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## ARTICLES

# ON THE MEASUREMENT OF TOTAL FACTOR PRODUCTIVITY: A LATENT VARIABLE APPROACH

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Despite the important role that total factor productivity (TFP) has played in the growth literature, few attempts have been made to change the methodology to estimate it. This paper proposes a methodology based on a state-space model to estimate TFP and its determinants. With this methodology, it is possible to reduce the measurement of our ignorance. As a by-product, this estimate yields the capital share in output and the long-term growth rate. When applied to Chile, the estimation shows a capital share around 0.5 and long-term growth of TFP around 1%. Capital accumulation tends to explain the growth rate in the fast growth periods under the econometric estimation more than the traditional growth accounting methodology.

**Keywords:** Productivity Measurement, Latent Variables

## 1. INTRODUCTION

Total factor productivity (TFP) has played a central role in the discussion of empirical growth. Since Solow (1957), many studies have tried to measure the contributions of production factors and technology to economic growth. Despite several criticisms, this methodology has experienced a revival, according to Klenow and Rodríguez-Clare (1997), and it has been widely used in the past 15 years, not only to decompose the growth rate of output per worker, but also to explain cross-country differences in income per capita.

Although the growth accounting methodology does not provide an explanation for economic activity, it is a simple first step in the search for the sources of growth. This line of research has provided strong evidence that TFP growth is an important source of overall growth [Easterly and Levine (2001); Bosworth and Collins (2003)]. Others have used this methodology to show that TFP level

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could explain a large part of the cross-country differences in per capita GDP [e.g., Mankiw et al. (1992); Hall and Jones (1999)].

But what is TFP after all? The first idea that comes to mind is technological progress.<sup>1</sup> This concept rests on models of endogenous technological change that explain growth on the grounds of endogenous technological progress [Romer (1990); Aghion and Howitt (1998)]. However, negative TFP growth for several years should not originate in negative technological progress. Therefore, many other explanations have come up, such as cost reduction or efficiency gains [Harberger (1998)], externalities and increasing returns [Romer (1986); Lucas (1988)], or policies favoring the adoption of new technologies [Parente and Prescott (1994); Prescott (1997)].

The importance of this methodology in the empirical growth literature is undeniable; however, there are a couple of caveats. First, TFP may be hiding measurement errors on factors of production. The growth rate of TFP is measured as a residual calculated as the difference between the growth rate of output and the combined growth rates of capital and labor weighted by their respective output-input elasticities. Therefore, the methodology imposes the need for an accurate measure of output, capital, labor, and the capital and labor shares<sup>2</sup> in total output. If improvements in the quality of labor and capital are not included as part of the measurement of these factors, then the contribution of TFP will be overestimated. Second, TFP and production factors will be moving together. For instance, TFP will increase the marginal productivity of capital, affecting capital accumulation. In this way, the contribution of TFP to growth will be understated, whereas the contribution of capital accumulation will be overestimated.

Given the importance of this variable for understanding growth, the efforts in the literature have concentrated on finding better ways to measure the factors of production, including corrections for quality of factors [Jorgenson and Griliches (1967); Greenwood and Jovanovic (2000)] and factor utilization [Costello (1993)], and better measures of labor share [Gollin (2002)] and the relationship between factor accumulation and TFP [Klenow and Rodriguez-Clare (1997)]. However, TFP is still computed as a residual, but not in the regression sense. In the methodology proposed here, the estimated residual is orthogonal to the variables included in the regression model.

This paper works with a natural methodology to estimate TFP using annual data from 1960 to 2005 for the Chilean economy. As with any unobservable, a latent-variable approach seems appropriate. The state-space model used here also allows estimation of the effect of several of the determinants proposed in the literature on TFP growth. In one step, using maximum likelihood estimation, it is possible to estimate TFP, disentangling the effect of its determinants and a residual source of growth. This last component is the true measure of our ignorance (the unexplained part of TFP growth).

