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C. Z. Woźniak, M. Woźniak

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Czeslaw Wozniak, Margaret Wozniak

ON THE MICROMODELLING OF DYNAMIC RESPONSE FOR THERMOELASTIC PERIODIC COMPOSITES

CZ. WOŹNIAK
Institute of Fundamental Technological Research
Świętokrzyska 21, PL 00-049 Warszawa, Poland

M. WOŹNIAK (Ms.)
Technical University of Łódź, Department of Geotechnical and Structure Engineering
Al. Politechniki 6, PL 93-590 Łódź, Poland

1. Introduction

As it is known, the asymptotic homogenization methods for micro-periodic composites, leading to the effective modulus theories, neglect inertial aspects of microstructural features related to the size of constituents (cf. [1,2] and the references therein). The main aim of this contribution is to propose a new approach to the formulation of macro-models for micro-periodic thermoelastic composite materials. This approach takes into account a length-scale effect on a dynamic response of a composite and is simple enough to be applied in analysis of engineering problems and for quasi-stationary processes reduces to the special effective modulus theory, [3,4]. Theories of this type for elastic composite materials and structures were discussed in [5-7] and are termed *refined macro-theories*. In this paper governing equations of the refined macro-thermoelastodynamics are formulated on the basis of heuristic hypotheses concerning the expected form of disturbances in displacement and temperature fields, caused by the micro-inhomogeneity of a composite. At the same time a special form of macro-modelling approximations is used. The resulting equations are obtained without any reference to a boundary value problem on the representative volume element, that is required in asymptotic homogenization approaches, [1,2]. The general considerations are illustrated by the simple example the aim of which is to compare results of the refined macro-theory and those of the effective modulus theory. It is shown that the microstructure length scale effects, described by the proposed macro-theory, play an essential role in investigations of the non-stationary behaviour of the composites.

The analysis will be carried out in the framework of the linear thermo-elastodynamics under assumption of the perfect bonding between constituents of the composite. The considerations are restricted to micro-periodic bodies, i.e., the maximum length dimension of the representative volume element is sufficiently small compared to the minimum characteristic length dimension of the body.

Denotations. The region in the reference space, occupied by the undeformed composite body, will be denoted by Ω. By $\mathbf{x} \equiv (x_1, x_2, x_3)$ and τ we denote points of Ω and a time coordinate, respectively, and x_1, x_2, x_3 are Cartesian orthogonal coordinates in the reference space. Subscripts i, j, k, l related to these coordinates run over the sequence 1, 2, 3. Superscripts a, b, run over 1,...,n being related to a certain micro-discretization of the representative volume element $V = (0, l_1) \times (0, l_2) \times (0, l_3)$ of the periodic composite structure. The summation convention holds both for i, j, k, l and a, b. For any V-periodic integrable function $f(\mathbf{x})$ we introduce the averaging operator

$$\langle f \rangle \equiv \frac{1}{l_1 l_2 l_3} \int_V f(\mathbf{x}) dv \quad ,$$

where $dv = dx_1 dx_2 dx_3$. The area element of the boundary $\partial\Omega$ will be denoted by da. The remaining basic denotations will be given at the beginning of the subsequent section.

2. Analysis

2.1. FOUNDATIONS

Foundations of the proposed approach are based on the governing equations of thermo--elastodynamics. Denoting by u_i, θ, s_{ij}, q_i, ρ, ε, b_i, α, s_i, q displacements, temperature, stresses, heat fluxes, mass density, specific energy, body forces, heat supply, boundary tractions and boundary heat supply, respectively, we shall postulate the principle of balance of momentum and that of balance of energy in the following weak form

