ON THE MECHANISM OF ACCRETION BY STARS

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Summary

The mechanism of accretion is investigated in detail for the case in which the interstellar material contains a sufficient proportion of molecules to ensure that the temperature is everywhere small. It is found that there is no unique steady-state solution and that the situation at any given time depends on the perturbations suffered by the system. The gravitational interaction between the interstellar material and the stars leads not only to an increase in the masses of the stars but also to a decrease in their peculiar velocities. The importance of these effects to stellar dynamics and to the evolution of the stars is briefly discussed.

1. *Introduction.*—The importance to stellar astronomy of a large rate of accretion of hydrogen by stars is well known. On the basis of certain physical and dynamical assumptions a simple argument was given by Hoyle and Lyttleton * for a rate of accretion according to the formula

$$\frac{4\pi\gamma^2 M^2 \rho_\infty}{V^3}, \tag{1}$$

where M is the mass of the star, γ is the constant of gravitation, V is the velocity of the cloud relative to the star, and ρ_∞ is the density of the cloud at very large distance from the star. The physical assumptions underlying this calculation were later shown to be satisfied † provided an appreciable proportion of the cloud is in molecular form, for in this case the temperature of the cloud remains small throughout the accretion process on account of the radiation of energy by the molecules. The existence of interstellar molecules throughout large tracts of the galaxy has now been confirmed by observation by McKellar and also by Adams.‡ Moreover, McKellar's observations provided direct evidence that the temperature of the interstellar material is far lower than is necessary to justify the assumptions of Hoyle and Lyttleton.† Thus there is important observational confirmation that the accretion problem can be simplified by regarding forces due to gas pressure as being in general small compared with gravitational forces. The object of the present paper is to obtain a rigorous solution of the accretion problem in this simplified form.

In section 2 an outline of the mechanism of accretion is given, and in section 3 the equations governing the steady state of accretion are formulated and discussed. It is found that the steady state equations together with the boundary conditions do not determine a unique solution to the problem, since it can be shown that a group of solutions can be found all giving an accretion rate greater than half the value given by (1). This remarkable property suggests that the situation at any given time must depend on the perturbations suffered by the system, and in section 4 a simple application of perturbation theory shows that perturbations will prevent the rate of accretion dM/dt exceeding the value given by (1). It is concluded, therefore, that the accretion rate at any given time

* *Proc. Cam. Phil. Soc.*, 35, 405, 1939.
† *Proc. Cam. Phil. Soc.*, 36, 424, 1940.
‡ *P.A.S.P.*, 52, 187, 1940; *H.A.C.*, 526, 1940.

must satisfy the inequality

$$\frac{2\pi\gamma^2M^2\rho_\infty}{V^3} < \frac{dM}{dt} < \frac{4\pi\gamma^2M^2\rho_\infty}{V^3},$$

the exact value depending upon the nature of previous perturbations. In section 5 the extreme case of perturbation is considered in which the star initially in empty space enters a cloud of uniform density possessing a plane boundary. The setting up of a steady state in this case is considered, and it is shown that the accretion rate will tend to a value close to

$$\frac{dM}{dt} = \frac{2{\cdot}5\pi\gamma^2M^2\rho_\infty}{V^3}. \tag{2}$$

It is plausible to suppose that (2) gives a lower limit to the accretion rate since it refers to a steady state set up after a particularly violent perturbation.

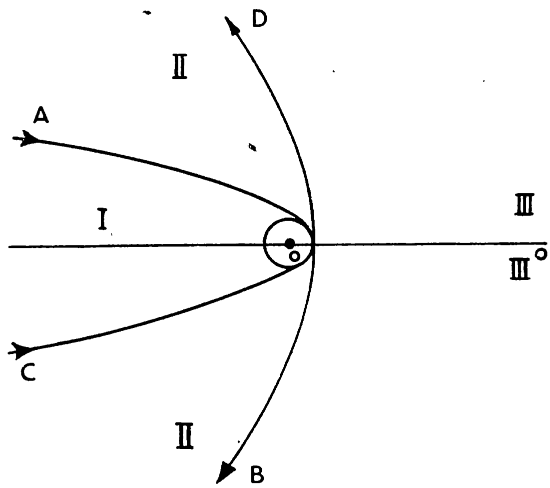

Fig. 1.