Section 2 presents the empirical model and the methodology that motivates the empirical approach, with a brief discussion of the determinants. This model is applied to Chilean time series data that are readily available. Section 3 presents the results and compares them with previous studies. Section 4 concludes.

## 2. METHODOLOGY

This section explains the alternative methodologies for estimating TFP. It presents the basic growth accounting approach, followed by some examples of the econometric estimation of TFP in the existing literature. It also proposes two possible specifications for the state-space model. TFP is modeled first as a pure AR process—as in the real business cycle (RBC) literature—and second as a function of exogenous variables in addition to lags of TFP.

### 2.1. Growth Accounting

Traditionally, growth accounting started with an aggregate neoclassical production function that exhibited positive and decreasing marginal productivity of all factors and constant returns to scale and satisfied Inada's conditions. Let the production function be written as

$$Y_t = F(K_t, h_t L_t, Z_t),$$

where  $Y$  represents total output,  $K$  physical capital,  $L$  raw labor,  $h$  human capital, and  $Z$  a TFP index. In this setting,  $Z$  could be interpreted as another factor of production that in the neoclassical framework is not remunerated. The traditional decomposition can be written as

$$\Delta \ln Y_t = \alpha_t \Delta \ln K_t + (1 - \alpha_t) \Delta \ln h_t L_t + \Delta \ln Z_t, \quad (1)$$

where  $\alpha$ —under perfect competition and profit maximization—is the share of capital cost in total revenues. Note that if labor is not corrected for human capital, the residual will be the change in  $Z$  plus the contribution of human capital.

In the case of a Cobb–Douglas production function, we can write the neoclassical production function as

$$y_t = Z_t k_t^\alpha = e^{z_t} (1 + \gamma)^t k_t^\alpha, \quad (2)$$

where  $y$  and  $k$  represent output and capital per worker (adjusted by human capital), respectively. Variable  $Z$  is modeled as a trend stationary variable that could be associated directly with a technological level plus a technological shock,  $z_t$ , which could be modeled as an AR( $q$ ) process. However, in equation (1), the Solow residual  $\Delta \ln Z$  will include technological change not involved in either capital or labor, the efficiency gains or losses that have not affected the marginal productivity of capital or labor, and the paths of these production factors. In this setting, TFP is explained by a deterministic trend that captures the technological change plus other variables capturing distortions<sup>3</sup> and supply shocks—for example, terms of trade shocks in the case of an open economy. If that is the case, we need to modify equation (2) to include those determinants.

Given that usually capital is not corrected for quality, the residual should include a measure of capital quality. However, Greenwood and Jovanovic (2000) suggested that the price of investment goods relative to consumption goods could be used as a proxy for the quality of capital.<sup>4</sup> Their idea is that technological progress is

embodied in the latest vintages of capital. So the observed decline in the price of capital goods relative to consumption goods in the postwar period is consistent with the idea of obsolescence of old capital due to the arrival of a new generation. However, this observation is also consistent with a capital good of the same quality but produced at less cost. If this technological change (which is specific to investment goods) is not captured in the measurement of capital, it will be implicitly included in the variation of TFP.

## 2.2. Econometric Estimation of Total Factor Productivity

But why estimate TFP by considering a deterministic aggregate production function, as in the growth accounting approach? The neoclassical production function represents the maximum output that could be obtained from a given combination of inputs; however, at the aggregate level, there could be several omitted “factors” that would make it impossible to achieve the production frontier. Examples of these excluded variables are adjustment costs for intersectoral reallocation of resources and diffusion of technology. The effect of these potentially omitted variables must then be captured by a stochastic disturbance term in the aggregate production function.

Starting from equation (1), and assuming that the TFP index  $Z$  takes the form of an exponential time trend, the stochastic function for growth in product per worker is given by

$$\Delta \ln y_t = \gamma + \alpha \Delta \ln k_t + \varepsilon_t,$$

where the constant term stands for the average TFP growth rate.

