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Classification
Physics Abstracts
25.20 — 21.10K

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(*Reçu le 14 janvier 1980, accepté le 19 mars 1980*)

Résumé. — Un terme de l'amplitude de diffusion de photons, qui est absent des résultats existants, est calculé et discuté en détail.

Abstract. — A term, overlooked in the previous treatments of the photon scattering amplitude, is computed and the consequences of its structure are discussed.

Low energy photon scattering on a nucleus has been the subject of numerous investigations both experimental and theoretical. Some of them are listed in the last and most exhaustive theoretical paper [1] and in a review of the subject [2].

One aspect of the problem is to disentangle the Delbrück scattering from the nuclear and atomic processes. For this purpose it is necessary to compute with precision the nuclear amplitude (dominant in the MeV energy range) in term of nuclear structure constants. These appear in the power expansion of the amplitude, which was computed in reference [1] up to the square of the photon energy. One piece of the amplitude said in this work to vanish is consequently missing in the final result. It involves the centre of mass (C.M.) velocity $(\mathbf{k}' - \mathbf{k})/Amc$ of the recoiling nucleus (A, Z). For low energy photon scattering, this velocity is small and the corresponding amplitude indeed negligible *versus* the other terms, but for the very light nuclei ($A = 2, 3, 4$).

The interaction hamiltonian between the transverse photon field $(\mathbf{k}, \hat{\varepsilon})$ and a spin zero nucleus is written explicitly in terms of external and internal coordinates :

$$\begin{aligned}\mathcal{H}_i = & -\frac{\hat{\varepsilon}.\mathbf{P}}{Amc}\exp i\mathbf{k}.\mathbf{R}\sum e_j \exp i\mathbf{k}.\boldsymbol{\rho}_j \\ & - \exp i\mathbf{k}.\mathbf{R}\hat{\varepsilon}.\sum e_j/c\dot{\boldsymbol{\rho}}_j \exp i\mathbf{k}.\boldsymbol{\rho}_j \\ & + \hat{\varepsilon}.\hat{\varepsilon}' \exp i(\mathbf{k}-\mathbf{k}').\mathbf{R}\sum e_j^2/mc^2 \exp i(\mathbf{k}-\mathbf{k}').\boldsymbol{\rho}_j\end{aligned} \tag{1}$$

where $\mathbf{P}$ and $\mathbf{R}$ are the C.M. coordinates, $\boldsymbol{\rho}_j = \mathbf{r}_j - \mathbf{R}$ the j nucleon coordinate relative to the C.M. The constraint $\sum \boldsymbol{\rho}_j = 0$ reduces to $A - 1$ the number of independent coordinates.

The process whose amplitude we compute hereafter consists in :

i) The transition from the internal initial state $| 0 \rangle$ to the intermediate internal state $| n \rangle$ (respective energies : ω_0 and ω_n) through the interaction with the internal current, and from the initial centre of mass state $| P_0 = 0 \rangle$ to the intermediate state $| P \rangle$ through the external translation operator (second term $\mathcal{H}_i^{\mathrm{II}}$ in eq. (1)).

ii) The transition from state $| n \rangle$ to the final state $| 0 \rangle$ through the internal form factor and to the final C.M. state $| \mathbf{k} - \mathbf{k}' \rangle$ through the interaction with the external current (first term $\mathcal{H}_i^{\mathrm{I}}$ in eq. (1)).

The corresponding matrix element is :

$$M = \sum_{n \neq 0} \left\{ \frac{\langle 0 | \langle \mathbf{k} - \mathbf{k}' | \mathcal{H}_i^{\mathrm{I}}(\hat{\varepsilon}', -\mathbf{k}') | \mathbf{k} \rangle | n \rangle \langle n | \langle \mathbf{k} | \mathcal{H}_i^{\mathrm{II}}(\hat{\varepsilon}, \mathbf{k}) | P_0 = 0 \rangle | 0 \rangle}{\omega_0 - \omega_n + k - k^2/2\,Am} + \frac{\langle 0 | \langle \mathbf{k} - \mathbf{k}' | \mathcal{H}_i^{\mathrm{I}}(\hat{\varepsilon}, \mathbf{k}) | -\mathbf{k}' \rangle | n \rangle \langle n | \langle -\mathbf{k}' | \mathcal{H}_i^{\mathrm{II}}(\hat{\varepsilon}', -\mathbf{k}') | P_0 = 0 \rangle | 0 \rangle}{\omega_0 - \omega_n - k' - k'^2/2\,Am} \right\}.$$

The first non-zero term in an expansion of M in power series of k is obtained if :

— k, k', $k^2/2\,Am$, $k'^2/2\,Am$ are neglected in the denominators.

— The internal current interaction is taken in its dipole limit, $\mathbf{k}_j.\boldsymbol{\rho}_j = 0$.

