On the limiting performance of broadcast algorithms over unidimensional ad-hoc radio networks

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Abstract— **Broadcast mechanisms are widely used in self–organizing wireless networks, for management, control and data exchange purposes. In general, broadcast algorithms are required to provide a *rapid*, *reliable* and *energy–efficient* way to diffuse information over a network of randomly distributed nodes.**

In this paper, we address such issues in the context of uni–dimensional networks, with nodes distributed according to a 1-dimension *inhomogeneous* Poisson process. We statistically derive the propagation dynamic of the optimum broadcast algorithm defined over the Minimum Connected Dominating Set (MCDS) of nodes. Hence, we derive the limiting broadcast performance over uni–dimensional networks, in terms of some useful measures, thus providing common reference values for comparing the effectiveness of different broadcast algorithms.

Keywords— **Ad hoc networks, broadcast, Minimum Connected Dominating Set, linear networks, inhomogeneous Poisson distribution**

I. Introduction

Broadcasting is a fundamental service for multi–hop wireless ad hoc networks, since it is commonly used to propagate user and control information over the network. For example, in the context of the car–networks [1], [2], broadcast may be used to signal hazards to the upcoming vehicles or, in a factory, to propagate sensors messages, in a multi–hop fashion, to some monitoring nodes scattered over the area. Furthermore, many routing algorithms for ad hoc networks rely on an efficient broadcast mechanism for path discovering and routing–table dissemination [3], [4]. It is then clear that the design of efficient broadcast mechanisms is an issue of primary importance for self–organizing wireless networks.

An efficient broadcast protocol shall minimize the number of retransmissions, preserving connectivity. Under ideal hypothesis (perfect packet reception within the transmission radius R, unloaded network, static network topology), this goal can be achieved by enabling only nodes in the *Minimum Connected Dominating Set* (MCDS) to relay the broadcast message. The MCDS is, in fact, defined as the subset of connected nodes with minimum cardinality and such that each node in the network is connected to a node in the subset.

The problem has been widely investigated for bi–dimensional networks, where nodes are randomly scattered over a plane [5]. The complexity of that scenario, however, makes the theoretical analysis rather difficult and, hence, most of the studies are based on computer simulations [6].

On the contrary, theoretical analysis turns out to be feasible for uni–dimensional networks, as in the case of Car-Networks [7], [1]. For a linear disposition of nodes, the MCDS can be recursively obtained by starting from the broadcast source and including in the MCDS, step by step, the farthest node within the coverage range of the previously inserted node. It is then clear that, enabling only nodes in the MCDS to relay broadcast messages, we guarantee optimal performance, i.e., maximum broadcast propagation speed along the network, minimum number of retransmissions, and collisions avoidance. Such a protocol will be referred to as MCDS-broadcast.

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The implementation of such a protocol requires each node to know the exact positions and status of surrounding nodes. The construction of an MCDS is, then, related to the connectivity properties of the network. Results on connectivity of linear networks can be found in [8], [9], [10], where only homogeneous disposition of nodes and asymptotic connection issues are considered. In particular, a closed form for the probability that two nodes at distance x are asymptotically connected (without any limitation on the number of needed hops) is derived. Such an index gives a reference value to test the *robustness* of any other broadcast algorithm, giving an upper bound to the maximum distance that a broadcast message can propagate over, and to the number of nodes that can be reached by the message. However, it does not capture the *dynamic* of message propagation and, thus, it does not provide any reference bound for the *speed* of the broadcast diffusion. A description of the message propagation, hop by hop, can be found in [11]. However, that result has been obtained under the assumption of homogeneous Poisson distribution of nodes only.

The aim of this work is to determine the performance of the MCDS–broadcast over ad hoc wireless networks with *inhomogeneous* and linear disposition of nodes. We derive a recursive and integral expression that gives a statistical characterization of the broadcast propagation dynamic. As a marginal result, we re–obtain the asymptotical results found in [9]. Furthermore, we derive reference values for some suitable performance measures, which can be used to compare the effectiveness of different broadcast strategies.