From a statistical standpoint, in order to allow for a changing productivity growth rate, Harvey et al. (1986) consider a stochastic trend for productivity, modeled as a structural time series, where the constant and slope of the trend are allowed to change over time, governed by a Markovian process. With a similar approach, French (2001) implements several alternative trend-cycle decomposition methods to extract the trend in TFP from the Solow residual, including the possibility of switching regimes to capture discrete changes in the estimated trend growth of TFP.

On the other hand, always considering productivity and technology as a latent variable, several studies apply the Kalman filter to estimate the rate and direction of change in technology at the micro level. Examples are Slade (1989) for the U.S. primary-metals industry and Esposti and Pierani (2000) for Italian agriculture; both use a state-space representation derived from a factor-demand system. However, the former decomposes TFP as a stochastic trend plus cyclical components, allowing correction of TFP estimates for measurement errors that induce procyclical bias.<sup>5</sup> In the latter paper, TFP growth is approximated as a stochastic trend, and the nonconventional inputs generating technical change are formally specified (R&D, human capital accumulation, spillover effects, etc.). In a related approach, Chen and Zadrozny (2009) consider—for U.S. manufacturing industries—capital and

TFP as latent variables determined as joint endogenous processes. Their method implies specifying a dynamic economic model for a representative firm<sup>6</sup> in the industry, and then obtaining Kalman-smoothed estimates of unobserved capital and TFP for the sample period.

Another possibility is the one followed by Fuentes et al. (2006) and Chumacero and Fuentes (2006), where TFP (in log) is computed by means of growth accounting in a first step, and then a regression of TFP on exogenous variables and lags of TFP is estimated in a second step. Although intuitively appealing, a wrong estimate of TFP from the first step could invalidate the statistical results and conclusions obtained from the second-step regression.

**2.3. A State–Space Model for Total Factor Productivity**

The state-space model is a useful tool to represent a dynamic system involving unobservable variables.<sup>7</sup> However, as far as we know, this method has never been applied to the simultaneous estimation of the GDP-TFP system.

Let the linear Gaussian state-space representation be defined as follows.

Signal equation:

$$\Delta \ln y_t = \Delta \ln Z_t + \alpha \Delta \ln k_t + \varepsilon_t, \tag{3}$$

where  $y$  is observable per worker GDP,  $k$  is the observable capital/labor ratio, and  $Z$  is the unobservable TFP. If we assume that the TFP growth is also determined by exogenous variables, then we have, uncorrelated with each other,

State equation,<sup>8</sup>

$$A(L)\Delta \ln Z_t = \gamma + \beta' (\Delta \ln X_t) + u_t, \tag{4}$$

and white noise disturbances,

$$\begin{pmatrix} \varepsilon_t \\ u_t \end{pmatrix} \sim NID \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_u^2 \end{pmatrix} \right], \tag{5}$$

with  $A(L)$  being a  $q$ -order polynomial on the lag operator  $L$ , and  $X$  a matrix of observable exogenous variables determining the TFP growth rate specified later.

The Kalman filter is an updating algorithm for the linear projection of the state vector (latent variables) based on observable variables that allow writing down—under the normality assumption—the likelihood function of the model based on the prediction error decomposition. Once the likelihood function is obtained, the coefficients are estimated by numerical optimization methods. In addition, a smoothed state-vector estimate for the full sample can be obtained if the values of the latent variables are of interest and permit a structural interpretation.

The filter requires meaningful startup conditions (parameter values) to achieve convergence. This is not a minor issue, given the potentially high nonlinearity in the likelihood function for the state-space representation of the GDP-TFP model.

A possible set of initial conditions can be extracted from a regression of the Solow residual (computed using growth accounting) on exogenous or predetermined variables (all as rates of change).

In order to see the results of this methodology, we apply it to Chilean annual data (1960–2005) used by Chumacero and Fuentes (2006). They analyze, from the empirical perspective, which variables are correlated with TFP and per capita GDP for Chile. The variables included in their study are terms of trade, government expenditures over GDP, the price of investment relative to consumption, and the inflation rate divided by one plus the inflation rate. These variables correspond to our matrix  $X$  in equation (4). Our one-step approach, compared to the approach of Chumacero and Fuentes (2006), has the advantage of producing more accurate estimates of the coefficients in the state equation, as well as reducing the measurement error for TFP growth coming from the use of the standard Solow residual methodology.

