working paper
department
of economics

/ON THE MISUSE OF THE PROFITS-SALES RATIO

TO INFER MONOPOLY POWER/

Franklin M. Fisher

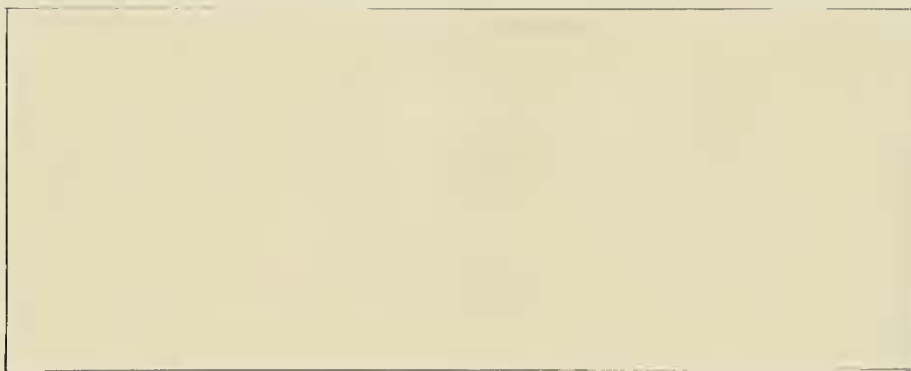Number 364   February 1985

massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139

/ON THE MISUSE OF THE PROFITS-SALES RATIO

TO INFER MONOPOLY POWER/

Franklin M. Fisher

Number 364   February 1985

On the Misuse of the Profits-Sales Ratio

to Infer Monopoly Power

Franklin M. Fisher
Department of Economics, E52-359
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

ABSTRACT

A large literature tests the structure-conduct-performance paradigm by regressions which use the profits-sales ratio as the dependent variable. That ratio is used because, under constant returns, it is generally supposed that profits/sales equals the Lerner measure of monopoly power. Profits/sales ratios are also used to make inferences about monopoly power for individual firms. This paper shows, however, that unless proper account is taken of the valuation of capital and the treatment of capital costs, profits/sales will be a very substantially inaccurate surrogate for the Lerner measure. Such proper account requires Hotelling valuation of capital goods which real firms do not use. The full analytics of the problem are worked out and some theorems proved relating the error to the cost of capital, the growth rate of the firm, and the depreciation method used. Examples are used to show that the error is potentially enormous, and the error is shown in general to be related to the variables used on the right hand side of the regression studies referred to above. The conclusion is reached that such studies are totally worthless. That conclusion will doubtless spark a certain amount of controversy as did the conclusion of the parallel paper on the accounting rate of return on assets or equity (Fisher and McGowan, American Economic Review, March 1983).

ON THE MISUSE OF THE PROFITS SALES RATIO

TO INFER MONOPOLY POWER

Franklin M. Fisher
Massachusetts Institute of Technology*

1. Introduction

It is popularly supposed that, under constant returns, the Lerner measure of monopoly power -- (price - marginal cost)/price -- is equal to the ratio of profits to sales. This paper shows that, once one leaves the simplest one-period model, this is generally not the case. Moreover, the errors can be quite large and are likely to be systematic.

The reason is not hard to find. Constant returns means constant returns to all factors including capital, so that unless proper account is taken of capital costs, the profits-sales ratio will simply misstate the Lerner measure.[1] Assuming, for simplicity that the prices of capital goods are constant, the capital costs involved will be depreciation and foregone interest (the opportunity cost of capital). The reported value of such costs depends on the way in which the firm values its capital stock. It should therefore come as no surprise that capital costs will generally not be correctly measured unless the firm uses Hotelling valuation in which capital goods are valued at the present value of the remaining net revenue streams they generate using the firm's risk-adjusted discount rate (Harold Hotelling). Real firms do not do this.

This capital-theoretic problem with the easy measurement of monopoly power has a familiar ring to it. Precisely the same difficulty occurs with the use of the accounting rate of return on capital value or on equity to infer monopoly profits. Indeed, the relations of the profits-sales ratio to the Lerner measure bear a striking resemblance to the relations of the accounting rate of return on capital to the economic rate of return studied in my earlier paper with John J. McGowan (Fisher and McGowan). Those who sought a refuge from our results in the use of the profits-sales ratio instead of the accounting rate of return have not found one (William Long and David Ravenscraft; Stephen Martin 1983, 1984). The analysis of monopoly power cannot be so cheaply accomplished.[2]

As with the issues surrounding the accounting rate of return, the existence of capital-measurement problems with the profits-sales ratio has been noticed in the literature. Leibowitz, in particular, presents a slashing attack, and Martin (1983, pp. 28-32) lays out some of what is involved. But, despite Martin's assertion that such problems are well known (Martin 1983, p. 32), they do not appear to be well understood. The full analytics of the problem appear not to have been spelled out nor has the importance of the problem been realized.

That importance is substantial. There is a very large literature using the profits-sales ratio as the dependent variable in regressions purporting to test various propositions of the structure-conduct-performance paradigm. (See, for example, Martin, 1983, Leonard Weiss, and David J. Ravenscraft.) Yet that

2

dependent variable is subject to substantial error from the capital-valuation problems studied here. Moreover, contrary to the facile assertion that one can assume such errors are distributed independently of the right-hand side variables used in such studies (See, for example, Martin 1983, p. 32), it is shown below that such independence cannot be assumed.

Of course, even the unwarranted assumption of independence would not justify the use of the profits-sales measure to make inferences about the monopoly power of individual firms. Yet such a use is sometimes made. The staff of the Federal Trade Commission's Line-of-Business program proposed to use the data generated by that program to identify targets of antitrust prosecution based on profits-sales ratios (Long et al), and consultants to the Antitrust Division of the Justice Department have pointed to high profits-sales ratios as convincing proof of monopoly power. Thus, F. M. Scherer (one of those whom Martin 1983 cites for the proposition that these problems are well known) states (Scherer p. 620):

> IBM's normal strategy was to set prices exceeding manufacturing costs by at least a factor of four. If this does not reflect monopoly power, what does?

