

ON THE MASSES OF CATAclySMIC VARIABLE STARS

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SUMMARY

Masses are derived for the components of 10 short period cataclysmic variable stars. With one exception, the masses of the white dwarf primaries lie in the range $1.2 \pm 0.2 M_{\odot}$, and are thus approximately three times heavier than isolated white dwarf stars.

1. INTRODUCTION

Although nearly all cataclysmic variable stars (novae, recurrent novae, dwarf novae and nova-like variables) are known to be short-period binary stars, and many are eclipsing, the determination of their masses is complicated by the complexities shown by these systems. The primaries are white dwarf stars, whose absorption line spectra—visible in only a few cases—are too wide to measure accurately for radial velocities. Surrounding each primary is a disc of emitting material. The emission from this disc is not uniform, which results in asymmetric line profiles that change around the orbital cycle. Spectra of the secondaries are visible in only a few cases. Eclipses are difficult to interpret because it is generally the case that the eclipse is of a bright spot, asymmetrically placed on the disc.

As a result, despite the wealth of observational data that has been accumulated for these objects, discussion of the masses obtained for two or three individual objects does not lead to results of much reliability. In this paper we will use a general discussion of the nature of the cataclysmic variable stars (CVs), together with empirically calibrated theoretical relationships, to provide a firm indication of the general region in which the masses of the white dwarf primaries lie. Such results can have important implications for detailed models of outbursts of white dwarfs during the nova phenomenon.

2. THE METHOD

The following discussion applies only to those CVs having main sequence secondaries. From the spectroscopic evidence accumulated by Kraft (1963) it is evident that the CVs having orbital periods from 6 hr up to 1 day have main sequence classifications. An exception is RU Peg, in which the luminosity class varies from V to III, which has been explained as a variation of effective gravity over the surface of the secondary due to the proximity of the primary. CVs with periods shorter than 6 hr do not show secondary spectra, but from the mean absolute magnitudes of CVs at normal light it is clear that the secondaries cannot be more luminous than main sequence stars, and there is at present no reason to believe that they are anything other than normal dwarfs. Thus only a few long period objects (e.g. TCrB) have evolved secondaries.

The general explanation of the presence of a disc and a bright spot in the CV systems is that the secondaries fill their Roche Lobes and are losing mass. Detailed models may be found in papers by Warner & Nather (1971) and Smak (1971). We will adopt this general model here.

A simplified treatment of the formation of a disc around the primary will be used. We assume that the secondaries have captured rotation and that particles are ejected from the inner Lagrangian point L_1 at thermal velocities. We will assume that after leaving L_1 a particle conserves its angular momentum about the primary and eventually takes up a circular orbit about the primary. Detailed calculations (Warner & Peters 1972) show that although angular momentum is not conserved about the primary, the difference in angular momentum, along a particle trajectory, as measured at L_1 and at the edge of the disc, is small. With the above procedure, and using Kepler's third law, for the case of circular orbits we find for the velocity v of particles in orbit around the primary

$$v = \frac{2\pi a_1 (1+q)^2}{Pf^2(q) q^3} \quad (1)$$

where a_1 is the semi-major axis of the orbit, P is the binary orbital period and q is the mass ratio M_2/M_1 . The function $f(q)$ is the distance from the centre of the primary to L_1 , in units of a_1 , and may be found from the tabulation given by Plavec & Kratochvil (1964).

The amplitude of the radial velocity curve of the primary (usually deduced from measurements made on the emission lines from the disc) is

$$K_1 = \frac{2\pi a_1 \sin i}{P} \quad (2)$$

where i is the orbital inclination. The projected rotational velocity of the disc, deduced from the width of the emission lines, is $v \sin i$. Therefore, from (1) and (2), we have

$$\frac{K_1}{v \sin i} = \frac{f^2(q)q^3}{(1+q)^2} \quad (3)$$

The observable quantity $K_1/v \sin i$ should therefore be a function only of the mass ratio q .

