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ON THE MAGNETOSPHERIC BEAT-FREQUENCY AND LENSE-THIRRING INTERPRETATIONS OF THE HORIZONTAL-BRANCH OSCILLATION IN THE Z SOURCES

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ABSTRACT

Three types of quasi-periodic oscillations (QPOs) have been discovered so far in the persistent emission of the most luminous neutron star low-mass X-ray binaries, the Z sources: ~10–60 Hz horizontal-branch and ~6–20 Hz normal/flaring-branch oscillations and ~200–1200 Hz kilohertz QPOs, which usually occur in pairs. Here we study the horizontal-branch oscillations and the two simultaneous kilohertz QPOs, which were discovered using the *Rossi X-Ray Timing Explorer*, comparing their properties in five Z sources with the predictions of the magnetospheric beat-frequency and Lense-Thirring precession models. We find that the variation of the horizontal-branch oscillation frequency with accretion rate predicted by the magnetospheric beat-frequency model for a purely dipolar stellar magnetic field and a radiation-pressure–dominated inner accretion disk is consistent with the observed variation. The model predicts a universal relation between the horizontal-branch oscillation, stellar spin, and upper kilohertz QPO frequencies that agrees with the data on five Z sources. The model implies that the neutron stars in the Z sources are near magnetic spin equilibrium, that their magnetic field strengths are $\sim 10^9$–10^{10} G, and that the critical fastness parameter for these sources is $\gtrsim 0.8$. If the frequency of the upper kilohertz QPO is an orbital frequency in the accretion disk, the magnetospheric beat-frequency model requires that a small fraction of the gas in the disk does not couple strongly to the stellar magnetic field at 3–4 stellar radii but instead drifts slowly inward in nearly circular orbits until it is within a few kilometers of the neutron star surface. The Lense-Thirring precession model is consistent with the observed magnitudes of the horizontal-branch oscillation frequencies only if the moments of inertia of the neutron stars in the Z sources are ~4–5 times larger than the largest values predicted by realistic neutron star equations of state. If instead the moments of inertia of neutron stars have the size expected and their spin frequencies in the Z sources are approximately equal to the frequency separation of the kilohertz QPOs, Lense-Thirring precession can account for the magnitudes of the horizontal-branch oscillation frequencies only if the fundamental frequency of the horizontal-branch oscillation is at least 4 times the precession frequency. We argue that the change in the slope of the correlation between the frequency of the horizontal-branch oscillation and the frequency of the upper kilohertz QPO, when the latter is greater than 850 Hz, is directly related to the varying frequency separation of the kilohertz QPOs.

Subject headings: accretion, accretion disks — binaries: close — X-rays: stars

1. INTRODUCTION

The discovery in the Z-type low-mass X-ray binaries (LMXBs) of ~10–60 Hz quasi-periodic oscillations (QPOs) with centroid frequencies that are positively correlated with the mass accretion rates (van der Klis et al. 1985; see van der Klis 1989 for a review) has led to a significant improvement in our understanding of such systems. These horizontal-branch oscillations (HBOs), which are named after the branch in X-ray color-color diagrams where they appear, were the first rapid-variability phenomena discovered in LMXBs and have played a key role in organizing the complex phenomenology of these sources that has emerged over the past decade (see, e.g., Hasinger & van der Klis 1989; van der Klis 1989).

Very soon after the discovery of the HBO, its centroid frequency was identified with the difference between the Keplerian frequency at the radius where the neutron star magnetosphere couples strongly to the gas in the accretion disk and the spin frequency of the neutron star (Alpar & Shaham 1985; Lamb et al. 1985; Shibazaki & Lamb 1987). This magnetospheric beat-frequency interpretation of the HBO was found to agree well with the observed properties of the HBOs, including the dependence of the HBO frequency on the X-ray count rate (Alpar & Shaham 1985; Lamb et al. 1985; Ghosh & Lamb 1992), the existence of correlated low-frequency noise (Lamb et al. 1985; Shibazaki & Lamb 1987), and the absence of any detectable QPO with

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a frequency equal to the Keplerian frequency at the magnetic coupling radius (Lamb 1988). The magnetospheric beat-frequency model predicted that the neutron stars in the Z sources have spin frequencies $\sim$200–350 Hz and magnetic field strengths $\sim 10^9$–10^{10} G (see Alpar & Shaham 1985; Ghosh & Lamb 1992; Wijnands et al. 1996), consistent with the hypothesis that these stars are the progenitors of the millisecond rotation-powered pulsars (Alpar & Shaham 1985; see Alpar et al. 1982; Radhakrishnan & Shrinivasan 1982). The neutron star properties inferred from the magnetospheric beat-frequency model have subsequently been shown to be consistent with the magnetic field strengths inferred from models of the X-ray spectra of the Z sources (Psaltis, Lamb, & Miller 1995; Psaltis & Lamb 1998) and with the 290–325 Hz spin frequencies inferred from the frequency separation of the two simultaneous QPOs with frequencies $\sim$1 kHz ("kilohertz QPOs") and from the high-frequency oscillations observed during type I X-ray bursts in other neutron star LMXBs (Strohmayer et al. 1996; Miller, Lamb, & Psaltis 1998b; Miller 1999). If the upper kilohertz QPO is an orbital frequency in the inner disk, the magnetospheric beat-frequency model of the HBO requires that a small fraction of the gas in the accretion disk must penetrate to radii smaller than the radius where it initially couples to the stellar magnetic field (van der Klis et al. 1997; see Miller et al. 1998b for a discussion), because observations show that the kilohertz QPOs are present at the same time as the HBO (see, e.g., Wijnands & van der Klis 1998). In addition to the HBOs, the magnetospheric beat-frequency model has been used to explain successfully similar QPOs observed in accretion-powered pulsars, where the neutron star spin frequency can be measured directly and the magnetic field strength can be estimated from the accretion torque, providing a stringent test of the model (see, e.g., Angelini, Stella, & Parmar 1989; Ghosh 1996; Finger, Wilson, & Harmon 1996).

Recently, Stella & Vietri (1998) have proposed an alternative HBO mechanism, motivated by concerns about whether orbiting gas can penetrate inside the magnetic coupling radius in the Z sources. In their model, the magnetic field of the neutron star plays no role in generating the HBO. Instead, the HBO observed in the Z sources and the power-spectral peaks with frequencies $\sim$20–60 Hz seen in some atoll sources (see, e.g., Ford & van der Klis 1998) are both generated by nodal (Lense-Thirring and classical) precession of a tilted ring of gas at a special radius in the inner disk. Stella & Vietri suggested that the nodal precession frequency of the ring is visible in X-rays because of the changes in the Doppler shift of radiation from blobs orbiting in the ring, changes in occultations by such blobs, or the changing aspect of the ring seen by an observer. Subsequently, Marković & Lamb (1998) studied the normal modes of the inner disk and showed that typically $\sim$10 high-frequency nodal precession modes are weakly damped. Nodal precession has also been proposed by Cui, Zhang, & Chen (1998; see also Ipser 1996) as an explanation for the QPOs observed in black hole candidates. If the HBO is generated by nodal precession at the same radius in the accretion disk where orbital motion generates the kilohertz QPO, as proposed by Stella & Vietri (1998), then the HBO frequency, the neutron star spin frequency, and the frequency of the upper kilohertz QPO should satisfy a specific relation. The shape of this relation was shown to be consistent with observations of the HBO and kilohertz QPO frequencies observed in the Z sources GX 17+2 and GX 5−1 (Stella 1997; Stella & Vietri 1997, 1998; Morsink & Stella 1999), although the predicted precession frequencies were found to be smaller than the observed HBO frequencies.

In this paper we use data on five Z sources obtained using the *Rossi X-Ray Timing Explorer* (*RXTE*) to investigate further the origin of the HBO. All of these data have been fully reported elsewhere. The sources we consider are GX 17+2 (Wijnands et al. 1997; Homan et al. 1998), GX 5−1 (Wijnands et al. 1998a), GX 340+0 (Jonker et al. 1998), Cyg X-2 (Wijnands et al. 1998b), and Sco X-1 (van der Klis et al. 1997). In all of these sources the HBO and two kilohertz QPOs have been observed simultaneously. In GX 349+2, the sixth originally identified Z source, no HBO has so far been detected simultaneously with the kilohertz QPOs (Kuulkers & van der Klis 1998; Zhang, Strohmayer, & Swank 1998). Therefore, we cannot include this source in the present study. We investigate the magnetospheric beat-frequency model in § 2 and the Lense-Thirring precession model in § 3, comparing their predictions with the available data. In § 4 we summarize our conclusions and their implications for the properties of the neutron stars in the Z sources. Finally, we characterize the correlations between the various frequencies in a model-independent way in the Appendix, in order to facilitate comparison of the present data with future data or other theoretical models.

