ON THE LIMITATIONS OF GOVERNMENT
BORROWING: A FRAMEWORK FOR
EMPIRICAL TESTING

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ABSTRACT

This paper seeks to distinguish empirically between two views on the
limitations of government borrowing. According to one view, nothing precludes
the government from running a permanent budget deficit, paying interest due
on the growing debt load simply by issuing new debt. An alternative perspec-
tive holds that creditors would be unwilling to purchase government debt un-
less the government made a credible commitment to balance its budget in present-
value terms. We show that distinguishing between these possibilities is mathe-
matically equivalent to testing whether a continuing currency inflation might
be fueled by speculation alone or is instead driven solely by economic funda-
mentals. Empirical tests which have been developed for this economic question
lead us to conclude that postwar U.S. deficits are largely consistent with
the proposition that the government budget must be balanced in present-value
terms.

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1. Introduction.

This paper addresses the question of whether the government budget must be balanced in present-value terms. This question holds profound implications for macroeconomic theory and practice. With a Keynesian consumption function, an affirmative answer can reverse the traditional conclusion that money-financed deficits are more expansionary than bond-financed deficits (Blinder and Solow, 1974). With a permanent-income consumption function, by contrast, it can imply that a bond-financed deficit has no effect on aggregate demand.[1] And if the only politically acceptable means of raising the requisite revenues is with money creation, it can imply a strong practical link between budget deficits and inflation, as argued by Sargent (1982) and Sargent and Wallace (1981). It of course also has serious implications for measures which may be necessary to adopt in the future as a consequence of budget deficits accumulating in the United States and many developing countries.

Why do some claim that government deficits must be balanced with future surpluses? Whenever the budget goes into deficit, new debt must be issued. If this is in the form of one-period bonds, then next year interest and principal on that debt will come due.[2] Accordingly, even if taxes in this following year were sufficient to cover spending on government programs, new debt would still have to be issued for that year, not only to replace the old debt which comes due, but further to cover the accumulated interest on this debt. If taxes are no greater than spending for yet a second year, then at the end of the second year outstanding debt will have grown by two years of accumulated interest. If taxes never exceed spending, debt would grow forever without bound as an infinite interest load accumulated as a consequence of a single year's deficit.

Whether rational households would in fact always be willing to buy this debt remains an unsettled theoretical question. If the government's borrowing cost equals or exceeds the economy's growth rate, then an unpaid deficit implies that debt must grow to become an infinite multiple of GNP.[3] Such an equilibrium seems counter-intuitive, though McCallum (1984) provides an optimizing example in which both households' interest income and the tax on that interest income indeed grow to become an arbitrarily large multiple of total production. On the other hand, if the interest rate (r) is less than the economy's growth rate (q), then an infinitely-lived government clearly could run a permanent deficit without having debt become a growing multiple of GNP, as stressed by Feldstein (1976) and Buiter (1984). While r < q is inconsistent with an optimizing equilibrium with infinitely-lived individuals in the standard neoclassical growth model (e.g., Cass 1965 or Dixit 1976, p. 109), it does appear to be allowed in certain overlapping-generations models,[4] and in models where private borrowing takes place at higher interest rates than public borrowing, e.g., if private debt is inherently riskier as in Eaton (1981).

It in any case seems desirable to supplement these theoretical considerations with empirical evidence. Seater (1982) and Aschauer (1985), among others, have tested the hypothesis that the present-value budget must be balanced jointly with a permanent-income efficient-markets hypothesis, and accepted. Barro (1984b) tested the hypothesis that the present-value budget must be balanced jointly with the assumption that taxation and deficit policies have historically been optimal, and again accepted. However, to our knowledge there has been no direct empirical test of the present-value constraint itself.

Such a test might at first glance seem straight-forward enough. The United States government, for example, has run more or less a chronic deficit since

1930, suggesting that permanent deficits are quite feasible and practical.
However, this simple argument ignores the potential role of debt retirement
through monetization and capital gains on bonds or tangible assets through
inflation. It further provides no formal test of whether these deficits were
rationally anticipated, nor allowance for what might rationally be expected to
happen out of sample.

In this paper we propose an empirical framework for testing the practical
limits to public borrowing which gets around these criticisms. We show that
the proposition that the government can accumulate ever-growing debt through
perpetual deficit-financing is mathematically equivalent to the proposition
that prices can rise continually in a self-fulfilling speculative bubble. Thus,
empirical tests which have been developed for the latter hypothesis may also
be fruitfully applied to study the limits of government borrowing.

Section 2 provides a formal statement of the present-value constraint to
be tested. Section 3 briefly discusses some issues of data and measurement.
In Section 4 we present the results of alternative empirical tests of whether
the postwar record of budget deficits in the United States could be consistent
with the present-value constraint under rational expectations, with brief con-
clusions offered in Section 5.