Apart from the fact that such a statement ignores IBM's substantial non-manufacturing costs, so far as I can tell, it at best glosses over the question of capital costs.[3] Evidently, there is a need for a systematic consideration of what is involved here.

2. Formal Model: Hotelling Valuation

To begin, suppose the firm produces output, x, from capital and labor inputs, K and L, respectively, according to a production function:

$$(1) \qquad\qquad x = F(K, L)$$

assumed once continuously differentiable. Here, K is a stock and L a flow, as is x. The time variable, t, has been omitted.

The firm faces an inverse demand curve, given by p = p(x), where p is price. The wage of labor (which stands for all variable inputs) is w, and the price of a unit of capital is normalized to be unity. The interest cost of capital is r.

Begin by considering the simplest case in which the firm invests in a single project starting at time 0. In that case, the firm maximizes its present value, given by:

$$(2) \qquad\qquad \int_0^{\infty} [pF(K, L) - wL]e^{-rt} \, dt \ - K \ .$$

Denoting differentiation by subscripts, the first-order conditions are:

$$(3) \qquad\qquad R'F_L = w \qquad\qquad \text{all } t$$

and

$$(4) \qquad\qquad \int_0^{\infty} R'F_K e^{-rt} \, dt = 1$$

where subscripts denote partial differentiation and:

(5) $R' \equiv xp'(x) + p$

is marginal revenue. Note that R', like the other variables,
depends on t.

Now, (4) gives the first-order condition with respect to
capital as of time 0, the moment at which the capital is bought.
It is not hard to see that a similar condition will hold for any
later time, with the right-hand side of (4) replaced by an appro-
priate value for a unit of older capital. Indeed, suppose that
the firm values a unit of capital of age θ according to a sche-
dule, $V(\theta)$, where $V(0) = 1$, and it is natural to assume $V(oo) = 0$. Then consistent planning will require that $V(.)$ satisfy:

(6) $$\int_{\theta}^{oo} R'F_K e^{-r(t-\theta)} \, dt = V(\theta) \quad ,$$

so that $V(\theta)$ is the present value as of θ of the future benefits
of a marginal unit of capital. This, of course, is Hotelling
valuation (or "economic valuation"), and I shall denote the $V(.)$
which satisfies (6) by $V^*(.)$, reserving the unstarred symbol for
general valuation functions. Obviously, $V^*(0) = 1$, and I shall
assume that $V^*(oo) = 0$.

Now assume that the firm does use Hotelling valuation, so
that $V(.) = V^*(.)$ and differentiate (6) with respect to θ,
obtaining:

(7) $R'F_K = -V^{*\prime}(\theta) + rV^*(\theta)$.

The right-hand side of (7) is readily seen to be the instanta-

5

neous cost of capital, consisting of depreciation plus the imputed interest on the funds tied up in a unit of capital.[4]

Since (6) and (7) are equivalent, given (4) and the fact that $V^*(0) = 1$, the firm (or the analyst) can use either one. If (7) is used, then optimal behavior is characterized totally in terms of values defined at time θ -- a property of "myopia" which Martin emphasizes (Martin 1983, p. 30), but which is quite misleading, since Hotelling valuation, $V^*(\theta)$, requires forecasting later marginal revenue products. In any event, what is represented on the left-hand side of (7) is the full marginal benefit at time θ of an additional unit of capital of age θ. If the firm could buy and sell capital of different ages according to the schedule $V^*(.)$, then the right-hand side of (7) would be the true long-run marginal cost of capital of age θ at time θ.

It is therefore not surprising that, if Hotelling valuation is used and constant returns assumed, the profits-to-sales ratio does in fact equal the Lerner measure. To see this, assume that $F(K, L)$ is homogeneous of degree one. Denoting profits/sales at t by $M(t)$:

$$(8) \qquad M(t) = \frac{px - wL - [rV^*(t) - V^{*'}(t)]K}{px} \qquad ,$$

where again it must be remembered that all the variables on the right-hand side of (8) depend on t (with the exception, for simplicity, of r). By (3) and (7) and Euler's Theorem:

$$(9) \qquad wL + [rV^*(t) - V^{*'}(t)]K = R'(LF_L + KF_K) = R'x \qquad ,$$

so that

(10) $$M(t) = \frac{p - R'}{p} = - 1/h$$

where h is the elasticity of demand. This is the Lerner measure assuming full exploitation of monopoly power on the part of the firm.

Note, however, that this assumes that Hotelling valuation is used. If (as is nearly always the case for real firms), any _other_ valuation schedule is used, then (10) will not hold and it will not follow that profits/sales equals the Lerner measure.

To go more deeply into this, it will aid to simplify notation somewhat. Let $G(t, \theta)$ be the marginal revenue product at time t of capital of age θ. Then, in the previous notation:

(11) $$G(t, t) \equiv R'F_K \qquad ,$$

recalling again that both marginal revenue and the marginal physical product of capital will generally depend on t. $G(t, t)$ is thus the stream of net benefits which flow from a marginal unit of capital acquired at time 0. I write $G(t, \theta)$ as having two arguments, because such a stream will generally depend both on the calendar date, t, at which the benefits are received and on the age of the capital involved, θ. We must now consider the nature of such dependence.

In full generality, the marginal product of capital at time t is not necessarily independent of the age of the capital. If it is -- a special case -- then capital is essentially of the one-hoss shay type with capital of different vintages perfect substitutes until one vintage evaporates. More generally, there

7

may be technical change embodied in the capital stock or there may be physical depreciation. Hence the net benefit stream from a particular unit of capital may depend on the capital's vintage, $t - \theta$, and thus on its age, θ, given the date, t.

At least as important for our purposes as such effects, however, is the fact that benefit streams will not generally depend only on capital age but also on calendar time. Thus, demand may change over time and so may wages. There may also be disembodied technical change. In response, the firm may choose different outputs and different employment levels at different times. This will make marginal revenue and the marginal product of capital vary over time. Only in the quite unrealistic case of a totally stationary environment will this not be so.