2. THE MAGNETOSPHERIC BEAT-FREQUENCY INTERPRETATION

2.1. *Model Predictions and Comparison with Observations*

In the magnetospheric beat-frequency model of the HBO (Alpar & Shaham 1985; Lamb et al. 1985), the centroid frequency $\nu_{\rm HBO}$ of the HBO is identified with the beat between the Keplerian frequency $\nu_{{\rm K},m}$ at the radius r_m where the neutron star magnetic field couples strongly to the gas in the accretion disk and the spin frequency ν_s of the neutron star. The frequency of this beat is

$$\nu_{\rm MBF} = \nu_{{\rm K},m} - \nu_s \; . \tag{1}$$

Ghosh & Lamb (1992) computed the dependence of $\nu_{\rm MBF}$ on the stellar mass and magnetic moment and the accretion rate for a variety of simple models of the inner accretion disk. They found that if the coupling radius is in an asymptotic region of the disk, then

$$\nu_{\rm MBF} \simeq \nu_{{\rm K},0}\, M^{\gamma} \mu_{27}^{\beta} \left(\frac{\xi \dot{M}}{\dot{M}_{\rm E}}\right)^{\alpha} - \nu_s \; , \tag{2}$$

where μ_{27} is the magnetic moment of the neutron star in units of 10^{27} G cm^3, M is its gravitational mass in units of solar masses, $\dot{M}$ is the mass accretion rate, and $\dot{M}_{\rm E}$ is the Eddington critical mass accretion rate onto a neutron star of 10 km radius; the proportionality constant $\nu_{{\rm K},0}$ and the exponents α, β, and γ in equation (2) are different for different models of the inner accretion disk and are listed in Table 1. The dimensionless parameter ξ describes the fraction of the mass flux through the inner disk that couples to the stellar magnetic field at r_m and is introduced here to allow for the possibility that some of the gas in the disk does not couple to the stellar magnetic field at r_m but instead penetrates to smaller radii, as required if the upper kilohertz QPO is an orbital frequency in the inner disk (Miller et al. 1998b); ξ may depend on the mass accretion rate.

TABLE 1

SCALING OF THE KEPLERIAN FREQUENCY AT THE COUPLING RADIUS

Disk Model[a]	$\nu_{K,0}$ (Hz)	α	β	γ
1G......	430	0.38	−0.87	0.82
1R......	210	0.23	−0.77	0.70
2B......	80	0.72	−0.86	0.43
2S......	50	2.55	−1.20	−0.60

NOTE.—From Ghosh & Lamb 1992.

[a] 1G: Optically thick, gas-pressure–dominated (GPD) disk. 1R: Optically thick, radiation-pressure–dominated (RPD) disk. 2B: Two-temperature, optically thin GPD disk with Comptonized bremsstrahlung. 2S: Two-temperature, optically thin GPD disk with Comptonized soft photons (for references to these disk models see Ghosh & Lamb 1992).

In the magnetospheric beat-frequency model of the HBO, the steep dependence of ν_{HBO} on the mass accretion rate inferred from the *EXOSAT* data implies that ν_{MBF} is small compared to $\nu_{K,m}$ and hence that the spin frequencies ν_s of the neutron stars in the Z sources are very close to, but less than, $\nu_{K,m}$ (Alpar & Shaham 1985; Lamb et al. 1985; Ghosh & Lamb 1992). Stated differently, the magnetospheric beat-frequency model of the HBO requires that the neutron stars in the Z sources be near magnetic spin equilibrium. Indeed, the 200–350 Hz spin frequencies predicted by the model (see Ghosh & Lamb 1992) are much larger than the ≈20–50 Hz HBO frequencies, as required. The similarity of the Z-source spin frequencies predicted by the magnetospheric beat-frequency model to the spin frequencies inferred from the separation frequencies of the kilohertz QPOs in the Z sources lends further support to the model (Miller et al. 1998b) and to its implication that the neutron stars in the Z sources are near magnetic spin equilibrium (White & Zhang 1997; Psaltis & Lamb 1998). If they are, then their spin frequencies are given by (Ghosh & Lamb 1979, 1992)

$$\nu_s \simeq \omega_c \, \nu_{K,0} \, M^{\gamma} \mu_{27}^{\beta} \left[\frac{\langle (\xi \dot{M})^{\alpha} \rangle}{(\dot{M}_E)^{\alpha}} \right] , \quad (3)$$

where ω_c is the critical fastness parameter and the angle brackets indicate an average over a time interval equal to the timescale on which the accretion torque changes the spin.

Combining equations (2) and (3) and identifying ν_{HBO} with ν_{MBF}, we find

$$\frac{\nu_{HBO}}{\nu_s} + 1 = \frac{1}{\omega_c} \left[\frac{(\xi \dot{M})^{\alpha}}{\langle (\xi \dot{M})^{\alpha} \rangle} \right] . \quad (4)$$

Equation (4) shows that the magnetospheric beat-frequency model of the HBO predicts that $\nu_{HBO}/\nu_s + 1$ should be $\approx 1/\omega_c$ and hence only slightly larger than unity. The Z sources are thought to be accreting at near-Eddington accretion rates (see Lamb 1989; Hasinger & van der Klis 1989). The inner accretion disks in these sources are therefore expected to be radiation-pressure dominated, in which case $\alpha \approx 0.2$ (see Table 1). Hence, if ξ depends only weakly on the instantaneous accretion rate $\dot{M}$, then $\nu_{HBO}/\nu_s + 1$ will also depend only weakly on $\dot{M}$ and possibly also on the magnetic field and mass of the neutron star, through the dependence of ω_c on these quantities.

According to the magnetospheric beat-frequency model, the HBO frequency ν_{HBO} is related to the frequency ν_2 of the upper kilohertz QPO only indirectly, through the dependence of both frequencies on the mass accretion rate. In all sources in which kilohertz QPOs have so far been discovered, ν_2 increases with inferred mass accretion rate (see, e.g., van der Klis et al. 1996; Strohmayer et al. 1996; van der Klis 1998). Here we explore the consequences of the simple *Ansatz* $\nu_2 = \nu_0 (\dot{M}/\dot{M}_E)^{\lambda}$, where ν_0 and λ are constants that are specific to each source and may depend on the mass and magnetic field strength of the neutron star. This relation, with $\lambda \approx 1$, is consistent with kilohertz QPO observations of several atoll sources, provided that the observed count rate from an atoll source is proportional to the mass accretion rate (see, e.g., Strohmayer et al. 1996; Ford et al. 1997). In the Z sources, ν_2 is consistent with being $\simeq 1200$ Hz when they are accreting at near-Eddington rates, which implies $\nu_0 \approx 1200$ Hz, independent of the expected modest differences in the masses and magnetic field strengths of these neutron stars. Using this simple ansatz, equation (2) can be written

$$\nu_{HBO} + \nu_s = A_1 \, \nu_2^{\alpha/\lambda} , \quad (5)$$

where

$$A_1 \equiv \xi^{\alpha} \nu_{K,0} \, M^{\gamma} \mu_{27}^{\beta} \, \nu_0^{-\alpha/\lambda} , \quad (6)$$

and equation (4) becomes

$$\frac{\nu_{HBO}}{\nu_s} + 1 = A_2^{-1} \left(\frac{\nu_2}{\nu_0} \right)^{\alpha/\lambda} , \quad (7)$$

where

$$A_2 \equiv \omega_c \left[\frac{\langle (\xi \dot{M})^{\alpha} \rangle}{(\xi \dot{M}_E)^{\alpha}} \right] . \quad (8)$$

The inferred value of A_2 therefore provides an estimate of the critical fastness ω_c.

In order to test the relations (5) and (7) predicted by the magnetospheric beat-frequency model, simultaneous measurements of ν_{HBO}, ν_2, and ν_s are needed. The HBO and kilohertz QPO frequencies are directly observed, but oscillations at the neutron star spin frequency have not yet been detected in the persistent emission of any Z source. However, comparisons of the frequencies of the two simultaneous kilohertz QPOs observed in the persistent emission of some atoll sources with the frequencies of the nearly coherent oscillations observed in these sources during type I X-ray bursts indicate that the neutron star spin frequency is nearly equal to the frequency separation between the two kilohertz QPOs (Strohmayer et al. 1996, 1998; Miller et al. 1998b; Psaltis et al. 1998; Miller 1999). Hence, for GX 17+2, GX 5−1, GX 340+0, and Cyg X-2 we set the spin frequency equal to the average frequency separation of the kilohertz QPOs. In Sco X-1, the frequency separation of the kilohertz QPOs is consistent with being constant at the lowest inferred accretion rates but decreases at higher rates (van der Klis et al. 1997). Sco X-1 is thought to be accreting at near-Eddington mass accretion rates when the frequency separation of the kilohertz QPOs decreases, and it is therefore plausible that this decrease is related to the effects of radiation forces on the dynamics of

TABLE 2

BEST-FIT PARAMETERS FOR THE MAGNETOSPHERIC BEAT-FREQUENCY MODEL

Source	ν_s (Hz)[a]	Normalization (A_1)	Exponent (α/λ)
GX 17+2	293.5 ± 5.5	365 ± 1	0.21 ± 0.01
GX 5−1	312.5 ± 9.2	378 ± 3	0.16 ± 0.02
GX 340+0......	324.5 ± 9.3	389 ± 4	0.15 ± 0.03
Cyg X-2	324.4 ± 44.9	393 ± 1	0.17 ± 0.02
Sco X-1	304.7 ± 1.3	354 ± 1	0.06 ± 0.03

[a] Inferred from the frequency separation of the kilohertz QPOs (see text).

the accretion flow near the neutron star (see, however, Méndez et al. 1998; Psaltis et al. 1998). Hence, in plotting the Sco X-1 data, we set the spin frequency equal to the nearly constant frequency separation of its kilohertz QPOs at low inferred mass accretion rates. The spin frequencies we have adopted are listed in Table 2.

