.[8] The distribution of wages accepted out of unemployment is a consistent estimate of the wage offer distribution $F(w)$, and can be estimated non-parametrically (Bontemps et al., 1999, 2000). The weekly employment-to-unemployment transition rate is a direct estimate of δdt. Given $F(w)$, the elasticity of search cost with respect to effort η is identified (up to the scale factor λ) from the unemployment-to-employment and job-to-job transition probabilities, conditional on assets and wages as they tell us directly about

$$\frac{\partial \lambda s(a,w)\overline{F}(w)}{\partial a} \text{ and } \frac{\partial \lambda s(a,w)\overline{F}(w)}{\partial w}.$$

Given δ, $F(w)$, and η, the age profile of employment rates is directly informative about λ through the equation describing the evolution of employment for the cohort

$$E_t = (1 - \delta) E_{t-1} + (1 - E_{t-1}) \lambda \int s(a,b)h_t(a)da,$$

where $h_t(a)$ is the distribution of asset holdings among the unemployed when the cohort is age t.

Finally, I turn to identifying the coefficient of relative risk aversion γ. Equation (2.11) indicates that the most informative moments here involve consumption growth. In the absence of panel data on consumption, I use panel data on assets. Consumption and asset growth are linked in a direct manner through the budget constraint. To pin down γ, I use the moments describing asset accumulation, $\overline{\Delta a_t}$, $var(\Delta a_t)$, $cov(\Delta a_t, \Delta a_{t-1})$, and the age profile for mean assets $\overline{a}_t$.

In addition to the moments described above, which are sufficient to identify all the parameters of the model, I also use the moments describing wage growth $\overline{\Delta w_t}$, $var(\Delta w_t)$, $cov(\Delta w_t, \Delta w_{t-1})$, which impose the over identifying restriction that the model is also consistent with the reduced form for wage dynamics. The information on wage levels is already captured in the non-parametric estimation of the wage offer distribution.

3.1. Indirect Inference

The data for this analysis are from the National Longitudinal Survey of Youth 1979 (NLSY). I use the white male sample for the years 1985 to 2002. The restriction of attention to white males is motivated by an attempt to create a relatively homogeneous subgroup that is well described by the model developed in the paper. The NLSY data provides weekly information on each individual's current employment status, wage, and whether the worker continued the next week at the same job, a new job, or transited to unemployment. The asset data are provided at the interview date, which is at most once a year. I use indirect inference (Smith, 1993; Gourieroux et al., 1993) to overcome the fact that assets are only partially observed, at irregularly spaced interview dates.[9] Indirect inference proceeds by estimating descriptive (as opposed to structural) models

---

8. In general, the job contact rate and the mass of offers below the reservation rate are not separately identified without adding further structure, for example, by modeling wage posting behavior of firms.

9. In a simpler model without on-the-job search or dispersed wages, Bayer and Wälde (2010) develop expressions for calculating the probability of observing, at any future point in time, employment-

on both the actual data and on data simulated from the structural model, and estimating the structural parameters by minimizing the distance between the coefficients from these auxiliary regressions. I choose the auxiliary models to capture the dynamics of assets and wages, and labor market transitions conditional on the state variables. Specifically, they correspond to the following twelve regressions (estimated separately by education group):

Employment Dynamics.

$$E2U_{i,m} = \beta_1 + \mathbf{x}'_{i,m}\alpha_{1,\mathbf{x}} + \alpha_{1,i} + \varepsilon_{1,i,m} \tag{3.15}$$

$$J2J_{i,m} = \alpha_2 + \beta_{2,a}a_{i,m} + \beta_{2,w}w_{i,m} + \mathbf{x}'_{i,m}\alpha_{2,\mathbf{x}} + \alpha_{2,i} + \varepsilon_{2,i,m} \tag{3.16}$$

$$U2E_{i,m} = \alpha_3 + \beta_{3,a}a_{i,m} + \mathbf{x}'_{i,m}\alpha_{3,\mathbf{x}} + \alpha_{3,i} + \varepsilon_{3,i,m} \tag{3.17}$$

Wage Dynamics.

$$\Delta w_{i,t} = \sum_t \beta_{4,t}d_t + \varepsilon_{4,i,t} \tag{3.18}$$

$$\left(\Delta w_{i,t} - \overline{\Delta w_t}\right)^2 = \sum_t \beta_{5,t}d_t + \varepsilon_{4,i,t} \tag{3.19}$$

$$\left(\Delta w_{i,t} - \overline{\Delta w_t}\right)\left(\Delta w_{i,t-1} - \overline{\Delta w_{t-1}}\right) = \sum_t \beta_{6,t}d_t + \varepsilon_{6,i,t} \tag{3.20}$$

Asset Dynamics.

$$\Delta a_{i,t} = \sum_t \beta_{7,t}d_t + \varepsilon_{7,i,t} \tag{3.21}$$

$$\left(\Delta a_{i,t} - \overline{\Delta a_t}\right)^2 = \sum_t \beta_{8,t}d_t + \varepsilon_{8,i,t} \tag{3.22}$$

$$\left(\Delta a_{i,t} - \overline{\Delta a_t}\right)\left(\Delta a_{i,t-1} - \overline{\Delta a_{t-1}}\right) = \sum_t \beta_{9,t}d_t + \varepsilon_{9,i,t} \tag{3.23}$$

$$\left(\Delta a_{i,t} - \overline{\Delta a_t}\right)\left(\Delta w_{i,t} - \overline{\Delta w_t}\right) = \sum_t \beta_{10,t}d_t + \varepsilon_{10,i,t} \tag{3.24}$$

Employment and Asset Levels.

$$E_{i,t} = \sum_t \beta_{11,t}d_t + \varepsilon_{11,i,t} \tag{3.25}$$

$$a_{i,t} = \sum_t \beta_{12,t}d_t + \varepsilon_{12,i,t}. \tag{3.26}$$

The subscript m refers to a week and t refers to a year. $E2U_{i,m}$, $J2J_{i,m}$, and $U2E_{i,m}$ are binary indicators for employment-to-unemployment, job-to-job, and unemployment-to-employment transitions between weeks m and $m+1$. d_t is a dummy variable for the year t. $E_{i,t}$ is equal to the number of weeks worked by individual i during year t. Details describing the NLSY, the sample used for estimation, and variable construction are presented in Appendix Appendix A.2.

wealth combinations conditional on initial employment status and wealth. These expressions could potentially be used for maximum likelihood estimation, although indirect inference is likely to be more robust to specification errors.

TABLE 1

Conditional Transition Probabilities

		High school			College		
		Data	I	II	Data	I	II
E2U	constant	0.0423	0.0417	0.0418	0.0190	0.0189	0.0171
		(0.0009)	[0.63]	[0.56]	(0.0007)	[0.04]	[2.41]
U2E	assets	-0.0810	-0.078	-0.0762	-0.0939	-0.0448	-0.0305
		(0.0321)	[0.10]	[0.15]	(0.0463)	[1.06]	[1.37]
J2J	assets	-0.0167	0.0046	0.0046	-0.0039	0.0115	0.0054
		(0.0019)	[11.17]	[11.14]	(0.0014)	[10.67]	[6.47]
	wage	-0.0149	-0.0255	-0.0256	-0.0134	-0.0537	-0.0197
		(0.0017)	[6.10]	[3.14]	(0.0017)	[22.94]	[3.55]

Note: The standard errors for the auxiliary model estimates from the data are in parenthesis. All coefficients and standard errors have been multiplied by 10. The t-statistic for a test that the each coefficient estimate from the data is equal to the estimate from the model is presented in square brackets.

The vector of β-coefficients in (3.15) to (3.26) are matched in estimation. The α-coefficients are treated as nuisance parameters used to control for unmodeled heterogeneity; the vector $\mathbf{x}_{i,m}$ includes controls for marital status, number of children, rural/urban and region of residence. Identical regressions are estimated on the NLSY data and data simulated from the model, with the exception that for the model $\alpha_{k,\mathbf{x}} = \alpha_{k,i} = 0$, $k = \{1,2,3\}$. [10]

Auxiliary models (3.15) to (3.17) capture job destruction plus the relationship between assets and wages and the probability of exiting unemployment or of making a job-to-job transition. The β-coefficients from these regressions are presented in the columns labeled Data in Table 1. There is a negative relationship between assets and the probability of exiting unemployment or changing jobs, and there is a negative relationship between wages and the probability of changing jobs. In the model this pattern can be replicated via search effort decreasing in both assets and wages.

The dynamics of wages and assets are captured by auxiliary models (3.18) to (3.24). In addition to the labor market transitions and wage and asset dynamics, I use auxiliary models (3.25) and (3.26) to capture the aggregate employment rates and asset accumulation as the cohort ages. In Figures 1 and 2, panels (a) to (i), I plot the coefficients that correspond to the age profiles for the mean, variance and autocovariance of wage and asset changes, for low and high education groups respectively, along with the the covariance between wage and asset changes, the employment rate and the age profile for assets. The wage moments are sufficient to identify the variances in a reduced form model of income dynamics with a permanent and transitory shock, where the variance of the permanent shock is time varying (see Meghir and Pistaferri 2011 for details). Between the ages of 23 and 39, mean wage growth is relatively low and stable for low educated workers, while it is slightly higher for young highly educated workers, and decreasing with age. For both education groups the variance of wage growth decreases with age, and the

