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On the Mean-Field and Classical Limits of Quantum Mechanics

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Abstract

Consider the N-body Schrödinger equation, with particle interaction given by a real-valued, even potential V such that ∇V is Lipschitz continuous. We assume a mean-field scaling, i.e. that the potential is scaled by a coupling constant of order $1/N$. We are concerned with the large N limit of this problem, assuming that the action of the typical particle is of order $\gg \hbar$. In other words, we seek to study the asymptotic behavior of the N-body Schrödinger equation in the mean-field (large N) and classical (small $\hbar$) limits simultaneously. Following Dobrushin's ideas for the mean-field limit in classical mechanics, we formulate both limits in terms of an estimate for a functional which can be viewed as a quantum analogue of the Monge-Kantorovich quadratic distance between phase-space probability densities. This functional controls the Monge-Kantorovich quadratic distance between the classical densities defined in terms of their quantum counterparts by the Husimi transform. We prove that the large N (mean-field) limit of the N-body linear Schrödinger equation leading to Hartree's nonlinear, mean-field equation for the single-particle density is uniform as $\hbar \to 0$. As an obvious consequence of this uniformity, the Vlasov equation with Lipschitz interaction force can be derived rigorously from the N-body Schrödinger equation in the limit as $N \to \infty$ and $\hbar \to 0$ jointly.

The results presented in this talk have been obtained in a joint work with Clément Mouhot and Thierry Paul [`arXiv:1502.06143`, to appear in Comm. Math. Phys.].