

and it has been conjectured that  $n$  may be replaced by  $m \leq n$  where  $m$  is the length of the longest circuit (with no repeated nodes or edges) in the graph of  $A$ . We show that when  $A$  is doubly stochastic this conjecture is correct not only for the eigenvalues of  $A$  but also for all elements of the field of values of  $A$ .

Note: The conjecture mentioned above has recently been proven by R. B. Kellogg and A. B. Stephens in a paper to appear in *Linear Algebra and Appl.*

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## On the largest prime factors of $n$ and $n + 1$

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Let  $P(n)$  denote the largest prime factor of  $n$ , let  $A(x, t)$  denote the number of  $n \leq x$  with  $P(n) \geq x^t$ , and let  $B(x, t, s)$  denote the number of  $n \leq x$  with  $P(n) \geq x^t$  and  $P(n+1) \geq x^s$ . A classical result of Dickman is that

$$a(t) = \lim_{x \rightarrow \infty} x^{-1} A(x, t)$$

is defined and continuous on  $[0, 1]$ . It is natural to guess that

$$b(t, s) = \lim_{x \rightarrow \infty} x^{-1} B(x, t, s)$$

is defined and continuous on  $[0, 1]^2$  and that  $b(t, s) = a(t)a(s)$ . Lending some support to these guesses, we prove that for each  $\epsilon > 0$ , there is a  $\delta > 0$  such that the number of  $n \leq x$  with

$$x^{-\delta} < P(n)/P(n+1) < x^{\delta}$$

is less than  $\epsilon x$ . Our proof entails Brun's method. A corollary is that the probability that  $P(n) > P(n+1)$  is positive (almost certainly this probability is  $1/2$ ).

Our methods allow us to say something about integers  $n$  which have the same sum of their prime factors as  $n+1$ . We prove the number of such  $n \leq x$  is  $O(x/(\log x)^{1-\epsilon})$  for every  $\epsilon > 0$ . We know a proof in the case  $\epsilon = 0$  as well, but it is more complicated and not presented.

In addition we present a brief discussion on the largest prime factors of 3 or more consecutive numbers.

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