$$
\begin{aligned}
\frac{d}{d\tau}\int_{\Omega}\rho(\mathbf{x})\dot{u}_i(\mathbf{x},\tau)\delta u_i(\mathbf{x})dv &= \oint_{\partial\Omega}s_i(\mathbf{x},\tau)\delta u_i(\mathbf{x})da - \int_{\Omega}s_{ij}(\mathbf{x},\tau)\delta u_{i,j}(\mathbf{x})dv + \\
&\quad + \int_{\Omega}\rho(\mathbf{x})b_i\delta u_i(\mathbf{x})dv , \\
\frac{d}{d\tau}\int_{\Omega}\varepsilon(\mathbf{x},\tau)\delta\theta(\mathbf{x})dv &= \oint_{\partial\Omega}q(\mathbf{x},\tau)\delta\theta(\mathbf{x})da - \int_{\Omega}q_i(\mathbf{x},\tau)\delta\theta_{,i}(\mathbf{x})dv + \\
&\quad + \int_{\Omega}[\alpha(\mathbf{x},\tau)+s_{ij}(\mathbf{x},\tau)\dot{u}_{i,j}(\mathbf{x},\tau)]\delta\theta(\mathbf{x})dv ,
\end{aligned}
\tag{1}
$$

where δu_i, $\delta\theta$ are sufficiently regular test functions. The constitutive equations will be assumed in the linearized form

$$
\begin{aligned}
\varepsilon(\mathbf{x},\tau) &= \frac{1}{2}C_{ijkl}(\mathbf{x})\, u_{(i,j)}(\mathbf{x},\tau)\, u_{(k,l)}(\mathbf{x},\tau) + B_{ij}(\mathbf{x})\, u_{(i,j)}(\mathbf{x},\tau)\,\theta(\mathbf{x},\tau) + \\
&\quad + \frac{1}{2}c(\mathbf{x})\,\theta^2(\mathbf{x},\tau) , \\
s_{ij}(\mathbf{x},\tau) &= C_{ijkl}(\mathbf{x})\, u_{(i,j)}(\mathbf{x},\tau) + B_{ij}(\mathbf{x})\,\theta(\mathbf{x},\tau) , \quad q_i(\mathbf{x},\tau) = K_{ij}(\mathbf{x})\,\theta_{,j}(\mathbf{x},\tau) ,
\end{aligned}
\tag{2}
$$

where the elastic modulae $C_{ijkl}(\cdot)$, the thermal conductivity modulae $K_{ij}(\cdot)$, the thermal expansion modulae $B_{ij}(\cdot)$ and the specific heat $c(\cdot)$ are V-periodic functions. Eqs (2) have to satisfy the known dissipation condition.

If Eqs (1) hold for arbitrary regular test functions δu_i, $\delta\theta$ then Eqs (1), (2) are equivalent to the well known equations of the linear thermo-elastodynamics, which for composite materials have to be considered together with the continuity conditions $[\sigma_{ij}]n_j = 0$, $[h_i]\, n_i = 0$ on the interfaces between constituents. However, due to the highly oscillating (V-periodic) form of Eqs (2), the aforementioned equations do not constitute the proper analytical tool for investigations of composite bodies. In order to formulate macro-modelling assumptions leading from Eqs (1), (2) to the equations of the refined thermo-elastodynamics we have to introduce certain preliminary concepts.

2.2. PRELIMINARY CONCEPTS

The first preliminary concept we are to introduce is related to the expected form of disturbances in displacement and temperature fields caused by the inhomogeneity of the medium. To this end we shall assume that from the qualitative viewpoint these disturbances can be described by a sequence of n linear independent functions $h^a(\mathbf{x})$, $\mathbf{x}\in\mathbf{R}^3$, which are V-periodic, continuous, have piece wise continuous first derivatives $h^a{}_{,i}$ suffering jump discontinuities across the interfaces between constituents and satisfy conditions: $\langle h^a \rangle = 0$, $\langle h^a{}_{,i} \rangle = 0$, $\langle \rho h^a \rangle = \langle \alpha h^a \rangle = \langle c h^a \rangle = \langle B_{ij} h^a \rangle = 0$. Moreover, we assume that $h^a(\mathbf{x})\in\mathcal{O}(l)$, where $l \equiv \max l_i$ is the microstructure length parameter, and that the values $h^a{}_{,i}(\mathbf{x})$ of the derivatives of h^a are independent of l. Functions $h^a(\cdot)$ are called *micro-shape functions*; their specification depends on the material structure of the representative volume element V of the periodic composite and can be also related to a certain discretization procedure of V; for particulars the reader is referred to [3-5].