Equation (5) describes adequately ($\chi^2 \lesssim 1.5$ per degree of freedom) the dependence of the sum of the HBO and inferred spin frequencies on the frequency of the upper kilohertz QPO in the five Z sources in our sample, considered separately. Table 2 lists the best-fit parameters and their 1 σ errors, and Figure 1 compares the best-fit relations with the frequency data on each source. If $\lambda \approx 1$, the power-law index α for all sources except Sco X-1 is $\approx$0.2. This value is consistent with the expectation that the Z sources are accreting at near-Eddington rates and hence that the inner accretion disk is optically thick and radiation-pressure dominated (see Table 1). For Sco X-1, however, a significantly weaker dependence of the HBO frequency on $\dot{M}$ is required, if λ is independent of the mass accretion rate (but see the Appendix and Psaltis, Belloni, & van der Klis 1999).

Figure 2 shows the quantity $\nu_{\rm HBO}/\nu_s + 1$ plotted as a function of the upper kilohertz QPO frequency ν_2 for the five Z sources in our sample. The frequency data on all the sources

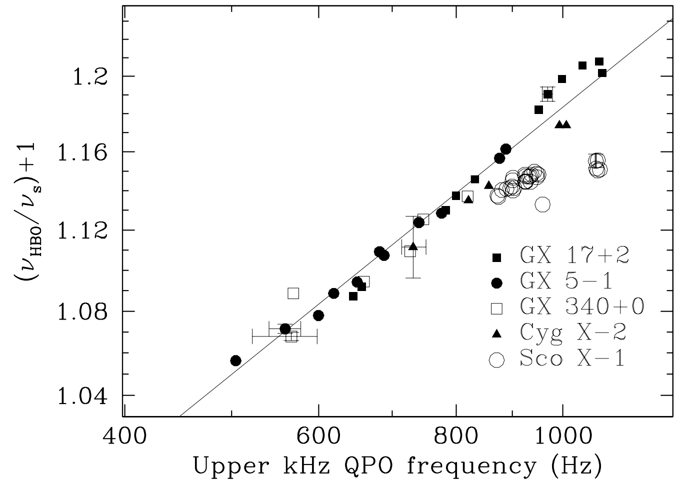

FIG. 2.—Correlation between $\nu_{\rm HBO}/\nu_s + 1$ and the frequency of the upper kilohertz QPO in five Z sources. The solid line is the best fit of the magnetospheric beat-frequency eq. (7) to the points with upper kilohertz QPO frequencies less than 850 Hz.

are consistent with a single, universal relation between $\nu_{\rm HBO}$, ν_s, and ν_2, as predicted by equation (7), when ν_2 is less than 850 Hz. This relation is shown as a solid line in Figure 2. Figure 3 shows the confidence contours for the power-law index α/λ and the coefficient A_2 in equation (7) obtained by fitting this relation to all the data with $\nu_2 < 850$ Hz. Assuming that ν_0 is $\approx$1200 Hz, the best-fit value of A_2 gives a lower bound on the critical fastness ω_c, because $\xi\dot{M}$ is expected to be a monotonically increasing function of $\dot{M}$ and hence $\langle(\xi\dot{M})^\alpha\rangle \leq (\xi\dot{M}_{\rm E})^\alpha$. If the magnetospheric beat-frequency model is the correct explanation of the HBO, then ω_c is $\gtrsim$0.8 for the magnetic field strengths and accretion rates of the Z sources.

When ν_2 is greater than 850 Hz, the HBO frequencies of GX 17+2 are up to 2% higher than predicted by extrapolating the universal relation that holds at lower frequencies,

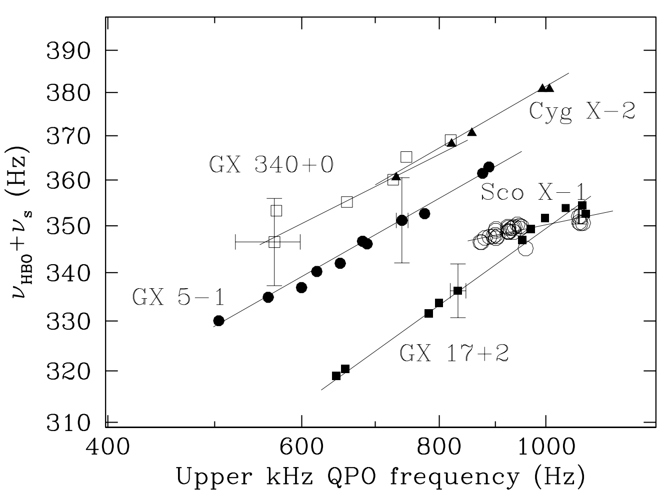

FIG. 1.—Correlations between $\nu_{\rm HBO} + \nu_s$, the sum of the HBO frequency and the inferred spin frequency, and the upper kilohertz QPO frequency in five Z sources. Only a single typical error bar is shown for each source, for clarity. The lines show the best fits of the magnetospheric beat-frequency model (eq. [5]) to the data. The error bars for the Sco X-1 points are smaller than the circles plotted. The error bars for the Cyg X-2 points are substantially larger than for the other points because of the large uncertainty in the Cyg X-2 spin frequency.

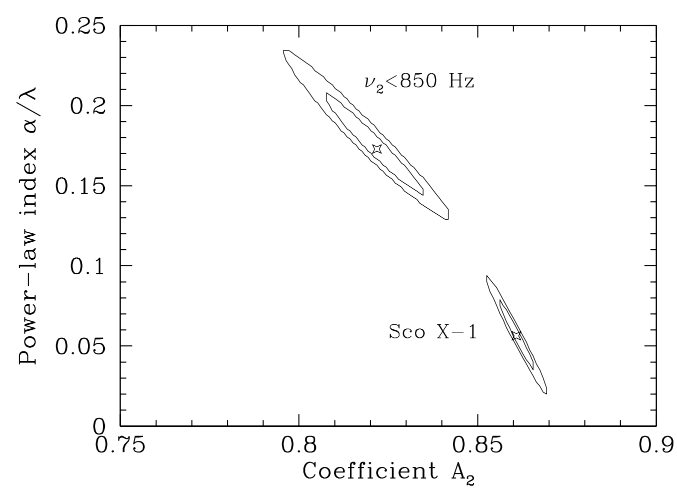

FIG. 3.—Confidence contours for the coefficient A_2 and the power-law index α/λ in the beat-frequency model (eq. [7]), obtained by fitting this relation to the Z-source frequency data with upper kilohertz QPO frequencies less than 850 Hz. The inner and outer contours show the 68% and 99% confidence limits, respectively. The star indicates the best-fit values. The best-fit relation is compared with the frequency data in Fig. 2. Within the beat-frequency model of the HBO, the value of A_2 is an upper bound on the critical fastness ω_c.

whereas those of Sco X-1 are as much as 5% lower. This indicates that there is at least one other important parameter that varies with ν_2. For example, the structure of the inner disk may change at high accretion rates, causing the exponent λ to vary from source to source. This conjecture cannot be tested without a specific model for the variation in λ, because if λ is chosen to reproduce the behavior of the data, equation (7) looses all predictive power.

The magnetospheric beat-frequency model of the HBO requires that the neutron stars in all the Z sources be near magnetic spin equilibrium. The tight, universal correlation between the HBO, spin, and upper kilohertz QPO frequencies in all the Z sources when $\nu_2 < 850$ Hz is explained by the model if $A_2 \nu_0^{\alpha/\lambda} = \omega_c \nu_0^{\alpha/\lambda} \langle (\xi \dot{M})^\alpha \rangle / (\xi \dot{M}_E)^\alpha$ is approximately the same in all of them. All the Z sources are thought to be accreting at very similar rates (comparable to the Eddington critical accretion rate; see, e.g., Lamb 1989; Hasinger & van der Klis 1989), and hence $\langle (\xi \dot{M})^\alpha \rangle / (\xi \dot{M}_E)^\alpha$ is not expected to differ much from one to another. The critical fastness ω_c is expected to be comparable to unity and may be a universal constant for a given inner disk structure (Ghosh & Lamb 1979, 1992). If so, then $\nu_0^{\alpha/\lambda}$ is nearly the same in the five Z sources in our sample, and hence the frequency of the upper kilohertz QPO is a good absolute measure of the mass accretion rate in these sources. Stated differently, *the relation between the upper kilohertz QPO frequency and the accretion rate appears to be very similar in all the Z sources.*

2.2. *Discussion*

The analysis presented in § 2.1 demonstrates that if (1) the neutron stars in the Z sources are spinning near their magnetic spin equilibrium rates, (2) the frequency separation between the upper and lower kilohertz QPOs is approximately equal to the neutron star spin frequency, (3) the inner accretion disks in the Z sources are optically thick and radiation-pressure dominated, and (4) the upper kilohertz QPO frequency is proportional to the mass accretion rate through the inner disk, then the magnetospheric beat-frequency model is consistent with the observed behavior of the HBO frequency and, in particular, with the tight, universal correlation (Fig. 2) between the HBO and kilohertz QPO frequencies in all the Z sources in our sample. All of these assumptions are expected to be satisfied, as discussed in § 2.1.