Now, consider the generalization of Hotelling valuation as in (6) and (7). If depreciation plus imputed interest are to match marginal benefits on all types of capital (a condition certainly sufficient for profits/sales to equal the Lerner measure under constant returns), then (7) must generalize to:

(12) $$G(t, \theta) = -V^*_\theta(t, \theta) + rV^*(t, \theta) \quad ,$$

so that Hotelling valuation, $V^*(t, \theta)$ is:

(13) $$V^*(t, \theta) = \int_\theta^\infty G(t, u)e^{-r(u-\theta)} \, du \quad .$$

Note that the depreciation term in (12) takes no account of the fact that when capital is one year older it will be a year later. In effect, the firm consistently planning at t behaves as though

it faced a schedule of capital prices, $V^*(t, \theta)$, at which it can buy or sell capital of differing ages.

The important feature of (12) and (13) is that Hotelling valuation in the model which leads to profits/sales equalling the Lerner measure, depends not merely on capital age, θ, but also on calendar time, t. This has the following implication.

Even when such dependence does not occur, we do not expect real firms to use Hotelling valuation and the depreciation schedules which correspond to it. For example, where the benefit stream has a positive bulge some time after the investment is made, Hotelling valuation would require the firm to write up the value of existing capital by taking negative depreciation as that bulge gets closer. Obviously, real firms do not do this sort of thing. In the present case, however, the fact that Hotelling valuation depends directly on calendar time would require firms to adopt different depreciation schedules for otherwise identical capital of different vintages solely because wage rates or demand changes over time. In principle, firms ought to do this; in practice, they obviously do not. The hope that firms in actual studies will use Hotelling depreciation and thus make profits/sales equal the Lerner measure under constant returns is plainly forlorn.

One more word must be added before proceeding. As we have seen, even the case of Hotelling valuation requires accounting profits to be adjusted by subtracting the imputed interest on capital if the profits-sales ratio is to equal the Lerner measure. In practice, this is seldom, if ever, done directly. Because r -- the risk-adjusted cost of capital -- is not known,

9

most studies which make any adjustment for the opportunity cost of capital do so by leaving it out of the computation of profits and putting a term in the value of capital divided by sales on the right-hand side of any regression. If Hotelling valuation were used and if the capital-output ratio had no other effect on the Lerner measure, then the coefficient of that variable would be r. In practice, as Leibowitz (p. 8) points out, that coefficient is often found to be significantly negative.

I shall comment later on a possible reason for such a finding. For the present, simply consider the procedure of leaving imputed interest out of costs and placing the value of capital divided by sales on with profits/sales as the dependent variable. Suppose that the resulting regression coefficient for that variable is b. This is equivalent to subtracting r times that variable from the dependent variable and obtaining a regression coefficient on the right-hand side of b - r. Such a subtraction, however, amounts to evaluating profits/sales allowing for imputed interest at the correct interest rate, r, but with the firm's own valuation of capital. In what follows, I assume that to have been done and consider the consequences for estimating the Lerner measure as though it were done explicitly. Studies or opinions which make no such adjustment are obviously hopeless.

3. A Stationary Environment: The Most Favorable Case

Suppose then that firms do _not_ use Hotelling valuation. Is it nevertheless possible that profits/sales will equal the Lerner measure under constant returns. To examine this question, it is useful to go to the most favorable case, to ignore the issues just raised, and to assume that the firm exists in an essentially stationary environment in which benefits do not depend on calendar time. (To make this consistent with net investment by the firm, one must suppose that demand shifts in a very special way and the firm invests to secure the same marginal revenue in all periods.) In this case, we can replace $G(t, \theta)$ by $f(\theta)$. Allowing for this change in notation:

$$(14) \qquad f(\theta) = - V^{*'}(\theta) + rV^*(\theta) \qquad .$$

Now denote the amount of investment which the firm makes at time u by $I(u)$. (For simplicity, I assume that the firm uses only one type of capital; generalization is easy.) Then, for any valuation schedule, $V(.)$, the sum of total depreciation and total imputed interest at time t is given by:

$$(15) \qquad A \equiv \int_{-\infty}^{t} I(u) [-V'(t-u) + rV(t-u)] \, du \qquad .$$

On the other hand, total net returns to capital, the depreciation and imputed interest corresponding to Hotelling valuation, are given by:

$$(16) \qquad B = \int_{-\infty}^{t} I(u) f(t-u) \, du \qquad .$$

Since capital costs are subtracted in calculating profits, an understatement of such costs (B > A) means that profits/sales overstates the Lerner measure, while an overstatement of such costs (B < A) means that profits/sales understates the Lerner measure. Only if B = A will profits/sales give the correct result.

4. The Error Formula

It is very instructive now to consider the precise expression for the difference between profits/sales and the Lerner measure in the stationary-environment case under consideration.

Let L* be the true value of the Lerner measure. Then L* $\equiv$ (p - R')p, where p is price. Equivalently,

$$(17) \qquad px = R'x/(1 - L*) \qquad .$$

Let L be the profits/sales ratio -- the supposed Lerner measure. Then, using (15), (16), and (17):

$$(18) \qquad L - L* = \frac{B - A}{px} = \frac{(B - A)(1 - L*)}{R'x} \qquad .$$

This shows that the error in the use of profits/sales as an estimate of the Lerner measure is very unlikely to be uncorrelated with variables which affect the Lerner measure. This is because L* itself appears on the right-hand side of (18).

Indeed, the only possibility of avoiding this result would be if the appearance of x in the denominator of the right-hand side of (18) somehow cancelled out the appearance of (1 - L*) in the numerator. This does not seem a very realistic possibility,

12

and we shall now see that it is definitely not true in the most interesting leading cases.