In section 6 the rate of change of V due to the drag of the cloud on the star is calculated. The value of dV/dt is found to be so large that the drag due to the cosmical cloud would appear to be of great importance in stellar dynamics, since it provides a mechanism whereby the interstellar material can control the peculiar motions of the stars. Such a mechanism introduces a stabilising influence into galactic dynamics that would appear to be of great significance. Moreover, the decrease of V with time reduces the time-interval required for the mass of a star to be increased by an appreciable factor. It is shown that the mass of a star may be significantly increased in about 1/10th of the period given in previous work.* This result implies that either the time-scale of the evolution of the stars is shortened by a factor of about 10, or that a smaller density of the cosmical cloud is sufficient to maintain the rate of evolution than was previously proposed.

2. In earlier work it was shown that provided the density ρ_∞ is sufficiently large for the mean free path of the particles to be small compared with $\gamma M/V^2$, then the process must be treated as a gas problem. The importance of the collisions between the particles can be seen by comparing the hypothetical case in which the collision cross-section is assumed to be negligibly small with the actual situation where the mean free path is small compared with $\gamma M/V^2$. The former case is illustrated in Fig. 1, in which the trajectories AB and CD are intended to represent the paths of particles just avoiding collision with the surface of the star. Then the paths AB and CD together with all similar trajectories divide space into three regions marked as I, II and III in Fig. 1. All particles moving

* Proc. Cam. Phil. Soc., 36, 325, 1940.

in region I strike the surface of the star. Region II is a single-stream region since there is only one trajectory passing through any point in the region, whilst region III is a multi-stream region since there are two trajectories passing through any point not on the accretion axis oo′ and there is an infinity of paths passing through any point on the axis.

If now the collision cross-section of the particles is restored to the gas-kinetic value, then owing to the fact that the temperature of the gas is small there will be no change in the properties of regions I and II since there is only one trajectory through every point in these regions. On the other hand, in region III particles will collide even though the temperature is small, and these collisions will tend to prevent the occurrence of two streams of material passing through each point not on the axis. Indeed it is clear that the multi-stream region cannot possess a dimension of much more than the mean free path in a direction measured along the trajectories. Now owing to the high density of material near the axis oo′, the mean free path * will then be much shorter than in regions where the density is comparable with ρ_∞. It follows, therefore, that the extension of the multi-stream region in a direction perpendicular to the axis must be very small compared with $\gamma M/V^2$. The multi-stream region must exist, however, since the stream-lines of the material cannot be diverted by gas pressure. Accordingly the mechanism of the

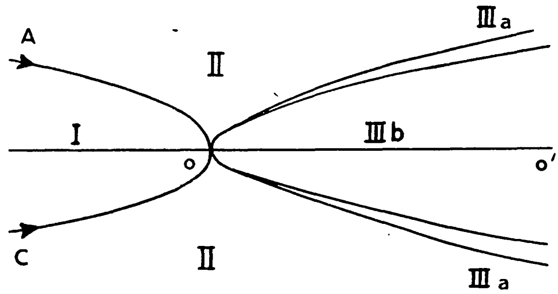

FIG. 2.

accretion process is determined by the four regions I, II, III*a* and III*b* shown in Fig. 2, where the region III*a* is a double-stream region with a thickness of the order of the mean free path and I, II and III*b* are all single-stream regions. The thickness of III*a* is clearly determined by the value of ρ_∞, and for large ρ_∞ this region is effectively a surface of discontinuity. In region III*b* although the temperature is low the pressure is nevertheless high on account of the very high density in this region. This pressure supplies an outward force on region III*a* that must balance the component of momentum transverse to oo′ of material entering III*a* from II. Indeed the transverse dimension of III*b* must adjust itself to make these two forces equal and opposite. The transverse dimension of III*b* accordingly depends on the temperature of the material in III*b* which, although small, is important in determining the size of the region.