The remaining of the paper is organized as follows. In Section II, definitions and mathematical models are given. In Section III we obtain probabilistic functions describing the dynamic of the broadcast diffusion. In Section IV we derive some related functions that can be used as performance indexes. These results are graphically shown in Section V. Finally, conclusions are given in Section VI.

II. Mathematical model

We consider a one–dimensional ad hoc network, where nodes are deployed following an inhomogeneous Poisson process with intensity $\lambda(x)$. Therefore, the number of nodes in an interval $[a, b)$ is a Poisson–distributed random variable, denoted by $\mathcal{N}[a, b)$, with probability mass function given by

$$P\left[\mathcal{N}[a,b)=k\right]=\frac{\bar{\lambda}(a,b)^k}{k!}e^{-\bar{\lambda}(a,b)}\ ;$$

where the value $\bar{\lambda}(a, b)$ is given by

$$\bar{\lambda}(a,b)=\int_a^b \lambda(\xi)d\xi\,.$$

Since the Poisson process is not homogeneous, the inter–arrival space statistic depends on the starting point. Hence, starting from a generic point x, the cumulative distribution function (cdf) of the distance d to the next node in the network, $F_d(a|x)$, is given by

$$F_d(a|x) = P[\mathcal{N}(x, x+a] > 0] = 1 - e^{-\bar{\lambda}(x,x+a)} \ .$$

The derivative of $F_d(a|x)$ with respect to a gives the probability density function (pdf) for d:

$$f_d(a|x) = \lambda(x+a) e^{-\bar{\lambda}(x,x+a)} \ .$$

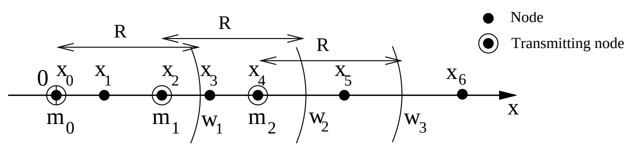

Fig. 1. Example of broadcast propagation.

We suppose each node is associated a transmission radius R, so that all and only nodes within a distance R from the transmitter receive the message. This assumption models a channel where a deterministic power–attenuation law is considered, only gaussian noise is taken into account as impairment, and successful packet reception is a step function of SNR. Let w_k denote the k–th broadcast wavefront, i.e., the maximum distance from the broadcast source node, reached by the k–th retransmission of the broadcast message (*rebroadcast* in the following). Furthermore, let m_k denote the position of the node that performs the k–th rebroadcast, so that $w_k = m_k + R$, as shown in Figure 1. Hence, m_k is the position of the k–th node of the MCDS rooted at the broadcast source. Clearly, the maximum number of rebroadcasting a message can undergo is equal to the size S of the MCDS, defined as the number of nodes that belong to the MCDS. Therefore, the k–th wavefront can exist only whether $S \geq k$.

III. BROADCAST PROPAGATION DYNAMIC

The broadcast propagation dynamic can be statistically described by the family $\mathcal{F}$ of probability density functions $\{f_k(x_k)\}_{k=1,2,\ldots}$, where $f_k(x_k)$ is defined as:

$$f_k(x_k) = \left(\frac{d}{da} P[w_k \leq a | S \geq k] \right) \Big|_{a=x_k} \ .$$

It may be worth remarking that the pdf $f_k(\cdot)$ for the generic k–th wavefront is conditioned on $S \geq k$, i.e., on the existence of the k-th wavefront. The following theorem defines a (not closed) form to derive such functions.

Theorem 1 *The family $\mathcal{F}$ of probability density functions, which describe the dynamic of the broadcast propagation along the network, can be recursively obtained as follows:*

$$f_1(x_1) = \delta(x_1 - R), \qquad \text{for } k = 1;$$
$$f_k(x_k) = \frac{P_{k-1}}{P_k} \lambda(x_k - R) \int_{x_k - R}^{x_k} e^{-\bar{\lambda}(x_k - R, x_{k-1})} f_{k-1}(x_{k-1}) dx_{k-1}, \qquad \text{for } k = 2, 3, \ldots, \quad (1)$$

where, for each k, P_k denotes the probability that $S \geq k$. For $k \geq 2$, the probability P_k can, in turn, be recursively derived as

$$P_k = P_{k-1} \int \lambda(x_k - R) \int_{x_k - R}^{x_k} e^{-\bar{\lambda}(x_k - R, x_{k-1})} f_{k-1}(x_{k-1}) dx_{k-1} dx_k$$

while, for $k = 1$, we have $P_1 = 1$.

