ON THE LAYERING PROBLEM OF MULTILAYER PWB WIRING*

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Abstract: This paper deals with the layering problem of multilayer PWB wiring, associated with single-row routing. The problem to be considered is restricted to the special case of street capacities up to two in each layer, and it is reduced to a problem of the interval graph by relaxing some restrictions in the original problem. Then, a heuristic algorithm is proposed for this problem.

1. Introduction

The single-row routing[1-4], first introduced for the backboard wiring[1], has been one of the fundamental routing methods for the multilayer high density printed wiring boards (PWB's)[5-7], due to "topological fluidity," that is, the capability to defer detailed wire patterns until all connections have been considered[6]. In the single-row routing, it is assumed that the multilayer board has fixed geometries; that is, the positions of pins and vias are restricted on nodes of a rectangular grid. Associated with this single-row routing the following problems are formulated: [Via-Assignment Problem]; to determine which vias are assigned for each net[7-9], [Layering Problem]; to decompose the interconnections on a single-row into the portions of each layer, and [Single-Row, Single-Layer Routing]; to lay out wire pattern on each layer[1-4].

Recent advance in the technology of microelectronics have changed the design rule for PWB's in such a way that the total amount of design for PWB's of four or more signal layers tends to grow rapidly, and hence

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the layering problem is of central importance. However, no specific development has been reported on this problem.

To attack the layering problem, we first have to seek a necessary and sufficient condition for a given net list to be realized by the single-row single-layer routing with the prescribed upper and lower street capacities. Concerning this, a specific development has been recently accomplished[3,4], and especially in the case of the upper and lower street capacities up to two, a necessary and sufficient condition is obtained[4], which can be easily checked. Noting that the case in which four etch paths are permitted to be laid out between two consecutive pins of an ordinary dual in line package corresponds to the single-row routing with the upper and lower street capacities both equal to two[7], we may assume that the upper and lower street capacities are up to two in each layer.

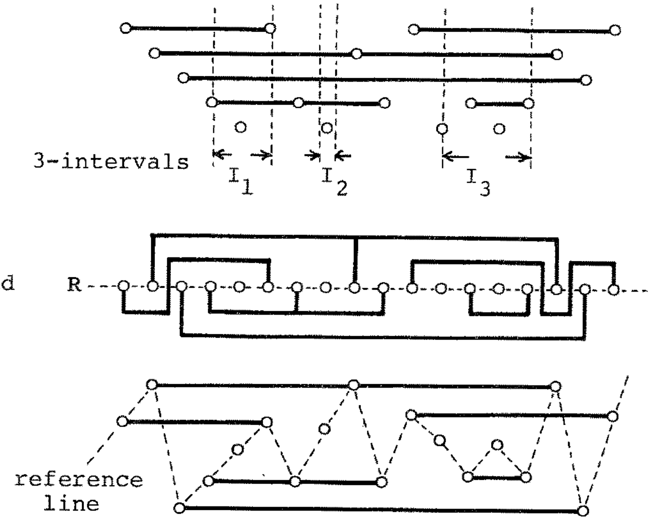
Thus, in this paper, we pay our attention to the layering problem such that in each layer the interconnections must be realized by single-row routing with the street capacities equal to two.

2. Difinitions and Formulation

Consider a set $\{v_1, v_2, \cdots, v_r\}$ of r nodes on the real line R, each of which corresponds to a pin or a via. A set of nodes on R to be interconnected is referred to as a net, and a set of nets is designated as a net list.

Given a net list $L = \{N_1, N_2, \cdots, N_n\}$ on R, the interconnection for each net N_i is to be realized by means of a set of paths on a certain number of layers, such that on each layer a path is constructed of horizontal and vertical line segments according to specifications. For example, consider a net list L as shown in Fig. 1 (a), where each net is represented by a horizontal line segment and each node denoted by a circle (note here that there exist nodes which are not used for any net). The interconnections of these nets using one layer are realized as shown in Fig. 1 (b). This way of realization for a given net list L on R is called single-row (in this example, single-layer) routing[1,2], where upward and downward zigzagging is allowed, but not forward and backward zigzagging.

In a realization, the space above the real line R on a layer is designated as the upper street on the layer, and the one below R as the lower street on the layer. The number of horizontal tracks available in the upper (lower) street on a layer is called the upper (lower) street



(a) Net list L.

(b) A realization of L, which is transformed from (c).

(c) Interval graphical representation.

Fig. 1 Single-row single-layer routing.

capacity on the layer. For example, if both the upper and lower street capacities are specified as two, then a net list L of Fig. 1 (a) can be realized on a sigle layer, as shown in Fig. 1 (b).

Using these terms, the problem to be considered in this paper is stated as follows: Given a net list L defined for r nodes on the real line R, and integers K_u and K_w, find a partition of L into the minimum number of subsets $L_1, L_2, \cdots, L_\ell$ such that each L_i $(i=1,2,\cdots,\ell)$ can be realized by single-row single-layer routing with the upper and lower street capacities K_u and K_w, respectively.

2.1 Single-Layer Case

In order to consider the layering problem stated above, we need a necessary and sufficient condition for each such L_i to be realized with prescribed street capacities on a sigle layer. Let us consider this in the following.

The single-row single-layer routing problem can be formulated with the use of the interval graphical representation[3,4]. For example, given a net list L of Fig. 1 (a), consider an ordered sequence s of nets of L and nodes not used for any net, then the interval graphical representation associated with s is dipicted as in Fig. 1 (c), where each horizontal line segment represents the interval covered by a net, and such line segments and nodes not used for any net are arranged according to the order in s.

In an interval graphical representation, let us define the reference line[3] as the continuous line segments which connect the nodes in succession from left to right. For example, in Fig. 1 (c), the reference line is shown by broken lines.

Now, let us stretch out the reference line and map it into the real line R. Associated with this topological mapping, let each interval line be transformed into a path composed of horizontal and vertical line segments so that the portions above and below the refernce line correspond to paths in the upper and lower streets, respectively. Then, this topological mapping yields a realization of a given net list. For example, by this topological transformation for the interval graphical representation of Fig. 1 (c), we obtain a realization as shown in Fig. 1 (b).