10. The model is one of homogeneous workers and is not designed in the current form to account for differences in outcomes across workers due to ex-ante worker heterogeneity. The interpretation of the results are conditional on the assumption of ex-ante homogeneous workers. I leave to future work the explicit modeling of potential heterogeneity in preferences, costs, or productivity, and the potential effects on aggregates of such heterogeneity.
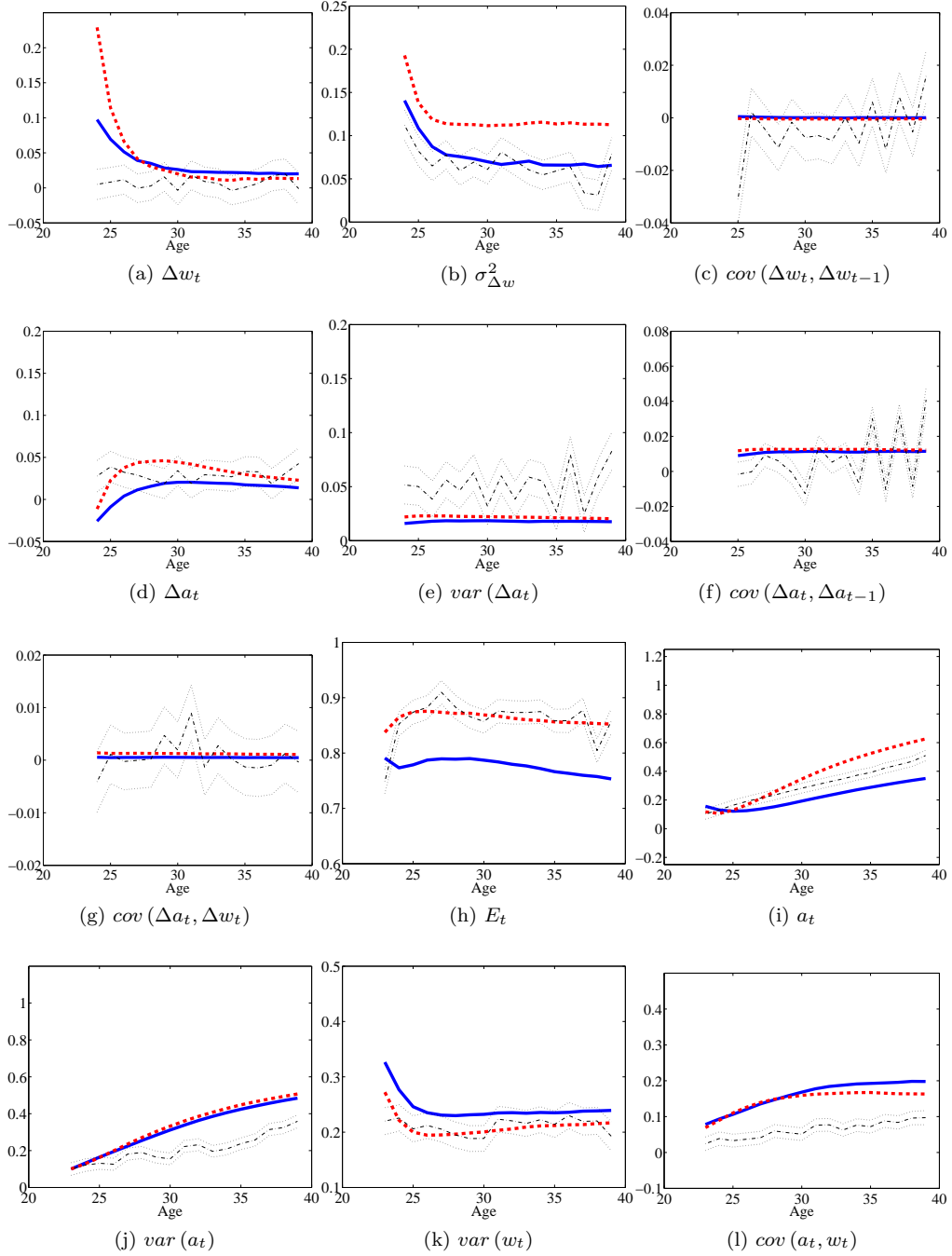
FIGURE 1

Data and Model Moments: Low Education

Note: The thin dashed line and the thin dotted lines are the data, plus and minus two standard errors.
The thick dashed lines are for model specification I, and the thick solid lines are model specification II.

FIGURE 2

Data and Model Moments: High Education

Note: The thin dashed line and the thin dotted lines are the data, plus and minus two standard errors.
The thick dashed lines are for model specification I, and the thick solid lines are model specification II.
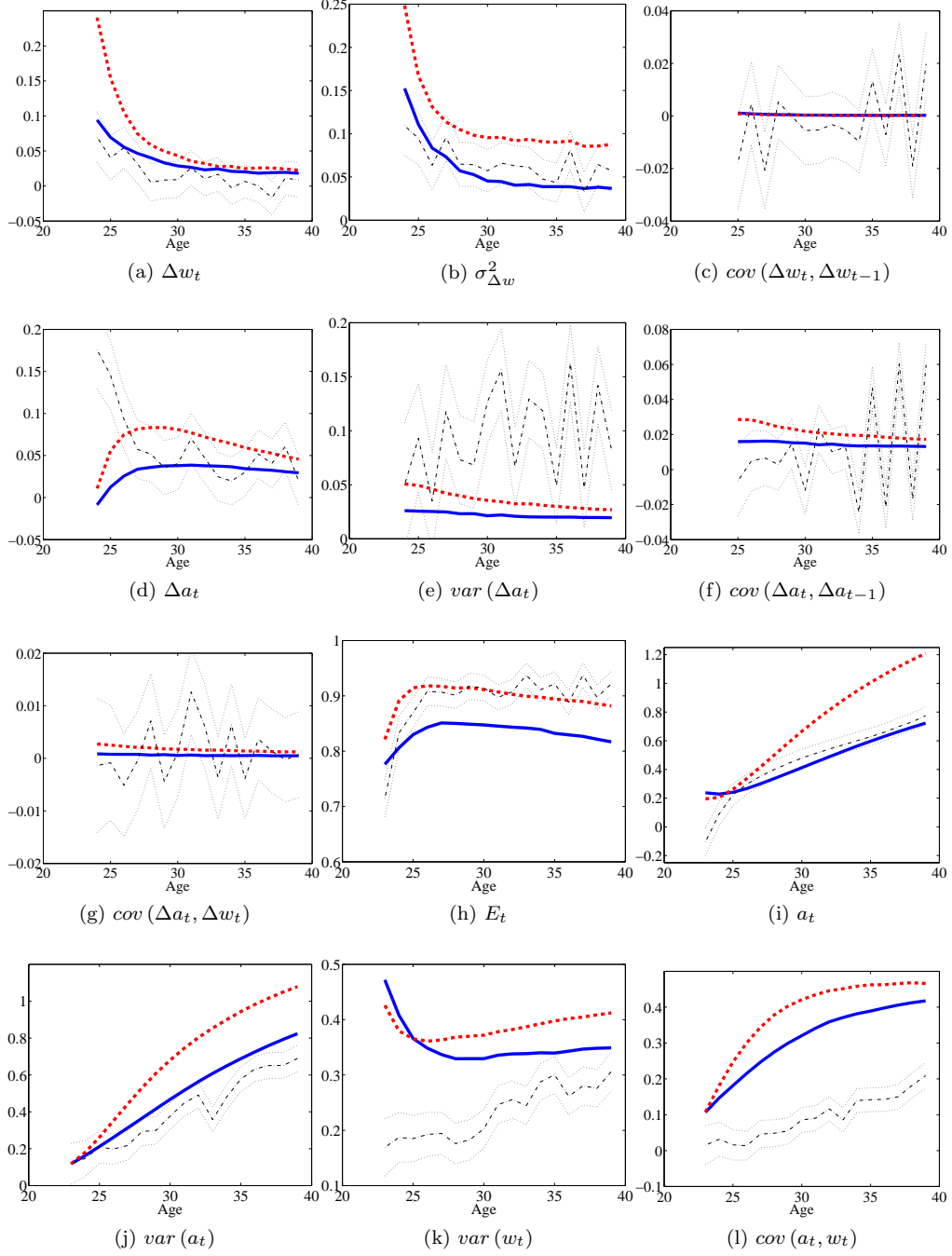
TABLE 2

Parameter Estimates

	High school		College	
	I	II	I	II
r: risk free rate		0.03		
ρ: time preference		0.05		
μ: search costs scale		1.0		
δ: job destruction rate	0.221 [0.219, 0.224]	0.216 [0.215, 0.219]	0.0989 [0.097, 0.101]	0.0893 [0.089, 0.090]
λ: job contact rate	0.615 [0.588, 0.643]	0.192 [0.184, 0.194]	0.407 [0.387, 0.434]	0.144 [0.141, 0.147]
η: elasticity of search costs w.r.t. effort	2.0 -	1.168 [1.166, 1.176]	2.0 -	1.268 [1.258, 1.274]
γ: relative risk aversion	2.0 -	1.455 [1.441, 1.487]	2.0 -	1.249 [1.204, 1.284]

Note: 95 percent confidence intervals in square brackets (2.5 and 97.5 quantiles of the quasi-posterior distribution). Specification I targets the age profile of employment and the job destruction rate (auxiliary models (3.15) and (3.25)). Specification II uses all the auxiliary models (3.15) to (3.26).

autocovariance is slightly negative and relatively stable. Turning to mean changes in assets, the age profile is positive and stable for the low education group, and positive and decreasing for the high education group. The variance and autocovariance of asset changes are quite noisy, and do not appear to have a strong age profile. The covariance between wage changes and asset changes is positive, but fairly noisy. For both education groups employment rises very rapidly at early ages, and then remains fairly stable, and mean assets increase with age.

Key to the estimation strategy is sampling from the model simulated data in a manner consistent with how the actual data were sampled. I do this by simulating data for the same number of individuals as I observe in the NLSY, and for the same number of years as recorded in the NLSY. To ensure the initial conditions are matched, I start the simulation using the initial employment status, wage (if employed) and asset levels of the NLSY workers at the beginning of 1985. Additionally, from the simulation, I sample assets at an annual frequency and wages at a weekly frequency. Further details regarding estimation are presented in Appendix Appendix A.3.

4. QUANTITATIVE RESULTS

I present in Table 2 estimates for two model specifications that differ, essentially, by the number of estimated parameters and the number of moments used to fit the parameters. The first set of estimates fix the coefficient of relative risk aversion at two and assume search costs are quadratic. This leaves only the job destruction rate δ, and the job contact rate λ, as free parameters. These are estimated by matching only the empirical employment-to-unemployment transition rate and the age profile of employment rates, E_t (auxiliary models (3.15) and (3.25)).

The model provides a near perfect fit to these targeted moments for both the low and high educated workers (see Table 1, the first row of column I and Figures 1 and 2, panel (h)). The ability of this restricted model to fit the other moments will be discussed below in relation to the second set of estimates.