3. If the temperature of the material is neglected and the collision cross-section of the particles of the material supposed to be negligibly small, then it is easy to show that the mass of material crossing the axis per unit length per unit time is

$$A = 2\pi\rho_\infty \frac{\gamma M}{V}, \tag{3}$$

whilst at the time the material crosses the axis the component of velocity parallel to the axis is V. Now since the transverse dimension of the region III*b* is small compared with $\gamma M/V^2$ (this must be the case on account of the low temperature due to radiation of energy by the molecules in the material), it is permissible to suppose that when gas-kinetic

* It can be shown that for distances from the axis small compared with $\gamma M/V^2$ the mean free path is proportional to the distance from the axis provided the temperature of the material can be neglected.

collisions take place the amount of material entering the region IIIb is still A per unit length per unit time and that the velocity of this material parallel to the axis is still V. Thus since A and V are independent of r, the only condition impressed on region IIIb that varies with r is the gravitational attraction of the star. The following equations can be set up if we assume that the pressure gradient parallel to the axis can be neglected compared with gravitational forces, and that the velocity v of material in IIIb is uniform over any cross-section and directed parallel to the axis.

$$\frac{d}{dr}(mv)=A \qquad \text{(conservation of mass)}, \tag{4}$$

$$\frac{d}{dr}(mv^2)=AV-\frac{Mm\gamma}{r^2} \qquad \text{(conservation of component of momentum in direction parallel to axis } oo'\text{)}, \tag{5}$$

where mdr is the mass of material in IIIb lying between r and $r+dr$.

The step of neglecting the longitudinal component of the gas-pressure gradient can be justified by noting that equation (4) integrates to give

$$mv=A(r-r_0), \tag{6}$$

where r_0 is a constant of integration that is later shown to be of order $\gamma M/V^2$. Equation (6) shows that the material flows inwards towards the star for $r<r_0$ and outwards to infinity for $r>r_0$. Now since V is the parabolic velocity corresponding to a distance $r=2\gamma M/V^2$, it follows that the time taken for material to move inward to the star from the neighbourhood of r_0 must be of order $\gamma M/V^3$, so that the value of m at $r=0{\cdot}5\, r_0$, say, must be of order $A\gamma M/V^3$. The pressure P in IIIb is to good approximation given by the equation

$$P=\frac{A}{2\pi s}\sqrt{\frac{2\gamma M}{r}}, \tag{7}$$

where s is the radius of the cross-section of IIIb at distance r. The equation (7) gives the first-order approximation to the condition that the region IIIa remains in a fixed position owing to the outward pressure from region IIIb being balanced by the momentum of particles with transverse velocity $\sqrt{2\gamma M/r}$ entering IIIa from II. To compare the longitudinal gas-pressure gradient with the gravitational force of the star, consider a region bounded by the cross-sections of IIIb at distances r and $r+dr$, and by the inner surface of IIIa. Then the force acting on the material in this region due to the longitudinal pressure gradient is

$$d(\pi s^2 P)=A\sqrt{\gamma M/2}\, d(s/\sqrt{r}),$$

which is of order $A\sqrt{\gamma M/2}\,.\,sdr/r^{3/2}$ since ds/s is of the same order as dr/r, whilst the gravitational force is $m\gamma M dr/r^2$ and is of order $A\gamma^2M^2dr/r^2V^3$. Thus since r is of order $\gamma M/V^2$ and s/r is small compared with unity, the ratio of the force due to gas-pressure gradient to the gravitational force is seen to be $<< 1$.