Let $I = [v_i, v_j]$ $(i \leq j)$ denote a closed interval between nodes v_i and v_j. Given an interval graphical representation, let us draw a vertical line at an inner point on interval $[v_i, v_{i+1}]$, and let us define the density $d(v_i, v_{i+1})$ as the number of interval lines cut by the vertical line[1,2]. Similarly, draw a vertical line at a node v_i, then define the cut number $c(v_i)$ as the number of interval lines cut by the vertical line, ignoring the one to which v_i belongs[2,3].

Let an interval $I = [v_i, v_j]$ such that $c(v_k) \geq h$ for all v_k on I and $c(v_{i-1}) = c(v_{j+1}) = h - 1$, be referred to as an h-interval. For an interval $I = [v_i, v_j]$, let $\overline{L}(I)$ denote a set of nets which have no node on I, but have two nodes v_a and v_b such that $a < i$ and $j < b$; and let $L(I)$ represent the union of $\overline{L}(I)$ and a set of nets having nodes on I.

By using the interval graphical representation, we can obtain necessary and sufficient conditions for a given net list to be realized with the upper and lower street capacities K_u and K_w[3,4]. However, only in the case of both K_u and K_w up to two, a simple necessary and sufficient condition is known[4], which is derived on the assumption that

(1) every net of a given net list contains at least two nodes,

(2) every nodes belongs to a net, and

(3) any net does not contain a pair of consecutive nodes v_i and v_{i+1}.

However, in the layering problem, there may possibly exist a node which does not belong to any net of subset L_i. Thus, the assumption of (2) is not satisfied in this case, and hence it should be removed.

Based on the necessary and sufficient condition derived in [4] on the assumption of (1), (2), and (3), we can describe another one when assumption (2) is removed, as follows.

THEOREM: A necessary and sufficient condition for a given net list L to be realized with the upper and lower street capacities K_u and K_w is as follows:

CASE A: $0 \le K_u + K_w \le 2$ $(0 \le K_u, K_w \le 1)$.

The maximum density $d_M \triangleq \max_{1 \le j < r} [\ d(v_j, v_{j+1})\ ]$ is not greater than $K_u + K_w$.

CASE B: $3 \le K_u + K_w \le 4$ $(1 \le K_u, K_w \le 2)$.

i) $d_M \le K_u + K_w$.

ii) For any (K_u+K_w-1)-interval I, $|\bar{L}(I)| \ge K_u + K_w - 2$.

iii) There do not exist two (K_u+K_w-1)-intervals I_1 and I_2 such that

$$|\bar{L}(I_1)| = |\bar{L}(I_2)| = K_u + K_w - 2,$$
$$|L(I_1) \cap L(I_2)| = K_u + K_w - 1, \text{ and}$$
$$\bar{L}(I_1) \ne \bar{L}(I_2).$$

Proof: The condition in CASE A can be easily verified, and henceforth we shall consider CASE B. The necessity of the conditions (i), (ii), and (iii) can be proved in a similar way as in [4]. Thus, the sufficiency is to be shown in the follwoing:

Let L be a net list satisfying conditions (i), (ii), and (iii), and let $L_{(2)}$ be a net list obtained from L by applying the following two operations repeatedly as far as possible.

[I] Delete every node not belonging to a net.

[II] Delete any one of two consecutive nodes which are contained in the same net.

Then, we can see that $L_{(2)}$ satisfies the assumption (1), (2), and (3), and also satisfies the necessary and sufficient condition for the realizability derived in [4]. Therefore, $L_{(2)}$ can be realized with the upper and lower street capacities K_u and K_w, respectively. Thus, the remaining task that we have to show is that from any realization of $L_{(2)}$ with the street capacities K_u and K_w, we can construct a realization of L with these street capacities, by adding nodes and nets deleted in the transformation from L to $L_{(2)}$. However, this can be easily done through the use of the condition (i), and the details are ommitted.

q.e.d.

For example, the net list shown in Fig. 1 (a) has three 3-intervals I_1, I_2, and I_3, and satisfies this necessary and sufficient condition. Thus, it has a realization with both the upper and lower street capacities equal to 2, as depicted in Fig. 1 (b).

2.2 Layering Problem

As can be verified from this theorem, it is easy to partition a given net list L into $L_1, L_2, \cdots, L_\ell$ so that each L_i can be realized with the upper and lower street capacities up to one. Thus, we shall pay attention to the layering problem in the case of $K_u = K_w = 2$, as follows.

[Layering Problem]: Given a net list L defined for r nodes on the real line R, find a partition of L into the minimum number of subsets $L_1, L_2, \cdots, L_\ell$ such that each L_i $(i = 1, 2, \cdots, \ell)$ satisfies the following conditions;

C1: the maximum density $d_M^i \leq 4$,

C2: for each 3-interval I, $|\bar{L}_i(I)| \geq 2$, and

C3: there do not exist two 3-intervals I_1 and I_2 with $|\bar{L}_i(I_1)| = |\bar{L}_i(I_2)| = 2$, $|L_i(I_1) \cap L_i(I_2)| = 3$, and $\bar{L}_i(I_1) \neq \bar{L}_i(I_2)$,

where $\bar{L}_i(I)$ and $L_i(I)$ are defined for net list L_i similarly to $\bar{L}(I)$ and $L(I)$, respectively.

Note here that the discussion for the case of $K_u = K_w = 2$ can be applied to the case of $K_u = 2$ and $K_w = 1$ with a slight modification, since the realizability condition in both cases are quite similar.

Let d_M be the maximum density of a given net list, then from condition C1, we have $\ell \geq \lceil d_M/4 \rceil$ where $\lceil x \rceil$ denotes an integer not less than x. On the other hand, if we partition a given net list L into subsets L_i such that each L_i has the maximum density equal to or less than 3, then each L_i satisfies C2 and C3 automatically. Thus, we have

$$\lceil d_M/4 \rceil \leq \ell \leq \lceil d_M/3 \rceil.$$

Namely, at least $\lceil d_M/4 \rceil$ layers are necessary, and at most $\lceil d_M/3 \rceil$ layers are sufficient to realize a net list under the constraint that both the upper and lower street capacities in each layer are equal to 2.