The second set of parameter estimates also estimate the elasticity of search costs with respect to effort η, and the coefficient of relative risk aversion γ. In this case I use the 12 auxiliary regressions (3.15) to (3.26). The estimated coefficients from the first three auxiliary models are presented in Table 1. The estimated coefficients from the last nine auxiliary models are presented in Figures 1 and 2, panels (a) to (i). There is little difference in terms of the estimate of δ between the two approaches, which is a direct result of this parameter being so tightly linked to the empirical employment-unemployment transition rate. In terms of search costs, the estimated version turns out to be substantially lower than quadratic; 1.17 and 1.27 for the low and high education groups respectively. The lower elasticity of search costs is offset by a smaller estimate of the job contact rate, implying more scope for endogenous search effort than what one obtains by assuming quadratic search costs. Estimating the search costs does very little to change the fit to the conditional transition probabilities other than improving the fit of the effect of the wage on the job-to-job transition rate, as indicated in Table 1. Similarly, the estimates for the coefficient of relative risk aversion are well below two for both low and high educated workers at 1.46 and 1.25 respectively. The higher estimated risk aversion for low relative to high educated workers is driven by the lower asset growth of the former (recall the pure rate of time preference is assumed to be common for both groups).

The model is well suited to jointly reproducing employment dynamics and the cross-sectional distribution of wages, but is somewhat less successful at also reproducing wage dynamics.[11] Turning to the model fit between the two specifications, we can see clearly where the model does well, and also where the tensions lie. Recall that when only targeting the employment transitions, the model matches the age profile of employment almost exactly (Panel (h) in Figures 1 and 2). This fit, however, comes at the cost of a poor representation of wage dynamics. Looking at panels (a) and (b) of these same figures we see that the model overstates both the mean and variance of wage growth. Conversely, when these moments are included in estimation, the model is able to describe well the wage dynamics, but at the cost of understating the employment rates (by approximately ten and five percentage points for the low and high educated worker, respectively). The tension here arises from the fact that the model puts a lot of structure on how wages can change. Indeed, the model requires wages to rise only when workers change jobs, and to fall only when workers experience a spell of unemployment. In order to match the low rate of wage growth in the data, the model requires a low job-to-job transition rate. Since the technology for changing jobs is assumed to be the same as the technology for finding jobs, this also results in workers leaving unemployment at a lower rate, resulting directly in a lower employment rate.

With the exception of the age profile of employment and mean asset holdings, the moments used in estimation are all in differences (individual labor market transitions and wage and asset changes). In panels (j), (k) and (l) I plot the implications for three macro moments that are not used in estimation: the age profiles of the variance of assets, the variance of wages and the covariance of assets and wages. Qualitatively, the model reproduces the rising variance of assets with age and the rising covariance between assets and wages, although to a slightly higher level than what we see in the data. The age

---

11. Recent work by Postel-Vinay and Turon (2010) finds that an on-the-job search model extended to feature firm specific productivity shocks and wages renegotiated when it is mutually agreeable to do so can jointly match the employment and wage dynamics. That said, theirs is is a model with risk neutral workers, and it is a non-trivial task to incorporate these types of wage contracts when workers are risk averse and can save.

profile of the variance of wages from the model is at odds with the increasing empirical profile. The model implies that the variance of wages falls during the first five years before increasing, while the data is more consistent with a monotonic increase with age.

The fit of the model to the data suggests that the logic of the job search model, where workers actively search for better opportunities and save to protect their standard of living in the event of job provides a very useful interpretation of the data. At this point it is useful to recall that the model is very parsimoniously parametrized. There are no time-varying parameters or shocks. The earnings process is completely characterized by a stationary wage offer distribution, a constant job destruction rate and a constant job contact rate. The age profiles of the moments generated by the model are purely the result of the cohort moving from an initial distribution toward the stationary distribution implied by the steady state of the model.

4.1. *Aggregate Implications for the Distribution of Earnings, Wealth and Consumption*

Having established that the model does a reasonably good job of matching employment and wage dynamics, I now turn to the model's implications for the aggregate distribution. The model generates simulated data that, suitably aggregated, provides a reasonable description of the cross-sectional distributions of wages, wealth, and consumption for the entire US population.

In order to approximate a cross-section from the economy I pool over education and age as follows. The model is simulated separately for low and high education groups for ages 25 to 65, starting with the initial conditions for employment, wages, and assets in the NLSY. The simulated data is then pooled over age and education, where the number of simulations within each education group is proportional to its size in the NLSY data. The pooled data can be viewed as approximating an overlapping generations economy with a constant age structure, where each new generation starts life with the same distribution of initial assets. The implications of the model in terms of the aggregate earnings, wealth, and consumption distributions are presented in Table 3. Here I present the share of earnings, wealth and consumption held by each quintile, plus the top decile broken into the 90-95th, 95th-99th and 99th-100th percentiles based on the model aggregated data. I also present these shares separately by education group to highlight the effect of aggregating over skill. The corresponding shares for the US economy are also presented, and are constructed from the SCF and CEX (average shares for the years 1989, 1992, 1995, 1998, and 2001). For the US data I present the shares calculated on four subsets of the data. The first row uses the full sample and includes all households. The shares calculated here are representative of the entire US population. The second row presents the shares for the sample of households in which the head is working for a wage (is not self employed, retired, or disabled). This is the population that that the model most closely represents. I further subdivide this population into high and low educated households (by education of the household head).

Comparing first earnings from the model with the full US sample, we see that the main discrepancy between the model simulation and the US cross sectional data is that the model is missing the very highest earners and those who have zero earnings. This is a direct implication of excluding the self employed, retired and disabled from the estimation sample. We see in Table 3, panel (a) that in the full US sample the entire bottom quintile has zero earnings. This quintile comprises both those who are not working due to unemployment but also the self employed with only capital income, the retired or disabled. Since the model assumes all workers are either employed or unemployed, the

TABLE 3

Distributions of Earnings, Wealth and Consumption: Data and Model

| | Quintile | | | | | Top Groups | | |
	First	Second	Third	Fourth	Fifth	90th–95th	95th–99th	99th–100th
(a) Distribution of Earnings								
Full sample	0.00	3.26	13.26	24.42	59.06	12.58	15.78	12.10
Wage earners	3.91	10.43	15.89	22.55	47.21	10.15	12.36	9.26
- Low education	3.83	11.37	17.48	25.08	42.24	9.83	10.75	5.33
- High education	4.64	10.70	15.56	21.89	47.21	9.91	12.71	9.69
Model pooled	1.01	10.62	16.50	23.21	48.66	10.77	13.39	8.38
- Low education	1.03	11.61	18.11	24.89	44.35	11.20	11.76	4.30
- High education	1.17	10.83	16.07	21.31	50.62	11.21	15.40	8.88
Augmented offer	1.32	2.28	11.70	18.87	65.83	16.21	21.07	10.88
(b) Distribution of Wealth								
Full sample	-0.20	1.38	5.15	12.58	81.08	12.19	23.67	32.19
Wage earners	-0.23	2.42	7.15	15.70	74.96	12.51	20.69	26.63
- Low education	-0.17	2.88	9.00	20.12	68.17	14.00	19.04	16.69
- High education	-0.14	2.84	7.43	15.63	74.24	12.33	21.62	25.59
Model Pooled	-6.72	-1.32	4.91	16.66	86.48	16.95	27.81	22.83
- Low education	-13.77	-5.03	4.85	22.30	91.66	21.71	29.67	14.83
- High education	-3.45	0.66	5.55	15.49	81.74	16.51	28.42	19.38
Augmented offer	-2.58	0.53	5.35	17.43	79.27	17.99	24.96	15.47
(c) Distribution of Consumption								
Full sample	7.75	13.16	17.48	22.93	38.67	9.34	9.90	4.50
Wage earners	8.79	13.70	17.65	22.71	37.16	8.98	9.37	4.23
- Low education	9.08	14.03	17.92	22.76	36.22	8.70	9.00	4.15
- High education	9.75	14.19	17.74	22.48	35.83	8.66	8.85	4.05
Model Pooled	11.51	15.03	17.53	20.84	35.09	7.99	9.14	5.10
- Low education	13.60	17.10	19.46	22.01	27.83	6.95	6.33	2.08
- High education	10.24	14.22	16.98	20.93	37.64	8.60	10.18	5.46
Augmented offer	11.62	14.86	17.04	20.45	36.03	8.23	9.54	5.39

Note: The cells represent the share of earnings/wealth/consumption held by the corresponding quantile; *i.e.*, the shares held by the five quintiles add to 100. The first four rows in each panel are based on the US SCF and the CEX data (average shares for the years 1989, 1992, 1995, 1998, and 2001). Consumption is expenditure on non durables plus imputed service flows of consumer durables. The rows labeled "Augmented offer" are based on a simulation where the wage offer distribution of the low education group is augmented to have a long right tail. Specifically, I replace the top one percent of wages in the empirical offer distribution with interpolated wages between the 99th percentile and ten times the maximum observed wage. This has the effect that even workers at the 99th percentile know there are wages substantially higher than their current wage still on offer.

share of earnings held by the lowest quintile could only be zero if the unemployment rate exceeded 20 percent. The share implied by the model is, however, very low at just over one percent. Looking at the top quintile, we see that in the entire US population this group has 59 percent of earnings, the model implies only 49 percent. When compared to the entire US population, the model implies that the lowest quintiles earn too much and the highest quintiles earn too little.

The model aggregates align much more closely with the data when we focus our attention on households that are loosely homogenous *ex ante* (by removing the self employed, retired and disabled). The share of total earnings held by the top four quintiles, as well as the shares within the 90-95th, 95-99th and 99-100th percentiles from the model all match to within 1.5 percentage points the corresponding shares in the subsample of wage earners in the data. The discrepancy now is that the bottom quintile in the data has 3.9 percent of total earnings, while the model implies only one percent. The model fits the subsamples of low and high educated workers equally well as it fits the pooled sample of wage earners.