In order to compare (3) with observations we must use double-line spectroscopic binaries in which an emission ring is seen. Table I contains a list of such objects, including three stars that are not CVs. These data are taken principally

TABLE I
Double-line binaries with emission rings

Star	P (days)	K_1 (km s ⁻¹)	$v \sin i$ (km s ⁻¹)	q	$K_1/v \sin i$
RU Peg	0.371	137	500	1.22	0.27
SS Cyg	0.276	122	500	1.06	0.24
RX Cas	32.3	34	150	0.94	0.23
Z Cam	0.290	144	700	0.74	0.21
GK Per	1.904	70	490	0.44	0.14
KU Cyg	38.4	17	184	0.17	0.092
AW Peg	10.6	17.5	210	0.16	0.083
WZ Sge	0.057	<40	860	0.071	<0.05

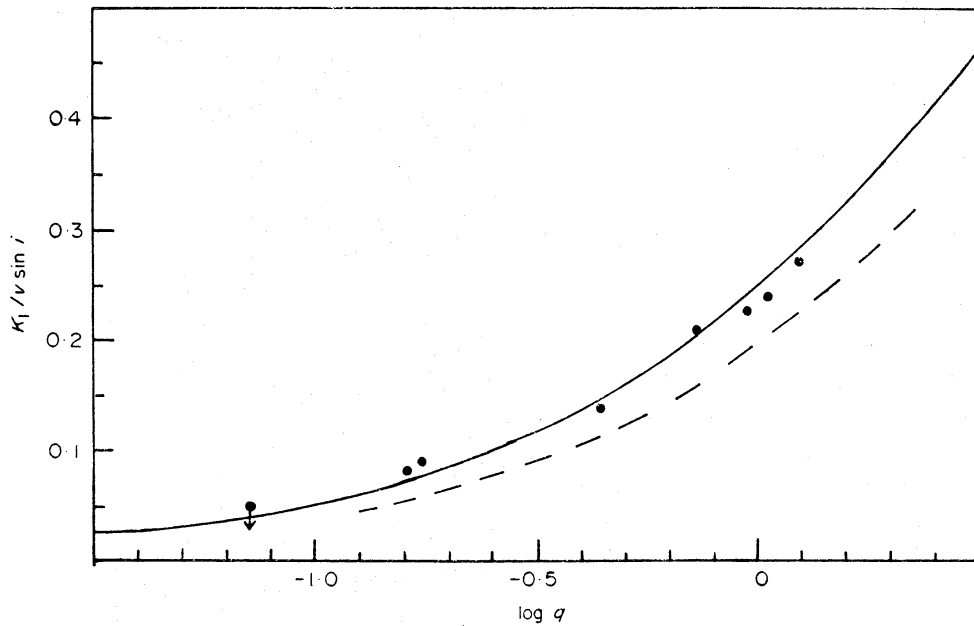


FIG. 1. Comparison of theoretical and observed variation of q with $K_1/v \sin i$. The solid line is calculated from equation (3) and the dashed line is that given by Kruszewski (1967).

from the paper by Kruszewski (1967), with the addition of some more recent material. Kruszewski computed particle trajectories from L_1 and was the first to point out the correlation between $K_1/v \sin i$ and q . A comparison between the observational data of Table I, the relationship given by equation (3), and Kruszewski's relationship is shown in Fig. 1. Kruszewski (1967) had already noted that his relationship fell systematically below the observational points. Smak (1969) attributed this to the fact that measured values of $v \sin i$ are weighted averages over a non-uniform emission ring. Our relationship (3) fits the observational data rather well, which must be a result of the fortuitous compensation of the approximation of angular momentum conservation about the primary, and the absence of any corrections of the sort advocated by Smak. For our present purposes we may regard (3) as a convenient interpolation formula representing the empirical calibration of $K_1/v \sin i$ against mass ratio q , and we use this rather than an arbitrary scaling of Kruszewski's relationship. With (3) we may now determine q for the CVs that are single-line binaries.

We will assume that the radius R_2 that the secondary *would* have, if the primary were removed, is given by the average of the three principal radii of the Roche surface. Plavec (1968) has given the following equation, which is accurate to about 1 per cent.

$$\frac{R_2}{a_1 + a_2} = 0.38 + 0.20 \log_{10} q. \quad (4)$$

Using Kepler's third law this gives

$$\frac{R_2}{M_2^{1/3}} = \left(\frac{G}{4\pi^2} \right)^{1/3} \left(\frac{1+q}{q} \right)^{1/3} P^{2/3} (0.38 + 0.20 \log_{10} q). \quad (5)$$

With q determined from (3) we can therefore determine $R_2 M_2^{-1/3}$ for all single-line CVs. The function of q appearing in (5) is only very weakly dependent on q for $0.1 < q < 1$. We note in passing that on cubing (5) we can find a $P\sqrt{\rho}$ relationship

for lobe-filling secondaries, similar to that found by Faulkner, Flannery & Warner (1972).