2. The present-value government borrowing constraint.

In the light of the disparate concepts of the correct measurement of real
government indebtedness prevalent in the literature (e.g., Jump 1980, Buiter
1983, Barro 1984a, and Eisner and Pieper 1984), it seems worthwhile to develop
from first principles the expression appropriate for the tests proposed in this
paper. One difficulty arises because at any time the public holds government

securities of a variety of terms and vintages. To understand the budget impli-
cations of capital gains and losses on these different issues, we allow each
dollar of debt potentially to have its own unique characteristics. Let the
integer J_t denote the nominal <u>face value</u> of interest-bearing government debt
at the end of period t, which we may think of as a collection of J_t different
$1 face-value bonds. Let $b_{j,t}$ (j = 1,2,..,J_t) denote the nominal <u>market value</u>
of the j'th (face value) dollar of debt, and $\theta_{j,t}$ its coupon paid between dates
t-1 and t. We further let P_t denote an aggregate price index for goods in the
economy and $\bar{r}$ the associated ex post real interest rate which is earned on
one-period government bonds during an average year. Finally, we define $v_{j,t}$
to be the real excess one-period holding yield of bond j relative to the average
earned on one-period bonds:

$$v_{j,t} \equiv \frac{b_{j,t} + \theta_{j,t}}{P_t} - \frac{(1+\bar{r})b_{j,t-1}}{P_{t-1}} \tag{1}$$

which implies

$$\frac{b_{j,t}}{P_t} \equiv \frac{(1+\bar{r})b_{j,t-1}}{P_{t-1}} - \frac{\theta_{j,t}}{P_t} + v_{j,t} \quad . \tag{2}$$

Of course, not all these bonds are held by the public, and the quantity
held by the public will increase or decrease in part as they are sold or pur-
chased by the Federal Reserve. Suppose that at the end of period t-1 the first
L_{t-1} bonds are held by the Fed and the remaining $J_{t-1}-L_{t-1}$ held by the public.
Consider an open market purchase during period t that acquires the next L_t-L_{t-1}
bonds:

$$\sum_{j=L_{t-1}+1}^{L_t} b_{j,t} = M_t - M_{t-1} + u_{1,t} \tag{3}$$

where M_t denotes the stock of high-powered money at the end of period t. In principle, equation (3) might be thought of as an accounting identity characterizing open-market operations. As a practical issue, however, we must append the measurement error term u_{1t}, since open-market purchases near the beginning of period t would actually take place at market prices closer to $b_{j,t-1}$, and since serious data difficulties arise in keeping track of the true market value of all the assets in the Fed's portfolio.

Additionally, some number of new bonds (say, $J_t - J_{t-1}$) might be issued during period t to cover any budget deficit, and these of course are likewise initially issued at market value:

$$\sum_{j=J_{t-1}+1}^{J_t} (b_{j,t}/P_t) = G_t + R_t - T_t + u_{2,t} \tag{4}$$

Here G_t denotes real government purchases of goods, R_t real interest expenditures,[5] and T_t real tax revenues. Once again, measurement error $u_{2,t}$ is introduced by issues of within-period timing of new debt issue and by the impossibility of accurately measuring the true market value of net government indebtedness.

Equations (2), (3) and (4) imply

$$\sum_{j=L_t+1}^{J_t} \frac{b_{j,t}}{P_t} = \sum_{j=L_{t-1}+1}^{J_{t-1}} \frac{(1+\bar{r})b_{j,t-1}}{P_{t-1}} - \sum_{j=L_{t-1}+1}^{J_{t-1}} \frac{\theta_{j,t}}{P_t} + \sum_{j=L_{t-1}+1}^{J_{t-1}} v_{j,t}$$

$$+ G_t + R_t - T_t - \frac{(M_t - M_{t-1})}{P_t} - \frac{u_{1,t}}{P_t} + u_{2,t} . \tag{5}$$

Next, define B_t to be the nominal market value of debt held by the public, i.e.,

$B_t = \sum\limits_{j=L_t+1}^{J_t} b_{j,t}$. Notice also $R_t = \sum\limits_{j=L_{t-1}+1}^{J_{t-1}} (\theta_{j,t}/P_t) + u_{3,t}$. Finally, let

S_t denote the real government surplus in year t (including money seignorage but excluding interest payments):

$$S_t \equiv T_t + \frac{(M_t - M_{t-1})}{P_t} - G_t \tag{6}$$

and let V_t denote the combined influence of time-varying real interest rates, capital gains and losses on government bonds, and measurement error:

$$V_t \equiv \sum\limits_{j=L_{t-1}+1}^{J_{t-1}} v_{j,t} - (u_{1,t}/P_t) + u_{2,t} + u_{3,t} \ . \tag{7}$$