3. Simplifications of the Problem

Since this Layering Problem seems too hard to be solved in its original form, we may have to simplify the problem. In the folowing, we relax conditions C2 and C3 so that the Layering Problem can be reduced to another one in terms of the so-called interval graph[10].

SIMPLIFICATION I: We first transform a given net list L into another L' such that each net of L' contains exactly two nodes, as follows: For each net N_a of L with more than two nodes $v_{a_1}, v_{a_2}, \cdots, v_{a_k}$ $(a_i < a_j$ for $i < j)$, split each v_{a_j} $(1 < j < k)$ into two nodes $v_{a_j}^-$ and $v_{a_j}^+$ such that $v_{a_j}^-$ is located at an inner point on $[v_{a_j-1}, v_{a_j}]$ and $v_{a_j}^+$ is located at an inner point on $[v_{a_j}, v_{a_{j+1}}]$, and replace N_a by k-1 nets N_{a_1}, N_{a_2}, $\cdots$, $N_{a_{k-1}}$ such that $N_{a_j} = \{ v_{a_j}^-, v_{a_{j+1}}^+ \}$ (let $v_{a_1}^- = v_{a_1}$ and $v_{a_k}^+ = v_{a_k}$).

By this transformation, we can disregard condition C3 in the Layering Problem, since any such L' does not have two 3-intervals I_1 and I_2 such that $|L'(I_1) \cap L'(I_2)| = 3$. Note here that the maximum density d_M' of L' increases by at most one from the maximum density d_M of L, i.e., $d_M' \le d_M + 1$. Moreover, we have the following proposition.

Proposition 1: If a subset L_i' of L' satisfies conditions C1 and C2, then the subset L_i of L, which is obtained from L_i' by merging every pair of splitted nodes v_j^- and v_j^+ into the original node v_j, satisfies conditions C1, C2, and C3.

Proof: To prove the proposition, we have only to show that the subset L_i of L can be realized with the upper and lower street capacities both equal to two. Since a subset L_i' of L' satisfying conditions C1 and C2 satisfies condition C3 automatically, L_i' can be realized with the upper and lower street capacities both equal to two. Therefore, there exists an interval graphical representation of L_i', which yields a realization of L_i' with these street capacities by means of the topological mapping stated in Section 2.1. From this interval graphical representation, we can construct an interval graphical representation of L_i which yields a realization of L_i with the upper and lower street capacities both equal to two, as follows.

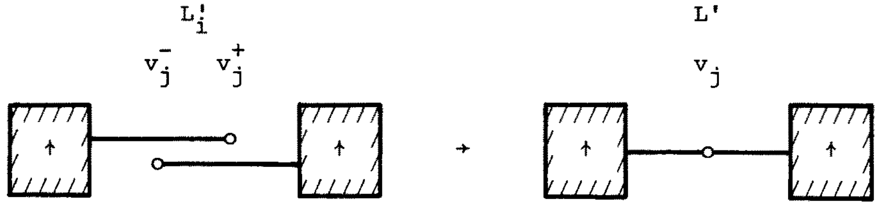
[a] In the case of $d(v_j^-, v_j^+) = 2$, the interval graphical representation of L_i' can be divided into two portions as illustrated in Fig. 2 (a). Merge v_j^- and v_j^+, and we can obtain a required interval graphical representation of L_i.

[b] In the case of $d(v_j^-, v_j^+) = 3$, suppose that two nets containing v_j^- and v_j^+ are adjacent in the interval graphical representation of L_i'. Then, merge v_j^- and v_j^+ as illustrated in Fig. 2 (b), and we can obtain a required interval graphical representation of L_i.

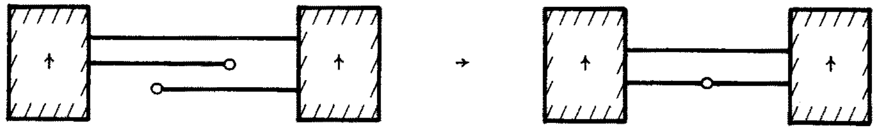
[c] In the case of $d(v_j^-, v_j^+) = 3$, suppose that two nets containing v_j^- and v_j^+ are not adjacent in the interval graphical representation of L_i'. Turn upside down the sequence of nets in the right-hand portion and merge v_j^- and v_j^+, as illustrated in Fig. 2 (c). Then, we can obtain a required interval graphical represenation of L_i.

[d] In the case of $d(v_j^-, v_j^+) = 4$, there exists an interval graphical representation of L_i' in which two nets containing v_j^- and v_j^+ are adjacent, as illustrated in Fig. 2 (d). Merge v_j^- and v_j^+, and we can obtain a required interval graphical representation of L_i. q.e.d.

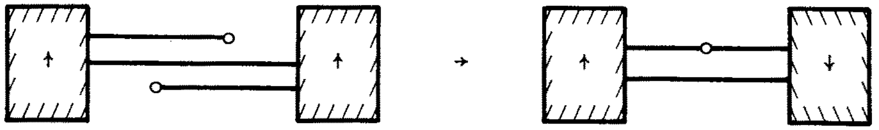
Thus, our problem is to find a partition of L' into subsets L_i' such that each subset L_i' satisfies conditions C1 and C2. Henceforth, unless



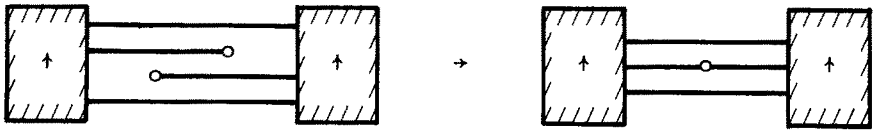
(a) The case of $d(v_j^-, v_j^+) = 2$.



(b) The case of $d(v_j^-, v_j^+) = 3$.



(c) The case of $d(v_j^-, v_j^+) = 3$.



(d) The case of $d(v_j^-, v_j^+) = 4$.

Fig. 2 The transformation from an interval graphical representations of L'_i into that of L_i.

otherwise specified, a given net list L' is assumed to contain only nets with exactly two nodes.