The cross-sectional distribution of wealth in the entire US population is much more unequal than earnings, as can be seen in Table 3, panel (b). The share of wealth of the bottom quintile is negative (-0.20 percent) while the top quintile holds 81 percent of wealth, with 32 percent of total wealth held by the top one percent. The model actually implies a distribution of wealth that is even more unequal than that in the data, with the exception of the very top. In the model the bottom two quintiles both hold a negative share; workers borrow much more in the model than in the data implying the bottom quintile holds -6.7 percent of wealth as opposed to the -0.20 percent observed in the data. The top quintile in the model holds an even larger share of wealth than in the data at 86 percent relative to 81 percent. Even though the model over predicts the share of wealth of the top quintile, it under predicts the share held by the top one percent by 9.4 percentage points. Focusing attention on only wage earners or wage earners by education we see that the model continues to over predict the dispersion in wealth (except, again, at the very top).

Looking next at the cross-sectional distribution of consumption presented in panel (c), the model aligns quite well with the data. Using a definition of consumption that includes non durables plus an imputed consumption flow from durables, the shares in the data look very similar for the entire population, the subsample of wage earners, and by education groups. The model matches the third quintile almost exactly at 17.5 percent, while implying slightly lower shares than the data for the top two quintiles (lower by two and 3.5 percentage points each) and implying slightly higher shares for the bottom two quintiles (higher by four and two percentage points).

Since the model implies a distribution of consumption that is slightly more equal than the data, while at the same time implying a similar earnings distribution and much less equal wealth distribution (looking at the quintile shares for wage earners), agents in the model are using borrowing and savings as a way to smooth consumption to a greater extent than appears to be the case in the data. It should also be noted that the model is constructed to represent *ex ante* identical workers (within education groups) and, given the infinite planning horizon, abstracts from any life cycle motive for saving (such as saving for a down payment, children's education, or retirement). In contrast, the raw cross-sectional distributions contain a substantial amount of individual heterogeneity, which will naturally lead to more dispersion than the homogeneous model can produce. Clearly adding more heterogeneity to the model, either by pooling over more skill groups, explicitly accounting for permanent differences between workers, modeling the lifecycle, or modeling entrepreneurs, would increase the model dispersion, aligning it closer to the data. I leave these extensions to future work.

4.1.1. Sensitivity to the Upper Tail of the Wage Offer Distribution. In this section I asses the sensitivity of the results to the upper tail of the wage offer distribution. As discussed in Section 2, in the model the savings motive of workers at

the top of the wage distribution is driven by the effective asymmetry in shocks to wages; the potential for wage growth has been exhausted and the only income shock that can occur is the very large shock of falling off the wage ladder and starting over again from unemployment. However, it may be that the true maximum wage is much higher than what I estimate based on wages accepted out of unemployment. Since the likelihood of being at the very top is low, it may simply be the case that the NLSY79 sample does not contain these observations.[12] Indeed, those workers at the top of the estimated wage offer distribution may still perceive substantial potential for increases, and behave accordingly. To assess the quantitative importance of the estimated upper bound on wages I conduct the following counterfactual experiment. I augment the estimated wage offer distribution by substantially extending the right tail. Specifically, I replace the top one percent of wages in the empirical offer distribution with interpolated wages between the 99th percentile and ten times the maximum observed wage. This has the effect that even workers at the top of the estimated wage offer distribution now perceive that their wage could still increase ten fold. The model is solved using this new offer distribution, and I present the implications for the distribution of earnings, wealth and consumption in the rows labeled augmented offer in Table 3.

This augmentation results in an earnings distribution that resembles more closely that of the full US sample. The implied distribution of wealth is also much more inline with the data, except for the share held by the top one percent, where the fit actually becomes worse. Under the augmented offer distribution there is less wealth concentrated among the top one percent of workers. The very strong precautionary motive for savings of this group has been dampened by the possibility of further wage increases. Indeed, the share of total wealth held by the top one percent falls to 15.5 percent, while it is 22.8 percent using the estimated wage offer distribution, and 26.6 percent for wage earners in the data. Finally, augmenting the wage wage offer distribution has little effect on the distribution of consumption; none of the shares differs by more than one percentage point when using the augmented offer as compared to the estimated offer distribution.

The results from this experiment are quite illuminating. On the one hand, the augmented wage offer distribution produces a distribution of earnings that more closely aligns with the US population as a whole; specifically it aligns better with the population that includes not just wage earners, but also the self employed, retired and disabled. It also provides an improved fit to the wealth shares by quintile for this population. However, the fit to the share of wealth held by the richest one percent actually worsens, since those workers near the top of the earnings distribution still have an expectation of further wage growth which offsets their precautionary motive of saving agains job loss.

4.2. *Further Implications and Related Literature*

The ability of the model to produce substantial dispersion in wealth is largely attributable to the effect on savings behavior of the wage ladder induced by on-the-job search. This mechanism, which arises endogenously in the model, can readily be related to the work of Krusell and Smith (1998) and Castañeda et al. (2003). There are several interesting cross-sectional implications that arise from the workers' consumption growth equation (2.11). Rewrite the consumption growth (equivalently, the asset accumulation) equation in terms

---

12. Castañeda, Díaz-Giménez, and Ríos-Rull (2003) argue that survey data consistently underestimates that incomes at the very top of the distribution, and this motivates their calibration strategy.

of the interest rate and an individual specific "effective discount rate" ρ_i, where i indexes individuals and

$$\rho_i = \rho + \lambda s_i \left(\overline{F}(w_i) - \int_{w_i}^{\overline{w}} \frac{u'(c(a_i,x))}{u'(c(a_i,w_i))} dF(x) \right) - \delta \left(\frac{u'(c(a_i,\underline{w}))}{u'(c(a_i,w_i))} - 1 \right).$$

Now the right hand side of equation (2.11) becomes $\gamma^{-1}(r - \rho_i)$, where individuals are all "discounting" at a different rate, and as a result, have very different savings behavior. Written in this form there is a close relationship to the stochastic discount rates used by Krusell and Smith (1998, KS), where individuals are heterogeneous in their rate of time preference, which evolves stochastically and leads to a very unequal wealth distribution. Here, the individual discount rates ρ_i jump at (random) employment transitions. There is, however, an important distinction between the two setups. In the KS setup, individuals are poor or rich (in terms of wealth) because they either have a high or low rate of time preference; in other words, because they prefer to be poor or rich. In contrast, in the current setup, all workers have identical preferences, and they would behave identically in the same circumstances; the differences across individuals arise from different sequences of good and bad luck in the labor market.

Krusell and Smith (1998) find they can get a very good fit to the wealth distribution with three quarterly discount factors, 0.9930, 0.9894, and 0.9858, which correspond to annual discount rates of 2.85, 4.35, and 5.89 percent. Individuals spend an average of 50 years with the same discount rate. Using the same pooled simulation discussed in Section 4.1, I present in Table 4 the first quartile, the median, and the third quartile of annual effective discount rates from the current model, which are 0.3, 2.5 and 6.2 percent. The dispersion between the first and third quartile is quite a bit larger than between the highest and lowest discount rates used by KS. The expected duration spent within the same quartile is between two and six years, which is substantially less than the 50-year durations in KS. The dispersion in wealth in KS results from small but very persistent differences in discounting across individuals, where in the present paper the differences in effective discount rates can be very large, but are not very persistent. While qualitatively the current model shares the flavor of stochastic discounting with KS, it does not seem to play same role quantitatively.

The wage ladder process for earnings, an endogenous result of on-the-job search, implies that expected wage growth is declining in the current wage, and the income loss associated with job destruction is increasing in the current wage. The exogenous earnings process (more precisely, productivity process) that Castañeda, Díaz-Giménez, and Ríos-Rull (2003, CDR) find is needed to account for the dispersion in wealth shares these qualitative features; the probability of obtaining a higher wage is decreasing in the current wage, and the income loss associated with exiting the highest wage is substantial. To get a feeling for how similar the wage ladder process is to the CDR process I replicate the corresponding transition matrix based on the current model. In Table 5, panel (a) I reproduce the transition matrix, relative wages and population shares from Tables 4 and 5 of CDR. Using data simulated from the current model, I bin wages into the same four population shares, comprising 61.1, 22.4, 16.5, and 0.04 percent of the population. The CDR process implies that the wages of the four groups, relative to the first group are 1.0, 3.2, 9.8 and 1061.0; the most productive 0.04 percent of workers earn wages over 1000 times those of the least productive 61 percent. The same calculation in the current model, presented in Table 5, panel (b), implies the average wages within each group, relative to the first, are 1.0, 2.8, 5.9, and 41.2. The wages among the top 0.04 percent

TABLE 4

Effective Discount Rates

				(a) Pooled Education				
Quantile	ρ_i	β_i				To		
			From	q1	q2	q3	q4	
0.25	0.003	0.997	q1	99.46	0.21	0.03	0.31	
0.50	0.025	0.975	q2	0.02	99.65	0.06	0.28	
0.75	0.062	0.940	q3	0.03	0.02	99.65	0.30	
			q4	0.52	0.14	0.26	99.09	

				(b) Low Education				
Quantile	ρ_i	β_i				To		
			From	q1	q2	q3	q4	
0.25	-0.002	1.002	q1	99.34	0.24	0.05	0.37	
0.50	0.026	0.974	q2	0.02	99.53	0.08	0.37	
0.75	0.066	0.936	q3	0.03	0.04	99.54	0.40	
			q4	0.63	0.21	0.32	98.84	

				(c) High Education				
Quantile	ρ_i	β_i				To		
			From	q1	q2	q3	q4	
0.25	0.009	0.991	q1	99.64	0.19	0.01	0.16	
0.50	0.025	0.975	q2	0.01	99.75	0.07	0.17	
0.75	0.052	0.949	q3	0.04	0.01	99.78	0.17	
			q4	0.33	0.07	0.13	99.46	

Note: The transition matrix is calculated at a weekly rate. q1 refers to workers with an effective discount rate in the lowest quartile and q4 refers to workers with an effective discount rate in the highest quartile. The transition matrix for the pooled group is created by pooling the low and high eduction simulations, in proportion to their representation in the NLSY, and refers to weekly transitions.

are substantially higher than the bottom, however they would still need to be 25 times greater to match the CDR calibration. In addition to the large difference in relative wage of the top group, there are also differences in the patterns of persistence in the groups. In the CDR calibration the probability of exiting the top group is significantly greater than the probability of exiting the other three groups. This is not the case in my model, where the probability of exiting the top group is only slightly greater than at the bottom, and is actually less than the probability of exiting the two middle groups. The idea of falling off the wage ladder is clear in this table: conditional on exiting the top group, a worker will end up in the bottom group with probability one.