To see this, we must explicitly account for the physical depreciation of capital and the time it takes for new machines to be installed and running. I do this by altering the description of the technology slightly and assuming that one unit of capital of age θ is equivalent to a fixed number of units, $a(\theta)$, of new capital. Then it is natural to measure capital in efficiency units, each of which has the same marginal product. (The case of a one-hoss shay -- no deterioration while the capital is in use -- corresponds to a constant $a(\theta)$ for $0 \leq \theta \leq$ the life of the capital good.) Consideration of (7) and (14) above, however, shows that the marginal revenue product of capital of age θ must be $f(\theta)$, so $a(\theta)$ is proportional to $f(\theta)$ and we might as well renormalize by weighting capital of age θ by $f(\theta)$. From (16), B is then the capital stock in the new efficiency units.

More precisely, suppose that, at time t, the labor assigned to capital of vintage u is $L(t, u)$ and that the output produced thereby is:

$$(19) \qquad x(t, u) = F(f(t - u)I(u), L(t, u)) \qquad .$$

Then (19) states that the effects of capital age can be captured as a capital-augmenting technical change. Since we are assuming constant returns and all labor has the same wage (and since marginal revenue is not changing), it follows from a well-known theorem on capital aggregation (see, for example, Fisher, pp. 559-60) that total output is given by the production function:

(20)   x = F(B, L)   ,

where B is defined in (16) and L is the total amount of labor employed by the firm. Note that (7) and (14) imply that we have measured capital so that the marginal <u>revenue</u> product of B is unity.

Now consider changes over time with w and R' held constant. Since F(., .) is homogeneous of degree one and L will be chosen to have marginal physical product equal w/R', the ratio of L to B will be constant. It follows that x/B will also be constant and that R'x = mB for some constant m. Substituting in (18) yields

$$(21) \qquad L - L^* = (1 - L^*) \frac{B - A}{mB} .$$

The constant, m, depends on w and on R', but, given w and R', it is determined by the technology. In particular, different firms with the same true Lerner measure, L*, will have different values of m, and different firms with the same value of m will have different values of L*. This means that the error in profits/sales as a substitute for the Lerner measure cannot be taken to be independent of the Lerner measure itself or of the variables assumed to influence that measure.

It is illuminating to evaluate m in a particular special case (and helpful in interpreting the numerical examples given below). Suppose the technology is Cobb-Douglas, so that

$$(22) \qquad F(B, L) = B^a L^{1-a} .$$

Then the fact that the marginal revenue product of B is unity implies that R'x = B/a, so that m = 1/a and (21) becomes[5]

$$(23) \qquad L - L^* = a\ (1 - L^*)\ \frac{B - A}{B} \qquad .$$

The appearance of the Cobb-Douglas parameter in this way is no accident. Returning to the more general case of any constant-returns production function, Euler's Theorem, profit-maximization, and the fact that the marginal revenue product of B is unity imply:

$$(24) \qquad B = R'x - wL \qquad ,$$

so that

$$(25) \qquad 1/m = 1\ - wL/R'x \qquad ,$$

or one minus labor's share of the value of output when that value is in terms of marginal revenue rather than of price. As in the Cobb-Douglas case, this will depend on the parameters of the production function, but it should come as no surprise that the errors which come from the mismeasurement of capital bulk larger when (true) capital costs are a large share of total costs than they do when that share is small.

In the case in which the aging of capital does not have a capital-augmenting effect on its productivity, simple formulae are less readily come by. It remains generally true, however, that the error depends on the Lerner measure and on the nature of the technology. It is obvious that, as in the capital-augmenting case, the error must be absolutely larger, _ceteris_ _paribus_ the more important capital costs are as a fraction of total costs.

I shall return to a discussion of the error formula below, and also to the use of the Cobb-Douglas example. For the pre-

sent, however, I want to put aside the leading but special case of capital-augmenting changes due to capital age and return to the general case to examine the question of whether <u>without</u> Hotelling valuation, it is possible that B = A and profits/sales equals the Lerner measure.

5. The Simplest Case: Exponential Growth

To examine the circumstances under which B = A, define

$$(26) \qquad K^* \equiv \int_{-\infty}^{t} I(u) \, du \qquad ,$$

the total value before depreciation of the firm's capital stock. Then A/K* is average depreciation and imputed interest per dollar of capital put in place in the past, while B/K* is a similar average of net returns to capital. For profits/sales to equal the Lerner measure, these two averages have to be the same. This obviously happens if each weight, I(u)/K*, applies to equal magnitudes in the two averages; that would be the case if the firm used Hotelling valuation. I now investigate the circumstances in which the two averages will be equal even though their components are not.

I shall consider only the simplest and most favorable case, that of exponential growth in which:

$$(27) \qquad I(u) = e^{gu} \qquad ;$$

this is at least a case in which one can imagine the stationarity assumption to hold with demand and all variables growing at the same rate g (which does not make it a realistic case, however).

16

As we shall now see, even in this unreasonably favorable case, it is not generally true that profits/sales equals the Lerner measure.

To see this, define:

$$(28) \qquad C(g) \equiv e^{-gt}(B - A) \qquad ,$$

so that $C(g) = 0$ is equivalent to profits/sales equalling the Lerner measure. Letting $\theta = t-u$, we see that, in the exponential growth case:

$$(29) \qquad C(g) = \int_0^{\infty} [rV^*(\theta) - V^{*\prime}(\theta) - rV(\theta) + V'(\theta)]e^{-g\theta} \, d\theta$$

$$= \int_0^{\infty} [f(\theta) - \hat{f}(\theta)]e^{-g\theta} \, d\theta \qquad ,$$

where

$$(30) \qquad \hat{f}(\theta) \equiv -V'(\theta) + rV(\theta) \qquad .$$

$\hat{f}(.)$ should be interpreted as that stream of benefits which would make $V(.)$ (the valuation actually used by the firm) the appropriate Hotelling valuation.

Theorem 1. In the exponential case, profits/sales equals the Lerner measure if $g = r$.