SIMPLIFICATION II: Let us now consider a relaxation of condition C2 as follows: Given a subset L'_i of L', let $\mathcal{J}(L'_i)$ be a set of intervals $[v_a, v_b]$ such that v_a and v_b are contained in some nets of L'_i. If $L'_i(I)$ for $I \in \mathcal{J}(L'_i)$, where $L'_i(I)$ for I is defined just as L(I) for I, is maximal and I is minimal, i.e., there does not exist an interval $I' \in \mathcal{J}(L'_i)$ such that $L'_i(I') \supsetneq L'_i(I)$, or $L'_i(I') = L'_i(I)$ and $I' \subsetneq I$, then interval $I \in \mathcal{J}(L'_i)$ is called a zone of L'_i. As can be readily seen from the definition, any two distinct zones do not overlap each other. By using this concept, we can introduce a condition C2' stronger than C2, as follows.

C2': For any two consecutive zones Z_j and Z_{j+1} of L'_i,

$$|L'_i(Z_j) \cap L'_i(Z_{j+1})| \le 2.$$

Proposition 2: If a net list L_i' satisfies conditions C1 and C2', then L_i' also satisfies condition C2.

Proof: Let $Z = [v_p, v_q]$ be an arbitrary zone of L_i'.

(i) If $|L_i'(Z)| \leq 2$, then there exists no 3-interval of L_i' which overlapps with zone Z.

(ii) If $|L_i'(Z)| = 3$, then even if there exists a 3-interval I of L_i' which overlapps with Z, we have $I \subset Z$, and moreover each node on I does not belong to any net of L_i'. Therefore, we have $L_i'(I) = L_i'(Z)$, and hence $|L_i'(I)| = 3$.

(iii) In the case of $|L_i'(Z)| = 4$, consider eight nodes belonging to four nets of $L_i'(Z)$, and denote them by v_a, v_b, v_c, v_p, v_q, v_x, v_y, and v_z $(a < b < c < p < q < x < y < z)$. Then, we can see from condition C2' that interval $I = [v_{c+1}, v_{x-1}]$ must be a 3-interval of L_i'. Moreover, among three nets which cover node v_{c+1}, at most one net has node v_q on I. Therefore, there holds $|L_i'(I)| \geq 2$. q.e.d.

Thus, through these simplifications I and II stated above, the Layering Problem can be reduced to the following problem.

[Simplified Layering Problem (SLP)]: Given a net list L' such that every net has exactly two nodes, partition L' into the minimum number ℓ' of subsets so that each subset satisfies conditions C1 and C2'.

For example, Fig. 3 shows a partition of a given net list L' into L_1' and L_2' each of which satisfies C1 and C2', where zones of L', L_1', and L_2' are also depicted. It can be seen from the reference lines drawn in the figure that both L_1' and L_2' are realized with the upper and lower street capacities equal to two.

Considering that condition C2' is concerned only with zones, to check whether or not C2' is satisfied, it is sufficient to know how many zones there are and which nets cover each zone. Thus, we define a zone representation, which indicates which nets cover which zones. For example, the zone representations associated with the net lists L', L_1', and L_2' of Fig. 3 are illustrated in Fig. 4.

Now, construct an interval graph G(L') from a given net list L' such that each vertex corresponds to a net and there exists an edge between vertices v and w if and only if the nets corresponding to v and w overlap each other. As can be readily seen, each zone and the maximum density of a given net list L' correspond to a maximal clique and the clique number[10] of G(L'), respectively. Therefore, problem SLP can be restated as a problem of the interval graph.

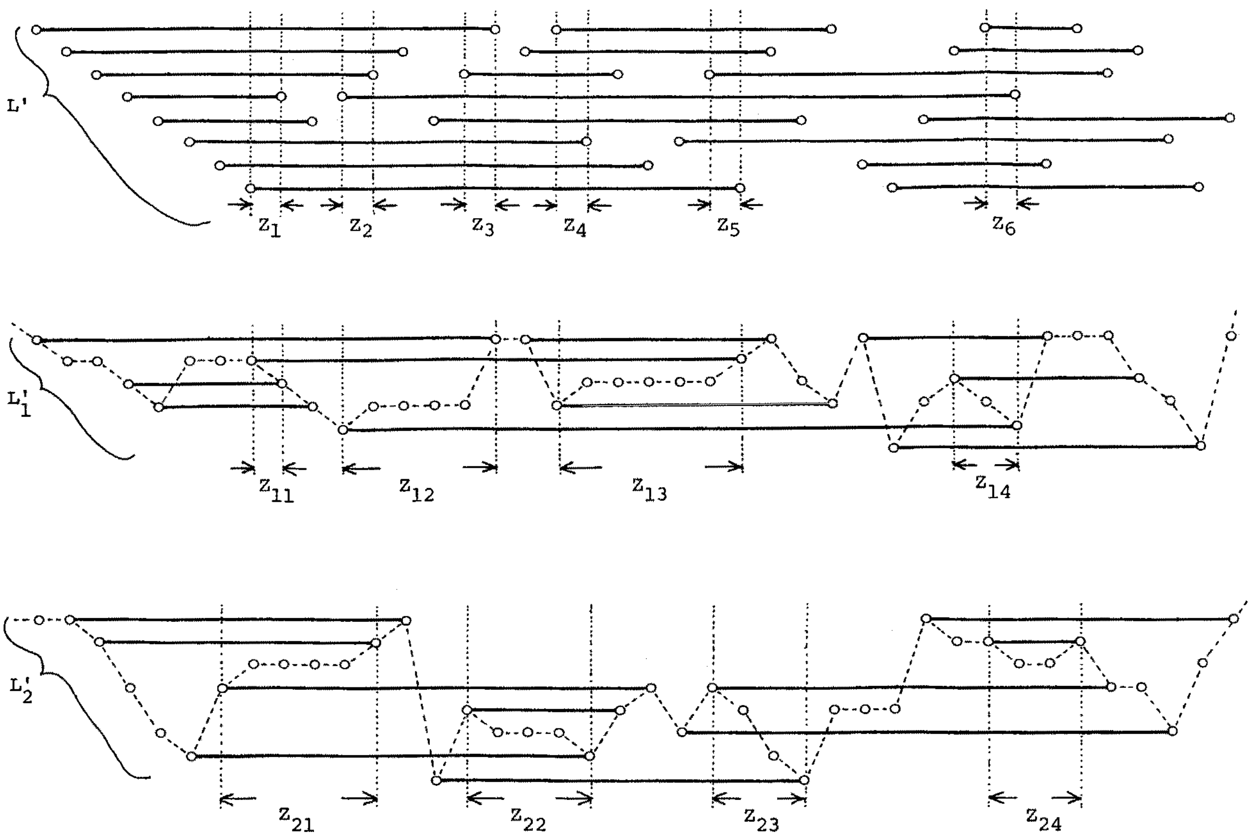


Fig. 3 Net list L' and its subsets L'_1 and L'_2.