As noted earlier, the magnetospheric beat-frequency model of the HBO is consistent with models for the upper kilohertz QPO that identify its frequency with an orbital frequency in the inner disk only if a small fraction of the accreting matter does not couple to the stellar magnetic field at the radius r_m but instead remains in a geometrically thin Keplerian disk down to the radius responsible for the upper kilohertz QPO (i.e., ξ must be less than unity; for discussions, see van der Klis 1998; Miller et al. 1998b; Alpar & Yilmaz 1997). In particular, the interpretation of both the HBO and the two simultaneous kilohertz QPOs as rotational beat phenomena requires that there be two distinct radii in the inner accretion disk at which beating of the neutron star spin frequency with the orbital frequency produces a QPO, as is the case, for example, in the sonic-point model (Miller et al. 1998b).

Assuming that the HBO is a magnetospheric beat-frequency phenomenon, we can use the observed HBO properties together with a general relation for the coupling radius to constrain the magnetic dipole moments of the neutron stars in the Z sources in a way that is largely independent of the structure of the inner accretion disks. In the Ghosh & Lamb (1979) model of disk-magnetosphere interaction, the radius r_m at which the stellar magnetic field strongly couples to the gas in the accretion disk is given implicitly by (see Ghosh & Lamb 1992)

$$r_m \simeq \left(\frac{B_\phi}{B_p}\right)^{2/7} \left(\frac{\Delta r}{r_m}\right)^{2/7} \left(\frac{\mu^4}{GMM_\odot \xi^2 \dot{M}^2}\right)^{1/7}$$
$$\simeq 3.3 \times 10^6 \left(\frac{B_\phi}{B_p}\right)^{2/7} \left(\frac{\Delta r}{r_m}\right)^{2/7}$$
$$\times\, \xi^{-2/7} \mu_{27}^{4/7} M^{-1/7} \left(\frac{\dot{M}}{\dot{M}_E}\right)^{-2/7} \text{ cm} \tag{9}$$

for any model of the inner accretion disk. Here B_ϕ/B_p is the mean azimuthal magnetic pitch in the annulus of radial width $\Delta r/r_m$ in the inner disk where the stellar field strongly interacts with gas in the disk.

If the stellar magnetic field is too weak, it cannot couple strongly to the gas in the accretion flow well above the stellar surface and hence cannot generate magnetospheric beat-frequency oscillations. Hence in the magnetospheric beat-frequency model, the coupling radius r_m must be larger than the neutron star radius R_{NS}, which requires

$$\mu_{27} \gtrsim 1.3 \xi^{1/2} M^{1/4} \left(\frac{B_\phi}{B_p}\right)^{-1/2} \left(\frac{\Delta r/r_m}{0.01}\right)^{-1/2}$$
$$\times \left(\frac{\dot{M}_{max}}{\dot{M}_E}\right)^{1/2} \left(\frac{R_{NS}}{10^6 \text{ cm}}\right)^{7/4}, \tag{10}$$

where $\dot{M}_{max}$ is the maximum mass accretion rate at which the HBO is detected. In deriving equation (10) we have neglected the contributions of any higher multipole moments of the stellar magnetic field that may be present near the neutron star surface or may be induced by the electrical currents flowing in the disk (see Psaltis, Lamb, & Zylstra 1996 for a discussion). For the Keplerian frequency at the coupling radius to exceed the neutron star spin frequency, which is also required in the magnetospheric beat-frequency model (see Ghosh & Lamb 1979), the magnetic dipole moment must satisfy

$$\mu_{27} \lesssim 10 M^{1/4} \left(\frac{B_\phi}{B_p}\right)^{-1/2} \left(\frac{\Delta r/r_m}{0.01}\right)^{-1/2} \left(\frac{\nu_s}{300 \text{ Hz}}\right)^{7/6}, \tag{11}$$

where we have used the fact that $\xi \dot{M} \lesssim \dot{M}_E$. These upper and lower bounds on the magnetic dipole moment depend only very weakly on the neutron star mass. Figure 4 shows the resulting lower (eq. [10]) and upper (eq. [11]) bounds on the magnetic dipole moments of the neutron stars in the Z sources, as a function of the relative width $\Delta r/r_m$ of the coupling region.

We can obtain an estimate of the magnetic dipole moments of the Z sources by using the value of A_1 obtained by fitting equation (5) to the frequency data and the optically thick, radiation-pressure–dominated model of the inner disk. The result is

$$\mu_{27} \simeq (0.8\text{–}1.0) \xi^{0.3} \left(\frac{\nu_0}{1200 \text{ Hz}}\right)^{-0.3} \left(\frac{M}{2\, M_\odot}\right)^{0.9}, \tag{12}$$

where ξ may depend on the magnetic field strength. Equation (12) shows that the HBO frequencies predicted by the

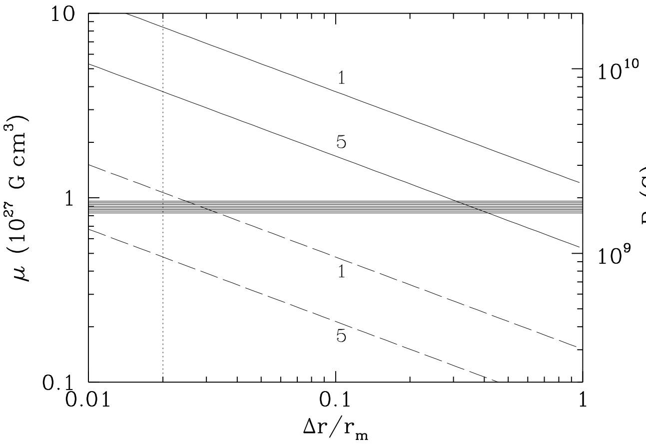

FIG. 4.—Bounds on the magnetic moments of the neutron stars in the Z sources, derived from the magnetospheric beat-frequency model of the HBO. Upper bounds on the magnetic dipole moment (*solid lines*) are derived from the requirement that the orbital frequency at the coupling radius be greater than the neutron star spin frequency. Lower bounds on the magnetic dipole moment (*dashed lines*) are derived from the requirement that the coupling radius be larger than the radius of the star. Solid and dashed lines are plotted for two values (1 and 5) of the mean azimuthal magnetic pitch in the coupling region. Lower bound on the fractional radial width of the coupling region (*dotted line*) is derived from the requirement that the predicted FWHM of the HBO be consistent with the observed width. The range of dipole magnetic moments (*shaded region*) is estimated from the fits shown in Fig. 1 of the magnetospheric beat-frequency model (eq. [5]) to the Z-source data. The polar magnetic field strength that corresponds to a given magnetic moment is shown on the right vertical axis. For simplicity, the neutron stars in the Z sources were all assumed to have masses, radii, and spin frequencies of 2 $M_{\odot}$, 10^6 cm, and 300 Hz.

magnetospheric beat-frequency model are consistent with the HBO frequencies observed if $\mu_{27} \approx 1$, which implies that the dipole magnetic fields of the Z sources have strengths at the magnetic poles of $\approx 10^9$ G for a 10 km neutron star. (Note that the estimated dipole magnetic moment depends only weakly on the unknown parameters ξ and ν_0.)

The relative width $\Delta r/r_m$ of the annulus where the stellar magnetic field strongly couples to the gas in the disk is expected to be greater than ~ 0.01 (Ghosh & Lamb 1992). Its value can be bounded below using the observed FWHM of the HBO (see also Alpar & Shaham 1985; Lamb et al. 1985). Assuming that all other QPO broadening mechanisms—such as lifetime broadening—are negligible, we can estimate the relative width of the annulus in the accretion disk in which the interaction at the beat frequency affects the X-ray luminosity, from the relative width of the HBO peak in power spectra. The width of this annulus is necessarily smaller than the width Δr of the layer where the magnetic field strongly interacts with the gas in the disk, and hence

$$\frac{\Delta r}{r_m} \gtrsim \frac{2}{3}\left(\frac{\delta\nu_{\rm HBO}}{\nu_{\rm HBO}+\nu_s}\right) = 0.02\left(\frac{\delta\nu_{\rm HBO}}{10\ {\rm Hz}}\right)\left(\frac{350\ {\rm Hz}}{\nu_{\rm HBO}+\nu_s}\right), \quad (13)$$

where $\delta\nu_{\rm HBO}$ is the FWHM of the HBO. Figure 4 displays this bound on $\Delta r/r_m$ and the constraint it imposes on the dipole moment of the stellar magnetic field.