Proof: Consider (29) at $g = r$. On the one hand, since the price of a unit of new capital has been normalized to be unity,

17

$$(31) \qquad \int_0^\infty f(\Theta) e^{-r\Theta} \, d\Theta = V^*(0) = 1 \qquad .$$

On the other hand,

$$(32) \qquad \int_0^\infty [-V'(\Theta) + rV(\Theta)] e^{-r\Theta} \, d\Theta = - V(\Theta) e^{-r\Theta} \Big|_0^\infty = 1 \qquad ,$$

for the same reason.

This result parallels that for the relation between the accounting rate of return and the economic rate of return in the exponential growth case (see Fisher and McGowan, p. 95), except that here the cost of capital, r, is involved instead of the economic rate of return. (Of course, r is the economic rate of return on the margin.) When the growth rate is equal to the cost of capital, everything comes out all right. In the present case, however, this is just about the only positive thing that can be said. It is not even true that the weak result that the accounting and economic rates of return are on the same side of the growth rate has a parallel here.

This can most easily be seen by observing that (29) shows $C(g)$ to be the present value of the difference between two benefit streams, $f(.)$ and $\hat{f}(.)$, discounted at interest rate g. Since there is nothing to prevent that difference from changing signs an arbitrary number of times, $C(g)$ can easily have several (indeed an infinite number of) roots, of which r must be one. Fur-

ther, C(g) will generally not be monotonic in g. So long as f(.) and $\hat{f}$(.) are unrestricted except by (31) and (32), this is all that can be said. These propositions are exemplified below.

In practice, however, it may be possible to get a bit further than this by considering one way in which the relation between f(.) and $\hat{f}$(.) is likely to be restricted. For obvious reasons, firms are likely to use depreciation schedules which accelerate depreciation. If they are able so to accelerate depreciation as to make the value of their capital stock never greater and sometimes less than would be the case under Hotelling depreciation, then a definite result does emerge. It is:

Theorem 2. Suppose that $V(\theta) \leq V^*(\theta)$ for all $\theta \geq 0$, with the strong inequality holding for some set of values of θ of non-zero measure.[6] Then, for growth rates sufficiently close to zero, profits/sales overstates the Lerner measure.

Proof. Evaluate (29) at g = 0. Then:

$$(33) \qquad C(0) = \int_0^\infty [V'(\theta) - V^{*'}(\theta)] \, d\theta + r \int_0^\infty [V^*(\theta) - V(\theta)] \, d\theta \ .$$

But the first integral is zero, since $V(1) = 1 = V^*(1)$ and $V(\infty) = 0 = V^*(\infty)$. Under the given assumption, the second integral must be positive. Hence, at g = 0, B > A and profits/sales overstates the Lerner measure. By continuity, the same result is true for growth rates sufficiently close to zero.

19

In essence, the proof depends on observing that, at zero growth rate, the firm is taking depreciation equal to the initial value of a single capital good, and this is the same using either valuation. It follows that all that matters is imputed interest, and accelerating depreciation must reduce that.

Note that essentially the same proof leads to:

Theorem 3. Suppose that one of two otherwise identical firms uses a depreciation schedule which results in a capital valuation always less than or equal than that of the other and less for non-trivial time periods. Then, for identical growth rates sufficiently close to zero, the firm using accelerated depreciation will have profits/sales greater than the other firm.

Note that, at low rates of growth, accelerating depreciation does not increase the profits/sales ratio. Of course this is because of the steady-state nature of what is going on. A similar result holds for the accounting rate of return (Fisher and McGowan, p. 86).

As one might expect, the opposite result holds at very high rates of growth, although this is less interesting. In particular, for high enough rates of growth, a firm accelerating depreciation relative to Hotelling valuation will have profits/sales understating the Lerner measure. To see this, observe that, as g goes to infinity, the only things that matter in C(g) will be the values of depreciation and foregone interest on newly-installed capital stock. But the value of new capital stock is the same in all valuation systems, so the sign of C(g) depends only on the difference in the depreciation being taken, and C(g)

will be negative under the stated assumption. Similarly, if one of two otherwise identical firms accelerates depreciation relative to the other, then at high enough rates of growth that firm will have a lower profits/sales ratio.

It is tempting to conclude from this that C(g) is monotonically decreasing in the growth rate in such accelerated-depreciation cases, but this need not be true. Accelerated depreciation means that depreciation is higher, but it also means that foregone interest is less. Even in such cases, therefore, [f(θ) − f̂(θ)] can change sign more than once, so that there can be several values of g at which C(g) = 0.

6. The Size and Behavior of the Error: Examples

It is not enough to know that such problems can exist, however. It is important to know whether they are likely to be large and whether they will tend to be correlated with variables used in regressions purporting to explain monopoly power. It is very instructive to begin by working out a set of examples. Consideration of the leading case discussed above in which the effects of capital aging are capital-augmenting shows that it is useful to do this by evaluating $(B - A)/B$ (which I shall sometimes denote by Z). For a Cobb-Douglas technology, we can then use (23) to obtain an order of magnitude for the errors.

Examples are fairly easy to generate. Each example requires a choice of f(.) subject to the restriction that

$$(34) \qquad \int_0^\infty f(\theta)\, e^{-r\theta}\, d\theta = 1 \qquad .$$

This is done in various ways as described below.

In addition, each of the examples below is worked out for three different types of depreciation schedule. The first -- straight line -- has:

$$(35) \qquad V'(\theta) = -1/T \qquad V(\theta) = (T - \theta)/T$$

for $0 \le T$, where T is the life of the capital good (the point at which $f(\theta)$ becomes and remains zero.

The second type of depreciation is the continuous-time equivalent of sum-of-the-years' digits. It has

$$(36) \quad V'(\theta) = - \frac{T - \theta}{T^2/2} \qquad V(\theta) = 1 - \frac{\theta(2T - \theta)}{T^2} \; .$$

The third type of depreciation is exponential with a switch to straight line when the latter method becomes faster. Define $d = 2/T$ and $\theta^* = T - 1/d$, so that $d = 1/(T - \theta^*)$. Then this method has

$$(37a) \quad V'(\theta) = - d \, e^{-d\theta} \qquad V(\theta) = e^{-d\theta}$$

for $O \le \theta \le \theta^*$ and

$$(37b) \quad V'(\theta) = - \frac{e^{-d\theta^*}}{T - \theta^*} \qquad V(\theta) = (T - \theta) \, d \, e^{1-dT}$$

for $\theta^* \le \theta \le T$.