A second relationship between M_2 and R_2 is obtained from the mass-radius relationship for main sequence stars. It is possible to show that the mean surface gravity on the Roche surface differs by only a few per cent from that corresponding to a star with no close companion, so we expect lobe-filling secondaries to follow a normal mass-radius relationship. This is poorly determined observationally for the lower main sequence so we must depend heavily on a theoretical relationship. The one adopted here has been computed by Bodenheimer (Faulkner *et al.* 1972). Some estimate of its applicability is afforded by the following comparison.

Van der Kamp (1959) gives masses and absolute magnitudes for a few lower main sequence binaries. Gliese (1969) has given a relationship between $B-V$ and M_V for the lower main sequence which, with the aid of Wesselink's (1969) calibration of surface brightnesses in the V band, provides an estimate of R as a function of M_V . Van der Kamp's masses and our estimated radii are given in Table II. The theoretical $M-R$ relationship and the empirical points are shown in Fig. 2, where

TABLE II
Lower main sequence stars

Star	M_V	$\log M/M_\odot$	$\log R/R_\odot$
Fu 46 A	10.96	-0.51	-0.55
Fu 46 B	11.34	-0.60	-0.60
Kr 60 A	11.84	-0.57	-0.67
Kr 60 B	13.39	-0.79	-0.89
Ross 614 A	13.34	-0.85	-0.87

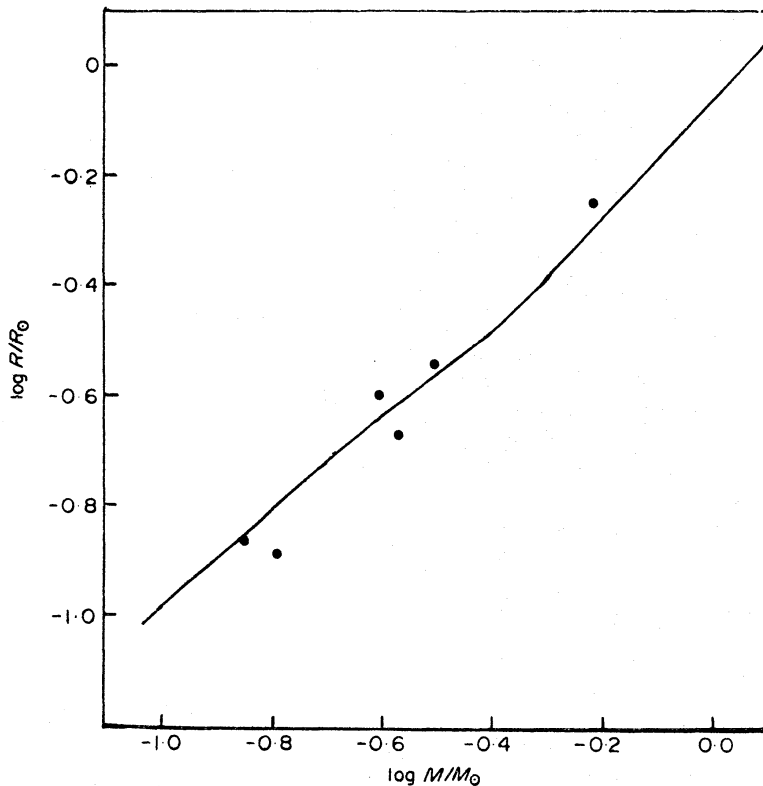


FIG. 2. Comparison of theoretical and empirical mass-radius relationship for the lower main sequence.

the mass and radius for YY Gem (Wesselink 1969) are also plotted. The agreement is as good as could be expected.

Thus provided with two independent relationships between M_2 and R_2 , and using the value of q found from equation (3), we may proceed to find M_1 and M_2 .