Figure 4 shows that these additional physical bounds on the dipolar magnetic fields of the Z sources derived from the magnetospheric beat-frequency interpretation of the HBO are consistent both with each other and with the field strengths $\approx 10^9$ G estimated in equation (12) and by modeling the X-ray spectra of the Z sources (Psaltis et al. 1995; Psaltis & Lamb 1998).

3. THE LENSE-THIRRING PRECESSION INTERPRETATION

3.1. *Model Predictions and Comparison with Observations*

In the nodal (Lense-Thirring and tidal) precession model of the HBO (Stella & Vietri 1998), a narrow ring or clumps of gas are assumed to be in a tilted orbit at the radius responsible for the upper kilohertz QPO and to precess with the frequency of a test particle in such an orbit (Stella & Vietri 1998). Alternatively, if the disk ends at this radius, one of the many weakly damped global precession modes localized near the inner edge of the disk (Marković & Lamb 1998) may be excited.

In the weak field limit, the nodal precession frequency of an infinitesimally tilted orbit at the radius where the frequency of a circular Keplerian orbit is $\nu_{\rm K}$ is (see Stella & Vietri 1998; Morsink & Stella 1999)

$$\nu_{\rm NP} \approx 13.2\left(\frac{I_{45}}{M}\right)\left(\frac{\nu_s}{300\ {\rm Hz}}\right)\left(\frac{\nu_{\rm K}}{1\ {\rm kHz}}\right)^2 - 4.7\left(\frac{I_{45}}{M^{5/3}}\right)\left(\frac{\eta}{0.01}\right)\left(\frac{\nu_s}{300\ {\rm Hz}}\right)^2\left(\frac{\nu_{\rm K}}{1\ {\rm kHz}}\right)^{7/3}, \quad (14)$$

where $\eta \equiv -(A/I_{zz})(\nu_s/300\ {\rm Hz})^{-2}$ in terms of A, the coefficient of the quadrupole moment of the gravitational field, and I_{zz}, the neutron star moment of inertia with respect to its spin axis. Equation (14) is derived by expanding the full expression for $\nu_{\rm NP}$ in a power series in ν_s and retaining only terms up to second order.

Lense-Thirring precession.—If the effects of the quadrupole component of the stellar gravitational field are negligible, the localized warping modes of the inner disk will precess with a frequency close to the Lense-Thirring frequency of a test particle (Marković & Lamb 1998), which is given by the first term in equation (14). Identifying the centroid frequency of the HBO with the Lense-Thirring frequency at the radius where the orbital frequency is equal to the frequency ν_2 of the upper kilohertz QPO gives (Stella & Vietri 1998)

$$\nu_{\rm HBO} = \frac{8\pi^2 I}{c^2 M}\,\nu_s \nu_2^2 = 13.2\left(\frac{I_{45}}{M}\right)\left(\frac{\nu_s}{300\ {\rm Hz}}\right)\left(\frac{\nu_2}{1\ {\rm kHz}}\right)^2\ {\rm Hz}, \quad (15)$$

where $I \equiv 10^{45} I_{45}$ g cm^2 is the moment of inertia of the neutron star and ν_2 is the orbital frequency at the radius responsible for the upper kilohertz QPO. The X-ray visibility as well as the excitation and damping of the precession modes of the inner disk have not yet been addressed (see Marković & Lamb 1998 for a discussion). Equation (15) predicts a relation between the HBO frequency, the spin frequency of the neutron star, and the frequency ν_2 of the upper kilohertz QPO that depends only on the structure of the neutron star, through the ratio I/M.

Figure 5 shows the HBO frequencies observed in the five Z sources in the present sample, plotted against the frequencies of their upper kilohertz QPOs. Separate fits of equation (15) to the data on the individual sources, using the neutron star spin frequency inferred from the frequency separation of the kilohertz QPOs and treating I/M as a free parameter, give values of χ^2 per degree of freedom of order unity for four of the Z sources but ~ 4 for Sco X-1. There is

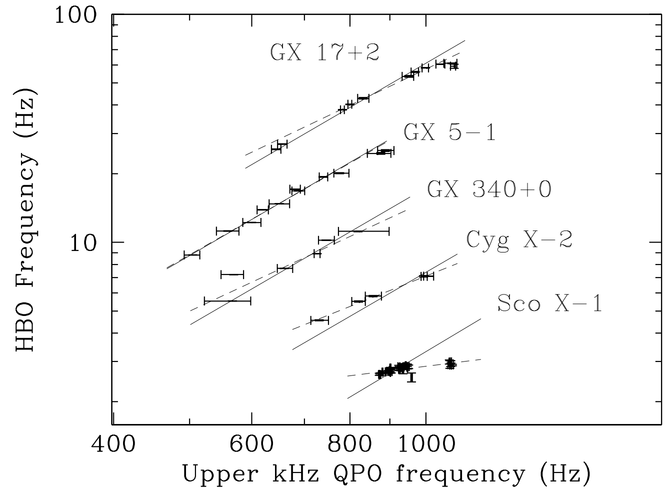

FIG. 5.—Correlations between HBO frequency and upper kilohertz QPO frequency in five Z sources. The solid lines show the best fits of the Lense-Thirring model (eq. [15]) to the data. The dotted lines show the best fits of the more general power-law eq. (A1), which is discussed in the Appendix. The HBO frequencies and lines for all the sources except GX 17+2 have been shifted downward by successive factors of 2 for clarity.

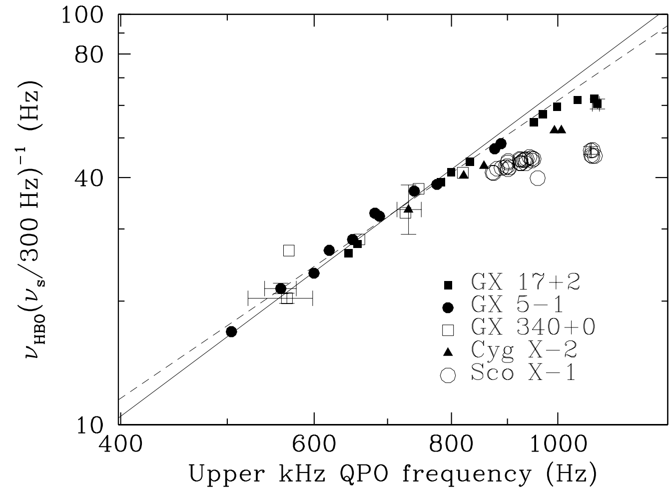

FIG. 6.—Correlation between the HBO frequency $\nu_{\rm HBO}$ divided by the inferred neutron star spin frequency ν_s (Table 2) and the upper kilohertz QPO frequency in five Z sources. Only a single typical error bar is shown for each source, for clarity. The solid line is the best fit of the Lense-Thirring model (eq. [15]) to all points with upper kilohertz QPO frequencies less than 850 Hz. The dotted line is the best fit of the more general power-law eq. (A2) discussed in the Appendix to the same points.

no other freedom in equation (15) and hence pure Lense-Thirring precession is not consistent with the HBO properties. Moreover, as Stella & Vietri (1998) noticed (see also Wijnands et al. 1998b; Jonker et al. 1998), the coefficients of $\nu_s \nu_2^2$ required to fit the data give values of $I_{45}/M \gtrsim 4$, which is $\sim$4–5 times larger than the largest ratios given by realistic equations of state for stars of any mass and about 2.5 times larger than the largest ratio given by the extremely stiff relativistic mean-field equation of state L (see Table 3).

In the Lense-Thirring precession model of the HBO, the relation between $\nu_{\rm HBO}/\nu_s$ and the upper kilohertz QPO frequency depends only on the mass of the star and the equation of state of neutron star matter (see eq. [15]). Data from similar neutron stars should therefore follow similar relations. Figure 6 shows how the ratio $\nu_{\rm HBO}/\nu_s$ scales with the frequency ν_2 of the upper kilohertz QPO; according to the Lense-Thirring precession model, this ratio should scale as ν_2^2. Figure 6 shows that the data with frequencies $\nu_2 < 850$ Hz are consistent ($\chi^2_{\rm d.o.f} \approx 1.1$) with a single relation of the form of equation (15). Again, however, the coefficient given by the fit requires neutron stars with $I_{45}/M \gtrsim 4$, which is implausibly large. Furthermore, the points that have $\nu_2 > 850$ Hz are inconsistent with the relation of the form of equation (15) that fits the points with lower values of ν_2.

TABLE 3

MAXIMUM I/M VALUES FOR ILLUSTRATIVE NEUTRON STAR EQUATIONS OF STATE

EOS[a]	Mass ($M_\odot$)	I_{45}/M
A	1.66	0.63
AU	2.13	0.93
FPS	1.80	0.76
L	2.70	1.73
M	1.80	0.91

NOTE.—The tabulated values are the maximum ratios found in a computational survey of all stable masses and spin rates for the equation of state indicated, using the code developed by Cook, Shapiro, & Teukolsky 1994.

[a] A, AU, and FPS are realistic equations of state; M illustrates the effects of a strongly repulsive tensor interaction; L is an extremely stiff equation of state produced by a simple relativistic mean-field theory. For a recent discussion of these equations of state, see Miller, Lamb, & Cook 1998a.