Let λ be a small macro-accuracy parameter related to the calculations of a certain real-valued function $F(\cdot)$ defined on Ω (F can also depend on the time coordinate τ). Function F will be called V-macro function if for every $\mathbf{x},\mathbf{y}\in\Omega$ such that $\mathbf{x}-\mathbf{y}\in V$ condition $|F(\mathbf{x}) - F(\mathbf{y})| < \lambda$ holds. If the condition of this form also holds for all derivatives of F then F will be referred to as the *regular V-macro function*. For any integrable V-periodic function $f(\cdot)$, micro-shape function $h^a(\cdot)$ and regular V-macro function $F(\cdot)$, we obtain

$$\int_\Omega f(\mathbf{x})F(\mathbf{x})dv = \langle f \rangle \int_\Omega F(\mathbf{x})dv + \mathcal{O}(\lambda) \ , \tag{3}$$

$$(h^a(\mathbf{x})F(\mathbf{x}))_{,i} = h^a{}_{,i}(\mathbf{x})F(\mathbf{x}) + \mathcal{O}(\lambda) \ ,$$

The concept of a regular V-macro function is strictly related to the macroscopic description of the behaviour of a composite in which the oscillations of functions within every single periodicity cell can be neglected, [5].

2.3. MACRO-MODELLING ASSUMPTIONS

The passage from Eqs (1), (2) of micromechanics to the proposed macro-model of a composite will be based on the following modelling assumptions.

Micro-Macro Localization Hypothesis. The displacement and temperature fields in the micro-periodic body can be expected in the form

$$u_i(\mathbf{x},\tau) = U_i(\mathbf{x},\tau) + h^a(\mathbf{x})V_i^a(\mathbf{x},\tau) \ , \tag{4}$$

$$\theta(\mathbf{x},\tau) = \Theta(\mathbf{x},\tau) + h^a(\mathbf{x})\Phi^a(\mathbf{x},\tau) \ , \qquad \mathbf{x} \in \Omega \ ,$$

where $U_i(\cdot,\tau)$, $V_i^a(\cdot,\tau)$, $\Theta(\cdot,\tau)$, $\Phi^a(\cdot,\tau)$ are arbitrary regular V-macro functions and $h^a(\cdot)$ are micro-shape functions, postulated in every problem under consideration.

Fields U_i and Θ will be called macro-displacement and macro-temperature field, respectively. Fields V_i^a, Φ^a are referred to as correctors and describe, from the quantitative point of view, the possible disturbances in displacements and a temperature caused by the micro-periodic inhomogeneous structure of a composite.

Macro-Balance Assumption. The balance equations (1) are assumed to hold for $\delta u_i = \delta U_i + h^a \delta V_i^a$, $\delta\theta = \delta\Theta + h^a\delta\Phi^a$, where δU_i, δV_i^a, $\delta\Theta$, $\delta\Phi^a$ are arbitrary linear independent regular V-macro functions.

Macro-Modelling Approximation. In the balance equations (1) terms $\mathcal{O}(\lambda)$ in integrals over Ω and terms $\mathcal{O}(l)$ in integrals over $\partial\Omega$ can be neglected.