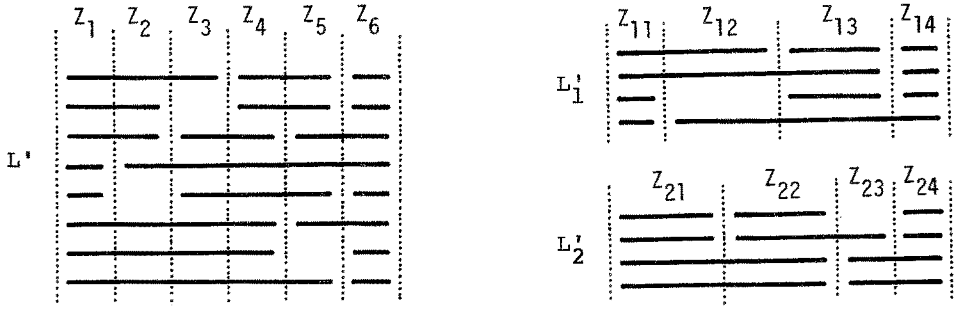


Fig. 4 Zone representations.

4. Lower Bound to the Number of Layers

Now, let us consider a lower bound to the minimum number ℓ' of subsets into which L' is partitioned in problem SLP. Let d'_M be the maximum density of a given net list L', then as can be readily seen from condition C1, we have $\lceil d'_M/4 \rceil$ as a lower bound to ℓ'. Moreover, let q'_M be the maximum number of nets which are common to two consecutive zones Z_j and Z_{j+1}, i.e., $q'_M \triangleq \max_j [\, |L'(Z_j) \cap L'(Z_{j+1})| \,]$. Then, we have the following proposition.

Proposition 3: There holds the following inequality.

$$\max[\, \lceil d'_M/4 \rceil,\ \lceil (q'_M + 2)/4 \rceil \,] \le \ell'.$$

Proof: Since $\lceil d'_M/4 \rceil \le \ell'$ can be readily verified, we have only to show $\lceil (q'_M + 2)/4 \rceil \le \ell'$. Let $q'_M \triangleq 4k + \alpha$, where k is a non-negative integer and $\alpha = 0$, 1, 2, or 3.

(i) If $\alpha \le 2$, then from the difinition of q'_M, there holds $d'_M \ge q'_M + 1 = 4k + 1 + \alpha$. Thus, $\ell' \ge \lceil d'_M/4 \rceil = k + 1 = \lceil (q'_M + 2)/4 \rceil$.

(ii) In the case of $\alpha = 3$, let Z_1 and Z_2 be zones of L' such that $q'_M = |L'(Z_1) \cap L'(Z_2)| = 4k + 3$. From the definition of a zone, we can see that $L'(Z_1) - L'(Z_2) \ne \phi$ and $L'(Z_2) - L'(Z_1) \ne \phi$. Therefore, for any partition of L' into k+1 subsets L'_i such that each L'_i satisfies C1, there exists a subset L'_h which has zones Z_1^h and Z_2^h satisfying $|L'(Z_1^h) \cap L'(Z_2^h)| = 3$. Thus, $\ell' \ne k + 1$. Moreover, similarly to (i), we have $\ell' \ge k + 1$. Hence, $\ell' \ge k + 2 = \lceil (q'_M + 2)/4 \rceil$. q.e.d.

Now, to obtain another lower bound, consider the case where the maximum density d'_M is a multiple of four, i.e., $d'_M = 4k$ (k : integer). Let $Z_1^{4k}, Z_2^{4k}, \cdots, Z_m^{4k}$ be zones of a net list L' arranged from left to right in this order such that $|L'(Z_j^{4k})| = 4k$ $(1 \le j \le m)$. For these zones, let us define

$$TR(Z_j^{4k}) \triangleq L'(Z_j^{4k}) - L'(Z_{j+1}^{4k}),$$

$$\mathrm{TL}(Z_j^{4k}) \triangleq \mathrm{L}'(Z_j^{4k}) - \mathrm{L}'(Z_{j-1}^{4k}),$$

where let $\mathrm{L}'(Z_0^{4k}) = \mathrm{L}'(Z_{m+1}^{4k}) = \phi$.

If a net list L' with $d'_M = 4k$ has a zone Z_j^{4k} such that $|\mathrm{TR}(Z_j^{4k})| = 2$ or 3, then in order to partition L' into subsets L'_1, $L'_2, \cdots$, L'_k each of which satisfies conditions C1 and C2', all the nets of $\mathrm{TR}(Z_j^{4k})$ have to be contained in a subset L'_i. In other words, if L' has such a zone Z_j^{4k} and can be partitioned into k subsets each of which satisfies C1 and C2', then such a partition contains all the nets of $\mathrm{TR}(Z_j^{4k})$ in a single subset. The reason is as follows: Assume that the nets of $\mathrm{TR}(Z_j^{4k})$ such that $|\mathrm{TR}(Z_j^{4k})| = 2$ or 3 are partitioned into two or more subsets. Then, there exists a subset L'_i which contains exactly one net of $\mathrm{TR}(Z_j^{4k})$, say N_h, and hence we have two consecutive zones Z_a $(\supset Z_j^{4k})$ and Z_b $(\supset Z_{j+1}^{4k})$ of L'_i such that $N_h \in L'_i(Z_a)$, $N_h \notin L'_i(Z_b)$, and $|L'_i(Z_a) \cap L'_i(Z_b)| = 3$, which do not satisfy C2'.