Effect of classical precession.—Stella & Vietri (1998; see also the extended discussion in Morsink & Stella 1999) suggested that the flattening of the $\nu_{\rm HBO}$–ν_2 correlation at high ν_2 might be caused by the increasing importance, as the disk penetrates closer to the star, of the classical precession caused the rotation-induced quadrupole component of the stellar gravitational field. This precession is retrograde but smaller than the prograde gravitomagnetic precession and therefore tends to reduce the nodal precession frequency. We can test this suggestion quantitatively using the data for Sco X-1 and GX 17+2, which deviate most strongly from the relation of the form of equation (15) that fits the points with low values $\nu_2 < 850$ Hz.

If the HBO is caused by nodal precession and classical precession is important, then $\nu_{\rm HBO}/\nu_2^2$ should decrease linearly with increasing $\nu_2^{1/3}$ (cf. Stella & Vietri 1998), because equation (14) can be rewritten as

$$\left(\frac{\nu_{\rm HBO}}{1\ {\rm Hz}}\right)\left(\frac{\nu_2}{1\ {\rm kHz}}\right)^{-2} = 13.2\left(\frac{I_{45}}{M}\right)\left(\frac{\nu_s}{300\ {\rm Hz}}\right) - 4.7\left(\frac{I_{45}}{M^{5/3}}\right)\left(\frac{\eta}{0.01}\right)\left(\frac{\nu_s}{300\ {\rm Hz}}\right)^2\left(\frac{\nu_2}{1\ {\rm kHz}}\right)^{1/3} . \quad (16)$$

Figure 7 plots $\nu_{\rm HBO}/\nu_2^2$ against $\nu_2^{1/3}$ for Sco X-1 and also shows the best-fit straight line with slope $\frac{1}{3}$, which has $\chi^2_{\rm d.o.f.} \approx 1$. The fact that the data in Figure 7 can be fit satisfactorily by a straight line with slope $\frac{1}{3}$ is not strong evidence for this scaling, because the range of measured $\nu_2^{1/3}$

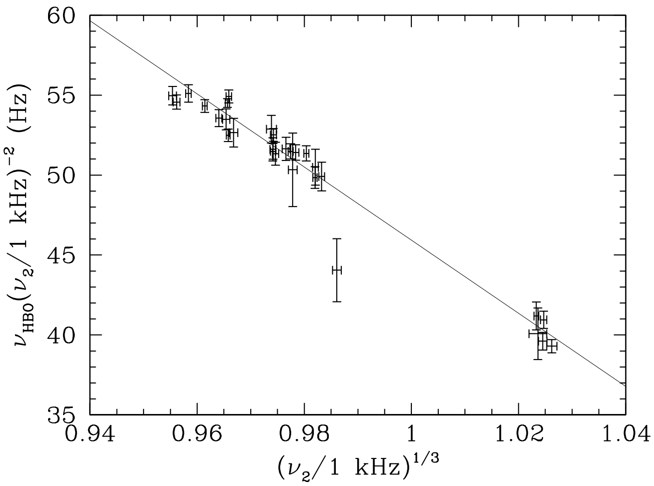

FIG. 7.—Correlation in Sco X-1 between $\nu_{\mathrm{HBO}}/\nu_2^2$ and $\nu_2^{1/3}$, where ν_{HBO} is the HBO frequency and ν_2 is the frequency of the upper kilohertz QPO. The solid line is the straight line that best fits the data.

values is very narrow. However, we can use the best-fit value of the intercept of the straight line with the vertical axis to estimate the value of the parameter η that characterizes the quadrupole moment of the gravitational field and to estimate I/M. The results are

$$\eta_{\mathrm{Sco}} \simeq 2.3 \times 10^{-2} M^{2/3} \left(\frac{\nu_s}{300\ \mathrm{Hz}}\right)^{-1} \quad (17)$$

and

$$\left(\frac{I_{45}}{M}\right)\left(\frac{\nu_s}{300\ \mathrm{Hz}}\right) = 20.8 \pm 2.1\ . \quad (18)$$

The value of I/M required by equation (18) is $\sim$20 time larger than the largest values given by realistic neutron star equations of state (see Table 3). The deviation of the GX 17+2 data from the power-law equation (15) at high frequencies (see Fig. 6) requires that the classical precession frequency be negligible for $\nu_2 < 850$ Hz but comparable to the Lense-Thirring precession frequency at slightly larger values of ν_2. This is not possible, because the exponents of ν_2 in the Lense-Thirring and classical precession terms of equation (14) are too similar. The data are therefore inconsistent with the predictions of the simple nodal precession model.

3.2. *Discussion*

In the Lense-Thirring precession model of the HBO, the HBO frequency, the spin frequency of the neutron star, and the frequency of the upper kilohertz QPO are related by equation (15). As discussed in the previous section, the data are consistent with this relation when the frequency of the upper kilohertz QPO is less than 850 Hz, but the inferred value of I/M is implausibly large (see also Stella & Vietri 1997, 1998; Morsink & Stella 1999). Aside from the rather unlikely possibility that neutron stars have values of I/M that are four times as large as the largest values for stellar models constructed with realistic equations of state, three other possibilities have been suggested for reducing this large discrepancy.

First, the frequency difference $\Delta\nu$ between the kilohertz QPOs might be equal to one-half the neutron star spin frequency ν_s rather than equal to it. This is very unlikely in any beat-frequency model of the kilohertz QPOs, because it would require a special direction that rotates with the neutron star but affects the inner accretion disk only once every two beat periods. However, $\Delta\nu = \frac{1}{2}\nu_s$ appeared possible, given the initial analysis of the data taken during the type I X-ray bursts of 4U 1636−536 (Zhang et al. 1997; Strohmayer et al. 1998), which showed a strong oscillation at about 580 Hz, approximately twice the frequency separation of the two kilohertz QPOs. However, further analysis of this data by Miller (1999) using a matched-waveform filtering technique has revealed the presence of a weak coherent oscillation at about 290 Hz, approximately equal to the frequency separation of the two kilohertz QPOs. Thus, it now appears very unlikely that the spin frequencies of these neutron stars are twice the frequency separation of their kilohertz QPOs.

Second, the observed HBO frequencies and their second harmonics might represent the second and fourth harmonics of the fundamental Lense-Thirring frequency (eq. [15]), rather than the first and second harmonics. Indeed, a precessing circular orbit has a twofold symmetry that could, in principle, produce even-order harmonics that are stronger than the odd-order harmonics. Moreover, power-density spectra of the Z sources show a relatively strong, broad-band noise component at frequencies comparable to the ones predicted by equation (15). This so-called low-frequency noise (Hasinger & van der Klis 1989) might inhibit detection of the fundamental of a low-frequency precession frequency (see, e.g., Fig. 6*a* of Kuulkers et al. 1994 for peaked features in the low-frequency noise component of GX 5−1). Determination of the upper limits on the amplitudes of any QPOs at these frequencies would significantly constrain this possibility. Note, however, that even if the HBO and its overtone are the second and fourth harmonics of the precession frequency, the I/M values required to explain the HBO observations would still be a factor $\gtrsim$2 larger than predicted by realistic neutron star equations of state.

A further difficulty with the Lense-Thirring precession interpretation of the HBO is that the observed correlation between the HBO and kilohertz QPO frequencies is significantly different from what is predicted by equation (15) when the frequency of the upper kilohertz QPO is greater than 850 Hz. As demonstrated in § 3.1, this difference cannot be explained by classical precession, nor can it be explained by strong-field corrections to equation (15) (Stella & Vietri 1998).

A third possibility is that radiation forces increase the ratio of ν_{NP} to ν_2 by the factor $\sim$2–5 required to bring it into agreement with the observed HBO and upper kHz QPO frequencies. The Z sources are thought to be accreting at near-Eddington mass accretion rates when the frequencies of the upper kilohertz QPOs are comparable to $\sim$1 kHz (see, e.g., Psaltis et al. 1995). Hence radiation forces, which were neglected in equation (14), almost certainly are important. At near-critical luminosities, both orbital and nodal precession frequencies can be altered by large factors compared to their values in the absence of radiation; hence, radiation forces might possibly explain the large discrepancy between the observed frequencies of the HBO and the frequencies predicted by the nodal precession model.

An explanation in terms of the combined effects of Lense-Thirring precession and radiation forces would, however, require the physically implausible result that radiation forces leave the variation with ν_2 basically unchanged while increasing the ratio of ν_{NP} to ν_2 by a factor $\sim$2–5. Such an explanation would also require that the QPO peaks not be significantly broadened by the radiation drag force at the same time that radiation forces are strong enough to change the orbital and precession frequencies by a factor $\sim$2–5. The HBO peaks in Sco X-1, for example, have fractional widths $\delta\nu/\nu \lesssim 0.5$ even when the inferred accretion rate is near the Eddington critical rate (van der Klis et al. 1997). More fundamentally, if radiation forces do change the orbital and precession frequencies of gas accreting onto the Z sources by large factors, as they may well do, the observed correlation between the HBO and upper kilohertz QPO frequencies would be explained primarily by the effect of the radiation forces and not by the gravitomagnetic torque.