Proof: Since we assume the MCDS contains at least the broadcast source node, the first transmission will reach a distance R from the source, so that $w_1 = R$ and $f_1(x_1) = \delta(x_1 - R)$.

For $k > 1$, the statistic of the distance reached by the k–th rebroadcast depends on w_{k-1} and w_{k-2}. Under the condition that $S \geq k$, it is easy to realize that m_k satisfies the following conditions: (a) $w_{k-2} < m_k \leq w_{k-1}$, (b) $\mathcal{N}(m_k, w_{k-1}] = 0$. Condition (a) means that m_k is reached, for the first time, by the $(k-1)$–th rebroadcast. Condition (b) means that m_k is the farthest node within the transmission range of m_{k-1}.

For easy of notation, throughout following we make use of some notation shortcuts for the events and functions that recur more often. In particular, by writing W_i we will intend the events $\{w_{k-i} = x_{k-i}\}$, while S_h will be used to denote the condition $S \geq h$.

Given that w_{k-2} and w_{k-1} wavefronts reach the distance x_{k-2} and x_{k-1}, respectively, and that $S \geq k$, i.e., the $k-1$–th rebroadcasting reaches new nodes, then the probability that m_k is greater than a is given by

$$P[m_k > a | W_1, W_2, S_k] = \quad (2)$$
$$= \begin{cases} 1, & a \leq x_{k-2} \ ; \\ P[\mathcal{N}(a, x_{k-1}) > 0 | W_1, W_2, S_k], & a \in (x_{k-2}, x_{k-1}] \ ; \\ 0, & a > x_{k-1} \ . \end{cases}$$

Let us focus on the expression obtained for $a \in (x_{k-2}, x_{k-1}]$, i.e.,

$$P[m_k > a | W_1, W_2, S_k] = P[\mathcal{N}(a, x_{k-1}) > 0 | W_1, W_2, S_k]. \quad (3)$$

Since $S \geq k$ implies $S \geq k-1$, we can write

$$P[\mathcal{N}(a, x_{k-1}) > 0 | W_1, W_2, S_k] = \quad (4)$$
$$= P[\mathcal{N}(a, x_{k-1}) > 0 | S_k, S_{k-1}, W_1, W_2] \ .$$

Applying Bayes rule to the probability on the righthand side of (4), we get

$$P[\mathcal{N}(a, x_{k-1}) > 0 | S_k, S_{k-1}, W_1, W_2] = \quad (5)$$
$$= \frac{P[\mathcal{N}(a, x_{k-1}) > 0, S_k | W_1, W_2, S_{k-1}]}{P[S_k | S_{k-1}, W_1, W_2]} \ .$$

Given W_1 and W_2, the condition $S \geq k$ can be removed from the numerator of the fraction in (5), since it is implicit in $\mathcal{N}(a, x_{k-1}) > 0$. Hence, (3) becomes

$$P[m_k > a | W_1, W_2, S_k] = \frac{1 - e^{-\bar{\lambda}(a, x_{k-1})}}{P[S_k | S_{k-1}, W_1, W_2]} \ . \quad (6)$$

The pdf of m_k, conditioned to W_1, W_2 and S_k, can be easily derived from (6). For $a \in (x_{k-2}, x_{k-1}]$ we get

$$f_{m_k}(a | W_1, W_2, S_k) = -\frac{d}{da} P[m_k > a | W_1, W_2, S_k]$$
$$= \frac{\lambda(a) e^{-\bar{\lambda}(a, x_{k-1})}}{P[S_k | S_{k-1}, W_1, W_2]} \ , \quad (7)$$

while, for $a \notin (x_{k-2}, x_{k-1}]$, the pdf is zero.