The second assumption used in deriving (4) and (5) is that v is constant over any cross-section of IIIb and directed parallel to the axis. This is seen to be the case since the ratio $s/r << 1$, so that a velocity of circulation of material transverse to the axis and small compared with V will produce this uniformity in a time $\gamma M/V^3$. The equations (4) and (5) are more conveniently treated by introducing the non-dimensional variables defined by

$$r=\frac{\gamma M}{V^2}x, \qquad v=Vy, \qquad m=Az\frac{\gamma M}{V^3}=z\,.\,2\pi\rho_\infty\left(\frac{\gamma M}{V^2}\right)^2.$$

Then we have

$$\left.\begin{aligned} y\frac{dy}{dx}+\frac{1}{x^2}&=\frac{y(1-y)}{x-a},\\ yz&=x-a, \end{aligned}\right\} \tag{8}$$

where $a=r_0V^2/\gamma M$. Since the velocity is towards the star for $x < a$, it follows that the rate of accretion is given by the amount of material entering the portion of III*b* lying between $x=a$ and the star. This gives

$$a\frac{\gamma M}{V^2}.\,A=a.2\pi\rho_\infty\frac{\gamma^2M^2}{V^3} \tag{9}$$

for the accretion rate. The determination of the parameter a is accordingly of great importance. The conditions on y are $y\rightarrow 1$ as $x\rightarrow\infty$ since the velocity must tend to V at great distances from the star, $y=0$ at $x=a$ and y must be bounded in any finite range of x not including $x=0$. It should be noticed that the above equations break down near the star as transverse motion of the material will become important for values of $x << 1$. But even if the motion of the material could be followed up to the surface of the star it would still not be possible to impose any further condition on y, for it seems certain that near the star the inward velocity of material in III*b* will be greater than the velocity of sound so that no influence from the star could travel outwards through the material. Now the above conditions on y can be satisfied for any value of a, so that the steady state equations (8) are not sufficient to define a unique solution to the problem. In fact these conditions permit an infinity of solutions for every value of a. If it is assumed that y is monotonic with x (that is, the magnitude of y *increases* with decreasing x from $x=a$ to the surface of the star, and y *increases* with increasing x from $x=a$ to infinity), which would seem to be an essential physical requirement, then it can be shown from (6) that a must be > 1. Thus for all values of $a > 1$ both the mathematical and physical requirements are satisfied and a unique solution cannot be determined. This means that the system may satisfy any one of a range of solutions, the particular solution satisfied at a given time being determined not only by the equations (8) and the boundary conditions but also by the perturbations experienced by the system. It is interesting to notice that all solutions for which $1 < a < 2$ satisfy $(dy/dx)_{(x=a)}=1/a^2$.

4. In considering perturbations acting on the system discussed in section 3 we shall consider only disturbances that preserve axial symmetry. The forms of the differential equations are changed since A and V are functions of time in the disturbed state. The modification of (8) to include this change is simple, however, for we have

$$\frac{\partial m}{\partial t}+\frac{\partial}{\partial r}(mv)=A, \qquad \frac{\partial(mv)}{\partial t}+\frac{\partial}{\partial r}(mv^2)=AV, \tag{10}$$

in place of (7). Eliminating m gives the equation

$$\frac{\partial}{\partial t}\frac{A(V-v)}{(\partial v/\partial t+v\partial v/\partial r+\gamma M/r^2)}+\frac{\partial}{\partial r}\frac{Av(V-v)}{(\partial v/\partial t+v\partial v/\partial r+\gamma M/r^2)}=A. \tag{11}$$

Again introducing the non-dimensional variables x, y and writing $t=\gamma M\tau/V^3$ the equation (10) becomes, at the point $x=a$,

$$\frac{\partial^2 y}{\partial\tau^2}+\frac{3}{a^2}\frac{\partial y}{\partial\tau}+\frac{2(2-a)}{a^4}y=\frac{1}{a^2A}\frac{\partial A}{\partial\tau}+\frac{1}{a^2}\left(\frac{\partial y}{\partial x}-\frac{1}{a^2}\right), \tag{12}$$

where A represents the mass per unit length added to III*b* from II *at the point* $x=a$. By restricting attention to disturbances for which $(\partial y/\partial x)_{x=a}=1/a^2$, it is easy to show from (11) that the system is stable only if $a < 2$. This result together with that of the previous section limits the value of a to the range $1 < a < 2$. The value that a will have at any particular time will depend on the perturbations suffered by the system at previous times. Accordingly to calculate a unique value of α it is necessary to consider the precise form of the disturbances experienced by the system. An investigation of this kind is attempted in the following section, where the steady state set up after a star passes into a uniform cloud of material possessing a plane boundary perpendicular to the direction of the relative velocity V is considered.