After the empirical characterization, Chumacero and Fuentes (2006) develop a stylized stochastic general equilibrium model that captures the main characteristics of this economy and simulate impulse-response functions that match the statistical results found in the empirical analysis. Their model assumes an economy inhabited by an infinitely lived representative agent that consumes leisure and an importable good. The production side is composed of two sectors; one produces the importable good (using a Cobb–Douglas production function with a stationary productivity shock) and the other (an endowment sector) produces the exportable good. This economy sells the exportable good abroad, taking the terms of trade as given. In this economy, a government levies taxes on capital and labor income and spends them as lump-sum transfers to the private sector plus a “loss” in government expenditures that does not provide services to the inhabitants. The government’s budget is always in equilibrium. The model does not have an analytical solution. Nevertheless, we can say that it features an open economy, a government that suffers losses in its expenditures, and technological improvement, à la Greenwood and Jovanovic (2000), and makes per capita GDP and TFP a function of terms of trade, government consumption (as a proxy for distortions), and the price of capital goods relative to consumption goods ( $p$ ). Additionally, we control in the empirical part for macroeconomic stability ( $\text{inf} \equiv \pi_t / (1 + \pi_t)$ , where  $\pi_t$  stands for inflation rate).

In the next section we estimate two systems, a pure AR( $q$ ) model composed of equations (3) and a special case of (4), without the  $X$  matrix of exogenous variables (which are those considered in Chumacero and Fuentes 2006), and the augmented model described by (3) and (4), using the Kalman filter algorithm.

### 3. ESTIMATION RESULTS

The sample includes annual observations for the Chilean economy from 1960 to 2005. This database is an extended sample of the one used by Fuentes et al. (2006), and by Chumacero and Fuentes (2006).

**TABLE 1.** Results of Kalman filter estimation

	AR( $q$ )	Augmented
Capital per worker	0.463 (0.139)	0.520 (0.054)
Constant	0.002 (0.004)	0.011 (0.028)
$\Delta \ln Z_{t-1}$	0.483 (0.230)	0.247 (0.089)
$\Delta \ln Z_{t-2}$	-0.635 (0.118)	-0.667 (0.081)
$\Delta \ln Z_{t-3}$	0.855 (0.189)	
$\Delta \ln T_t$		0.179 (0.012)
$\Delta \ln T_{t-1}$		0.093 (0.030)
$\Delta \ln g_{t-1}$		-0.108 (0.044)
$\Delta \ln \inf_t$		-0.013 (0.009)
$\Delta \ln \inf_{t-1}$		0.020 (0.009)
$\Delta \ln p_{t-1}$		-0.522 (0.044)
$\Delta \ln p_{t-2}$		0.524 (0.016)

*Note:* Standard errors in parentheses. The long-run coefficients for  $\Delta \ln(\inf)$  and  $\Delta \ln(p)$  are not statistically significant.

The analysis of time graphs (see the Appendix) for all the growth series tells us that nonstationarity should not be a concern, even though all of them present significant variability and particularly the price of capital to consumption goods exhibits significant autocorrelation.

Table 1 presents the estimated coefficients for the two alternative state-space representations proposed above. The included explanatory variables for the “augmented model” are selected according to a general-to-specific approach, based on individual significance, considering up to two lags of each explanatory variable in the state equation. To select the lag order for both models, the Akaike information criterion (AIC) is minimized. The likelihood function is maximized using the Berndt–Hall–Hall–Hausman (BHHH) algorithm, under the assumption of Gaussian errors. Starting values for the parameters come from the regression of the Solow residual on the independent variables considered for each state-space model. In addition, we compare growth accounting and Kalman filter estimates for the augmented model, as well as the estimates from the two alternative state-space specifications, for TFP growth and level index.