Then equation (5) may be written:

$$\frac{B_t}{P_t} = \frac{(1+\bar{r})B_{t-1}}{P_{t-1}} - S_t + V_t \ . \tag{8}$$

By recursive substitution forwards, equation (8) is seen to imply

$$\frac{B_{t-1}}{P_{t-1}} = \sum\limits_{i=1}^{N} \frac{(S_{t+i-1} - V_{t+i-1})}{(1+\bar{r})^i} + \frac{B_{t+N-1}}{P_{t+N-1}(1+\bar{r})^N} \ . \tag{9}$$

Equation (8) and its implication (9) can not be a point of serious controversy, for they do little more than summarize the equations of motion characterizing monetary and fiscal policy. What is of economic interest (and subject in principle to empirical refutation) is what happens to the second term in (9) as N gets large. This term registers whether the supply of bonds held by the public grows faster on average than the rate of interest. It accordingly reflects the feasibility of the government running a permanent budget deficit, paying in-

terest on the growing debt load simply by issuing new debt. In this paper we are therefore interested in testing

$$H_0: \lim_{N\to\infty} \frac{B_{t+N}}{P_{t+N}(1+\bar{r})^N} = 0 \tag{10a}$$

or equivalently

$$H_0: \frac{B_t}{P_t} = \sum_{i=1}^{\infty} \frac{(S_{t+i} - V_{t+i})}{(1+\bar{r})^i} . \tag{10b}$$

Buiter (1984) calls (10) the "present value budget constraint," while West's (1984) study of stock price volatility refers to a condition analogous to (10) as the "transversality condition." Note that condition (10) does not imply that "the national debt would eventually be paid off" (e.g., Dornbusch and Fischer, 1984, p. 525). In fact, (10) is consistent with a constantly increasing stock of debt, as long as the rate of increase is less than the government's borrowing rate. Rather, the question we pose is whether interest on this debt is to be paid with future tax increases or instead with continual issue of new debt.

If (10) represents the null hypothesis that the government budget must be intertemporally balanced, how may we usefully frame the alternative possibility that government deficits could continue forever? The answer to this question is obtained by now solving equation (8) <u>backwards</u> to get:

$$B_t/P_t = (1+\bar{r})^t(B_0/P_0) + \sum_{j=1}^{t} (1+\bar{r})^{t-j}(V_j - S_j) . \tag{11}$$

In what follows we assume that the infinite sum

$$\sum_{j=1}^{\infty} (1+\bar{r})^{-j}(S_j - V_j) \tag{12}$$

converges. Note this assumption makes sense even when debt itself $(1+\bar{r})^{-N}(B_{t+N}/P_{t+N})$ does not converge, because by definition S_t excludes the ballooning interest payments on this debt. Adding and subtracting $(1+\bar{r})^t$ times expression (12) to the right-hand side of (11) yields

$$(B_t/P_t) = (1+\bar{r})^t(B_0/P_0 - \sum_{j=1}^{\infty} (1+\bar{r})^{-j}(S_j - V_j)) + \sum_{j=t+1}^{\infty} (1+\bar{r})^{t-j}(S_j - V_j)$$

or

$$B_t/P_t = A_0(1+\bar{r})^t + \sum_{j=1}^{\infty} (1+\bar{r})^{-j}(S_{t+j} - V_{t+j}) \tag{13}$$

where $A_0 \equiv B_0/P_0 - \sum_{j=1}^{\infty} (1+\bar{r})^{-j}(S_j - V_j)$. Comparing equation (13), which is the completely general solution to the difference equation (8), to (10b), the restricted solution under the hypothesis H_0 that the government budget must be balanced in present-value terms, we observe the following:

H_0 is true if and only if $A_0 = 0$ in equation (13).

Equation (13) is mathematically equivalent to models used to study self-fulfilling fads or speculative bubbles first explored by Flood and Garber (1980). We accordingly propose that such tests might also be fruitfully applied to understanding the limits of government borrowing. The next section discusses the data on which such tests might be based, while section 4 summarizes our results.

3. Issues of data and measurement.

Eisner and Pieper (1984) and Barro (1984a) have shed substantial light on the relation between officially reported budget figures and the appropriate measures of true economic interest. Here we briefly discuss how the theoretical magnitudes appearing in our equation (13) are related to the budget figures

actually reported in the United States. Complete details on data sources and methods used in our study are available in a data appendix available from the authors on request.

a. <u>Interest payments</u>.

The theoretical measure of government spending (G_t) in the above derivation excludes outlays for interest expenditures. Correcting the officially reported surplus $(\hat{T}_t - \hat{G}_t)$ to exclude interest payments requires three steps. (1) Add to $(\hat{T}_t - \hat{G}_t)$ total interest payments (some $28.1 B for fiscal year 1974). (2) For data prior to 1982, subtract back out that part of this Treasury interest (some $6.6 B in FY 1974) that was paid directly into federal government trust funds such as social security. (Prior to 1982, this sum was already included as an expenditure offset in the officially reported surplus, and accordingly would be counted twice if we just added total reported interest payments to $\hat{T}_t - \hat{G}_t$. After 1982, it has furthermore already been subtracted before reporting the total interest figure, so just adding the reported interest figure to $\hat{T}_t - \hat{G}_t$ in this case gives the correct measure.) (3) Likewise subtract the deposit of Federal Reserve earnings (some $4.8 B in FY 1974), which again are already included in $\hat{T}_t$ under miscellaneous receipts. A sample calculation is provided in Table 1, with details for each fiscal year in the data appendix.