5. The boundary of the cloud is supposed plane when the star is at great distances, but will become distorted as the star approaches. The form of the boundary at any time is easily calculated. Thus consider the trajectory of a particle lying at the surface of the cloud with angular momentum pV about the star. The particle moves in a hyperbolic path which is given by the following equation until it crosses the accretion axis:—

$$\frac{p^2}{rl} = 1 + \cos\theta + \frac{p}{l}\sin\theta, \tag{13}$$

where $l = \gamma M/V^2$, and r, θ are polar co-ordinates with axis along the accretion axis (the equation of the path is normally referred to the line joining the star to the vertex of the hyperbolic path, but the present procedure of choosing the line through the star parallel to the accretion axis is more convenient here). The distance of the star from the vertex of the path is $r_1 = \sqrt{p^2 + l^2} - l$, and if t_1 is the time at which the particle is at the vertex, then it is easy to show that

$$\left.\begin{aligned} V(t-t_1) &= \sqrt{r^2+2lr-p^2} + l\log\frac{r+l-\sqrt{r^2+2lr-p^2}}{\sqrt{l^2+p^2}} \qquad \text{if } t > t_1 \\ V(t-t_1) &= -\sqrt{r^2+2lr-p^2} - l\log\frac{r+l-\sqrt{r^2+2lr-p^2}}{\sqrt{l^2+p^2}} \qquad \text{if } t < t_1 \end{aligned}\right\} \tag{14}$$

If the particle were at distance r_0 at time $t_0 < t_1$, then it follows immediately from (14) that

$$V(t-t_0) = r_0 + \sqrt{r^2+2lr-p^2} + l\left\{1 + \log\frac{r+l-\sqrt{r^2+2lr-p^2}}{2r_0}\right\} \qquad t > t_1, \quad t_0 < t_1 \tag{15}$$

provided r_0 is very large. Now define t_s by means of the equation

$$V(t_s - t_0) = r_0 + l + l\log(l/2r_0), \tag{16}$$

so that $(t_s - t_0)$ is the time taken by a particle with $p = 0$ to fall to the star from distance r_0. Thus we have

$$V(t-t_s) = \sqrt{r^2+2lr-p^2} + l\log\frac{r+l-\sqrt{r^2+2lr-p^2}}{l}, \tag{17}$$

where $(t - t_s)$ is the difference between the times taken by a particle with angular momentum pV to fall to distance r, and by a particle with zero angular momentum to fall to the star, both particles starting from distance r_0 (which is very large). Accordingly, the form of the boundary of the cloud for $t > t_1$ (this condition is easily seen to be equivalent to $\tan\theta < p/l$, $r > l(1/\cos\theta - 1)$) is given by

$$\left.\begin{aligned} &\sqrt{r^2+2lr-p^2} + l\log\frac{r+l-\sqrt{r^2+2lr-p^2}}{l} = V(t-t_s), \\ &p^2 - rp\sin\theta - rl(1+\cos\theta) = 0. \end{aligned}\right\} \tag{18}$$

Solving for p from the second of these equations and substituting in the first equation gives, after some reduction,

$$V(t-t_s) = r\left(1 - \frac{\theta^2}{4}\right) + \text{higher terms in } \theta \tag{19}$$

for the equation of the boundary near $\theta = 0$.