In both the CDR model and the current model the top end of the wealth distribution is driven by the behavior of those who are lucky enough to receive the highest wage. The relative difference between the top and bottom wage in these models is very large, and it is the desire to smooth the marginal utility of consumption in the expectation of losing this wage that drives this small group in the population to accumulate such a high degree of savings. In Table 5, panel (c) I present the same statistics, but with equally sized groups.

TABLE 5

Relative Wages and Transition Probabilities

(a) Castañeda et al. (2003)

From w	To w'				Relative w	Share
	w'_1	w'_2	w'_3	w'_4		
w_1	96.24	1.14	0.39	0.006	1.00	61.11
w_2	3.07	94.33	0.37	0.000	3.15	22.35
w_3	1.50	0.43	95.82	0.020	9.78	16.50
w_4	10.66	0.49	6.11	80.51	1061.00	0.0389

(b) Model, Pooled Education

From w	To w'				Relative w	Share
	w'_1	w'_2	w'_3	w'_4		
w_1	91.03	5.08	3.89	0.008	1.00	61.11
w_2	12.66	86.81	0.53	0.000	2.77	22.35
w_3	12.33	0.38	87.29	0.002	5.92	16.50
w_4	10.14	0.00	0.00	89.86	41.22	0.0389

(c) Model, Pooled Education

From w	To w'				Relative w	Share
	w'_1	w'_2	w'_3	w'_4		
w_1	65.38	13.81	10.28	10.52	1.00	25.0
w_2	11.42	82.59	2.93	3.05	24.53	25.0
w_3	11.11	1.32	86.47	1.10	38.85	25.0
w_4	11.52	0.73	0.51	87.24	84.72	25.0
max w					785.70	-

Note: panel (a) is taken directly form Castañeda, Díaz-Giménez, and Ríos-Rull (2003, Tables 4 and 5). The model data is computed by pooling a simulation of low and high skilled workers (weighted according to representation in the NLSY), and pooling over ages 25 to 65. This simulated economy can be interpreted as an overlapping generations economy with a constant age structure. The transition matrices in panels (b) and (c) are created by first finding the wage quantiles that correspond to the shares in the last column, and then finding the fraction of yearly transitions between these quantiles in the simulated data. The relative wage is calculated as the mean wage in the specified quantile relative to the mean wage in the first quantile. The last row, labeled max w, contains the maximum wage relative to the average wage in the first quantile.

Comparing again relative wages, the amount of dispersion becomes clearer; the average wage in the top quartile is 84.7 times that of the bottom quartile, and the very top wage is 785.7 times the average in the bottom quartile. The model also implies substantial movement between the quartiles. Persistence decreases as we move up the quartiles since the probability of receiving a higher wage offer is declining in wages; search effort is declining as the workers have both higher wages and more assets; and the probability of job loss remains constant.

5. CONCLUSIONS

In this paper, I show that a model of the labor market with on-the-job search and saving can generate substantial dispersion in both earnings and wealth. In a labor

market characterized by informational frictions and the possibility of job destruction, workers with different wages will exhibit very different savings behavior. The specific earnings process generated by the search model implies that only a very few lucky individuals will ever earn the highest wage and, once there, a job loss means falling all the way back to the bottom. Faced with such a large expected income loss, these workers save at a very high rate and accumulate substantial assets. Qualitatively, the equation characterizing consumption growth provides a direct and intuitive link between the labor market frictions and the motives for saving or dis-saving at various points in the earnings and asset distribution.

Quantitatively, the model performs well on many dimensions, including wage dynamics and the cross-sectional distributions of earnings, wealth, and consumption. There is somewhat of a tension when fitting employment dynamics and wage dynamics simultaneously, as the parsimony of the model places strong restrictions on the joint evolution of wages and employment. Understanding these joint dynamics within an equilibrium search model is central to my research agenda, and substantial progress has already been made in Lise et al. (2012). The model misses somewhat the extremes of the earnings and wealth distributions, suggesting that the fit could be substantially improved by the incorporation of entrepreneurs in the model. In principle it is possible to use the firm side from an equilibrium version of the model to attribute earnings to this group. That said, one would need data on firm profits in order to put some discipline on this aspect of the model. Given the increasing availability of matched employee-employer data this should prove a fruitful line of inquiry, as already demonstrated in papers such as Cahuc et al. (2006) and Lentz and Mortensen (2010).

APPENDIX A. APPENDIX

Appendix A.1. *Proofs*

Proof (Proposition 1).   Since for all asset levels a, the value of being employed $W(a, w)$ is increasing in w, an employed worker always accepts any wage higher than his current wage. Since at each asset level a, the value of being unemployed $U(a)$ is independent of w, then for any asset level a there is a unique reservation wage $\tilde{w}(a)$ above which the value of employment is higher than the value of unemployment. This reservation wage is the unique solution to

$$W(a, \tilde{w}(a)) = U(a).$$

Expanding this relationship gives

$$\rho U(a) = u(c(a,b)) - e(s(a,b)) + U_a(a)[ra + b - c(a,b)] + \lambda s(a,b) \int_{\tilde{w}(a)}^{\overline{w}} [W(a,x) - U(a)]\, dF(x)$$

$$= u(c(a, \tilde{w}(a))) - e(s(a, \tilde{w}(a))) + W_a(a, \tilde{w}(a))[ra + \tilde{w}(a) - c(a, \tilde{w}(a))]$$

$$+ \lambda s(a, \tilde{w}(a)) \int_{\tilde{w}(a)}^{\overline{w}} [W(a,x) - W(a, \tilde{w}(a))]\, dF(x) + \delta[U(a) - W(a, \tilde{w}(a))]$$

$$= \rho W(a, \tilde{w}(a))$$

Substituting $W(a, \tilde{w}(a)) = U(a)$ using the reservation wage property, and substituting $u'(c) = U_a = W_a$ using the first order conditions for consumption we have

$$u(c(a,b)) - e(s(a,b)) - u(c(a,\tilde{w}(a))) + e(s(a,\tilde{w}(a)))$$

$$+ u'(c(a,b))[ra + b - c(a,b)] - u'(c(a,\tilde{w}(a)))[ra + \tilde{w}(a) - c(a,\tilde{w}(a))]$$

$$+ [\lambda s(a,b) - \lambda s(a,\tilde{w}(a))] \int_{\tilde{w}(a)}^{\overline{w}} [W(a,x) - W(a,\tilde{w}(a))]\, dF(x) = 0.$$

We can directly verify that the solution occurs at

$$s(a, \tilde{w}(a)) = s(a,b), \quad c(a, \tilde{w}(a)) = c(a,b), \quad \text{and} \quad \tilde{w}(a) = b.$$

The reservation wage is independent of assets and equal to the unemployment benefits.

Proof (Proposition 2).   To prove this we need to show that for all $w_2 > w_1$ we have $u'(c(a, w_2)) \leq u'(c(a, w_1))$. In other words, we need to show that consumption is non-decreasing in the wage. Equivalently, by the first order condition in equation (2.8), we have $u'(c(a, w_2)) \leq u'(c(a, w_1)) \Leftrightarrow W_a(a, w_2) \leq W_a(a, w_1)$. To establish this it is convenient to work in discrete time and then let the time interval shrink to zero. Throughout I will assume that the time interval Δ is sufficiently short such that $(1 - \lambda \overline{F}(w)\Delta - \delta\Delta) > 0$. The value function is[13]:

$$W(a, w) = \max_c \left\{ u(c(a,w))\Delta + \frac{1}{1 + \rho\Delta}\left[\lambda\Delta \int_w^{\overline{w}} W(a', x) dF(x) \right.\right.$$

$$\left.\left. + \delta\Delta W(a', \underline{w}) + [1 - \lambda\Delta\overline{F}(w) - \delta\Delta]W(a', w) + o(\Delta t) \right] \right\}, \quad \text{(A27)}$$

subject to $a' = (1 + r\Delta)a + (w - c)\Delta$. The shadow price of assets is

$$W_a(a, w) = \frac{1 + r\Delta}{1 + \rho\Delta}\left[\lambda\Delta \int_w^{\overline{w}} W_a(a', x) dF(x) + \delta\Delta W_a(a', \underline{w}) + (1 - \lambda\Delta\overline{F}(w) - \delta\Delta) W_a(a'w) \right].$$
$$\text{(A28)}$$

Lemma A4.   *The value function is strictly concave and increasing in assets and the wage.*

Proof.   Consider $(a_1, w_1) \neq (a_2, w_2)$, with $(a_i, w_i) \in [\underline{a}, \overline{a}] \times [\underline{w}, \overline{w}]$. Let $a'_i \in [\underline{a}, \overline{a}]$ be defined by $a'_i = (1 + r)a_i + (w_i - c_i)$, where c_i is optimal consumption associated with (a_i, w_i) in equation (A27). For $0 \leq \theta \leq 1$, let $a_\theta = \theta a_1 + [1 - \theta]a_2$ and $w_\theta = \theta w_1 + [1 - \theta]w_2$, and $a'_\theta = \theta a'_1 + (1 - \theta)a'_2 \in [\underline{a}, \overline{a}]$. It

---

13. See Web Appendix for a derivation of equation (2.5) as the limit of the discrete time Bellman equation (A27).

follows that[14]

$$W(a_\theta, w_\theta) \geq u\left((1+r)a_\theta + w_\theta - a'_\theta\right)$$
$$+ \frac{1}{1+\rho}\left(\lambda \int_{w_\theta}^{\overline{w}} W(a'_\theta, x)dF(x) + \delta W(a'_\theta, \underline{w}) + [1 - \lambda \overline{F}(w_\theta) - \delta]W(a'_\theta, w_\theta)\right)$$
$$> \theta\left[u\left((1+r)a_1 + w_1 - a'_1\right)\right.$$
$$+ \frac{1}{1+\rho}\left(\lambda \int_{w_1}^{\overline{w}} W(a'_1, x)dF(x) + \delta W(a'_1, \underline{w}) + [1 - \lambda \overline{F}(w_1) - \delta]W(a'_1, w_1)\right)\right]$$
$$+ [1-\theta]\left[u\left((1+r)a_2 + w_2 - a'_2\right)\right.$$
$$+ \frac{1}{1+\rho}\left(\lambda \int_{w_2}^{\overline{w}} W(a'_2, x)dF(x) + \delta W(a'_2, \underline{w}) + [1 - \lambda \overline{F}(w_2) - \delta]W(a'_2, w_2)\right)\right]$$
$$= \theta W(a_1, w_1) + [1-\theta]W(a_2, w_2),$$

where the weak inequality follows from the fact that a'_θ does not necessarily attain $W(a_\theta, w_\theta)$ and the strict inequality follows from the strict concavity of u. That the value function is increasing in a and w follows directly from the fact that increasing a or w is a relaxation of the budget constraint.