Some debt is issued by various federal agencies in addition to that issued by the Treasury. Outlays for the service of such debt are presently included as expenditures of that agency, and should be subtracted from $\hat{G}_t$ to arrive at our measure of G_t. Unfortunately, such data are not readily available. Since agency debt is small (e.g., $12 B in 1974 compared with $475 B in Treasury debt), little harm can come from <u>assuming</u> that the present value relation holds

for agency debt, in which case we could approximate the present value of later

agency interest payments by the current market value of new agency debt issue.

Thus, we subtract new issue of agency securities ($0.9 B in FY 1974) from $\hat{G}_t$,

and do not count the market value of agency securities in our measure of public

holdings of government debt, B_t.[6]

b. Trust funds.

In the unified budget system used in the U.S. since 1969 (and extended

retroactively to 1959), two separate accounts are maintained. Federal funds

are general purpose revenues and programs. Trust funds collect specified taxes

and spend on specified programs such as social security.

In FY 1974, the Federal funds deficit was $17.4 B, whereas the Trust funds

surplus was $14.0 B. The difference, -$3.4 B, is the deficit officially reported

for FY 1974. This is obviously the correct concept, and needs to be extended

to debt accounting as well. Treasury debt went up from $457.3 B at the end of

FY 1973 to $474.2 B at the end of FY 1974, but clearly this does not represent

an equivalent increase in net government indebtedness, since most of these new

bonds were simply held by a different branch of the government. The correct

measure would subtract the stock of investments in government accounts ($140.2 B

at the end of FY 1974) from total Treasury debt.

Some might argue that these Trust fund holdings are to be used against the

government's liabilities implied by future social security benefit payments.

It seems to us that this is an inaccurate interpretation. Such programs are not

a current liability in the sense that they can be associated with any concrete

number. Rather, they represent the outcome of an uncertain political process,

and the correct way to represent this "liability" is by the discounted cash

flow of an entry on current account rather than any dubious imputation to capital

account. That is, we follow the official accounts in registering net social se-

curity inflows or outflows on the deficit account, and not as figures imputed

to the debt itself.

c. Off-budget items.

Starting in 1971, the activities of certain agencies such as the Postal

Service Fund and Rural Electrification and Telephone Revolving Fund are charac-

terized as "off-budget."[7] Any deficits of such operations require the issue

of Treasury bonds, though they do not count in the officially measured deficit

of the U.S. government.

Suppose that these funds were indeed primarily conveyors of sound loans.

Imagine the agency issuing a $1 loan financed through a $1 sale of Treasury bonds,

and so running an off-budget $1 deficit for that year. In the following year,

the agency receives $r as interest payments from the private sector, but the

Treasury pays $r back to the public as interest on the T-bond. For this year,

the official budget and off-budget items would accordingly sum to a zero net

deficit. Thus, a net intertemporal deficit would be measured if one combined

the official budget and off-budget items into a single account, whereas the oper-

ation itself is clearly fiscally neutral— the government issued a $1 Treasury

bond but acquired a $1 private bond, and has simply sqapped like assets with

the public. A correct measure would be obtained in this case if we adopt our

convention of excluding Treasury interest payments from G_t and simply ignore

the off-budget surplus or deficit altogether.

Of course, these programs are not pure market loans but in fact have a substantial subsidy aspect. Our only justification for ignoring this is that it is difficult to quantify and presumably small relative to the complete budget.

d. Tangible assets.

Eisner and Pieper (1984) have begun the difficult task of quantifying the market value of various tangible assets owned by the government. The question for purposes of the present study is, do government bond-holders believe that future interest payments will really be met through sale of such assets rather than by more conventional means such as tax revenues or debt retirement through monetization? One's position on this question is perhaps a matter of taste as much as anything else. Our own view is that, with one important exception, the promise of substantial liquidation of government tangible assets is not a politically credible backing for U.S. Treasury debt, and should not realistically be expected to substitute for future tax revenues or money seignorage.