Each of the tables giving the results for $Z \equiv (B - A)/B$ is in two parts. In the first half of each table, the interest rate, $r = .2$; in the second half, $r = .15$. These seem reasonable

22

enough for illustrative purposes.   In practice, of course, firms
differ in the rate r that should be used, for r must be inter-
preted as the cost of capital including a risk premium.[7]

As we should expect from the analysis above, $Z \equiv (B - A)/B$
is most often greater in absolute value for growth rates far from
r than it is for growth rates close to r although this is not a
universal property.   Since growth rates as high as 15% are not
common, this suggests that the problems with profits/sales as an
estimate of the Lerner measure are likely to be relatively less
severe when interest rates are low than when they are high,
although it would be a mistake to conclude that those problems
become absolutely small.   In any event, the results for the
exponential case are given for growth rates from zero to 30% in
steps of 2 percentage points each.

I   come   then to the question of how to judge whether the
values of Z given in the tables are high or low.   As already
indicated, a rough rule of thumb can be given using the error
formula (23) for the Cobb-Douglas-capital-augmenting case.   (In
view of (21) and (25), other capital-augmenting cases will yield
roughly similar results.) Dividing (23) by L* yields

$$(38) \qquad \frac{L - L^*}{L^*} = a \left(\frac{1 - L^*}{L^*}\right)\left(\frac{B - A}{B}\right) \quad .$$

Obviously, this means that a given value of $(B - A)/B$ corresponds
to a greater percentage error for low values of L* than for high
ones, a fact I shall discuss below.

Ravenscraft (p. 31) gives a mean value for L from the Line-
of-Business data of .0648, excluding imputed interest, while
Cobb-Douglas estimates for the United States suggest an average

value of a about .25. If one were to believe that profits/sales actually measure L* on average and were to forget about the exclusion of imputed interest, this would mean that the values given in the tables below should be multiplied by about 3.6 to obtain an order of magnitude for the errors as a fraction of the Lerner measure. Because of the exclusion of imputed interest, this number is an underestimate.[8]

The whole point of this paper, however, is that profits/sales is far from being a trustworthy estimate of the Lerner measure. It is therefore of some interest to judge the magnitude of the error without assuming that L and L* are close together on the average. This is easy to do and leads to roughly the same orders of magnitude. Solving (23) for L*, one can express the error as a function of L. Dividing by L, one obtains:

$$(39) \qquad \frac{L - L^*}{L} = a \left(\frac{1 - L}{L}\right)\left(\frac{Z}{1 - aZ}\right) \quad ,$$

where, again, $Z \equiv (B - A)/B$. This gives the error as a fraction of the observable profits/sales ratio, L. For the values of Z in the tables below and for a approximately .25, however, aZ is sufficiently small that the (underestimated) multiplier of 3.6 given above remains roughly correct (as do the other multipliers given in footnote 8); somewhat greater values are required when Z is positive, and somewhat smaller ones when Z is negative.

In sum, the Line-of-Business data suggest that the values in the tables below should be multiplied by at least 3.6 to be interpreted as fractions of observed average profits/sales ratios. If

one believes that the average profits/sales ratio is also the average true Lerner measure (a dangerous assumption), then the same multiplier gives the error as a fraction of the average true Lerner measure

Table 1 gives the values of $(B - A)/B$ for a one-hoss shay case in which $f(\theta)$ is constant for 10 years and zero thereafter. (As in all the examples, the maximum value of $f(\theta)$ is chosen to satisfy (34).) Here the values of $(B - A)/B$ exhibit the simplest pattern. They are all positive for growth rates near zero and decline monotonically, passing through zero, as they must, at $g = r$. This is to be expected, given Theorems 1 and 2 above. As we should also expect, for low growth rates, the use of sum-of-the-years' digits depreciation yields the absolutely highest value of $(B - A)/A$, with straight line depreciation yielding the absolutely lowest values and exponential depreciation values somewhere in between the two.

[TABLE 1 HERE]

It is evident, given a multiplier of 3.6, that the values given in Table 1 for low rates of growth are quite large enough to be worth attention, and the magnitudes will increase as we progress to later tables. Certainly this is true for the values generated with sum-of-the-years' digits depreciation, but any comfort in the notion that straight line depreciation leads to relatively small errors (on the order of "only" 20% or so) will soon disappear. Table 1 happens to be one of the more favorable cases for the use of profits/sales.

Table 2 shows that even what appears to be a relatively minor change in the benefit profile, $f(.)$, can make an enormous

difference in the magnitude of the errors. That table, like Table 1, presents the results for a one-hoss shay case. The difference is that now the benefits from the capital good only begin two years after the expenditure for that good is made. In other words, the benefit profile of Table 1 is shifted two years into the future. At low growth rates, this results in roughly tripling the values of (B - A)/B generated by straight-line and exponential depreciation and somewhat less than doubling those for sum-of-the-years' digits. Even in Table 1, at four percent growth, the results correspond to lower bounds for errors in profits/sales at the latter's average value of from about 24% to about 80%. Now, in Table 2, they correspond to lower bounds ranging from about 76% to about 145%. Even with a much lower multiplier than 3.6, these are large errors.

[TABLE 2 HERE]

The results in Table 3 also correspond to (absolutely) large percentage errors, but they illustrate a different phenomenon. In that table, f(θ) has been chosen to decline exponentially for 10 years at a unit rate of decay (in other words, f(θ) = Ce$^{-\theta}$ for 0 $\leq$ 10) after which f(θ) becomes zero. Again, the maximum value of f(θ) is chosen to satisfy (34). The new feature in this table is that the values of (B - A)/B are negative at low growth rates and increase, rather than decrease monotonically with g.