4. CONCLUSIONS

In §§ 2 and 3 we have studied in detail the behavior of the HBO frequencies observed in five Z sources and in particular their correlation with the frequencies of the kilohertz QPOs, comparing the observed behavior with the behaviors predicted by the magnetospheric beat-frequency (Alpar & Shaham 1985; Lamb et al. 1985) and Lense-Thirring precession (Stella & Vietri 1998) models of the HBO.

In § 2 we showed that the magnetospheric beat-frequency model is consistent with the observed correlation between the HBO and upper kilohertz QPO frequencies in the five Z sources studied here if, as expected, the neutron stars in these sources are spinning near their magnetic spin equilibrium rates, the frequency separation between the upper and lower kilohertz QPOs is approximately equal to the neutron star spin frequency, the inner part of their accretion disks are optically thick and radiation-pressure–dominated, and the frequency of the upper kilohertz QPO is approximately proportional to the mass accretion rate. The model predicts a universal relation between the horizontal-branch oscillation, stellar spin, and upper kilohertz QPO frequencies that agrees well with the data on five Z sources. The spin rates predicted by the model are consistent with the range of the spin frequencies of the Z sources inferred from the frequency separation of their kilohertz QPOs if they are all accreting at similar, near-critical rates and all have 10^9–10^{10} G dipole magnetic fields. Such magnetic fields are consistent with models of Z-source X-ray spectra. The inferred value of the critical fastness for the accretion rates and magnetic field strengths of the Z sources is $\gtrsim 0.8$. If the frequency of the upper kilohertz QPO is an orbital frequency in the accretion disk, the magnetospheric beat-frequency model requires that a fraction of the accreting gas does not couple strongly to the stellar magnetic field until it has penetrated to within a few kilometers of the neutron star surface.

In § 3, we showed that the trend of the correlation between the HBO frequency and the upper kilohertz QPO frequency observed at upper kilohertz QPO frequencies $\nu_2 < 850$ Hz agrees with the trend predicted by the Lense-Thirring precession model. However, the observed trend is inconsistent with the model for $\nu_2 > 850$ Hz. The observed magnitudes of the HBO frequencies are $\gtrsim$4–5 times larger than the magnitudes predicted by the Lense-Thirring precession model for realistic neutron star equations of state. Thus, in order to be consistent with the observed magnitudes, either I/M must be $\gtrsim$4–5 times larger than expected or the principal frequency of the X-ray oscillation generated by nodal precession must be $\gtrsim$4–5 times the nodal precession frequency.

We thank Greg Cook for making available the numerical code used to compute the suite of neutron star models used in this work. D. P. acknowledges Charles Gammie for many useful discussions on the physics of warped accretion disks. D. P. also thanks L. Stella, V. Kalogera, R. Narayan, A. Esin, C. Gammie, K. Menou, and E. Quataert for discussions on the observational evidence for the Lense-Thirring effect. This work was supported in part by a post-doctoral fellowship of the Smithsonian Institute (D. P.), by the Netherlands Foundation for Research in Astronomy (ASTRON) grant 781-76-017 (R. W., J. H., P. J., M. v. d. K.), by NSF grant AST 96-18524 (F. K. L.), by NASA grants NAG 5-2925 (F. K. L.), NAG 5-2868 (M. C. M.), NAG 5-3269 and NAG 5-3271 (J. v. P.), and by several *RXTE* observing grants. W. H. G. L. gratefully acknowledges support from NASA. M. v. d. K. gratefully acknowledges the Visiting Miller Professor Program of the Miller Institute for Basic Research in Science (UCB).

APPENDIX

AN EMPIRICAL DESCRIPTION OF THE CORRELATION OBSERVED BETWEEN THE HBO AND UPPER-KILOHERTZ QPO FREQUENCIES

In this appendix we show that the correlations observed between the HBO frequency ν_{HBO}, the upper kilohertz QPO frequency ν_2, and the frequency separation $\Delta\nu$ between the two kilohertz QPOs can be characterized by simple power-law relations among the frequencies involved. Our purpose is to facilitate comparison of the present data with future data (see, e.g., Psaltis et al. 1999) or other theoretical models.

The frequency correlations in the five Z sources in our sample can be described adequately by the power-law relation

$$\nu_{HBO} = 13.2 a_1 \left(\frac{\nu_2}{1\ \mathrm{kHz}}\right)^{b_1}\ \mathrm{Hz}\,, \tag{A1}$$

where the constants a_1 and b_1 are different for each source. The confidence contours obtained by fitting this relation to the measured HBO and upper kilohertz QPO frequencies of the five Z sources in the present sample are shown in Figure 8. The power-law index that describes the Sco X-1 data is significantly smaller than the index that describes the data on the other four sources. The best-fit relations for each source are the dashed lines shown in Figure 5.

For all the Z sources except Sco X-1, the best-fit value of the parameter a_1 is approximately proportional to the spin frequency inferred from the frequency separation of the two kilohertz QPOs (see Fig. 6). Indeed, when the upper kilohertz

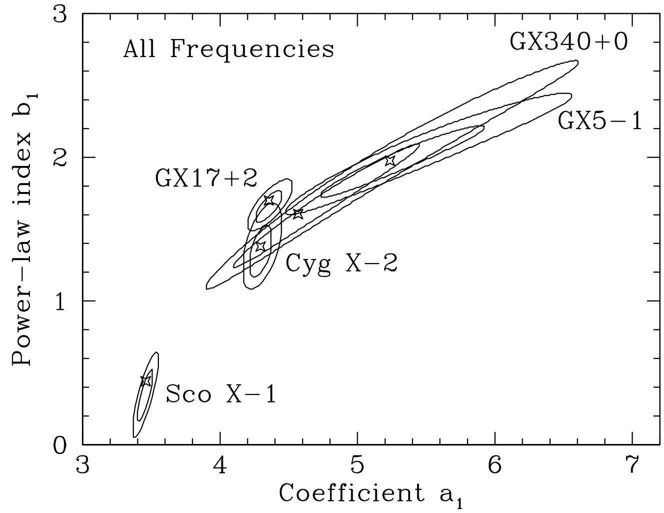

FIG. 8.—Confidence contours for the coefficient a_1 and the power-law index b_1, obtained by fitting eq. (A1) to the frequency data on each of five Z sources, considered individually. The inner and outer contours show the 68% and 99% confidence limits, respectively, while the stars indicate the best-fit values for each source. The best-fit relations are shown as dashed lines in Fig. 5.

QPO frequency is less than 850 Hz, the correlation between $\nu_{\rm HBO}$, ν_s, and ν_2 is described adequately by the relation

$$\nu_{\rm HBO} = 13.2 a_2 \left(\frac{\nu_s}{300\ {\rm Hz}}\right)\left(\frac{\nu_2}{1\ {\rm kHz}}\right)^{b_2}, \tag{A2}$$

with $a_2 \approx 4.6$ and $b_2 \approx 1.8$. The confidence contours obtained by fitting this relation to the Sco X-1 points and to the points on the other four sources for which $\nu_2 < 850$ Hz are shown in Figure 9. As Figure 6 shows, the frequency correlation is significantly flatter when ν_2 is greater than 850 Hz.

In Sco X-1, the HBO and kilohertz QPOs were simultaneously detected mostly when it was on the normal branch. In the other four Z sources, the HBO and kilohertz QPOs were simultaneously detected mostly when they were on their horizontal branches. The transition from the horizontal to the normal branch is thought to take place when the mass accretion rate increases to within a few percent of the Eddington critical rate (see Lamb 1989; Psaltis et al. 1995). If so, the resulting change in the accretion flow pattern (see Lamb 1989) might be responsible for the different dependences of the HBO frequency $\nu_{\rm HBO}$ and the instantaneous frequency separation $\Delta\nu$ of the two kilohertz QPOs on the upper kilohertz QPO frequency ν_2 in Sco X-1, compared to the dependences in the other four Z sources in our sample.

The ratio of $\nu_{\rm HBO}$ to $\Delta\nu$ in Sco X-1 increases more steeply with ν_2 than does the ratio of $\nu_{\rm HBO}$ to the (constant) inferred spin frequency ν_s. This is demonstrated most clearly by the correlation plots shown in Figure 10. (For all the Z sources in the

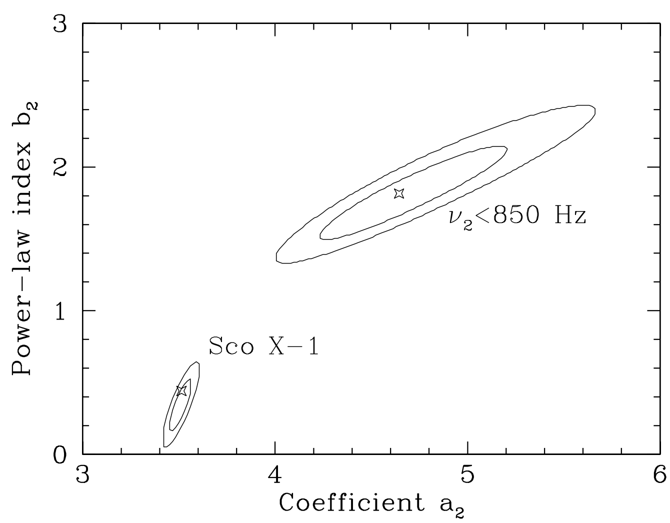

FIG. 9.—Confidence contours for the coefficients a_2 and power-law indices b_2 derived by fitting eq. (A2) to two disjoint sets of frequency data. The contours marked "Sco X-1" were obtained by fitting this relation to the Sco X-1 data. The contours marked "$\nu_2 < 850$ Hz" were obtained by fitting this relation to the frequency data on all the other sources, when the upper kilohertz QPO frequency is less than 850 Hz; the best-fit version of this relation is shown by the dashed line in Fig. 6. The inner and outer contours show the 68% and 99% confidence limits, respectively, while the stars indicate the best-fit values for each source.