The one exception is the government's gold holdings. There is abundant historical evidence that governments willingly draw down or deplete these stocks in the wake of fiscal crises; this indeed is presumably the primary purpose of holding such stocks in the first place. To see the budget implications of such gold sales, let Au_t denote government's gold holdings in ounces, P_t^{Au} the price per ounce of gold, and B_t the nominal market value of debt, all measured at the beginning of period t. Let S_t denote the correctly measured surplus during period t (excluding interest payments from spending but making no correction for gold flows), and let i_t be the one-period interest rate, which for simplicity we assume is the same for all bonds. If at the end of period t the government sells off some amount of gold ($Au_t - Au_{t+1} > 0$) at price P_{t+1}^{Au} and uses the

proceeds to retire debt, then next period's debt will be given by

$$B_{t+1} = (1+i_t)B_t - S_t - (Au_t - Au_{t+1})P_{t+1}^{Au}$$

which can be written as

$$(B_{t+1} - P_{t+1}^{Au}Au_{t+1}) = (1+i_t)(B_t - P_t^{Au}Au_t) + (i_t - \pi_t^{Au})P_t^{Au}Au_t - S_t$$

where $\pi_t^{Au} \equiv (P_{t+1}^{Au} - P_t^{Au})/P_t^{Au}$. Thus, if we define true government indebted-

ness as the stock of Treasury debt held by the public less the current market

value of the government's gold holdings (i.e., as $B_t - P_t^{Au}Au_t$), the equation

governing the evolution of true government indebtedness is obtained by sub-

tracting $(i_t - \pi_t^{Au})P_t^{Au}Au_t$ from the officially measured surplus $(\hat{T}_t - \hat{G}_t)$.

(Note, however, that revenues should not include changes in the gold stock,

i.e., ignore $Au_{t+1} - Au_t$ in calculating $T_t - G_t$).

For fiscal year 1974, gold prices increased by 17.16%, whereas the nominal

interest rate on one-year government bonds was 7.56%. Based on a market value

of the government's gold holdings of $33.8 B, a sum of $(.1716 - .0756)(33.8)$

= $3.2 B should be added to the surplus for FY 1974 to represent capital gains

from gold.

e. Money seignorage.

When the Federal Reserve acquires a bond, future interest payments on that

bond accrue to the Fed and are counted as part of tax revenues under "Miscel-

laneous Receipts" in the official budget. Since the present value of this tax

benefit is equal to the Fed's initial cost of the bond, seignorage is in this

sense theoretically already included in the budget. In practice, however,

any finite-sample estimate of the discounted value of these miscellaneous receipts

must be less than the discounted value of $M_{t+i} - M_{t+i-1}$, because open market

purchases at the end of the sample period have not yet been amortized. For this

reason, we exclude deposits of Fed interest from our measure of T_t and include

the seignorage measure $(M_t - M_{t-1})$, where M_t denotes high-powered money (the

sum of currency in circulation plus reserves in member banks).

f. Other adjustments.

Painstaking calculations of the true market value of government debt,

based on actual market quotations of outstanding securities, have been provided

by Seater (1981) and Cox and Hirschhorn (1983) and updated by Cox (1985). At

the end of FY 1974, outstanding government debt was trading at a market value

of only 95% of its par value.

We also need to subtract Federal Reserve assets from our measure of the

government debt. Since these are all carried on the books at par value, the

simplest way to construct the correct measure closer to market value of these

assets is from the liabilities side, namely, by sub tracting high-powered money

from outstanding government debt to get a measure of net interest-bearing debt

held by the public.

Finally, the Treasury operating cash balance and market value of gold

holdings must be subtracted to arrive at the figure B_t appearing in equation

(8). Sample calculations for the 1974 deficit and debt are provided in Table 1.

Table 2 details the actual series used in the calculations to follow.

4. Empirical tests.

We are interested in the question of whether the U.S. government's credi-

tors could rationally expect that the government budget would be balanced in

present-value terms. In the light of the discussion of section 2, we inter-

pret this as the restriction $A_o = 0$ in the formulation

$$B_t/P_t = A_o(1+\bar{r})^t + E_t \sum_{j=1}^{\infty} (1+\bar{r})^{-j}S_{t+j} + n_t \qquad (14)$$

where B_t/P_t and S_t are the adjusted debt and surplus series reported in Table 2 and n_t is a regression disturbance term reflecting expected changes in real short-term interest rates, the term structure of long rates, and measurement error. The operator E_t denotes the expectations of creditors, which we assume are formed rationally.

Equation (14) is mathematically equivalent to the model proposed by Flood and Garber (1980) for studying self-fulfilling hyperinflations. However, Hamilton and Whiteman (1984) expand on Flood and Garber's caution that their technique implicitly imposes strong restrictions on the variables used by agents in forming expectations E_t or on the dynamics allowed for n_t, and note that a more general test should first be considered[8] which is more robust with respect to such restrictions. In particular, for <u>any</u> stationary process for $(n_t, E_t \sum_{j=1}^{\infty} (1+\bar{r})^{-j}S_{t+j})$, when $A_o = 0$, B_t/P_t will be stationary, whereas for $A_o > 0$, B_t/P_t will not be stationary. We accordingly initially examine two simple tests.