Recalling that $w_k = m_k + R$, it immediately follows

$$f_{w_k}(x_k | W_1, W_2, S_k) = \frac{\lambda(x_k - R) e^{-\bar{\lambda}(x_k - R, x_{k-1})}}{P[S_k | S_{k-1}, W_1, W_2]} \ , \quad (8)$$

for $x_k \in (x_{k-2} + R, x_{k-1} + R]$, and 0 otherwise.
In order to remove the conditioning on w_{k-1} and w_{k-2}, we apply the marginal rule. We first multiply both sides of (8) for $f(W_1, W_2 | S_k)$, which denotes the joint pdf of w_{k-1} and w_{k-2} conditioned to $S \geq k$. Hence, integrating in dx_{k-2} and dx_{k-1} we obtain

$$f_k(x_k) = \int \int f_{w_k}(x_k | W_1, W_2, S_k) f(W_1, W_2 | S_k) dx_{k-1} dx_{k-2}$$
$$= \lambda(x_k - R) \int_{x_k - R}^{x_k} e^{-\bar{\lambda}(x_k - R, x_{k-1})} \cdot \quad (9)$$
$$\cdot \int_{x_k - R}^{x_{k-1}} \frac{f(W_1, W_2 | S_k)}{P[S_k | W_1, W_2, S_{k-1}]} dx_{k-1} dx_{k-2} \ .$$

Then, applying the Bayes rule to the probability that appears at the denominator of the inner integral expression, after some algebra we get

$$P[S_k|W_1,W_2,S_{k-1}] = \frac{f(W_1,W_2|S_k)P_k}{f(W_1,W_2|S_{k-1})P_{k-1}}\,, \qquad (10)$$

where $P_k = P[S \geq k]$. Putting (10) into (9) we finally get

$$\begin{aligned} f_k(x_k) &= \frac{P_{k-1}}{P_k}\lambda(x_k-R)\int_{x_k-R}^{x_k} e^{-\bar{\lambda}(x_k-R,x_{k-1})}\cdot \qquad (11)\\ &\cdot \int_{x_k-R}^{x_{k-1}} f(W_1,W_2|S_{k-1})dx_{k-1}dx_{k-2}\\ &= \frac{P_{k-1}}{P_k}\lambda(x_k-R)\int_{x_k-R}^{x_k} e^{-\bar{\lambda}(x_k-R,x_{k-1})} f_{k-1}(x_{k-1})dx_{k-1}, \end{aligned}$$

which proves the first part of the theorem.

The second part of the theorem follows immediately applying the normalization condition to each the pdf $f_k(x_k)$.

For numerical calculation, we note that, starting from (f_1, P_1) it is possible to calculate (f_2, P_2) and so on. ■

Theorem 1 defines a recursive way to determine the complete statistic description of the distance reached by each rebroadcasting, under the condition that the broadcast propagation has not stopped before. This statistical characterization of the broadcast dynamic opens the way to the definition and evaluation of a number of performance measures. In the next section, we introduce some possible performance indexes that can be derived from the family of functions $\mathcal{F}$.

IV. Performance Measures

In general, the performance of a broadcast algorithm can be evaluated with respect to some quality of service indexes, which depend on the specific scenario considered. For instance, when broadcasting is used to propagate network topology information, then the focus is on reliability, and control traffic generated. On the other hand, when the information carried by the broadcast message is subject to strict delivery time constraints, as in the case of propagation of alarm or hazard messages, the most relevant measures are the propagation delay and the maximum distance reached by the message within a given time period.