Lemma A5. *Consider $a_1 \neq a_2$. If $a_2 > a_1$ then $a'_2 > a'_1$, where a'_i attains $W(a_i, w)$.*

Proof. By Lemma A4, $W(a, w)$ is increasing and strictly concave in a and w, direct inspection of equation (A28) implies that $a'_2 > a'_1$.

Lemma A6. *The shadow price of assets is non-increasing in the wage; for all $w_2 > w_1$ we have $W_a(a, w_2) - W_a(a, w_1) \leq 0$.*

Proof. Let Ω denote the set of continuous, bounded functions defined over $[\underline{a}, \overline{a}] \times [\underline{w}, \overline{w}]$ that are non-increasing in a and w. Under the sup norm, Ω is a closed subset of the complete metric space of continuous, bounded functions defined over $[\underline{a}, \overline{a}] \times [\underline{w}, \overline{w}]$. Now let $\omega \in \Omega$ and define operator $\mathcal{T}$ as:

$$\mathcal{T}\omega(a, w) = \frac{1+r\Delta}{1+\rho\Delta}\left[\lambda\Delta \int_w^{\overline{w}} \omega(a', x)dF(x) + \delta\Delta\omega(a', \underline{w}) + (1 - \lambda\Delta\overline{F}(w) - \delta\Delta)\omega(a', w)\right],$$

with $a' := (1 + r\Delta)a + (w - c)\Delta$, where c is optimal consumption associated with (a, w) in equation (A27). The operator $\mathcal{T}$ satisfies Blackwell's sufficient conditions for a contraction; monotonicity and discounting ($\rho > r$ by assumption). The shadow price of assets W_a is $\mathcal{T}$'s fixed point.

We can now verify that $\mathcal{T}$ maps non-increasing functions into itself. Let $a_2 > a_1$ and denote $a'_i := (1 + r\Delta)a_i + (w - c_i)\Delta$:

$$\mathcal{T}\omega(a_2, w) - \mathcal{T}\omega(a_1, w) = \frac{1+r\Delta}{1+\rho\Delta}\left[\lambda\Delta \int_w^{\overline{w}} \left[\omega(a'_2, x) - \omega(a'_1, x)\right]dF(x)\right.$$
$$\left. + \delta\Delta\left[\omega(a'_2, \underline{w}) - \omega(a'_1, \underline{w})\right] + (1 - \lambda\Delta\overline{F}(w) - \delta\Delta)\left[\omega(a'_2, w) - \omega(a'_1, w)\right]\right]$$
$$\leq 0,$$

where by Lemma A5 we have $a'_2 > a'_1$, and the weak inequality follows from the hypothesis that $\omega(a, w)$ is non-increasing in a. Now, let $w_2 > w_1$ and denote $a'_i := (1 + r\Delta)a + (w_i - c_i)\Delta$:

---

14. I assume the time interval is short and suppress the Δ notation for clarity.

$$\mathcal{T}\omega\left(a,w_2\right) - \mathcal{T}\omega\left(a,w_1\right) = \frac{1+r\Delta}{1+\rho\Delta}\left[\lambda\Delta\int_{w_2}^{\overline{w}}\omega\left(a_2',x\right)dF\left(x\right) - \lambda\Delta\int_{w_1}^{\overline{w}}\omega\left(a_1',x\right)dF\left(x\right)\right.$$

$$+\delta\Delta\left[\omega\left(a_2',\underline{w}\right) - \omega\left(a_1',\underline{w}\right)\right] + \left(1 - \lambda\Delta\overline{F}\left(w_2\right) - \delta\Delta\right)\omega\left(a_2',w_2\right)$$

$$\left. - \left(1 - \lambda\Delta\overline{F}\left(w_1\right) - \delta\Delta\right)\omega\left(a_1',w_1\right)\right]$$

$$\leq \frac{1+r\Delta}{1+\rho\Delta}\left[\lambda\Delta\int_{w_2}^{\overline{w}}\omega\left(a_2',x\right)dF\left(x\right) - \lambda\Delta\int_{w_1}^{\overline{w}}\omega\left(a_2',x\right)dF\left(x\right)\right.$$

$$+\delta\Delta\left[\omega\left(a_2',\underline{w}\right) - \omega\left(a_2',\underline{w}\right)\right] + \left(1 - \lambda\Delta\overline{F}\left(w_2\right) - \delta\Delta\right)\omega\left(a_2',w_2\right)$$

$$\left. - \left(1 - \lambda\Delta\overline{F}\left(w_1\right) - \delta\Delta\right)\omega\left(a_2',w_1\right)\right]$$

$$= \frac{1+r\Delta}{1+\rho\Delta}\left[-\lambda\Delta\int_{w_1}^{w_2}\omega\left(a_2',x\right)dF\left(x\right)\right.$$

$$\left. + \left(1 - \lambda\Delta\overline{F}\left(w_2\right) - \delta\Delta\right)\omega\left(a_2'w_2\right) - \left(1 - \lambda\Delta\overline{F}\left(w_1\right) - \delta\Delta\right)\omega\left(a_2',w_1\right)\right]$$

$$\leq \frac{1+r\Delta}{1+\rho\Delta}\left[-\lambda\Delta\int_{w_1}^{w_2}\omega\left(a_2',w_2\right)dF\left(x\right)\right.$$

$$\left. + \left(1 - \lambda\Delta\overline{F}\left(w_2\right) - \delta\Delta\right)\omega\left(a_2',w_2\right) - \left(1 - \lambda\Delta\overline{F}\left(w_1\right) - \delta\Delta\right)\omega\left(a_2',w_1\right)\right]$$

$$= \frac{1+r\Delta}{1+\rho\Delta}\left[-\lambda\Delta\left[\overline{F}\left(w_1\right) - \overline{F}\left(w_2\right)\right]\omega\left(a_2',w_2\right)\right.$$

$$\left. + \left(1 - \lambda\Delta\overline{F}\left(w_2\right) - \delta\Delta\right)\omega\left(a_2',w_2\right) - \left(1 - \lambda\Delta\overline{F}\left(w_1\right) - \delta\Delta\right)\omega\left(a_2',w_1\right)\right]$$

$$= \frac{1+r\Delta}{1+\rho\Delta}\left(1 - \lambda\Delta\overline{F}\left(w_1\right) - \delta\Delta\right)\left[\omega\left(a_2',w_2\right) - \omega\left(a_2',w_1\right)\right]$$

$$\leq 0.$$

To obtain the first inequality we replace a_1' by a_2' everywhere on the right hand side, by Lemma A5 we have $a_2' > a_1'$, and the weak inequality follows from the hypothesis that $\omega(a,w)$ is non-increasing in a. To obtain the second inequality we replace $\omega\left(a_2',x\right)$ by $\omega\left(a_2',w_2\right)$ in the integral, and the inequality follows from the hypothesis that $\omega(a,w)$ is non-increasing in w. The last inequality follows from the hypothesis that $\omega(a,w)$ is non-increasing in w (and the assumption that the time interval is sufficiently short to ensure $1 - \lambda\Delta\overline{F}\left(w_1\right) - \delta\Delta > 0$).

Now, by the first order condition in equation (2.8), for all $w_2 > w_1$ we have $W_a\left(a,w_2\right) \leq W_a\left(a,w_1\right) \Leftrightarrow u'\left(c\left(a,w_2\right)\right) \leq u'\left(c\left(a,w_1\right)\right)$. In equation (2.11), the term multiplying λ is therefore positive and non-increasing in the wage, while the term multiplying δ is positive and non-decreasing in the wage:

$$\frac{\partial}{\partial w}\left(\overline{F}(w) - \int_w^{\overline{w}}\frac{u'(c(a,x))}{u'(c(a,w))}dF(x)\right) = \int_w^{\overline{w}}\frac{u'(c(a,x))u''(c(a,w))c_w(a,w)}{u'(c(a,w))^2}dF(x) \leq 0,$$

$$\frac{\partial}{\partial w}\left(\frac{u'(c(a,\underline{w}))}{u'(c(a,w))} - 1\right) = -\frac{u'(c(a,\underline{w}))u''(c(a,w))c_w(a,w)}{u'(c(a,w))^2} \geq 0.$$

For any wage level w, there is a target level of assets, $a^*(w)$, at which point the savings rate is zero. The target level of assets is implicitly defined by:

$$\frac{1}{\gamma}\left(r - \rho - \lambda\left(\overline{F}(w) - \int_w^{\overline{w}}\frac{u'\left(c\left(a^*(w),x\right)\right)}{u'\left(c\left(a^*(w),w\right)\right)}dF(x)\right) + \delta\left(\frac{u'\left(c\left(a^*(w),\underline{w}\right)\right)}{u'\left(c\left(a^*(w),w\right)\right)} - 1\right)\right) = 0.$$