Noting this fact, let us introduce a binary relation $\mathcal{R}^*$ into a set L* of nets defined by

$$L^* \triangleq \bigcup_{j=1}^{m} \mathrm{L}'(Z_j^{4k}),$$

such that $N_x \mathcal{R}^* N_y$ if and only if nets N_x and N_y in L* have to be contained in the same subset, so that L' can be partitioned into k subsets each of which satisfies conditions C1 and C2'.

In the following, we list up cases in which we can easily find a pair of nets in relation $\mathcal{R}^*$.

1°: If there exist zones Z_j^{4k} and Z_{j+1}^{4k} such that $|\mathrm{TR}(Z_j^{4k})| = |\mathrm{TL}(Z_{j+1}^{4k})| = 2$ or 3, then as discussed above, we have $N_x \mathcal{R}^* N_y$ for any pair of nets N_x and N_y in $\mathrm{TR}(Z_j^{4k}) \cup \mathrm{TL}(^{4k}_{j+1})$.

Similarly to 1°, we can find a pair of nets satisfying relation $\mathcal{R}^*$ in the following.

2°: If there exists a zone Z_j^{4k} such that $|\mathrm{TR}(Z_j^{4k})| = 4$ and $N_a \mathcal{R}^* N_b$ for N_a and $N_b \in \mathrm{TR}(Z_j^{4k})$, then we have $N_x \mathcal{R}^* N_y$ for N_x and $N_y \in \mathrm{TR}(Z^{4k}) - \{N_a, N_b\}$.

3°: The case similar to 2° with $\mathrm{TR}(Z_j^{4k})$ replaced by $\mathrm{TL}(Z_j^{4k})$.

4°: If there exists a zone Z_j^{4k} such that $|\mathrm{TR}(Z_j^{4k})| = 5$ and there hold $N_a \mathcal{R}^* N_b$ and $N_b \mathcal{R}^* N_c$ for N_a, N_b, and $N_c \in \mathrm{TR}(Z_j^{4k})$, then we have $N_x \mathcal{R}^* N_y$ for N_x and $N_y \in \mathrm{TR}(Z_j^{4k}) - \{N_a, N_b, N_c\}$.

5°: The case similar to 4° with $\mathrm{TR}(Z_j^{4k})$ replaced by $\mathrm{TL}(Z_j^{4k})$.

Let $N \mathcal{R}^* N$ for any net $N \in L^*$, then we can readily see that relation $\mathcal{R}^*$ is an equivalence relation. Thus, we can partition L* into equivalence classes S_i $(i = 1, 2, \cdots)$ by $\mathcal{R}^*$. Using these equivalence classes, we can find other pairs of nets, for which there holds relation $\mathcal{R}^*$, as in the following.

6°: If there exists a zone Z_j^{4k} satisfying the following conditions;

i) there exists exactly one equivalence class S_x such that $|TR(Z_j^{4k}) \cap S_x| = 1$,

ii) there exists exactly one equivalence class S_y other than S_x such that $TR(Z_j^{4k}) \cap S_y \neq \phi$ and $|L'(Z_j^{4k}) \cap S_y| \leq 4 - |L'(Z_j^{4k}) \cap S_x|$, and

iii) for any equivalence class S_i exclusive of S_x and S_y such that $TR(Z_j^{4k}) \cap S_i \neq \phi$, there holds $|L'(Z_j^{4k}) \cap S_i| > 4 - |L'(Z_j^{4k}) \cap S_x|$,

then we have $N_x \mathcal{R}^* N_y$ for any pair of nets $N_x \in S_x$ and $N_y \in S_y$.

7°: The case similar to 6° with $TR(Z_j^{4k})$ replaced by $TL(Z_j^{4k})$.

8°: If there exists a zone Z_j^{4k} satisfying the following conditions;

i) there exist exactly two equivalence classes, say S_x and S_y, such that $|TR(Z_j^{4k}) \cap S_x| = |TR(Z_j^{4k}) \cap S_y| = 1$, and

ii) there do not exist two equivalence classes S_a and S_b other than S_x and S_y such that $TR(Z_j^{4k}) \cap S_a \neq \phi$, $TR(Z_j^{4k}) \cap S_b \neq \phi$, $|L'(Z_j^{4k}) \cap S_a| \leq 4 - |L'(Z_j^{4k}) \cap S_x|$, and $|L'(Z_j^{4k}) \cap S_b| \leq 4 - |L'(Z_j^{4k}) \cap S_y|$,

then we have $N_x \mathcal{R}^* N_y$ for any $N_x \in S_x$ and $N_y \in S_y$.

9°: The case similar to 8° with $TR(Z_j^{4k})$ replaced by $TL(Z_j^{4k})$.

10°: If there exists a zone Z_j^{4k} satisfying the folloiwng conditions;

i) there exist exactly three equivalence classes, say S_x, S_y, and S_z, such that $|TR(Z_j^{4k}) \cap S_x| = |TR(Z_j^{4k}) \cap S_y| = |TR(Z_j^{4k}) \cap S_z| = 1$, and

ii) there does not exist an equivalence class S_i different from S_x, S_y, and S_z such that $TR(Z_j^{4k}) \cap S_i \neq \phi$ and $|L'(Z_j^{4k}) \cap S_i| \leq 4 - A$, where $A \triangleq \min_{h=x,y,z} [\, |L'(Z_j^{4k}) \cap S_h| \,]$,

then we have $N_x \mathcal{R}^* N_y$ and $N_y \mathcal{R}^* N_z$ for any $N_x \in S_x$, $N_y \in S_y$, and $N_z \in S_z$.

11°: The case similar to 10° with $TR(Z_j^{4k})$ replaced by $TL(Z_j^{4k})$.

Now, given a net list L', check whether or not L' satisfies any condition of 1° - 11°, and seek as many pairs of nets in relation $\mathcal{R}^*$ as possible. Let S_i^* $(i = 1, 2, \cdots)$ be equivalence classes thus obtained (namely, S_i^* are the equivalence classes associated with the coarsest partition of L* by $\mathcal{R}^*$ through the use of 1° - 11°). From the definition of $\mathcal{R}^*$ and S_i^*, we can easily verify the following proposition.

Proposition 4: Given a net list L' with $d'_M = 4k$, if there holds one of the following conditions I, II, and III, then we have $\ell' \geq k + 1$.