It follows, therefore, that the point at which the boundary intersects the accretion axis moves with velocity V, and that the tangent plane to the boundary at this point is perpendicular to the axis. Behind this point collisions between the particles will tend to build up a column of the type discussed in section 3, this process being governed by equation (11), which can be written in the following non-dimensional form:—

$$\frac{\partial}{\partial\tau}\frac{1-y}{(\partial y/\partial\tau+y\partial y/\partial x+1/x^2)}+\frac{\partial}{\partial x}\frac{y(1-y)}{(\partial y/\partial\tau+y\,\partial y/\partial x+1/x^2)}=1, \tag{20}$$

where $\tau=V^3t/\gamma M$ and $x \leqslant \tau$. Equation (2) has the interesting property that, although a second-order equation, the single boundary condition that $y=1$ at $x=\tau$ is sufficient to determine a solution, since it can be seen from the nature of the problem that $y=1$, $x=\tau$ is a characteristic of the equation which is of the parabolic type.*

A solution of (20) of the form

$$y=1+\sum_{n=1}^{\infty}(\tau-x)^{n/2}f_n(x)$$

has been tried and the first few coefficients determined. The form of the power series is

$$y=1-\frac{1}{x}\sqrt{\frac{2}{3}}(\tau-x)-\frac{2}{21}\frac{\tau-x}{x}+\sqrt{\frac{2}{3}}\left\{\frac{25}{4.147x}+\frac{1}{4x^2}\right\}(\tau-x)^{3/2}$$
$$+\left\{\frac{-23}{7.441x}+\frac{1}{18x^2}\right\}(\tau-x)^2+\sqrt{\frac{2}{3}}\left\{\frac{1429}{16.490.441x}-\frac{2671}{8.70.441x^2}-\frac{431}{7.480x^3}\right\}(\tau-x)^{5/2}+\cdots \tag{21}$$

which converges so well for $x > 1{\cdot}5$, $(\tau-x) < 1$, that it seems sufficient to regard y as being given to good approximation by (21). In the following table the values of y corresponding to a series of values of $(\tau-x)$ are given for $x=2$ and $x=3$.

TABLE I

$\tau-x$	$x=2$	$x=3$
0·0	1	1
0·25	0·7827	0·8603
0·49	0·7144	0·8054
0·64	0·6773	0·7788
0·81	0·6417	0·7530
1·00	0·6078	0·7280
1·1025	0·5910	0·7156

Table I shows that y tends to a steady value as $(\tau-x)$ increases to a value near unity. If $y_{\text{lim}}(x=2)$ and $y_{\text{lim}}(x=3)$ denote these steady values of y, then a unique solution of the steady-state equation (8) can be obtained giving these values of y at $x=2$ and $x=3$ respectively. Thus for any given value of α there is one solution of (8) satisfying the condition $y=0$, $x=\alpha$; $y=y_{\text{lim}}(x=2)$; $y\to 1$ as $x\to\infty$. There is, however, only one value of α for which the solution so determined also satisfies the additional condition $y=y_{\text{lim}}(x=3)$, $x=3$. This may be illustrated by the following numerical case, in which we put $y_{\text{lim}}(x=2)=0{\cdot}5$, $y_{\text{lim}}(x=3)=0{\cdot}6$. (It is important to notice that these particular numerical values of $y_{\text{lim}}(x=2)$ and $y_{\text{lim}}(x=3)$ have been chosen for the sake of definiteness, and that the present argument would be equally applicable if somewhat different values had been chosen.) The following three numerical solutions satisfy the conditions $y=0$, $x=\alpha$; $y=0{\cdot}5$, $x=2$; $y\to 1$ as $x\to\infty$.

* Hadamard, *Problème de Cauchy*, Hermann et Cie, 1932.