3. APPLICATIONS

Values of both K_1 and $v \sin i$ are available for 10 short periods CVs. The published values of K_1 may be taken at face value, but two different types of $v \sin i$ measurement have been made and they are not always equivalent. The majority of CVs show broad emission lines, and published values of $v \sin i$ are based on the total line widths (corrected for instrumental broadening). A few stars, however, show *doubled* emission lines, arising from the high inclination of the disc, and in these cases the peak separations are given. The data for the calibration stars given in Table I have all been based on total emission width, so for consistency we must use this throughout. In cases where each component of a doubled emission line is no broader than the instrumental width the two methods are clearly equivalent.

Table III lists published values of K_1 and $v \sin i$ for CVs with $P < 10$ hr. In addition, for some objects for which Kraft has given K_1 but not $v \sin i$ it has been necessary for the author to measure $v \sin i$ from published spectrograms. These latter are indicated by colons in Table III. Any necessary corrections to values of $v \sin i$ given for line separations have also been made from photographs.

TABLE III
Spectroscopic data for cataclysmic variables

Star	P (10^4 s)	K_1 (km s^{-1})	$v \sin i$ (km s^{-1})	q	M_2/M_\odot	M_1/M_\odot
RU Peg	3.21	137	500	1.15	1.3	1.1
Z Cam	2.51	144	700	0.73	1.0	1.4
SS Cyg	2.38	122	500	0.86	1.0	1.2
RX And	1.83	77	410:	0.64	0.75	1.2
DQ Her	1.67	150	680	0.72	0.70	1.0
SS Aur	1.56	85	450:	0.64	0.65	1.0
U Gem	1.50	265	850	1.50	0.98	0.65
V 603 Aql	1.20	37	240	0.46	0.50	1.1
EX Hya	0.59	≤ 50	700	< 0.15	0.17	≥ 1.1
WZ Sge	0.49	≤ 38	860	< 0.076	0.11	≥ 1.4

From inspection of Table III we see that, with the exception of U Gem, there is a well-defined correlation between mass ratio q and orbital period P . This is shown in Fig. 3 to be linear except perhaps for the smallest values of q . It is important to realize that this correlation has not been forced by the method of analysis. The quantity q has been derived from Fig. 1 by using the independent observed quantities K_1 and $v \sin i$. The orbital period is unconnected with either of these. The correlation seen in Fig. 3 is not followed by the four longer period objects listed in Table I; thus only the short period CVs have $q \propto P$.

The other striking feature of Table III, again with the exception of U Gem, is the near constancy of the primary masses M_1 . It is easy to show that this is demanded by the observed $q \propto P$ relationship: the weak dependence of equation

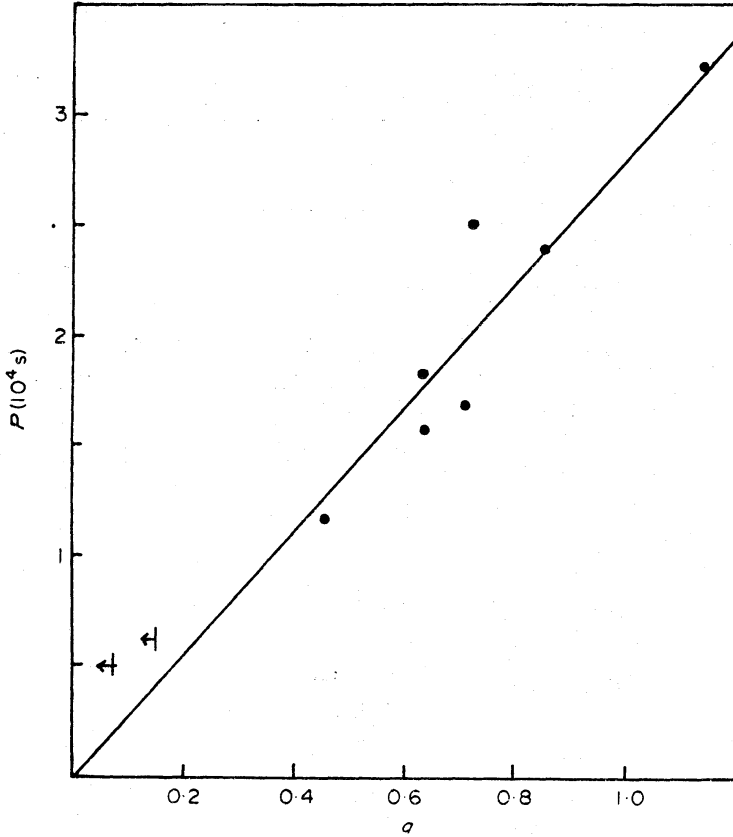


FIG. 3. Variation of mass ratio q with orbital period. Arrows indicate upper limits.