I: There exist an equivalence class S_i^* and a zone Z (not necessarily $|L'(Z)| = 4k$) such that $|L'(Z) \cap S_i^*| \geq 5$.

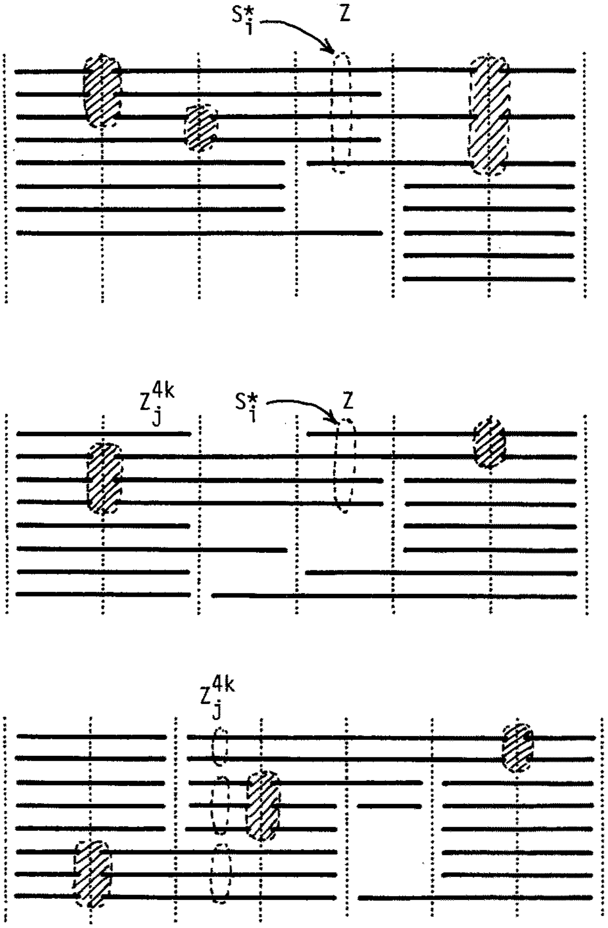
II: There exist an equivalence class S_i^* and zones Z (not necessarily $|L'(Z)| = 4k$) and Z_j^{4k} such that $|L'(Z) \cap S_i^*| = 4$ and $|L'(Z) \cap S_i^* \cap L'(Z_j^{4k})| = 3$.

III: There exists a zone Z_j^{4k} such that

i) there exists an equivalence class S_x^* satisfying $|L'(Z_j^{4k}) \cap S_x^*| < 4$, and

ii) for any equivalence class S_i^* with $L'(Z_j^{4k}) \cap S_i^* \neq \phi$ exclusive of S_x^*, there holds $|L'(Z_j^{4k}) \cap S_i^*| > 4 - |L'(Z_j^{4k}) \cap S_x^*|$.

For example, zone representations of net lists which satisfy conditions I, II, and III are shown in Figs. 5 (a), 5 (b), and 5 (c), respectively, and we can see that for these net lists, we have $\ell' \geq k+1 = 3$.



(a) Condition I.

(b) Condition II.

(c) Condition III.

Fig. 5 Examples of net lists with $\ell' > 2$.

5. Outline of Algorithm

In what follows, we describe a heuristic algorithm for problem SLP.

The algorithm tries to seek subsets L'_i of a given net list L' through a number of stages such that at each stage a subset L'_i satisfying C1 and C2', is taken out from L'. In this process, relation $\mathcal{R}^*$ is made use of in such a way that if the current subset L'_i contains any net in an equivalence class S^*_h, then let L'_i contain all the nets in S^*_h; if the union of L'_i and S^*_h does not satisfy condition C1 or C2', then let any net of S^*_h be not added to L'_i.

Before describing the algorithm, let us consider the case in which any pair of nets in relation $\mathcal{R}^*$ have not been found. Then, let us provide p ($\geq d'_M$) tracks, and allocate all nets of L' on these tracks without overlapping. If we can choose four tracks among them such that a set L'_0 of nets allocated on these four tracks satisfies condition C2', then this L'_0 can be a subset L'_i of L'. Thus, the problem here is how to find such four tracks, on which we touch in the following.

First, construct a directed bipartite graph $G = [T,B;E,D]$ such that

i) each vertex $t_i \in T$ corresponds to a track,

ii) each $b^i_j \in B$ corresponds to a break b^i_j of track t_i, where a break of a track indicates an interval $[v_a, v_b]$ such that there are two nets on the track; one starting at v_a to the left and the other starting at v_b to the right, and there is no net on the track between v_a and v_b,

iii) $E \triangleq \{ (b^i_j, t_i) \}$, where (b^i_j, t_i) denotes an edge incident from b^i_j into t_i, and

iv) there exists an edge $(t_h, b^i_j) \in D$ if and only if on track t_h there does not exist any net passing over break b^i_j.

For a set X of vertices on this graph G, let $\Gamma^+(X) \triangleq \{ v \mid (x,v) \in E \cup D,\ x \in X \}$ and $\Gamma^-(X) \triangleq \{ v \mid (v,x) \in E \cup D,\ x \in X \}$. Then, a subset $T_0 \subset T$ such that $|T_0| = 4$ and $\Gamma^-(T_0) \subset \Gamma^+(T_0)$, yields desired four tracks, and hence a set of nets on these four tracks satisfies conditions C1 and C2'.

<ALGORITHM>

Input : A net list L' with the maximum density d'_M.

Output : A subset L'_0 of L' satisfying conditions C1 and C2'.

Step 1: Using Propositions 3 and 4, seek a lower bound k to ℓ'. If $d'_M = 4k$, then go to Step 2; else go to Step 4.

Step 2: If there exists an equivalence class containing more than one net, which is generated in Step 1 to find a lower bound by Proposition 4, then go to Step 3; else go to Step 4.