— In the expansion of the internal form factor the second term only is kept : $i\mathbf{k}.\sum e_j \boldsymbol{\rho}_j$, the first one, $\sum e_j$, giving no contributions to transitions

$$| 0 \rangle \to | n \rangle \neq | 0 \rangle .$$

Using $\langle n | \dot{\boldsymbol{\rho}}_j | 0 \rangle = (1/i\hbar)(\omega_0 - \omega_n) \langle n | \boldsymbol{\rho}_j | 0 \rangle$ and closure over internal states, and denoting

$$\sum e_j \boldsymbol{\rho}_j = \mathbf{D} ,$$

the internal dipole momentum, one gets finally :

$$M = \frac{1}{Amc^2}\left(\frac{k}{\hbar c}\right)^2 \langle 0 | (\hat{\boldsymbol{\varepsilon}}'.\hat{\mathbf{k}})(\hat{\mathbf{k}}'.\mathbf{D})(\hat{\boldsymbol{\varepsilon}}.\mathbf{D}) + (\hat{\boldsymbol{\varepsilon}}.\hat{\mathbf{k}}')(\hat{\mathbf{k}}.\mathbf{D})(\hat{\boldsymbol{\varepsilon}}'.\mathbf{D}) | 0 \rangle .$$

(The condition $k^2/2\,Am \ll k$ enables to write $\mathbf{k} = k\hat{\mathbf{k}}$ and $\mathbf{k}' = k\hat{\mathbf{k}}'$.)

With the help of the transversality condition :

$$\hat{\boldsymbol{\varepsilon}}.\hat{\mathbf{k}} = \hat{\boldsymbol{\varepsilon}}'.\hat{\mathbf{k}}' = 0 ,$$

this expression is cast in the more convenient form :

$$M = \frac{1}{Amc^2}\left(\frac{k}{\hbar c}\right)^2 \langle 0 | - (\hat{\boldsymbol{\varepsilon}} \times \hat{\boldsymbol{\varepsilon}}')(\hat{\mathbf{k}} \times \hat{\mathbf{k}}')\,\mathbf{D}^2 + [(\hat{\boldsymbol{\varepsilon}} \times \hat{\boldsymbol{\varepsilon}}').\mathbf{D}]\,[(\hat{\mathbf{k}} \times \hat{\mathbf{k}}').\mathbf{D}] | 0 \rangle . \quad (2)$$

Among the four combinations of suitably chosen orthogonal polarization states of the incident and scattered photons, one only provides strictly non-vanishing matrix element, namely $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\varepsilon}}'$ coplanar with $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}'$. After averaging over spatial orientation, it yields

$$M = -\frac{2}{3}\sin^2\theta\left(\frac{k}{\hbar c}\right)^2 \frac{e^2}{Amc^2} \langle 0 | D^2/e^2 | 0 \rangle . \quad (3)$$

When $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\varepsilon}}'$ are both normal to $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}'$, the operator in eq. (2) vanishes. In the remaining two cases where one of the polarization vectors is coplanar with $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}'$ and the other one is normal to $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}'$, the operator in (2) reduces to the form (Oz is the normal to $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}'$) :

$$(aD_x + bD_y)\,D_z .$$

Its expectation value in a state of definite parity is zero. Consider now an object which, like a left or right-handed form, is not superposable by a continuous transformation to its image in the mirror (xoy). The operator written above changing its sign by this reflection, its expectation value is now non vanishing. Therefore the observation of the scattering of a photon with the particular combinations of polarization vectors should indicate a certain degree of handedness in the scattering object. Nothing prevents a nucleus to exhibit such a feature. However, to stabilize it in right or left-handed form requires either parity violating forces, or a symmetry breaking effect, like the one operating on large stereoisomers. It should be stressed that a parity violating force is not sufficient to lead to handedness.

As inferred from the small value of the recoiling nucleus velocity, the amplitude (3) is rather small. In the case of light nuclei ($A = 2, 3, 4$) whose ground states are almost spatially symmetric, Foldy has derived [3] the relation :

$$\langle D^2/e^2 \rangle = \frac{NZ}{A-1}\langle r^2 \rangle \quad (4)$$

where $\langle r^2 \rangle$ is the mean square radius of the nucleon distribution. Therefore :

$$M = -\frac{2}{3}\sin^2\theta\left(\frac{k}{\hbar c}\right)^2 \frac{e^2}{mc^2}\,\frac{NZ}{A(A-1)}\langle r^2 \rangle$$

which is of the order of 1 % of the Thomson amplitude at an energy k of a few MeV, and of same order as the remaining terms given in ref. [1].

For heavier nuclei, whose wave functions are not spatially symmetric, relation (4) does not necessarily hold. There are indications that $\langle D^2/e^2 \rangle$ increases with A more slowly than relation (4) indicates, namely as $A^{4/3}$ only [4]. The amplitude (3) is then completely negligible (this is also true for the part proportional to $\langle D^2 \rangle$ in the expression for the diamagnetic susceptibility given in reference [1]).

A final comment is in order here. In reference [1], the authors showed that the classical and the quantum-mechanical treatments of the scattering lead to the same result. Clearly, the amplitude computed here has no counterpart in the classical result. The reason is that the amplitude computed in this paper implies the transfer a finite momentum and energy from the electromagnetic field to the scattering object and this at a rate independent of the intensity of the incident field. That is properly a character of a quantized field.

The author wishes to thank the referee for his criticism which helped to improve this article.

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