3. Refined theory

Substituting the right-hand sides of Eqs (2) into Eqs (1) and using the aforementioned macro-modelling assumptions (we apply formulae (3)), after some manipulations and introducing the following V-macro fields

$$
\begin{aligned}
S_{ij} &= \langle C_{ijkl} \rangle U_{k,l} + \langle C_{ijkl} h^a{}_{,k} \rangle V^a_l + \langle B_{ij} \rangle \Theta + \underline{\langle B_{ij} h^b \rangle \Phi^b} \quad , \\
H^a_i &= \langle C_{ijkl} h^a{}_{,j} \rangle U_{k,l} + \langle C_{ijkl} h^a{}_{,j} h^b{}_{,l} \rangle V^b_k + \langle B_{ij} h^a{}_{,j} \rangle \Theta + \\
&\quad + \underline{\langle B_{ij} h^a{}_{,j} h^b \rangle \Phi^b} \quad , \qquad (5) \\
Q_i &= \langle K_{ij} \rangle \Theta_{,j} + \langle K_{ij} h^b{}_{,j} \rangle \Phi^b \quad , \\
G^a &= \langle K_{ij} h^a{}_{,j} \rangle \Theta_{,i} + \langle K_{ij} h^a{}_{,i} h^b{}_{,j} \rangle \Phi^b \quad ,
\end{aligned}
$$

we obtain

$$
\begin{aligned}
&\langle \rho \rangle \ddot{U} - S_{ij,j} = \langle \rho \rangle b_i \quad , \\
&\underline{\langle \rho h^a h^b \rangle \ddot{V}^b_i} + H^a_i = 0 \quad , \qquad (6) \\
&\langle c \rangle \dot{\Theta} - Q_{i,i} + \langle B_{ij} \rangle \dot{U}_{i,j} + \langle B_{ij} h^b{}_{,j} \rangle \dot{V}^b_i = \langle \alpha \rangle \quad , \\
&\underline{\langle c h^a h^b \rangle \dot{\Phi}^b} + \underline{\langle B_{ij} h^b{}_{,j} h^a \rangle \dot{V}^b_i} + G^a = 0 \quad ,
\end{aligned}
$$

and $S_{ij}n_j=s_i$, $Q_in_i=q$ on $\partial\Omega$. Substituting the right-hand sides of Eqs (5) into Eqs (6) we arrive at the system of 4+4n equations for macro-displacements U_i, macro--temperature Θ and correctors V_i^a, Φ^a. These equations have constant coefficients and hence, represent a certain macro-model of the periodic body under consideration. The underlined constants in Eqs (5), (6) depend on the microstructure length parameter l and describe the effect of the microstructure on the behaviour of the composite. Hence, Eqs (5), (6) represent the refined macro-thermoelastodynamics of composite materials and will be called macro-constitutive equations and local macro-balance equations, respectively. It has to be emphasized that the equations for correctors (the second and the last from Eqs(6)) are ordinary differential equations and hence, the correctors play a role of certain internal dynamical variables, i.e., they do not enter boundary conditions. For a homogeneous body from Eqs (5), (6) we obtain $V_i^a = 0$, $\Phi^a = 0$, provided that initial values of V_i^a, $\dot{V}_i^a$ and Φ^a are equal to zero. Hence, we see that correctors describe the effect of inhomogeneity on the macro-behaviour of the body.

4. Effective Modulus Theory

Scaling the microstructure down in Eqs (5), (6) by means of $l \to 0$, we arrive at a certain asymptotic theory; in this case the underlined terms are equal to zero and we arrive at conditions $H_i^a = 0$, $G^a = 0$ representing the systems of linear algebraic equations for the correctors V_i^a, Φ^a. Hence, eliminating correctors from Eqs (5), (6) we obtain equations of a certain special effective modulus theory, given by

$$
\begin{aligned}
&<\rho>\ddot{U}_i - S_{ij},_j = <\rho> b_i \ , \qquad c^{eff}\,\dot{\Theta} - Q_i,_i + B_{ij}^{eff}\,\dot{U}_i,_j = <\alpha> \ , \\
&S_{ij} = C_{ijkl}^{eff}\,U_k,_l + B_{ij}^{eff}\,\Theta \ , \qquad Q_i = K_{ij}^{eff}\,\Theta,_j \ .
\end{aligned} \tag{7}
$$