Step 3: Define a weight $w(S^*_i)$ of each equivalence class S^*_i by an ordered pair such that $w(S^*_i) \triangleq (|S^*_i|, \max_Z [\, |L'(Z) \cap S^*_i| \,])$, and a weight

$w(N_h)$ of each net N_h in $L' - L^*$ by the length of the interval covered by N_h, i.e., $w(N_h) \triangleq |a-b|$ for $N_h = \{v_a, v_b\}$. Then, let L_0' be an equivalence class with a lexicographically maximum weight. While L_0' satisfies conditions C1 and C2', add to L_0' as many equivalence classes as possible in lexicographically descending order of weight. After this, conduct the similar process for nets in $L' - L^*$ according to the weight $w(N_h)$ of $N_h \in L' - L^*$. Then, go to Step 9.

Step 4: Provide 4k tracks, and assign all the nets in L' to these tracks, so that the nets assigned to a track do not overlap each other. This assignment is done as follows: Pick out a net with the leftmost node among unassigned nets, and assign it to the one among 4k tracks such that the rightmost node of nets on it is located at the leftmost position. In case there exist any tracks to which no net is assigned, choose one of them arbitrarily.

Step 5: Construct a directed bipartite graph $G = [T,B;E,D]$ mentioned above, and define a weight of each vertex $t \in T$ by an ordered pair such that

$$W(t) \triangleq \begin{cases} \left(\min_{b \in \Gamma^-(t)} [\, |\Gamma^-(b)| \,],\ [\, \sum_{b \in \Gamma^-(t)} |\Gamma^-(b)| \,] / |\Gamma^-(t)| \right); & \\ & \text{if } \Gamma^-(t) \neq \phi, \\ (\infty, \infty);\ \text{otherwise.} & \end{cases}$$

Let $t_0 \in T$ be a vertex with a lexicographically minimum weight $W(t_0)$. Then, set $T_0 \leftarrow \{t_0\}$, and add vertices in T to T_0 in lexicographically ascending order of weight, until T_0 satisfies $|T_0| \leq 4$ and $\Gamma^-(T_0) \subset \Gamma^+(T_0)$. If such T_0 can be found, then go to Step 7; else go to Step 6.

Step 6: Choose three vertices of T in ascending order of weight, and let L_0' be a set of nets contained in the corresponding three tracks. Then, go to Step 8.

Step 7: If $|T_0| = 4$, then let L_0' be a set of nets contained in the tracks corresponding to the vertices in T_0, and go to Step 8. Otherwise, try to find a set T_0' such that $T_0 \subset T_0' \subset T$, $|T_0'| \leq 4$, and $\Gamma^-(T_0') \subset \Gamma^+(T_0')$, similarly to Step 5. If $|T_0'| < 4$ and there exists a vertex t of weight (∞, ∞), then add each such vertex to T_0', unless $|T_0'| = 4$.

i) If $|T_0'| = 4$, then let L_0' be a set of nets contained in the tracks corresponding to the vertices in T_0', and go to Step 8.

ii) If $|T_0'| = 3$, then conduct (iv).

iii) If $|T_0'| \leq 2$, then add to T_0' the vertices in $T - T_0'$ with a lexicographically minimum weight, unless $|T_0'| = 3$.

iv) Let L_0' be a set of nets contained in the tracks corresponding to the vertices in T_0', then go to Step 8.

Step 8: Add to L_0' as many nets in L' as possible in descending order of weight defined for nets in $L' - L_0'$ similarly to $w(N_h)$ for $N_h \in L' - L^*$,

while L_0' satisfies conditions C1 and C2'.
Step 9: Terminate by setting $L' \leftarrow L' - L_0'$.

By repeated applications of this algorithm, we can partition a given net list L' into subsets satisfying conditions C1 and C2'. Moreover, it should be noted that we can introduce into Steps 3 and 5 - 7, a procedure to find pairs of nets in relation $\mathcal{R}^*$ by using 6° - 11°, so that the current execution of the algorithm may not decrease the possibility in the next execution that the remaining net list L' may be partitioned into a minimum number of subsets.

6. Concluding Remarks

In this paper, we have described an approach to the layering problem in multilayer PWB wiring. We have paid attention only to the case of $K_u = K_w = 2$, since the discussion on it can be applied to the case of $K_u = 2$ and $K_w = 1$ with a slight modification. However, there still remain a number of problems, among which of primary importance is a necessary and sufficient condition (or non-trivial sufficient condition) for a net list to be realized with a given number of layers.

In what follows, we point out another approach to problem SLP, which is applied only to the case of $K_u = K_w = 2$.

A set of pairwise disjoint pairs of distinct nets is called a matching M of a given net list L'. For two nets $N_1 = \{v_a, v_b\}$ and $N_2 = \{v_c, v_d\}$, the following operation is called a merging of nets N_1 and N_2: Replace two nets N_1 and N_2 by a new net $N_{12} = \{v_x, v_y\}$ defined by $x = \min[a, c]$ and $y = \max[b, d]$. Given a net list L' and a matching M of L', the net list L" obtained from L' by merging every pair of nets in M is denoted by L'[M]. Let ρ be the maximum density of L" = L'[M], and consider a partition of L" into $\lceil \rho/2 \rceil$ subsets $L_1'', L_2'', \cdots, L_{\lceil \rho/2 \rceil}''$ such that each subset L_i'' has the maximum density not greater than 2. Based on this partition, we can generate a partition of the original net list L' into subsets L_i' such that each L_i' of L' is obtained from L_i'' by decomposing every merged net in L_i'' into two original nets. Then, we can readily see that each subset L_i' satisfies conditions C1 and C2', and hence we can use such a partition of L' as an approximate solution to problem SLP. Noting that it is easy to find a partition of L" into $\lceil \rho/2 \rceil$ subsets, in this approach, the following problem has to be solved.

[Matching Problem]: Given a net list L', find a matching M of L' such that the maximum density ρ of L'[M] is minimized.

With respect to this problem, we have the following propositions;

Proposition 5: If there holds $\mathcal{Z}(N_j) = \mathcal{Z}(N_h)$ for two distinct nets N_j and N_h, then there exists an optimum matching M* containing pair $\{ N_j, N_h \}$, where $\mathcal{Z}(N)$ is a set of zones which have net N, i.e.,

$$\mathcal{Z}(N) \underline{\Delta} \{ Z \mid N \in L'(Z) \}.$$

Proposition 6: The Matching Problem is polynomially transformable[11] to problem SLP.

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