TABLE II

x	$\alpha=1{\cdot}15$	$\alpha=1{\cdot}25$	$\alpha=1{\cdot}5$
2·0	0·500	0·500	0·500
2·1	0·508	0·515	0·541
2·2	0·5145	0·523	0·570
2·35	0·523	0·537	0·603
2·5	0·530	0·549	0·625
2·75	0·541	0·564	0·650
3·00	0·551	0·577	0·667

It is clear from Table II that a value of α close to 1·25 will give a solution satisfying the additional condition $y=0{\cdot}6$, $x=3$. Accordingly it is to be expected that a steady state will be set up giving an accretion rate close to

$$\frac{2{\cdot}5\pi\gamma^2M^2\rho_\infty}{V^3}.$$

It should be noticed that these considerations give an estimate for α only for the steady state set up after a special form of perturbation. The perturbation considered above is of a particularly violent character, and it would seem that for smaller perturbations α could hardly be less than the value obtained above.

6. *The Rate of Change of V.*—It is easy to show from (8) that for any steady-state solution $y \sim 1-\log x/x$ as $x \to \infty$. Now in any steady state the mass of material crossing the section of III*b* at distance $x>\alpha$ is given by

$$A(x-\alpha)\frac{\gamma M}{V^2}=(x-\alpha)\,.\,2\pi\rho_\infty\frac{\gamma^2M^2}{V^3} \text{ grams per sec.}$$

This material before approaching the star is moving with velocity V relative to the star, whilst after passing the star the velocity is modified to $V\left(1-\dfrac{\log x}{x}\right)$ provided $x>>\alpha$. Accordingly the momentum of the star relative to the cloud must be reduced by

$$2\pi\rho_\infty\frac{\gamma^2M^2}{V^2}(x-\alpha)\frac{\log x}{x} \sim 2\pi\rho_\infty\frac{\gamma^2M^2}{V^2}\log x \text{ grams per sec.,}$$

due to the gravitational action of the star on material striking the axis between α and $x>>\alpha$.

The equation of conservation of momentum is therefore given by

$$\frac{d}{dt}(MV)=-2\pi\rho_\infty\frac{\gamma^2M^2}{V^2}\log x. \tag{22}$$

The right-hand side of (22) $\to\infty$ as $x\to\infty$ on account of the term in $\log x$, but for astronomical reasons an upper limit can be assigned to x. The presence of neighbouring stars will introduce a cut-off effect analogous to that which arises in the calculation of the collision cross-section of a charged particle in an ionised gas. It is important to notice that since dV/dt depends only on $\log x$ the cut-off value of x need not be known at all accurately. Indeed it is clear that the average interstellar distance may be used as the cut-off value of $(x\,.\,\gamma M/V^2)$ without introducing any appreciable error in dV/dt. The average interstellar distance is about 3.10^{18} cm., and $\gamma M/V^2$ is of order 10^{16} cm. for stars of large mass and is of order 10^{13} cm. for stars of small mass. It follows, therefore, that $\log x$ lies between 5 and 15 for almost all stars. The equation (22) may be solved in conjunction with equation (9), which gives

$$\frac{dM}{dt}=\alpha\,.\,2\pi\rho_\infty\frac{\gamma^2M^2}{V^3}.$$

The solution for M, V at time t is given by

$$\frac{M}{M_0}=\left(\frac{V_0}{V}\right)^{\frac{a}{\log x}}, \quad \frac{V}{V_0}=\left[1-\frac{6\pi\rho_\infty \gamma^2 M_0}{V_0^3}\log x.t\right]^{\frac{1}{3}}, \tag{23}$$

where $V=V_0$, $M=M_0$ at time $t=0$.

From (23) it is seen that M increases significantly in a time given by

$$\frac{V_0^3}{6\pi\rho_\infty \gamma^2 M_0}\log x.$$

This formula may be compared with the estimate of $V_0^3/(4\pi\rho_\infty \gamma^2 M_0)$ given by Hoyle and Lyttleton.* It is seen from this comparison that the time-scale of the evolution of the stars due to accretion is shortened below earlier estimates by a factor of $1\cdot5 \log x$, which, as seen above, is of order 10. This is an important result, since it would seem that the dynamical evolution of the stars can be achieved in a period of less than 10^{10} years. For example, if we take

$$V_0=5 \text{ km./sec.}, \qquad \rho_\infty=10^{-22} \text{ grm. per cm.}^3, \qquad M_0=2.10^{34} \text{ grm.}$$

as representing a typical star of Class B, then putting $\log x=10$, the time required for the mass of the star to increase by an appreciable factor is about $2\cdot5 . 10^9$ years.