(5) on q means that we may write $R_2^3 \propto M_2 P^2$; on the main sequence in the vicinity of $1 M_\odot$ we have $R_2 \propto M_2$; hence $M_2 \propto P$ which leads to $M_1 \approx \text{const.}$ if $P \propto q$.

Two entries in Table III deserve special attention. The masses of the components of DQ Her derived by the method used in this paper are ~ 6 times greater than those advocated by Kraft (1963). However, Kraft based his estimates on the assumption that the 71-s pulsations of the primary in DQ Her arise from radial oscillations, which require $M_1 = 0.12 M_\odot$. The recent demonstration (Warner *et al.* 1972) that the pulsations arise from non-radial oscillations removes any strong constraints on the mass of the primary.

The other object to be noted is U Gem. The separation of the emission components in this star gives $v \sin i = 580 \text{ km s}^{-1}$ (Kraft 1962). Our estimate of the total width of the emission lines, made from the original spectrograms kindly loaned by Dr Kraft, gives $v \sin i = 850 \text{ km s}^{-1}$. The former value would give $q = 3.0$, a value totally at variance with the mass ratios found for other CVs. Our value of $q = 1.5$ is in agreement with that found by Warner & Peters (1972) from theoretical considerations of the position of the bright spot in U Gem. As can be seen from Table III, however, even $q = 1.5$ leads to U Gem being the only object which does not fall close to the relationship shown in Fig. 3. We are currently engaged in accumulating photometric observations with high time resolution of eclipses of U Gem, and will analyse these in conjunction with a complete re-measurement of Kraft's spectrograms, in order to produce a more reliable model of U Gem.

Because of the insensitivity of equation (5) to the value of q , the estimates of M_2 given in Table III are essentially independent of K_1 and $v \sin i$. Therefore the accuracy of the values of M_1 depend almost entirely on the reliability of the estimates of q . With a typical observational uncertainty of ~ 15 per cent in the values of $K_1/v \sin i$ we find (from Fig. 1) that q will be uncertain by ~ 20 per cent. The values of M_1 given in Table III are therefore uncertain by $\pm 0.2 M_\odot$. As a result, with the possible exception of U Gem, the spread in masses may not be real and it is possible that all the white dwarf primaries have masses close to $1.2 M_\odot$.

4. DISCUSSION

The white dwarf masses derived in this paper, with the possible exception of U Gem, all lie just below the Chandrasekhar limiting mass for degenerate stars. Thus the primaries of CVs have masses some three times greater than those of isolated white dwarfs (Warner 1972). Two possible explanations may be sought for this significant result.

On the one hand we know that the lobe-filling secondaries of CV systems are transferring mass to the primaries. If the primaries start out as $\sim 0.4 M_\odot$ white dwarfs, and if they are initially dwarf novae (for which there is no observational evidence for mass ejection during eruptions), then they will retain all of their accreted material (converting the hydrogen into helium during the outbursts). We might then hypothesize that it is not until an accreting white dwarf approaches the Chandrasekhar limit that the outbursts are sufficiently energetic to cause mass ejection in the form of classical novae, this producing a natural limiting process that prevents the stars actually attaining the critical mass. Against this must be levelled the fact that Table III contains mostly dwarf novae in which the primaries are close to the limiting mass, which would appear statistically unlikely. Also, in this picture, assuming that the binary orbital periods decrease with age, there should be a correlation between orbital period and type of outburst (the dwarf novae having longer orbital periods than the classical novae). One of the striking features of CVs is that no such correlation exists (Mumford 1967).

The second suggestion, which appears more probable, is that the process of formation of white dwarf stars is greatly altered by the presence of a close companion, leading to more massive objects. Detailed study of the evolution of close binaries, especially the contact systems (W UMa stars), will be necessary to test this hypothesis.

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