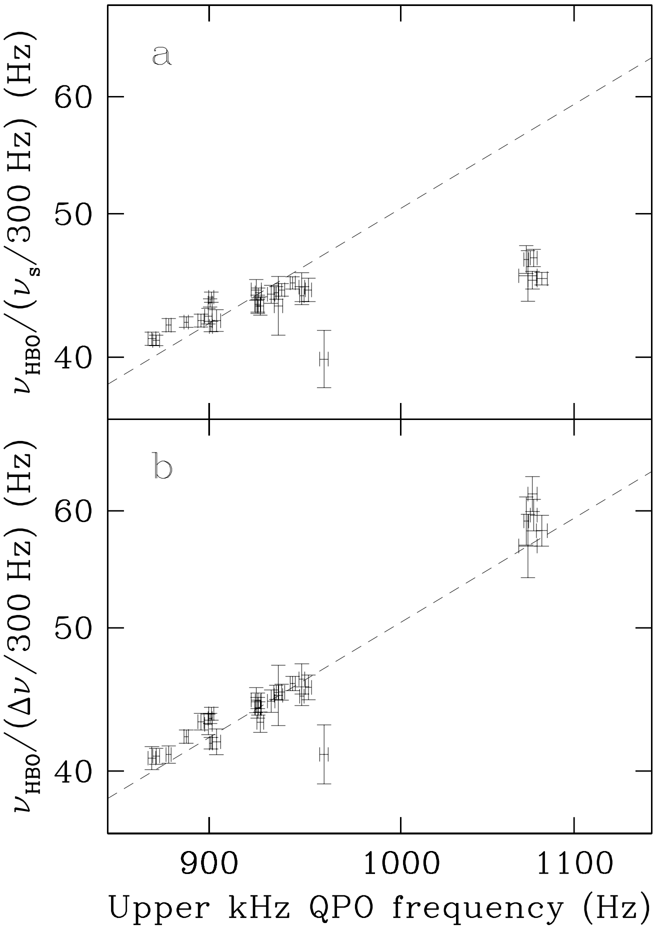

FIG. 10.—(*a*) Correlation observed in Sco X-1 between ν_{HBO}/ν_s, the HBO frequency divided by the neutron star spin frequency inferred from the frequency separation of the two kilohertz QPOs at low count rates, and the frequency of the upper kilohertz QPO. (*b*) Correlation between $\nu_{HBO}/\Delta\nu$, the HBO frequency divided by the instantaneous frequency separation of the kilohertz QPOs, and the frequency of the upper kilohertz QPO. In both panels the dashed line shows the best fit of eq. (A3) to the data.

sample except Sco X-1, $\Delta\nu$ is consistent with being constant or with varying in the same way as it does in Sco X-1; Psaltis et al. 1998; see also Wijnands et al. 1997, 1998a; Jonker et al. 1998. This is true mostly because of the relatively large uncertainties in the measured kilohertz QPO frequencies.) The dependence of $\nu_{HBO}/\Delta\nu$ on ν_2 in Sco X-1 is more consistent with the behavior seen in the other four Z sources and suggests that we plot $\nu_{HBO}/\Delta\nu$ against ν_2 for all five of the Z sources in our sample. The

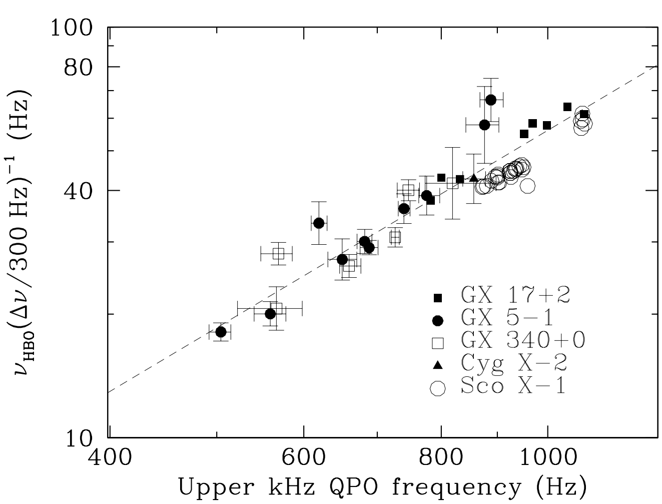

FIG. 11.—Correlation between the HBO frequency ν_{HBO} divided by the instantaneous frequency separation $\Delta\nu$ of the kilohertz QPOs and the frequency of the upper kilohertz QPO in five Z sources. The uncertainties in the GX 17+2 and Sco X-1 points are typically smaller than the sizes of the symbols plotted for these sources and have therefore been omitted. The dashed line is the best fit of eq. (A3) to the frequency data on all the sources except Sco X-1.

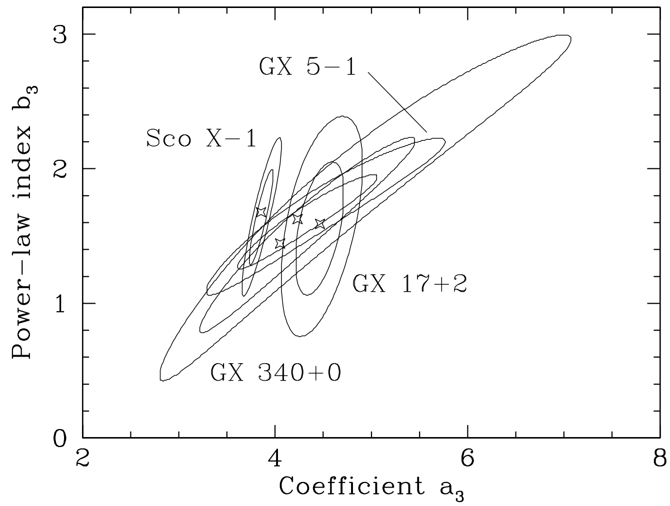

FIG. 12.—Confidence contours for the coefficients a_3 and power-law indexes b_3 derived by fitting eq. (A3) separately to the frequency data on four of the five Z sources in our sample. Cyg X-2 was not considered because there is only one data point for it. The inner and outer contours show the 68% and 99% confidence limits, respectively, while the stars indicate the best-fit values for each source.

result is shown in Figure 11. In this plot we have included only points derived from simultaneous observations of the HBO and kilohertz QPO frequencies. The larger scatter of the points in Figure 11 at $\nu_2 < 850$ Hz compared to the scatter of the points in Figure 7 corresponding to the same values of $\nu_{\rm HBO}$ and ν_2 is caused by the large uncertainties in $\Delta\nu$. The points plotted in Figure 11 are consistent with power-law relations of the form

$$\nu_{\rm HBO} = 13.2 a_3 \left(\frac{\Delta\nu}{300\ {\rm Hz}}\right)\left(\frac{\nu_2}{1\ {\rm kHz}}\right)^{b_3} . \tag{A3}$$

The confidence contours obtained by fitting this relation to the data on each of the sources except Cyg X-2 are shown in Figure 12. Cyg X-2 was not included because its HBO and kilohertz QPO frequencies were measured simultaneously only once. These contours show that a single universal relation of the form of equation (A3) with $a_3 = 4.2$ and $b_3 = 1.6$ is consistent with the data on all the sources except Sco X-1. In fact, $b_3 = 1.6$ is consistent with all the data. The Sco X-1 and GX 17+2 data have relatively small uncertainties and give best-fit coefficients a_3 that are slightly but significantly different. The uncertainties in the data on GX 5−1 and GX 340+0 are sufficiently large that their contours allow values of a_3 that are consistent with either the Sco X-1 or the GX 17+2 value.

The surprising correlation between the HBO frequency, the frequency of the upper kilohertz QPO, and the instantaneous frequency separation of the kilohertz QPOs shown in Figure 11 may be coincidental. The relatively large uncertainties in the currently measured kilohertz QPO frequencies in all the Z sources except Sco X-1 prevent us from drawing any firm conclusions about the significance of this correlation. However, the strikingly similar relation between the lower and upper kilohertz QPO frequencies in all LMXBs that show kilohertz QPOs (Psaltis et al. 1998) together with the correlation shown in Figure 11 suggests that the varying frequency separation of the kilohertz QPOs in Sco X-1 is a general property of the kilohertz QPOs and is related, directly or indirectly, to the frequency of the HBO. Additional data are needed to test directly this conjecture (see Psaltis et al. 1999 for an alternative possibility).

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