Proof (*Proof of Proposition 3*).   Setting equation (2.11) equal to zero gives

$$0 = r - \rho - \lambda s \left(\overline{F}(w) - \int_w^{\overline{w}} \frac{u'(\hat{c})}{u'(c)} dF(x) \right) + \delta \left(\frac{u'(\underline{c})}{u'(c)} - 1 \right),$$

$$\rho + \lambda s \overline{F}(w) + \delta - r = \left(\lambda s \int_w^{\overline{w}} u'(\hat{c}) dF(x) + \delta u'(\underline{c}) \right) \frac{1}{u'(c)},$$

$$u'(c) = \frac{\lambda s \int_w^{\overline{w}} u'(\hat{c}) dF(x) + \delta u'(\underline{c})}{\rho + \lambda s \overline{F}(w) + \delta - r},$$

$$c = \phi \left(\frac{\lambda s \int_w^{\overline{w}} u'(\hat{c}) dF(x) + \delta u'(\underline{c})}{\rho + \lambda s \overline{F}(w) + \delta - r} \right), \tag{A29}$$

where ϕ is the inverse function of the marginal utility of consumption $u'(c)$. Setting $\dot{a} \equiv da/dt = 0$ (the asset accumulation equation) gives

$$c = w + ra. \tag{A30}$$

For the existence of a stable saddle-path equilibrium, it is necessary that $\rho > r - \delta - \lambda s \overline{F}(w)$, which collapses to $\rho > r - \delta$ since it must hold at all $w \in [\underline{w}, \overline{w}]$. The existence of a finite upper bound on assets requires more, specifically that $\rho > r$. This can be seen by equating equations (A29) and (A30), and evaluating at $w = \overline{w}$:

$$\overline{w} + r\overline{a} = \phi \left(\frac{\delta u'(c(\overline{a}, \underline{w}))}{\rho + \delta - r} \right). \tag{A31}$$

Equation (A31) can be rewritten as

$$(\rho - r) u'(c(\overline{a}, \overline{w})) = \delta \left(u'(c(\overline{a}, \underline{w})) - u'(c(\overline{a}, \overline{w})) \right),$$

where $c(\overline{a}, \overline{w}) > c(\overline{a}, \underline{w})$ implies the right hand side is strictly positive for any finite $\overline{a}$, implying $\rho - r$ is strictly positive for finite c.

In the limit, as $(\rho - r)/\delta$ tends to zero from above, equation (A31) tends to

$$\overline{w} + r\overline{a} = c(\overline{a}, \underline{w}).$$

Appendix A.2. *Data*

The data for the main analysis are from the National Longitudinal Survey of Youth 1979 (NLSY).[15] The NLSY consists of 12,686 individuals who were 14 to 21 years of age as of January, 1979. The NLSY contains a nationally representative random sample, as well as an over-sample of black, Hispanic, the military, and poor white individuals. A complete labor market history, by week, can be constructed for each individual in the sample. The labor market history provides the potential of over 1,300 weekly observations per individual, including the current weekly earnings, transitions to and from unemployment and between jobs. Since 1985, the NLSY contains detailed questions on the asset holdings of each individual. The asset data are not observed at the same frequency as the labor market data; asset data are collected at interview dates, providing at most one observation on assets per year. I discuss the estimation issues arising from this partially observed state variable in Section 3.

1.2.1. Construction of NLSY Sample Used in Estimation.   I use the white male sample from the NLSY data for the years 1985 to 2002. The restriction of attention to white males is motivated by an attempt to create a relatively homogeneous subgroup that is well described by the model developed in the paper. Since the schooling decision is exogenous to the model, I only include data for individuals once they have completed their education. I also drop individuals who have served in the military, or have identified their labor force status as out of the labor force. The majority of individuals who are not in the labor force report being disabled and are clearly not searching for employment. I subdivide the data into two education groups: those with a high school degree, and those with a college degree. I do not use data on high school dropouts and college dropouts. The summary statistics for these groups are too different from either included group to pool them, and the sample sizes are too small to use on their own. Since not all individuals finished school at the same time, and some have dropped out of the survey, I am left with an unbalanced panel of 792 high school graduates (567,895 person weeks), and 581 college graduates (340,264 person weeks). With working and unemployed as the only two labor force states in the model, I need to choose a cutoff for the number of hours that qualify as employed. I follow Bowlus et al. (2001) and define employment as working 35 hours or more a week. The moments used in estimation are not very sensitive to an alternative definition of 40 hours (see Tables 7 and 8 in the Web Appendix).

15. This data is publicly available at: http://www.bls.gov/nls/nlsy79.htm.

Monetary variables are adjusted for inflation using the GDP deflator. To reduce the influence of outlying observations, I trim the top and bottom one-half-of-one percent of the wage and asset observations. I follow the definition of total assets used by Keane and Wolpin (2001) and Imai and Keane (2004). I construct total assets (net worth) by adding up the following variables in the NLSY: "Total market value of vehicles including automobiles r/spouse own," "Total market value of farm/business/other property r/spouse own," "Market value of residential property r/spouse own," "Total market value of stocks/bonds/mutual funds," "Total amount of money assets like savings accounts of r/spouse," "Total market value of all other assets each worth more than $500." From this I subtract the total of "Total amount of money r/spouse owe on vehicles including automobiles," "Total amount of debts on farm/business/other property r/spouse owe," "Amount of mortgages and back taxes r/spouse owe on residential property," "Total amount of other debts over $500 r/spouse owe." As a sensitivity exercise, I also present the moments based on financial assets only (see Tables 7 and 8 in the Web Appendix).

The model developed in Section 2 assumes that the wage offer distribution is stationary. Since the NLSY data is based on a cohort, there are two sources of non-stationarity in observed wages. The first is that we observe this cohort as they are moving toward the stationary distribution, since they begin life out of employment and are slowly moving up the wage ladder. This aspect of the non-stationarity can be fully accounted for by thinking of the cohort as a sample of unemployed workers who we follow forward. The second source of non-stationarity comes from sectoral growth. Since the model does not have growth, I detrend the data using the following procedure, designed to ensure a stationary wage offer distribution. Let w_{it}^o be the log of wages accepted out of unemployment. I estimate the growth in the wage offer distribution by the regression $w_{it}^o = a + gt + u_{it}$. I then detrend all wages using $\hat{g}$. Thus, the wage offer distribution has the same mean every year by construction, and any increases over time in the mean of accepted wages are attributed to the effect of on-the-job search. The estimates for sectoral growth are 1.87 percent for low educated workers and 4.31 percent for high educated workers, suggesting substantial skill bias over this period.

The model is written assuming workers are single individuals. To approximate this fiction in the data I project assets off of controls for marital status and number of children, removing the deterministic component associated with family size. In all the empirical work I define wages as log average annual detrended wages and assets are transformed using the inverse hyperbolic sine transformation, $\log\left(a + \sqrt{1+a^2}\right)$. This is a log-type transformation that admits zero or negative values (see Burbidge et al. (1988) for a discussion of the desirability of this transformation when working with wealth data). When matching moments, the same transformations are done on both the NLSY data and the model generated data.

The lower bound on wealth used in the model is the self-imposed borrowing constraint: $-\underline{w}/r$. I estimate the lowest possible wage, $\underline{w}$, using the lowest 0.5 percentile of the observed wage distribution by education. This value is also used for the flow income of unemployed workers. The implied lower bounds on wealth are -85,068 and -169,025 for low and high educated workers respectively. These values line up reasonably well with the corresponding lowest 0.5 percentile of the observed asset distributions by education: -75,358 and -153,120 with respective 95 percent confidence intervals (-76,880, -73,398) and (-181,499, -134,999). It seems that the self imposed borrow limit provides a reasonable approximation to the empirical lower bound on assets.

In the auxiliary regressions (3.15) to (3.17), I define the variables as follows: $E2U_{i,m}$ equals one if the worker is employed in week m and unemployed in week $m+1$, and equal to zero if employed in both weeks m and $m+1$. $J2J_{i,m}$ equals one if the worker is employed at job j in m and employed at job $k \neq j$ in week $m+1$, and equal to zero if the worker is employed at job j in both weeks m and $m+1$. In addition, if the worker experiences a spell of unemployment of two weeks or less between job changes this is coded as $J2J_{it} = 1$. This is done in an attempt to avoid misclassifying short vacation breaks between jobs as unemployment spells. $U2E_{i,m}$ equals one if the worker is unemployed in week m and employed in week $m+1$, and equals zero if the worker is unemployed in both weeks m and $m+1$.

1.2.2. Construction of SCF and CEX Samples used in Table 3. The data on earnings, wealth and consumption presented for the entire US population and the subsamples of wage earners by education group are from the US SCF (earnings and wealth) and the US CEX.[16] The shares are calculated as the average shares for the years available in the SCF: 1989, 1992, 1995, 1998, and 2001. Earnings are defined as wage and salary income in the SCF, and wealth is defined as net worth (real and financial assets minus debts). For the SCF subsample of wage earners I drop those with occupation code given as self employed, retired or disabled. Consumption is defined expenditure on non durables plus an imputed service flow from consumer durables (see Krueger and Perri (2006) for details). For the subsample of wage earners I keep only those who report zero business income.

16. The SCF data is from the Board of Governors of the Federal Reserve System and is publicly available at: http://federalreserve.gov/econresdata/scf/scfindex.htm. The CEX data is from Krueger and Perri (2006) and is publicly available at: http://www.fperri.net/data/datadocum_web.txt.

Appendix A.3. *Estimation*

Conditional on the first step non-parametric estimate of $F(w)$, the structural parameters $\theta = \{\gamma, \eta, \lambda, \delta\}$ can be estimated by minimizing the distance between the regression coefficients estimated on the actual data, and the average the same regression coefficients estimated on R simulated data sets:

$$\hat{\theta} = \arg \min_{\theta} \mathcal{L}(\theta),$$

where

$$\mathcal{L}(\theta) \equiv \left(\hat{\beta} - \frac{1}{R} \sum_{r=1}^{R} \tilde{\beta}^r(\theta)\right)^{\top} \Omega(\hat{\beta})^{-1} \left(\hat{\beta} - \frac{1}{R} \sum_{r=1}^{R} \tilde{\beta}^r(\theta)\right),$$

and $\Omega(\hat{\beta})$ is the diagonal of the covariance matrix for the regression coefficients estimated from the NLSY data. In practice this extremum estimator is difficult to work with since for any finite R the objective function is not smooth due to simulation error and the discrete jumps that occur as a result of search.