From equation (3), we obtain an estimate for the capital–output elasticity shown in the first row of Table 1. The coefficient obtained is 0.46 for the autoregressive model and 0.52 for the augmented model. These values seem a bit high compared with Gollin (2002) and those used in theoretical models. However, they are consistent with the share of capital in total income in the Chilean National Accounts (0.51) and with the one used by Elias (1990) for the case of Chile (0.52).

The constant reported in Table 1 corresponds to the estimate for parameter  $\gamma$  of state equation (4). Note that  $\gamma/A(L)$  is the estimator of the long-term growth rate. The values are similar for the AR( $q$ ) model (0.7%) and the augmented model (0.8%), and both are similar to those estimated using the growth accounting methodology for Chile.<sup>9</sup>



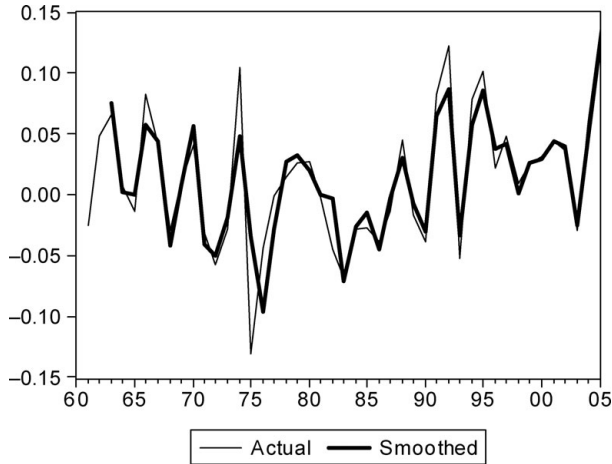


FIGURE 1. Growth of GDP per worker (actual vs. smoothed-augmented).

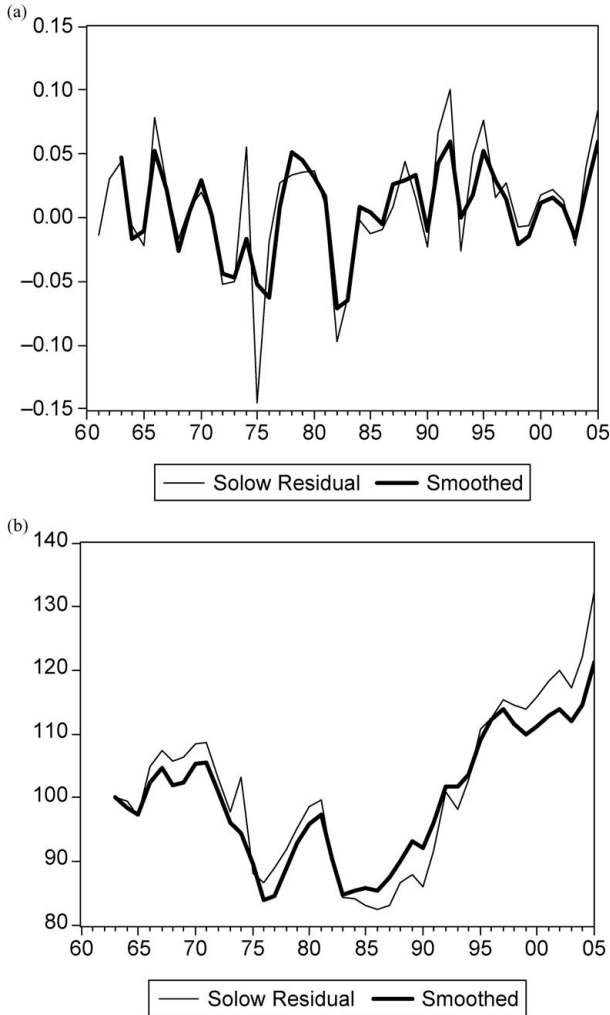
The coefficients of the explanatory variables have the expected signs according to the theoretical model and the simulations in Chumacero and Fuentes (2006). The long-run effect of the terms of trade on growth rate is positive, whereas our proxy for distortions (government consumption) has a negative effect.<sup>10</sup> On the other hand, the cumulative effects of macroeconomic stability and price of investment relative to consumption are both statistically equal to zero.