Let us focus on a target node a, placed at a distance x_a from the broadcast source. We start deriving $c_k(x_a)$, for node a, defined as the probability that node a is reached *for the first time* at k–th rebroadcast. Notice that, this metric is not conditioned on the existence of the k–th rebroadcast. Hence, in order for node a to be reached at the k-th rebroadcast, the following conditions have to hold: a) $S \geq k$, b) $\{w_{k-1} < x_a, w_k \geq x_a\}$; so that

$$c_k(x_a) = P[w_{k-1} < x_a, w_k \geq x_a|S_k]P[S_k]. \qquad (12)$$

This function can be obtained by opportunely integrating the conditioned joint probability density function of w_k and w_{k-1}, as follows

$$c_k(x_a) = P_k\int_{x_a}^{x_a+R}\int_{x_k-R}^{x_a} f(W_0,W_1|S_k)dx_kdx_{k-1}\,, \qquad (13)$$

where, for easy of notation, we have resorted again to the notation shortcuts introduced before. The integration limits of (13) are obtained by the intersection of the intervals defined by (12) and the regions where the integrand function is not zero. The joint probability density function, in turn, can be derived as stated by the following corollary.

Corollary 1 *The function $f_{w_k,w_{k-1}}(x_k,x_{k-1}|S_k)$ is given by*

$$\begin{aligned} f_{w_k,w_{k-1}}(x_k,x_{k-1}|S_k) &= \frac{P_{k-1}}{P_k}\int\lambda(x_k-R)e^{-\bar{\lambda}(x_k-R,x_{k-1})}\cdot\\ &\cdot f_{w_{k-1},w_{k-2}}(x_{k-1},x_{k-2}|S_{k-1})dx_{k-2}\,; \qquad (14) \end{aligned}$$

where the recursion starts from $f_{w_1,w_0}(x_1,x_0|S_0) = \delta(x_1 - R, x_0)$.

Proof: The initial condition directly derives by the assumption that the MCDS contains at least the source broadcast node, which is placed in position $x_0 = 0$.
By using the marginal rule, we can express the conditioned joint probability as follows:

$$f(W_0,W_1|S_k) = \int f(W_0,W_1,W_2|S_k)dx_{k-2}\,. \qquad (15)$$

Applying the Bayes rule, (15) can be written as

$$f(W_0,W_1|S_k) = \int f(W_0|W_1,W_2,S_k)f(W_1,W_2|S_k)dx_{k-2}\,. \qquad (16)$$

Finally, replacing (8) into (16) and, hence, using (10), the proof is completed. ■

From $c_k(x_a)$, we can easily derive the so–called k–hop connectivity, $C_k(x_a)$, which is defined as the probability that node a, placed at a distance x_a from the broadcast source, is reached by the message within no more than k rebroadcasts. In other terms, we have

$$C_k(x_a) = P[\exists h \leq k : S \geq h, w_h > x_a] = \sum_{h=1}^{k} c_h(x_a)\,. \qquad (17)$$

Notice that, this probability measure permits to determine the probability that a node will receive the broadcast message within a limited time delay.

In [9], a closed form for $C_\infty(a)$ for a homogeneous disposition of nodes is derived. This result can be numerically obtained by (17) for $k \gg$. Furthermore, by using (17), it is easy to prove that whenever $x_a < \lceil \frac{k}{2} \rceil R$, we have $C_k(x_a) = C_h(x_a)$ for each $h \geq k$, so that

$$\lim_{k\to\infty} C_k(x_a) = C_h(x_a)\;;$$

with h satisfying $\lceil \frac{h}{2} \rceil > \frac{x_a}{R}$. Therefore, by using the recursive method described, we can obtain $C_\infty(x_a)$ after a finite number of iterations, also for inhomogeneous Poisson distributions.

Another measure of interest is the farthest distance reached by the broadcast message after k rebroadcasts, denoted as z_k, and the number of nodes reached. Let f_{z_k} be the pdf of z_k. From the definition of $C_k(x_a)$, it immediately follows

$$C_k(x_a) = P[z_k \geq a]\;,$$

and, consequently,

$$f_{z_k}(a) = \frac{d(1 - C_k(x_a))}{da}\,.$$

Therefore, the mean distance covered by k–hop rebroadcasts is given by

$$\overline{z_k} = \int_0^\infty a f_{z_k}(a)da = \int_0^\infty C_k(a)da.$$

The mean number of k–hop connected nodes, N_k^C, can be obtained taking the statistical expectation of the number of nodes in $(0, z_k]$, i.e.,

$$\begin{aligned} N_k^C &= E[\bar{\lambda}(z_k)]\\ &= \int_0^\infty \bar{\lambda}(x)f_{z_k}(x)dx\,. \end{aligned}$$

Recalling the definition of $\bar{\lambda}(\cdot)$ and of $f_{z_k}(\cdot)$, after some algebra, we obtain

$$N_k^C = \int_0^\infty C_k(x)\lambda(x)dx.$$

Clearly, the mean number N_k of nodes reached exactly at the k–th hop can be obtained as $N_k = N_k^C - N_{k-1}^C$.