(1) <u>Dickey-Fuller test for unit roots</u>. Dickey and Fuller (1979, p. 431) suggest the following test of the null hypothesis that a series z_t is nonstationary with unit roots. Estimate

$$z_t - z_{t-1} = \theta_o + \theta_1 z_{t-1} + \theta_2(z_{t-1} - z_{t-2}) + e_t$$

by ordinary least squares, and calculate $\hat{\theta}_1/\hat{\sigma}_1$ where $\hat{\sigma}_1$ is the OLS standard error for $\hat{\theta}_1$. The null hypothesis (nonstationarity) says this statistic should be zero; the alternative (stationarity) says less than zero.

Using the data in Table 2 we estimated the following equation by OLS for

t = 1962 to 1984 (standard errors in parentheses):

$$S_t - S_{t-1} = \underset{(3.27)}{-0.53} - \underset{(0.24)}{0.70} S_{t-1} + \underset{(0.24)}{0.38} (S_{t-1} - S_{t-2})$$

$$B_t/P_t - B_{t-1}/P_{t-1} = \underset{(28.13)}{79.63} - \underset{(0.17)}{0.48} B_{t-1}/P_{t-1} + \underset{(0.22)}{1.02} (B_{t-1}/P_{t-1} - B_{t-2}/P_{t-2})$$

The Dickey-Fuller test statistics are -2.92 in the case of the surplus and -2.82 in the case of debt, to be compared with a 5% critical value of -3.00 and 10% value of -2.63 reported in Table 8.5.2., p. 373 of Fuller (1976). The data thus favor rejection of the null hypothesis of nonstationarity in both cases; that is, the data seem fully compatible with the assertion that investors rationally expected the budget to be balanced in present value terms.

(2) Generalized Flood-Garber test. If expectations of future surpluses are conditioned in part on past surpluses and if we include lagged debt to elimi- nate the serial correlation of the resulting error term, then equation (14) takes the form

$$B_t/P_t = c_o + A_o(1+\bar{r})^t + c_1(B_{t-1}/P_{t-1}) + \ldots + c_p(B_{t-p}/P_{t-p})$$
$$+ b_o S_t + b_1 S_{t-1} + \ldots + b_{p-1} S_{t-p+1} + \varepsilon_t \ . \tag{15}$$

If one were willing to impose stronger restrictions on the dynamics of n_t and on the information set used by creditors, then one would want to estimate (15) jointly with cross-equation restrictions on the process followed by S_t, as in Flood and Garber's (1980) study of money demand. In the absence of such restrictions, Hamilton and Whiteman (1984) show that the coefficients c_j, b_j are unrestricted, and a more general test of H_o is obtained by simple OLS[9] estimation of (15):

$$B_t/P_t = 48.41 - 22.68 (1+\bar{r})^t + 0.69 B_{t-1}/P_{t-1} + 0.20 B_{t-2}/P_{t-2}$$
$$(26.40) \quad (21.29) \quad\quad (0.21) \quad\quad\quad (0.24)$$

$$- 1.30 S_t - 0.63 S_{t-1} \quad .$$
$$(0.13) \quad\quad (0.31)$$

We took $\bar{r} = 0.0112$, the average ex post real rate over 1960-1984. This equation clearly yields no indication that government debt tends to be growing at rate $\bar{r}$; the coefficient $\hat{A}_o$ is statistically insignificant, and, if anything, negative in sign.

(3) <u>Restricted Flood-Garber test</u>. The test actually used by Flood and Garber (1980) would be valid for our application only under the further restrictions that n_t follows a white noise process

$$n_t = k + \varepsilon_{1t} \tag{16}$$

and that creditors' expectations of future surpluses are based solely on past realizations of surpluses.[10] A straight-forward manipulation of the formula derived by Hansen and Sargent (1981, p. 99) yields that for

$$S_t = k_2 + a_1 S_{t-1} + a_2 S_{t-2} + a_3 S_{t-3} + \varepsilon_{2t} \tag{17}$$

then

$$\hat{E}_t \sum_{j=1}^{\infty} b^j S_{t+j} = \frac{bk_2}{(1-b)(1-a_1 b-a_2 b^2-a_3 b^3)} + \frac{(a_1 b+a_2 b^2+a_3 b^3)S_t}{(1-a_1 b-a_2 b^2-a_3 b^3)}$$

$$+ \frac{(a_2 b+a_3 b^2)S_{t-1}}{(1-a_1 b -a_2 b^2-a_3 b^3)} + \frac{(a_3 b)S_{t-2}}{(1-a_1 b -a_2 b^2-a_3 b^3)} \tag{18}$$

where $\hat{E}_t(Y) \equiv E(Y|S_t,S_{t-1},S_{t-2},...)$ and $b \equiv 1/(1+\bar{r})$. Substituting (16) and (18) into (14) yields

$$B_t/P_t = A_o(1+\bar{r})^t + k_1 + \frac{(a_1b+a_2b^2+a_3b^3)S_t}{(1-a_1b-a_2b^2-a_3b^3)} + \frac{(a_2b+a_3b^2)S_{t-1}}{(1-a_1b-a_2b^2-a_3b^3)}$$

$$+ \frac{(a_3b)S_{t-2}}{(1-a_1b-a_2b^2-a_3b^3)} + \varepsilon_{1t} . \tag{19}$$

We estimated equations (17) and (19) jointly by nonlinear least squares for t = 1963 to 1984. Parameter estimates and standard errors are summarized in Table 3.