[TABLE 3 HERE]

Table 3 shows that it is a mistake to conclude that profits/sales overestimates the Lerner measure at low rates of growth. As Theorem 2 suggests, if Hotelling depreciation is

26

accelerated relative to the depreciation methods actually used instead of the other way round, then underestimation will result. (Note in this regard that Ravenscraft (p. 31) finds that the minimum value of the profits/sales ratio in the Line-of-Business data is substantially negative.)

As one should expect from Theorem 3 in these circumstances, it is now the least rapid depreciation method, straight line, that yields the (absolutely) largest errors at low rates of growth. It should also come as no surprise that shifting the function, f(.), which corresponds to Table 3 by delaying its start by two years this time reduces rather than increases the absolute values of the errors for low rates of growth. Such a shift moves Hotelling depreciation closer to the three depreciation schedules examined. It makes the errors for some methods positive and those for others negative, but while some of the errors naturally become absolutely small, not all of them do, and the size of the errors (or their sign) cannot be predicted without knowledge of the benefit profile, f(.). These results are given in Table 4.

[TABLE 4 HERE]

Table 4 also illustrates a new phenomenon. The results for sum-of-the-years' digits depreciation are not monotonic in the growth rate and change sign at rates of growth other than r. This is because we have now reached cases in which {f(θ) - $\hat{f}$(θ)} can change sign more than once (cf. (29)).

It may be objected, of course, that the values of (B - A)/B given in Table 3 are too large (absolutely) because the example used to generate them is extreme. That example involves a very

27

rapid exponential decline in benefits, so that benefits are close to zero for a good deal of the ten-year life of the capital goods involved while the firm is forced to depreciate the goods at a non-negligible rate over the entire ten years.

There are three reasons to be careful before offering such an objection. First, the example can be though of as one with benefits after taxes, including the effects of the chosen depreciation method.[9] Second, extending the life of the capital good, as in Table 4 makes things better, not worse. Third, as this suggests, the general principles being examplified do not rest on the choice of particular examples. Table 5 presents the results for an exponential decay case identical with that of Table 3 but with a life of five years rather than ten. The resulting values of (B - A)/B for low rates of growth are smaller in absolute value than those in Table 3, but they still translate into enormous errors in the use of profits/sales.[10]

[TABLE 5 HERE]

It is possible to continue with many more examples. (Indeed, only the desire to keep benefit shapes, f(.), relatively -- and probably unrealistically -- simple keeps one from illustrating even more complex behavior.) The conclusion to be drawn should now be clear, however. Profits/sales is a totally unreliable estimate of the Lerner measure. The errors involved in using it can vary from very small to frighteningly large. Which they will be, what sign they will have, and how they will behave as functions of the growth rate cannot be determined without knowledge of the benefit profile, f(.). No conclusion as to the

monopoly power of a firm or group of firms can be drawn from such a measure. Further, all these results come from the unreasonably favorable stationary-environment case of exponential growth.

7. Regression Studies and "Random" Errors

This does not end the matter, however, for profits/sales is also used as a measure of monopoly power in regression studies in which it is the dependent variable. In such studies, errors in the dependent variable become part of the error term. This has led some practitioners (e.g. Martin 1983, p. 32) to assume that such errors create no problem because one might just as well assume them uncorrelated with the variables used in the regression. For brevity, I shall refer to this as the assumption of "independence".

This is, at best, a dangerous procedure. The assumption that error terms are uncorrelated with regressors is often made. Usually, as here, it has no better foundation than the assertion that the investigator has been unable to think of a reason why it should not hold. Where errors are small, such an independence assumption may do little harm. Where, as here, they can be very large indeed, one is risking a great deal. The very fact that regression studies appear to recover systematic results in the presence of errors which can obviously have a very large variance should make one suspect (although it certainly does not prove) that those errors are systematically related to the variables used, even though one would wish to assume otherwise.

In any event, even such a wishful-thinking assumption cannot be made in the present case. The error involved in the use of

the profits/sales ratio is systematically related to the variables used in regression studies, and the independence assumption is therefore false.

To see this, consider the error formula (23) (or (21) or (18)). It contains a term in $(1 - L^*)$ which does not cancel out. This means that errors will be absolutely larger for firms with low values of the Lerner measure than for high ones (and this effect will be even more pronounced in percentage terms). This says that it will not generally be the case that the error can be taken to be independent of the variables used to explain the Lerner measure.

More precisely, suppose that L^* is supposed to be related to a set of variables, X, by:

$$(40) \qquad L^* = X\beta + \epsilon$$

in the usual notation for regression. Assume that ϵ has mean zero and is distributed independently of X. We know that $L - L^*$ is in the form $(1 - L^*)u$. Suppose that u is in fact distributed independently of X; indeed, for convenience, take X to be non-stochastic. For convenience, suppose further that u and ϵ are independently distributed. Then b, the least-squares estimate obtained from regressing L on X has the property:

$$(41) \qquad E(b) = E\{(X'X)^{-1}X'L\} = \beta(1 - E(u)) \qquad .$$

Only if u has mean zero can one assume that this creates no bias (and no inconsistency -- the problem does not disappear for large sample sizes).

No doubt there will be some willing to assume that $E(u) = 0$,

but the lack of basis for such an assumption now lies exposed. Those who make it are asserting that because a possibly large number has an unknown sign, one might as well assume it to be zero. There is no basis for believing that the benefit profiles characteristic of capital goods in the American economy just happen to be distributed relative to the depreciation methods used so as to make the average value of $u \equiv a(B - A)/B$ zero. The fact that $(B - A)/B$ can be either positive or negative is not a sufficient basis for such a belief.

In the absence of such an unwarranted assumption, even the (also unwarranted) assumption that u is distributed independently of the variables used in regression analyses will not rescue such studies. Assuming that $1 - E(u) > 0$, then such studies will have their coefficients biased towards zero if $E(u) > 0$ and away from zero if $E(u) < 0$. This sort of problem would only be avoided if the dependent variable were $\log (1 - L)$.[11]

Even such a choice of dependent variable -- not commonly made in the literature -- will not rescue studies using profits/sales as a surrogate for the Lerner measure. This is because it is clear that u itself is correlated with the variables used in such studies. I comment briefly on some of the effects involved.