Finally, another useful performance metric is the mean number N_{hop} of rebroadcasts (hops) before the broadcast propagation stops. Recalling that $P_k = P[S \geq k]$, it can be obtained by

$$N_{hop} = \sum_{k=1}^{\infty} P_k.$$

V. Limiting Performance

In this section we evaluate the performance measures defined in the previous section, for the MCDS–broadcast. As stated, MCDS–broadcast is the optimum broadcast strategy, under ideal conditions. Therefore, the results we provide in this section represent the limiting performance of any other broadcast algorithm over linear networks.

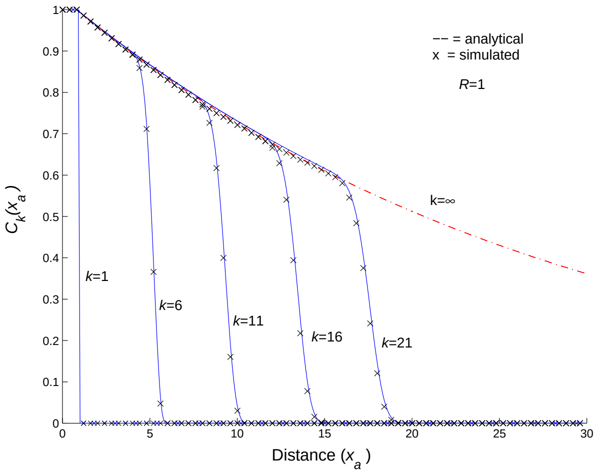

Fig. 2. $C_k(x_a)$ for $\lambda(x_a) = 5$.

Figure 2 shows $C_k(x_a)$, obtained for different k values and with a homogeneous Poisson distribution of nodes. As can be seen from the figure, continuous lines, obtained by the analytical method, perfectly interpolate the points ($\times$) obtained by simulations, thus validating the theoretical analysis. Note that, as a marginal result, we reobtain the curve for $C_\infty(a)$ that upper bounds all other curves.

In Figure 3 we show some results obtained for an inhomogeneous Poisson distribution of nodes. More in detail, we considered a distribution of nodes with a sinusoidal intensity $\lambda(x_a)$, as shown in the upper part of the figure. The middle and lower parts of the same figure show $C_k(a)$ and $f_k(a) \cdot P_k$ for various k, respectively. For $C_k(x_a)$, both theoretical (continuous line) and simulation (crossed line) results are plotted, showing a very good agreement that validates the proposed analysis also for inhomogeneous distributions of nodes. The plot of $f_k(a) \cdot P_k$ offers a visual perspective of the broadcast propagation along the linear network.

In Figure 4 we show P_k for the same inhomogeneous disposition of nodes as shown in the upper part of Figure 3 ($\circ$) and for an homogeneous distribution with constant intensity $\lambda(x_a) = 4$ nodes/R ($\times$). As expected, being the average node density the same, inhomogeneous distributions lead to a lower reliability.

Finally, Figure 5 shows the mean number N_{hop} of reached nodes versus the number of hops, for the same inhomogeneous disposition considered before.

VI. Conclusion

In this paper, we presented an analysis of the limiting performance of broadcast over linear ad–hoc networks, with nodes distributed according to an inhomogeneous Poisson process. The analysis was carried out by considering the connectivity properties of a Minimum Connected Dominating Set (MCDS) defined over the linear network. Results were used to derive the performance of the optimum broadcast protocol, which enables only nodes belonging to the MCDS rooted at the broadcast source to relaying the broadcast message. The so–called MCDS–broadcast is the optimum broadcast protocol, under ideal conditions, since it guarantees maximum reliability and speed for the broadcast diffusion.

