On the maximum $\mathcal{X}$ entropy negation of a complex-valued distribution

Fuyuan Xiao, *Member, IEEE*

***Abstract*—In this paper, we propose a generalized model of the negation function, so that it can has more powerful capability to represent the knowledge and uncertainty measure. In particular, we first define a vector representation of complex-valued distribution. Then, an entropy measure is proposed for the complex-valued distribution, called $\mathcal{X}$ entropy. After that, a transformation function to acquire the negation of the complex-valued distribution is exploited. Finally, we verify that the proposed negation method has a maximal entropy.**

***Index Terms*—Knowledge representation, Uncertain information, Negation function, Complex-valued distribution, $\mathcal{X}$ entropy, Uncertainty measure, Decision-making.**

I. Introduction

In this paper, we propose a generalized model of the negation function, so that it can has more powerful capability to represent the knowledge and uncertainty measure. In particular, we first define a vector representation of complex-valued distribution. Then, an entropy measure is proposed for the complex-valued distribution, called $\mathcal{X}$ entropy. After that, a transformation function to acquire the negation of the complex-valued distribution is exploited on the basis of the newly defined $\mathcal{X}$ entropy. Finally, we verify that the proposed negation method has a maximal entropy.

II. Vector representation of complex-valued distribution

Definition 1 *(Complex-valued distribution vector)*

Let $\mathbb{C}_k$ be a complex-valued distribution (CvD) vector on the space $\Psi = \{\psi_1, \psi_2, \ldots, \psi_j, \ldots, \psi_n\}$, denoted by

$$\mathbb{C}_k = [c_{k1}, c_{k2}, \ldots, c_{kj}, \ldots, c_{kn}], \tag{1}$$

where c_{kj} represents the complex value of the occurrence of ψ_j, denoted by

$$c_{kj} = x_{kj} + y_{kj} i, \tag{2}$$

where x_{kj} and y_{kj} are real numbers and i is the imaginary unit, satisfying $i^2 = -1$.

In Eq. (2), for c_{kj}, it satisfies the conditions:

$$\begin{aligned} & x_{kj} \geq 0, \\ & \sqrt{x_{kj}^2 + y_{kj}^2} \in [0, 1], \\ & \sum_{j=1}^{n} c_{kj} = 1. \end{aligned} \tag{3}$$

Corresponding author: Fuyuan Xiao.

F. Xiao is an Associate Professor at the School of Computer and Information Science, Southwest University, No.2 Tiansheng Road, BeiBei District, Chongqing, 400715, China (e-mail: xiaofuyuan@swu.edu.cn).

Definition 2 *(The inner product between CvDs)*

Let $\mathbb{C}_k$ and $\mathbb{C}_h$ be two complex-valued distribution vectors on the space Ψ. The inner or dot product of $\mathbb{C}_k$ and $\mathbb{C}_h$ is defined by

$$\langle \mathbb{C}_k, \mathbb{C}_h \rangle = \mathbb{C}_k \cdot \mathbb{C}_h = \sum_{j=1}^{n} c_{kj} \bar{c}_{hj}. \tag{4}$$

Then, the norm of the complex-valued distribution vector is defined as

$$\|\mathbb{C}_k\| = \sqrt{\langle \mathbb{C}_k, \mathbb{C}_k \rangle}. \tag{5}$$

III. Entropy of the complex-valued distribution

Definition 3 *(Entropy for CvD)*

Let $\mathbb{C}_k$ be a complex-valued distribution vector on the frame of discernment Ψ. The entropy of $\mathbb{C}_k$, denoted as $\mathcal{X}(\mathbb{C}_k)$ is defined as

$$\mathcal{X}(\mathbb{C}_k) = 1 - \frac{\|\mathbb{C}_k\|^2}{3} = 1 - \frac{\sum_{j=1}^{n} |c_{kj}|^2}{3}, \tag{6}$$

where

$$0 \leq \mathcal{X}(\mathbb{C}_k) \leq 1 - \frac{1}{3n}. \tag{7}$$

IV. Negation of a complex-valued distribution

Definition 4 *(The negation of the CvD)*

The negation of the complex-valued distribution is defined by

$$\overline{\mathbb{C}}_k = [\bar{c}_{k1}, \ldots, \bar{c}_{kj}, \ldots, \bar{c}_{kn}], \tag{8}$$

where

$$\bar{c}_{kj} = \frac{1 - c_{kj}}{n - 1}, \tag{9}$$

which satisfies:

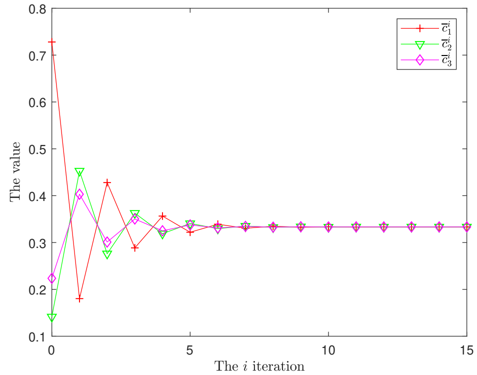
$$\sum_{j=1}^{n} \bar{c}_{kj} = \frac{1}{n-1} \sum_{j=1}^{n} (1 - c_{kj}) = 1. \tag{10}$$

When $\mathbb{C}_k$ turns into a probability distribution, the proposed negation method degrades into Yager's [1] negation method.

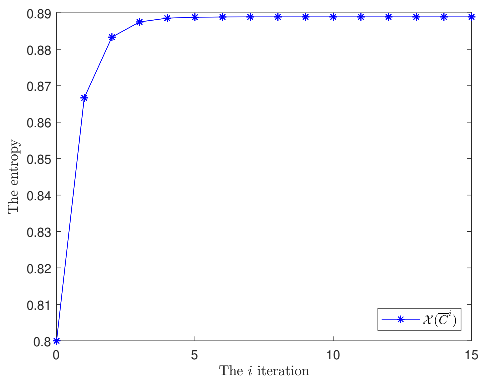
V. On the view from the entropy

Example 1 Assume there is a CvD $\mathbb{C} = [0.7 - 0.2i, 0.1 + 0.1i, 0.2 + 0.1i]$.

Fig. 1(a) shows the variation of $\bar{c}_j^i$ with i times negation iteration. Meanwhile, Fig. 1(b) shows the entropy measure of $\overline{\mathbb{C}}^i$. These results verify that the proposed negation method has a maximal entropy.



(a) The variation of $|\overline{c}_j^i|$.



(b) The variation of $\mathcal{X}(\overline{\mathbb{C}}^i)$.

Fig. 1. The measure of entropy in Example 1.

VI. Conclusions

In this paper, a generalized model of negation function was proposed for complex-valued distributions. In particular, when the complex-valued distributions turn into probability distributions, the proposed negation method degraded into the traditional one.

References

[1] R. R. Yager, "On the maximum entropy negation of a probability distribution," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 5, pp. 1899–1902, 2014.