The constant coefficients in Eqs (7) are termed effective modulae and defined by:

$$
\begin{aligned}
C_{ijkl}^{eff} &\equiv <C_{ijkl}> - <C_{ijmn}h^a,_m> D_{np}^{ab} <C_{klrp}h^b,_r> \ , \\
B_{ij}^{eff} &\equiv <B_{ij}> - <B_{kl}h^a,_l> D_{kp}^{ab} <C_{prij}h^b,_r> \ , \\
K_{ij}^{eff} &\equiv <K_{ij}> - <K_{ik}h^a,_k> D^{ab} <K_{jl}h^b,_l> \ , \\
c^{eff} &\equiv <c> - <B_{ij}h^a,_j> D_{ik}^{ab} <B_{kl}h^b,_l> \ ,
\end{aligned}
$$

where D^{ab}_{ik} and D^{ab} represent linear transformations inverse to those given by $<C_{ijkl}h^a{}_{,j}h^b{}_{,l}>$ and $<K_{ij}h^a{}_{,i}h^b{}_{,j}>$, respectively. The aforementioned results have been derived independently in [3], without any reference to the refined theory.

5. Example of Application

Let us consider a laminated body made of two orthotropic constituents. In this case we introduce one micro-shape function $h(x_1)$ (periodic in a direction x_1 normal to the lamina interfaces, cf. [1]), denoting by V_k, Φ the pertinent correctors related to $h(x_1)$. For the sake of simplicity let us neglect the body forces b_i and heat supply α. The aim of this example is to show a difference between results obtained from the refined theory and those derived from the effective modulus theory. To this end we shall consider the homogeneous boundary conditions for the macro-displacements and macro--temperature: $U_i = 0$, $Q_i n_i = 0$ on $\partial\Omega$, homogeneous initial conditions for the macro--displacements and correctors: $U_i = 0$, $\dot{U}_i = 0$, $V_i = 0$, $\dot{V}_i = 0$, $\Phi = 0$ at $\tau = 0$ and the initial condition for the macro-temperature in the form: $\Theta = \Theta_0$ at $\tau = 0$, $\Theta_0 =$ const. Then in the framework of the refined theory we obtain $U_i = 0$, $V_2 = V_3 = \Phi = 0$ for every $\tau > 0$, $\mathbf{x} \in \Omega$, and

$$\Theta = \Theta_0 K \left(1 - \frac{<B_{11}h_{,1}>^2}{<c><C_{1111}(h_{,1})^2>} \cos\omega\tau\right) , \tag{8}$$

$$V_1 = -\Theta_0 K \frac{<B_{11}h_{,1}>}{<C_{1111}(h_{,1})^2>} (1 - \cos\omega\tau) ,$$

where

$$K \equiv \frac{<c><C_{1111}(h_{,1})^2>}{<c><C_{1111}(h_{,1})^2> - <B_{11}h_{,1}>^2} , \quad \omega^2 \equiv \frac{<c><C_{1111}(h_{,1})^2> - <B_{11}h_{,1}>^2}{<\rho h^2><c>} .$$

At the same time, the effective modulus theory yields the constant values of Θ and V_1 for every $\tau \geq 0$:

$$\Theta = \Theta_0 \quad , \qquad V_1 = -\Theta_0 \frac{< B_{11} h_{,1} >}{< C_{1111} (h_{,1})^2 >} \quad . \tag{9}$$

From Eqs (8) it follows that the inhomogeneity of the medium and the coupling between temperature and deformations produce highly oscillating character of the macro-temperature field; this fact is not described by the effective modulus theory leading to (9). Thus we conclude that in investigations of non-stationary processes in thermo-elastic composites, the refined macro-elastodynamics has to be used instead of the effective modulus theory.

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