In particular, we provided a recursive expression that describes the dynamic of broadcast propagation, that is, the probability that a node receives the broadcast message in function of the distance to the broadcast source and the number of rebroadcasts (hops). From this main result, many other useful indexes were deduced, such as the mean distance and number of nodes reached after a given number of hops, and the mean number of hops before the broadcast stops. Evaluating these metrics on the MCDS–broadcast, we provided the limiting performance for any other broadcast protocol.

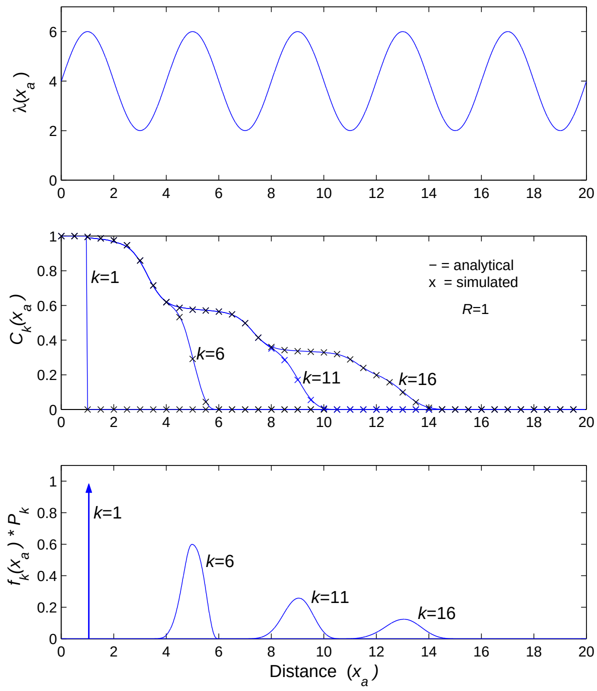

Fig. 3. $C_k(x_a)$ and $f_k(x_a) \cdot P_k$ for inhomogeneous λ.

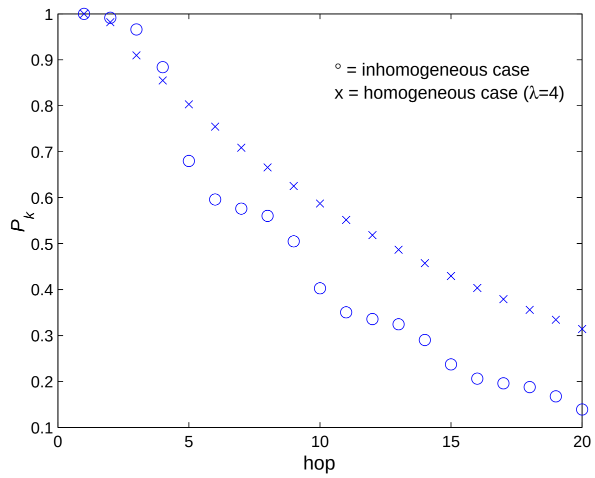

Fig. 4. P_k for inhomogeneous λ.

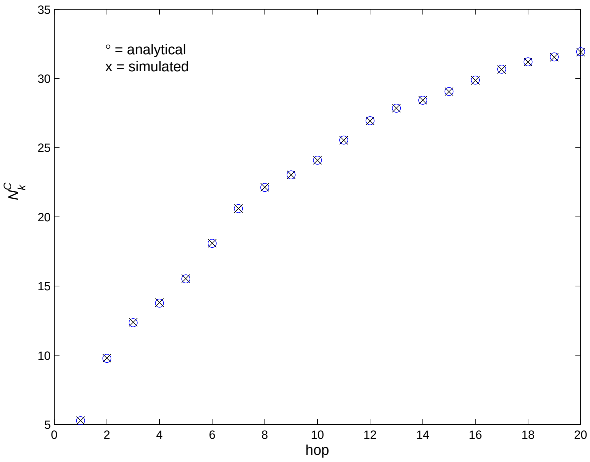

Fig. 5. Mean number of reached nodes.

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