Mathematical
Inequalities
& Applications
Volume 10, Number 3 (2007), 499–516

ON THE LOG–CONVEXITY OF TWO–PARAMETER HOMOGENEOUS FUNCTIONS

ZHEN-HANG YANG

Abstract. Suppose $f(x,y)$ is a positive homogeneous function defined on $\mathbb{U}(\subseteq \mathbb{R}_+ \times \mathbb{R}_+)$, then call $\left(\frac{f(a^p,b^p)}{f(a^q,b^q)}\right)^{\frac{1}{p-q}}$ two-parameter homogeneous function and denote by $\mathcal{H}_f(a,b;p,q)$. If $f(x,y)$ is third differentiable, then the log-convexity with respect to parameters p and q of $\mathcal{H}_f(p,q)$ depend on the sign of $J=(x-y)(xI)_x$, where $I=(\ln f)_{xy}$. As applications a group of chains of inequalities for homogeneous means are established, which generalize, strengthen and unify Tong-po Ling 's and Stolarsky's inequalities, and a reversed chain of inequalities for exponential mean (identic mean) is derived, which contains a reversed Stolarsky's inequality. Several estimations of lower and upper bounds of extended mean are presented.

Mathematics subject classification (2000): 26B25, 26D07, 26E60,26A48.
Key words and phrases: two-parameter homogeneous function, homogeneous mean, logarithmically convexity, chain of inequalities, estimation of lower and upper bounds.

REFERENCES

[1] B. C. CARLSON, *The logarithmic mean*, Amer. Math. Monthly, **79**, (1972), 615–618.
[2] P. CZINDER, Z. PÁLES, *An extension of the Hermite-Hadmard inequality and an application for Gini and stolarsky means*, Journal of Inequalities in Pure and Applied Mathematics, **5**, (2) (2004) [ONLINE] Available online at http://jipam.vu.edu.au/images/167_03_JIPAM/167_03.pdf.
[3] C. GINI, *Diuna formula comprensiva delle media*, Metron, **13**, (1938), 3–22.
[4] P. A. HÄSTÖ, *A montonicity property of ratios of symmetric homogeneous means*, Journal of Inequalities in Pure and Applied Mathematics, **3**, (5) (2002) [ONLINE] Available online at http://jipam-old.vu.edu.au/v3n5/013_02.pdf.
[5] GAO JIA, JINDE CAO, *A new upper bound of the logarithmic mean*, Journal of Inequalities in Pure and Applied Mathematics, **4**, (4) (2003) [ONLINE] Available online at http://jipam.vu.edu.au/images/088_03/088_03_www.pdf.
[6] E. B. LEACH, M. C. SHOLANDER, *Extended mean values*, Amer. Math. Monthly, **85**, (1978), 84–90.
[7] E. B. LEACH, M. C. SHOLANDER, *Extended mean values II*, J. Math. Anal. Appl., **92**, (1983), 207–223.
[8] T.-P. LIN, *The power mean and the logarithmic mean*, Amer. Math. Monthly, **81**, (1974), 879–883.
[9] Z. PÁLES, *Inequalities for differences of powers,* J. Math. Anal. Appl. , **131**, (1988), 271–281.
[10] F. QI, *Logarithmically convexities of the Extended Mean values*, Proc. Am. Math. Soc. **130**, (6) (2002), 1787–1796.
[11] F. QI, *Generalized weighted mean values with two parameters*, Proceedings of the Royal Society of London, Series, Mathematical, Physical and Engineering Sciences, **454**, (1978) (1998), 2723–2732.
[12] F. QI, *On a two-parameter family of nonhomogeneous mean values*, Tamkang Journal of Mathematics, **29**, (2) (1998), 155–163.
[13] F. QI, *Generalized Abstracted Mean Values*, Journal of Inequalities in Pure and Applied Mathematics, **1**, (1) (1999) [ONLINE] Available online at http://jipam-old.vu.edu.au/v1n1/013_99.pdf.
[14] K. B. STOLARSKY, *Generalizations of the Logarithmic Mean,* Math. Mag., **48**, (1975), 87–92.
[15] K. B. STOLARSKY, *The power and generalized Logarithmic Means,* Amer. Math. Monthly., **87**, (1980), 545–548.

[16] MING-BAO SUN, *Inequality for two-parameter mean of convex function*, Mathematics in Practice and Theory, **27**, (3) (1997), 193–197.
[17] ZHEN-HANG YANG, *Exponential mean and logarithmic Mean*, Mathematics in Practice and Theory, **4**, (1987), 76–78.
[18] ZHEN-HANG YANG, *Inequalities for power mean of convex function*, Mathematics in Practice and Theory, **20**, (1) (1990), 93–96.
[19] ZHEN-HANG YANG, *Inequalities for general mean of convex function,* Mathematics in Practice and Theory, **33**, (8) (2003), 136–141.
[20] ZHEN-HANG YANG, *Simple discriminance for convexity of homogeneous functions and applications*, Study in College Mathematics, **7**, (4) (2004), 14–19.