In terms of goodness of fit, the  $R^2$  for the  $AR(q)$  model is .52 whereas the augmented model exhibits a coefficient of .82. The significantly better fit for this latter model gives support to the idea of TFP including additional components beyond just technological progress, as usually assumed in real business cycle theories.

Figure 1 depicts the growth rate of actual GDP per worker and the Kalman smoothed GDP per worker growth (obtained by using all the sample data to smooth the Kalman filter estimate of the unobserved TFP growth). The smoothed series closely follows the actual series, but has difficulties matching the extreme realizations of the growth rate.

Figure 2a compares the TFP growth rate from the state-space model with the one computed using growth accounting. Even though both series move closely together, they depart from each other at all the spikes in the growth rate of TFP series.

Figure 2b presents the level of TFP constructed using the growth rates estimated from the Solow residual and from the fitted value of the Kalman filter (smoothed). The two series tend to move together, but with some important differences. Compared to growth accounting TFP, the smoothed series goes below in the 1960s and 1970s, above from the mid-1980s to the mid-1990s, and then below again. As our series is smoother, it does not exhibit spikes such as the one in 1974, where the



**FIGURE 2.** (a) Estimated TFP growth and (b) estimated TFP index (growth accounting vs. smoothed-augmented).

TFP were declining and the Solow residual-based series shows a spike that has little or no justification. According to the growth accounting methodology, TFP fell very hard in the 1982–1983 crises and did not begin to recover until 1987, whereas with the new estimate the recovery seems to have started slowly in 1984, right after the crisis.

Figures 3 and 4 compare the series estimated under the  $AR(q)$  model and the augmented model. Given the structure of the lags in the  $AR(q)$  model, the growth rate and the level of TFP have a peculiar evolution over time. The growth rates

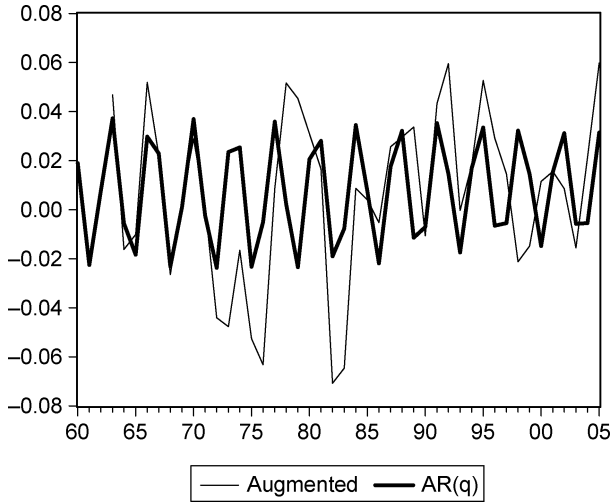


FIGURE 3. Estimated TFP growth (smoothed-AR( $q$ ) vs. smoothed-augmented).

from the AR( $q$ ) model fluctuate a lot less and the level of the TFP always goes above the one obtained with the augmented model. The series from the AR( $q$ ) model does not follow the cyclical pattern observed in the TFP series obtained with the augmented model or with growth accounting. On the other hand, the series from the augmented model follows the cycle better, because it includes variables like terms of trade that are very procyclical.

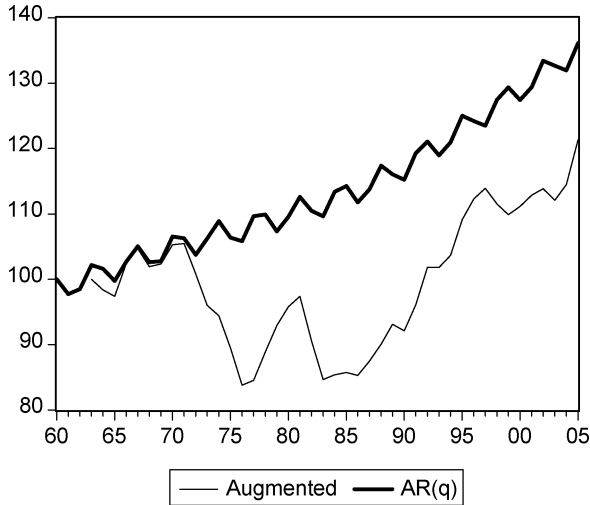


FIGURE 4. Estimated TFP index (smoothed-AR( $q$ ) vs. smoothed-augmented).