The analysis given above neglects the temperature of the interstellar material. The observations of McKellar † indicate that the average thermal velocity u of interstellar hydrogen is of order 10^4 cm./sec. It will be realised that the analysis breaks down for material that passes the star at distances of the order of, or greater than, $\gamma M/u^2$, and a much more complicated investigation is required to describe the motion in this case since the effects of gas pressure can no longer be neglected. For $u=10^4$ cm./sec., $M=2.10^{33}$ grm., the distance $\gamma M/u^2$ is $1\cdot33 . 10^{18}$ cm., which is of the same order as the average interstellar distance. Thus for values of u of order 10^4 cm./sec. the effect of gas pressure on the motion of material at large distances from the star can have no appreciable effect on the value of dV/dt. Moreover, it is clear that since dV/dt depends only on $\log x$, the value of u could be appreciably in excess of 10^4 cm./sec.$^{-1}$ without any possibility of a change in the order of magnitude of dV/dt arising.

7. The process discussed in section 6 would appear to be of considerable importance, and it seems desirable to conclude the present paper with a few general remarks concerning this question. The problem of the resistance experienced by a gravitating body moving through a gas was considered by Jeffreys ‡ who drew attention to the fact that the characteristic linear dimension occurring in the problem is $\gamma M/V^2$ and not the radius of the body (except of course when the mass of the body is so small that $\gamma M/V^2$ is comparable or less than the radius). Jeffreys regarded the drag due to the cloud as being capable of rounding up the initially elliptic orbits of the planets into approximately circular paths, and also of introducing a stabilising influence into the planetary system. This application of the process is analogous to the investigation of section 6, since the reduction of the velocities V of the stars relative to the interstellar material means that the peculiar motions of the stars are being continually reduced and that their paths are being rounded up into circular orbits.

The evolution of a star is dominated by a struggle between opposing forces. On the one hand we have the resistance of the interstellar material constantly tending to reduce the value of V, whilst on the other hand the perturbing force due to the changing gravitational field of the galaxy as a whole tends to increase the value of V (the exact nature of the changing gravitational field is a question that is outside the scope of the present

* *Proc. Cam. Phil. Soc.*, 35, 592, 1939.
† *P.A.S.P.*, 52, 187, 1940.
‡ *The Earth*, Cambridge, 1929.

paper, and the detailed discussion of this matter has to be left over for investigation in future work). The evolution of the star depends on the outcome of the struggle between these opposing tendencies. Thus if the changing gravitational field should become the dominating effect, then the orbit of the star in the galaxy will become elliptic and the star will cease to increase in mass on account of the large value of V that must arise; whilst if the opposing tendency should become predominant and the resistance of the interstellar material reduces V to small values, then the mass and luminosity of the star will increase to large values. Indeed it would appear that the question of whether the rate of accretion of hydrogen by a star is sufficient to balance the hydrogen destroyed by the energy generation in its interior is determined by the battle between the opposing forces mentioned above. Thus it would appear that the physical evolution of the stars is directly associated with the dynamical considerations governing the variation of V with time.

Although it is outside the scope of the present paper to discuss in any detail the problems of stellar dynamics, it is of interest that the interaction described above between the interstellar material and the stars may have an important bearing on stellar dynamics. Thus by assuming that an appreciable proportion of the mass of the galaxy is in the form of interstellar material (the existence of large tracts of interstellar hydrogen is known from observation; the observational data, however, are not at present sufficiently complete to enable an estimate to be given for the fraction of the mass of the galaxy in diffuse gaseous form), the whole subject of stellar dynamics would appear to be greatly simplified. This simplification is due to the fact that the interaction between the interstellar material and the stars enables the stellar system to be controlled by the diffuse gaseous material in the galaxy.