As in the less restrictive tests above, we note that there seems to be no role whatever for the "bubble" term; $\hat{A}_o$ is statistically insignificant, and, if anything, negative. The assumption that bond-holders rationally expected the debt to be paid back in present value terms fits the data better than the assumption that debt has simply accumulated with an ever-growing interest load. Moreover, for these parameter estimates, equation (19) has an R^2 of 0.53; that is, more than half of the observed variance in the market value of real government debt could be explained from a rational-expectations forecast of the discounted value of future surpluses. Note such high explanatory power is achieved despite the fact that all of the other parameters which characterize the dynamics of outstanding debt in equation (19) are tied down by the univariate process for surpluses (17), and such parameters appear in (19) only to the extent that they could characterize rational-expectations forecasts of future surpluses. The high R^2 is also achieved despite the omission of all of the additional variables besides past surpluses which would be used by agents to forecast future surpluses.

Of course, to achieve such a fit to the data, the regression is forced to fit strongly negative coefficients in the autoregressive process for surpluses at two- and three- year lags, as the values in Table 3 indicate. Bond-holders must assume that big deficits typically only last a year or two, and will later be balanced out with surpluses. There is indeed moderate support for this position even in a completely unconstrained OLS estimate of the latter regression:

$$S_t = -0.30 + 0.62 \, S_{t-1} - 0.30 \, S_{t-2} - 0.20 \, S_{t-3}$$

$$(3.51) \quad (0.23) \quad\quad (0.28) \quad\quad (0.27)$$

The restricted rational-expectations estimates of Table 3 simply exaggerate this feature in the data. Overall, then, the present value hypothesis seems to hold up quite well.

5. Conclusions.

In this paper we have examined the proposition that in order to be able to issue interest-bearing debt, a government must promise to balance its budget in expected present-value terms. We suggested a battery of empirical tests of this proposition, some of which are quite robust with respect to assumptions about the dynamics of variables which are seen by agents but not the econometrician, and others which are highly restrictive. The conclusion from all our tests, however, is the same— the proposition that the government must promise creditors that it will balance the budget in expected present-value terms seems largely consistent with postwar U.S. data.

This result might seem surprising since the official budget accounts register essentially uninterrupted deficits for the United States from 1960 to 1981. However, the real value of government debt held by the public actually

fell during this period, indicating that the continuing reported deficits grosssly misstated the true fiscal posture of the government. We suggested an alternative measure of the government deficit which takes into account revenues from monetization and capital gains on gold but excludes interest payments. From the time-series properties of this adjusted deficit series, one can construct a rational-expectations forecast of the present value of future government budget surpluses. Such a forecast series can account for 53% of the observed variance of real government debt under the assumption that the government budget must be balanced in present-value terms.

If our conclusion on the limitations of government borrowing is correct, then the prevailing sentiment in Washington that current deficits can continue forever is wrong; the adjusted deficit series must soon turn to surplus. One policy change which could turn the adjusted series to surplus would be a resurgence of money growth.

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ton University, 1984.

FOOTNOTES

We are indebted to Charles Engel, Ron Michener, and John Taylor for help-
ful comments, and to Michael Cox for generously providing us with his data.

[1]If government taxes (T_t), spending (G_t) and debt (D_t) are related by

$$D_t + \sum_{j=1}^{\infty} (1+r)^{-j} G_{t+j} = \sum_{j=1}^{\infty} (1+r)^{-j} T_{t+j}$$

then any proposal which changes taxes but leaves spending the same must be such
that the right-hand side of the above expression remains constant. Thus, per-
manent disposable income defined by $\sum_{j=1}^{\infty} (1+r)^{-j} (W_{t+j} L_{t+j} - T_{t+j})$ would be com-
pletely unaffected by any such tax policy. See for example Barro (1984a, pp.
380-381) or Blanchard (1985) for additional details.

[2]The more general case of a deficit financed by a mixture of money and
different-term bonds is analyzed formally below.

[3]To take a simple example, let output $Q(t)$ grow at the rate q ($Q(t)$ =
$Q_o e^{qt}$), and let spending $gQ(t)$ and taxes $xQ(t)$ be fixed multiples of GNP. If
r is the cost of borrowing and $D(t)$ is the debt, then debt accumulates accord-
ing to

$$dD/dt = (g-x)Q(t) + rD(t)$$

implying

$$D(t) = \{D_o + tQ_o(g-x)\}e^{rt} \qquad \text{if } q = r$$
$$= \{(g-x)/(q-r)\}Q_o(e^{qt} - e^{rt}) + D_o e^{rt} \qquad \text{otherwise}$$

which explodes relative to $Q_o e^{qt}$ whenever $g > x$ and $q \leq r$.