**TABLE 2.** Descriptive statistics for estimated TFP growth

	Solow residual	Smoothed-augmented	Smoothed-AR( <i>q</i> )
Mean	0.0086	0.0061	0.0078
Median	0.0135	0.0087	0.0075
Maximum	0.1008	0.0598	0.0372
Minimum	-0.1452	-0.0707	-0.0237
Std. dev.	0.0468	0.0345	0.0210

Table 2 summarizes the descriptive statistics of all the measures and the correlation for the growth rate. The mean and the median of the growth rate obtained with the Solow residual are higher than the other two, but also more volatile.

Tables 3 and 4 present the decomposition of GDP growth, identifying the contributions of capital, labor, and total factor productivity implied by growth accounting versus the state-space augmented model proposed above.

It is important to mention that the reasons for the differences in alternative decompositions are threefold.

First, the state-space representation allows the direct econometric estimation of the elasticity of capital, instead of imposing it to compute the Solow residual.

Second, for the signal equation, the econometric estimation gives a residual that is orthogonal to the explanatory variables in the model, including the determinants of TFP growth. This is an important improvement over the growth accounting methodology, where the Solow residual includes all that is not related to growth in the production factors. These two elements above imply a significant reduction of the measurement error for TFP growth. The relationship between the TFP growth from the growth accounting and the proposed state-space approach is given by

$$\Delta \ln Z_t^* = \Delta \ln Z_t + [(\alpha - \alpha^*) \Delta \ln k_t + \varepsilon_t] = \Delta \ln Z_t + v_t,$$

**TABLE 3.** Factor contribution to total growth

	Growth accounting				Augmented			
	GDP growth	Share of <i>K</i>	Share of <i>Lh</i>	Share of TFP	Share of <i>K</i>	Share of <i>Lh</i>	Share of TFP	Share of residual
1963–2005	3.87%	1.47%	1.53%	0.86%	1.92%	1.23%	0.62%	0.11%
1963–1973	3.07%	1.46%	1.36%	0.25%	1.90%	1.09%	0.10%	-0.02%
1974–1989	2.90%	0.92%	2.49%	-0.51%	1.20%	1.99%	-0.11%	-0.18%
1990–2005	5.39%	2.04%	0.70%	2.66%	2.64%	0.56%	1.70%	0.49%
1990–1997	7.35%	2.12%	1.69%	3.54%	2.76%	1.35%	2.58%	0.66%
1998–2005	3.43%	1.95%	-0.28%	1.77%	2.53%	-0.23%	0.82%	0.31%
2003–2005	5.05%	1.93%	-0.30%	3.42%	2.51%	-0.24%	2.19%	0.60%

**TABLE 4.** Factor contribution to total growth (in percent)

	GDP growth	Growth accounting			Augmented			
		Share of <i>K</i>	Share of <i>Lh</i>	Share of TFP	Share of <i>K</i>	Share of <i>Lh</i>	Share of TFP	Share of residual
1963–2005	3.87	38.1	39.6	22.3	49.5	31.7	15.9	2.9
1963–1973	3.07	47.7	44.2	8.1	61.9	35.4	3.2	−0.5
1974–1989	2.90	31.8	85.8	−17.6	41.3	68.7	−3.9	−6.1
1990–2005	5.39	37.7	13.0	49.3	49.0	10.4	31.5	9.1
1990–1997	7.35	28.9	22.9	48.2	37.5	18.3	35.1	9.0
1998–2005	3.43	56.7	−8.2	51.5	73.7	−6.6	23.8	9.1
2003–2005	5.05	38.2	−5.9	67.7	49.6	−4.7	43.3	11.8

where (\*) denotes figures related to the growth accounting methodology. Measurement error could explain a large amount of growth that would be attributed to TFP if we followed the traditional approach.