To address this issue I use the Markov Chain Monte Carlo (MCMC) method for classical estimators proposed by Chernozhukov and Hong (2003). Estimation proceeds by simulating a chain of parameters that (once converged) has the quasi-posterior density

$$p(\theta) = \frac{e^{\mathcal{L}(\theta)} \pi(\theta)}{\int e^{\mathcal{L}(\theta)} \pi(\theta) \, d\theta}.$$

A point estimate for the parameters is obtained as the average of the MCMC chain:

$$\hat{\theta}_{MCMC} = \frac{1}{B} \sum_{j=1}^{B} \theta^j,$$

and confidence intervals are constructed from the empirical quantiles of the sequence of θ^j. To simulate a chain that converges to the quasi posterior, I use the Metropolis-Hastings algorithm. The algorithm generates a chain $(\theta^0, \theta^1, ..., \theta^B)$ as follows. First, choose a starting value θ^0. Next, generate ξ from a proposal density $q(\xi|\theta^j)$ and update θ^{j+1} from θ^j for $j = 1, 2, ...$ using

$$\theta^{j+1} = \begin{cases} \xi & \text{with probability} & d(\theta^j, \xi) \\ \theta^j & \text{with probability} & 1 - d(\theta, \xi), \end{cases}$$

where

$$d(x, y) = \min\left(\frac{e^{\mathcal{L}(y)} \pi(y) q(x|y)}{e^{\mathcal{L}(x)} \pi(x) q(y|x)}, 1\right).$$

This procedure is repeated many times to obtain a chain of length B that represents the ergodic distribution of θ. Choosing the prior $\pi(\theta)$ to be uniform and the proposal density to be a random walk ($q(x|y) = q(y|x)$), results in the simple rule

$$d(x, y) = \min\left(e^{\mathcal{L}(y) - \mathcal{L}(x)}, 1\right).$$

The main advantage of this estimation strategy is that it only requires function evaluations, and thus the discontinuous jumps do not cause the same problems that would occur with a gradient based extremum estimator. Additionally, the converged chain provides a direct way to construct valid confidence intervals for the parameter estimates. The drawback of the procedure is that it requires a very long chain, and consequently a very large number of function evaluations, each requiring the model to be solved and simulated. In practice, I simulate 100 chains in parallel, each of length 10,000, and use the last 2000 elements (pooled over the 100 chains) to obtain parameter estimates and confidence intervals. Details pertaining to tuning the MCMC algorithm, a parallel implementation, and related methods in statistics can be found in Robert and Casella (2004), Vrugt et al. (2009), and Sisson and Fan (2011).
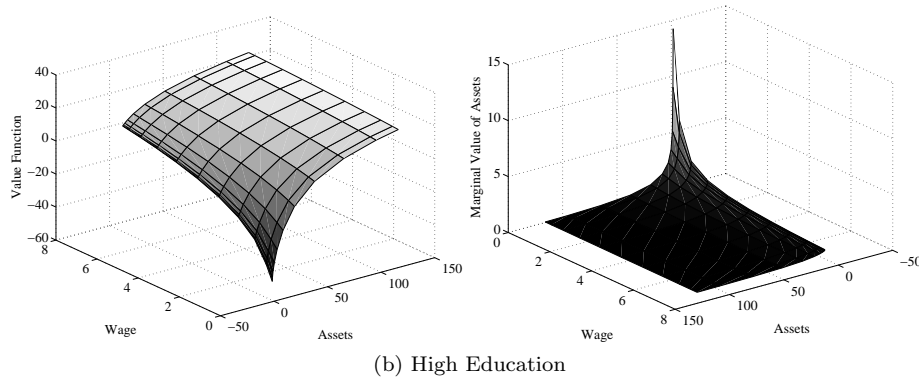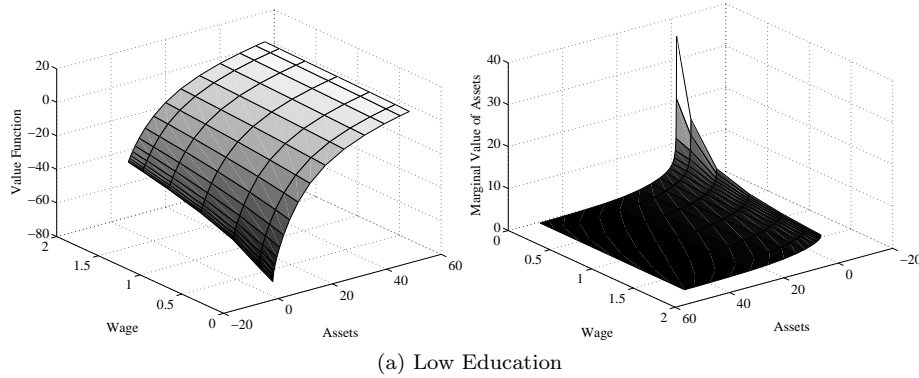
(a) Low Education

(b) High Education

FIGURE 3

$W(a, w)$ and $W_a(a, w)$ at estimated parameters by education

REFERENCES

Acemoglu, D. and R. Shimer (1999). Efficient unemployment insurance. *Journal of Political Economy 107*(5), 893–928.

Aiyagari, S. R. (1994). Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics 109*, 659–684.

Bayer, C. and K. Wälde (2010). Matching and Saving in Continuous Time: Theory. unpublished manuscript.

Bils, M., Y. Chang, and S.-B. Kim (2011). Worker Heterogeneity and Endogenous Separations in a Matching Model of Unemployment Fluctuations. *American Economic Journal: Macroeconomics 3*, 128–54.

Bontemps, C., J.-M. Robin, and G. van den Berg (1999). An Empirical Equilibrium Job Search Model with Search on the Job and Heterogeneous Workers and Firms. *International Economic Review 40*, 1039–1074.

Bontemps, C., J.-M. Robin, and G. van den Berg (2000). Equilibrium Search with Continuous Productivity Dispersion: Theory and Nonparametric Estimation. *International Economic Review 41*, 305–358.

Bowlus, A. J., N. M. Kiefer, and G. R. Neumann (2001). Equilibrium Search Models and the Transition from School to Work. *International Economic Review 42*, 317–343.

Browning, M., T. F. Crossley, and E. Smith (2007). Asset Accumulation and Short Term Employment. *Review of Economic Dynamics 10*, 400–423.

Burbidge, J. B., L. Magee, and A. L. Robb (1988). Alternative Transformations to Handle Extreme Values of the Dependent Variable. *Journal of the American Statistical Association 83*, 123–127.

Cahuc, P., F. Postel-Vinay, and J.-M. Robin (2006). Wage Bargaining with On-the-Job Search: Theory and Evidence. *Econometrica 74*, 323–364.

Card, D., R. Chetty, and A. Weber (2007). Cash-On-Hand and Competing Models of Intertemporal Behavior: New Evidence from the Labor Market. *Quarterly Journal of Economics 122*(4), 1511–1560.

Carroll, C. (2004). Theoretical Foundation of Buffer Stock Saving. NBER Working Paper No. 10867.

Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (2003). Accounting for the US earnings and wealth inequality. *Journal of Political Economy 111*(4), 818–857.

Chernozhukov, V. and H. Hong (2003). An MCMC approach to classical estimation. *Journal of Econometrics 115*(2), 293–346.

Costain, J. (1999). Unemployment Insurance with Endogenous Search Intensity and Precautionary Saving. Unpublished Manuscript.

Crossley, T. and H. Low (2011). Is the elasticity of intertemporal substitution constant? *Journal of the European Economic Association 9*, 87–105.

Danforth, J. P. (1979). On the Role of Consumption and Decreasing Absolute Risk Aversion in the Theory of Job Search. In S. A. Lippman and J. J. McCall (Eds.), *Studies in the Economics of Search*, Volume 123 of *Contributions to Economic Analysis*, Chapter 6, pp. 109–131. North-Holland.

Gourieroux, C., A. Monfort, and E. Renault (1993). Indirect Inference. *Journal of Applied Econometrics 8*, S85—-S118.

Imai, S. and M. Keane (2004). Intertemporal Labor Supply and Human Capital Accumulation. *International Economic Review 45*, 601–641.

Keane, M. P. and K. I. Wolpin (2001). The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment. *International Economic Review 42*, 1051–1103.

Krueger, D. and F. Perri (2006). Does Income Inequality Lead to Consumption Inequality? Evidence and Theory. *Review of Economic Studies 73*, 163–193.

Krusell, P., T. Mukoyama, and A. Sahin (2010). Labor-Market Matching with Precautionary Savings and Aggregate Fluctuations. *Review of Economic Studies 77*, 1477–1507.

Krusell, P. and A. A. Smith (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy 106*, 867–896.

Lentz, R. (2009). Optimal Unemployment Insurance in an Estimated Job Search Model with Savings. *Review of Economic Dynamics 12*, 37–57.

Lentz, R. and D. T. Mortensen (2010). Labor Market Friction, Firm Heterogeneity, and Aggregate Employment and Productivity.

Lentz, R. and T. Tranaes (2005). Job Search and Savings: Wealth Effects and Duration Dependence. *Journal of Labor Economics 23*, 467–489.

Lise, J., C. Meghir, and J.-M. Robin (2012). Matching, Sorting and Wages. Unpublished Manuscript.

Meghir, C. and L. Pistaferri (2011). Earnings, Consumption and Lifecycle Choices. In O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics* (4b ed.)., Chapter 9, pp. 773–1823. Elsevier.

Postel-Vinay, F. and H. Turon (2010). On-the-job Search, Productivity Shocks, and the Individual Earnings Process. *International Economic Review 51*, 599–629.

Rendón, S. (2006). Job Search and Asset Accumulation Under Borrowing Constraints. *International Economic Review 47*, 233–263.

Robert, C. P. and G. Casella (2004). *Monte Carlo Statistical Methods* (2 ed.). New York: Springer.

Sisson, S. A. and Y. Fan (2011). Likelihood-free markov chain monte carlo. In S. P. Brooks, A. Gelman, G. Jones, and X.-L. Meng (Eds.), *Handbook of Markov Chain Monte Carlo*, pp. 319–341. Chapman and Hall.

Smith, A. (1993). Estimating Nonlinear Time-Series Models Using Simulated Vector Autoregressions. *Journal of Applied Econometrics 8*, S64—-S84.

Vrugt, J., C. ter Braak, C. Diks, D. Higdon, B. Robinson, and J. Hyman (2009). Accelerating Markov chain Monte Carlo simulation by differential evolution with self-adaptive randomized subspace sampling. *International Journal of Nonlinear Sciences and Numerical Simulation 10*(3), 273–290.