[4]An example of this was provided by Carmichael (1982). See, however, the
discussion by Burbidge (1983, 1984) and Buiter and Carmichael (1984).

[5] Here we refer to the dollar flow of nominal interest payments made to the public divided by the aggregate price level, as distinct from the real user cost of capital which would include capital gains (i,e., use the real interest rate). We use the former interpretation because we are interested in the stock of outstanding government debt, for which purpose the correct expression is the cash-flow identity (4) governing debt accumulation.

[6] To take a simple example, suppose that in year 1 spending exceeds taxes by $1, with the shortfall made up by $1 issue of new agency debt. In all subsequent years, interest spending is $r and taxes are correspondingly higher by $r. In the accounts as actually reported, this policy would be associated with a $1 deficit in year 1 and no surplus in subsequent years. In our proposed measure, by contrast, the deficit is zero in all years: i.e., agency debt has no effect on the present value calculation for Treasury debt, as it should not in this case.

[7] The Export-Import Bank was also originally off-budget, though the accounts were later revised to reflect its current on-budget status.

[8] This stationarity test also forms the basis for Diba and Grossman's (1984) investigation of gold price behavior.

[9] Flood and Garber rightly note in their footnote 18 on page 754 that caution must be exercised in interpreting the usual t-test in this application. A related issue of course arises with our Dickey-Fuller tests; if (10) fails, the nonstationary root is not unity as the Dickey-Fuller tests assume but rather $(1+\bar{r})$.

[10] The second assumption in particular is admittedly unrealistic. Indeed, it can be shown that expectations <u>must</u> be based on additional information besides S_t (namely, on the exogenous shocks to which the endogenous policy variable S_t

responds) if equation (14) is to hold, because the forecast errors $(E_t - E_{t-1})S_{t+j}$ satisfying (14) cannot be fundamental for S_t. It nevertheless seems of interest to see how good an approximation one gets to the data by ignoring this difference between the true forecasts of creditors $(E_t S_{t+j})$ and our econometric forecasts $(\hat{E}_t S_{t+j})$ based on a univariate autoregression for S_t.

Table 1

Illustrative summary of adjustments to officially reported surplus for fiscal year 1974 and for debt at end of fiscal year 1974, in millions of current dollars

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SURPLUS:

Officially reported surplus	-3,460
Plus: interest	+28,072
Minus: interest paid to government trust funds	-6,583
Minus: deposit of Fed earnings	-4,845
Plus: change in agency securities	+903
Equals: Measure of surplus in which interest payments are excluded	+14,087
Plus: net capital gains on gold stock	+3,250
Plus: money seignorage	+11,331
Equals: true surplus (current dollars)	+28,668
Divided by: consumer price index	÷1,469
Equals: true surplus (1967 dollars)	+19,515

DEBT:

Officially reported debt	474,235
Minus: investments in government accounts	-140,194
Equals: par value of Treasury debt	334,041
Multiplied by: market-par ratio	0.951
Equals: market value of Treasury debt	317,740
Minus: currency in circulation	-73,833
Minus: member bank reserves	-30,086
Equals: market value of net interest-bearing debt held by public	213,821

Table 1 (continued)

Minus: Treasury operating balance   -9,159

Minus: market value of gold holdings   -39,951

Equals: adjusted government debt (current dollars)   164,711

Divided by: consumer price index   ÷1,469

Equals: adjusted government debt (1967 dollars)   112,124

Table 2

Adjusted values for surplus and debt, millions of 1967 dollars

Fiscal year	Adjusted surplus for fiscal year	Adjusted debt for end of fiscal year
1960	+8,533	167,954
1961	+921	170,826
1962	+2,117	176,187
1963	+3,697	177,944
1964	+3,968	176,036
1965	+9,604	173,319
1966	+11,514	160,103
1967	+7,997	157,441
1968	-3,262	162,227
1969	+14,142	144,302
1970	+6,529	136,877
1971	-3,631	145,977
1972	+1,812	151,836
1973	+13,451	136,272
1974	+19,515	112,124
1975	-13,152	134,673
1976*	-35,480	185,230
1977	-4,162	191,707
1978	+3,022	183,956
1979	+28,211	148,641
1980	+20,506	121,432
1981	-27,747	145,895
1982	-22,147	209,445
1983	-38,336	265,164
1984	-28,675	303,205

*Data for fiscal year 1976 include the transition quarter (July 1, 1976 through September 30, 1976).

Table 3

Estimates of parameters in equations (17) and (19)

Parameter	Estimate	(Standard error)
A_o	-61.52	(58.20)
k_1	241.51	(68.87)
k_2	0.90	(3.83)
a_1	0.15	(0.19)
a_2	-0.47	(0.22)
a_3	-0.51	(0.20)