Third, the advantage of the proposed approach over the two-step methodology used by Chumacero and Fuentes (2006) has to do with a more efficient estimation of the coefficients for the explanatory variables in the state equation, given the heteroskedasticity exhibited by the disturbance term when the Solow residual is used as the dependent variable for the second-step regression

$$A(L)\Delta \ln Z_t^* = \gamma + \beta' (\Delta \ln X_t) + [u_t + v_t] = \gamma + \beta' (\Delta \ln X_t) + u_t^*.$$

The OLS estimation of the second-step regression will give inefficient estimates of the coefficients, given that the variance of the error term grows with capital and with the estimation error for the output elasticity of capital.

In summary, the augmented state-space model, by separating TFP growth from the residual term, provides an additional “source” of growth, which is really the unexplained part of growth (just good or bad fortune). In this way, we could say that this methodology reduces the “measurement of our ignorance.”

Under the Kalman filter estimation, periods of high growth are also characterized by significant contributions from both capital and TFP. However, the contribution from the fortune factor is also significant during those periods.

#### 4. CONCLUDING REMARKS

Growth accounting methodology allows computing TFP as a residual from the difference between growth in output and growth in inputs. The problem with this approach is that TFP includes measurement error in the production factors and omitted explanatory variables of growth, such as technological change and efficiency improvement. This paper proposes the use of a state-space model to estimate the level of TFP, its long-term growth rate and other relevant parameters such the capital–output elasticity. The advantage of the proposed methodology is

the depuration of TFP between observed factors and a true measure of our ignorance. For this reason, the augmented model that includes explanatory variables gives meaningful results from an economic point of view. This methodology could be extended to test any model that attempts to explain TFP growth.

The state-space model is applied to Chilean data available from previous studies. We find that the capital share in total income is around 0.5, which is consistent with the National Account value. The decomposition of growth under this methodology—compared to the growth accounting results—shows that the importance of capital in explaining growth increases, the importance of TFP declines, and a small part of the growth rate is due to an additional factor, which is the measure of our ignorance.

## NOTES

1. Another interpretation for TFP comes from the real–business cycle literature, where this parameter is associated with technological shocks that drive the cycle. Nevertheless, they do not take growth accounting to estimate TFP; they rather assume that this variable has a trend plus stationary shocks.

2. If the production function is homogenous of degree 1, the output-input elasticity corresponds to the share of each factor cost in total revenue. Under the same assumption, capital share will be equal to 1 minus labor share.

3. Parente and Prescott (1994) argue that barriers to adoption explain growth miracles and income disparities. In a sense these barriers are taxes on adoption activities, and therefore we call them distortions, in general.

4. Chumacero and Fuentes (2006) found find that this variable is positively correlated with TFP in the case of Chile. In the next section we extend their database to estimate TFP and to compare the results.

5. This will be the case when trend and cycle go in opposite directions, partially offsetting one another.

6. Solving a dynamic optimization problem, where adjustment costs on capital and technology are derived from a parameterized production function.

7. See Durbin and Koopman (2001) and Hamilton (1994) for discussions of methods and applications.

8. As pointed out by an anonymous referee, it is possible to consider the following version of the state equation, with an AR representation for the error term:

$$\Delta \ln Z_t = \gamma + \beta'(\Delta \ln X_t) + u_t/A(L),$$

with the advantage of imposing over identified restrictions on  $\gamma$  and  $\beta$  and obtaining potentially sharper estimates of these parameters.

9. Fuentes et al. (2006) found a growth rate around 0.7% for the 1960–2005 time period.

10. By long-run effect we mean the sum of current and lagged coefficients of each variable divided by polynomial  $A(L)$  in equation (4).

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## APPENDIX: TIME SERIES GRAPHS OF MODEL VARIABLES