(a) Capital Intensity. As already observed, investigators who recognize that the opportunity cost of capital -- imputed interest -- needs to be included frequently attempt to take it into account by using the capital/output ratio as a regressor. We saw above, however, that relatively capital-intensive firms

31

will have relatively high (absolute) errors. While we cannot say what sign those errors are likely to have, it is again foolhardy to suppose that this means they can be assumed zero.

In fact, there is some evidence here as to the sign of such errors. If the book value of capital were also the Hotelling value (and Hotelling depreciation subtracted from profits), the coefficient of the capital-intensity variable should equal the risk-adjusted cost of capital (assuming that capital intensity has no direct effect on the Lerner measure). Leibowitz (p. 8) points out, however, that such coefficients are often found to be significantly negative. A distinct possibility is that this results from negative values of (B - A)/B, as in Table 3.[12]

(b) Depreciation Methods. It is obvious that the choice of depreciation method influences the errors, with faster depreciation methods producing algebraically larger errors than slower ones at low enough rates of growth. Gerald Salamon points out, however, that large firms tend to use accelerated depreciation methods. This will lead to an upward bias in the coefficient of a firm size variable.

(c) Growth Rate. As we have seen, the size of the error is influenced by the growth rate of the firm. Studies which do not include the growth rate as an explanatory variable will have errors correlated with any independent variable correlated with growth rate. Firm size or market share appear candidates here, since they will be influenced by past growth.

Even the inclusion of the growth rate requires the assumption that the environment is stationary in the sense described above. (Departures from stationarity can also be correlated with

included variables, but it is hard to say anything specific about this.)

(d) Risk. The value of r used obviously makes a sizeable difference in the errors. Yet r differs over firms because it is the cost of capital including a risk premium. At low growth rates, it seems likely that firms engaged in relatively risky activities will have relatively larger errors (absolutely) than will relatively risk-free firms. Some variables, such as research and development expenditures, may be related to participation in risky activities; others, such as the variance of firm sizes or the Hirschman-Herfindahl index may relate to the risky nature of the industry involved.

8. Conclusion

The conclusion to be drawn from all this must be clear. I can do no better than to paraphrase the conclusion of Fisher and McGowan (p. 91) on the companion issue of the relations of the accounting and economic rates of return. Economists (and others) who believe that analysis of the profits-sales ratio will tell them much are deluding themselves. The literature which supposedly relates concentration and the Lerner measure does no such thing, and examination of the profits-sales ratio to draw conclusions about monopoly power is a totally misleading enterprise.

Appendix: A Note on the Use of Tobin's q

Several recent papers have used Tobin's q -- the ratio of the market value of a company's stock to the value of its capital assets -- as the dependent variable in regressions intended to test hypotheses about monopoly power. (See Eric Lindenberg and Stephen Ross, Michael A. Salinger, and Michael Smirlock, Thomas Gilligan, and William Marshall.) Such a use has much to recommend it relative to the use of the accounting rate of return or the profits/sales ratio; in particular, problems relating to risk disappear as the variable itself includes the market's evaluation of the required risk premium. Nevertheless, the capital-theoretic problems which eviscerate the meaning of studies using the other two variables also affect those using Tobin's q.

In the first place, the value of the capital assets which appears in the denominator of q typically does not include the value of intangible assets such as past research and development activities. This tends to bias q upwards in industries where such activities are important and may lead to the erroneous conclusion that such activities are correlated with monopoly power.

More directly related to the problems considered in the present paper, even the valuation of physical assets in the denominator of q raises serious problems. Firms that have historically taken accelerated depreciation will show a lower book value for their capital stock, ceteris paribus, than firms that have used slower depreciation methods. Further, the size of such an effect depends on the rate of growth of the firms involved. Yet depreciation methods may be correlated with variables such as

firm size.

It is certainly true, as pointed out by Salinger (p. 160) that the capital-valuation problem affects only the denominator of Tobin's q and not both the numerator and denominator as it does in the case of the accounting rate of return. Hence a one percent error in capital valuation relative to Hotelling valuation corresponds to a one percent error in the measurement of q. It is not fully clear whether that makes q more or less sensitive to such errors than are the accounting rate of return and the profits/sales ratio. Even if, as I suspect is true, the sensitivity is less, the errors in valuation seem likely to be quite large enough to pose a very serious problem and possibly to vitiate the use of the measure altogether.

Not even Tobin's q fully escapes the problem that firms invest in capital goods for the future stream of benefits those goods will generate. While valuation of assets at true replacement cost would help, firms' book values usually do not provide an appropriate valuation for analytical purposes.

References

Fisher, Franklin M., "The Existence of Aggregate Production Functions," _Econometrica_, October 1969, _37_, 553-77.

_____ and McGowan, John J., "On the Misuse of Accounting Rates of Return to Infer Monopoly Profits." _American Economic Review_, March 1983, _73_, 82-97.

Hotelling, Harold, "A General Mathematical Theory of Depreciation," _Journal of the American Statistical Association_, September 1925, _20_, 340-53.

Leibowitz, S. J., "The Measurement and Mismeasurement of Monopoly Power," mimeo, March 1984.

Lindenberg, Eric and Ross, Stephen, "Tobin's q Ratio and Industrial Organization," _Journal of Business_, January 1981, _54_, 1-32.

Long, William F., Lean, David F., Ravenscraft, David J., and Wagner, Curtis L., III, _Benefits and Costs of the Federal Trade Commission's Line of Business Program, Volume I: Staff Analysis_, Federal Trade Commission, Bureau of Economics, September 1982.

Long, William F. and Ravenscraft, David J., "The Misuse of Accounting Rates of Return: Comment." _American Economic Review_, June 1984, _74_, 494-500.

Martin, Stephen, _Market, Firm, and Economic Performance_, Salomon Brothers Center for the Study of Financial Institutions, Graduate School of Business Administration, New York University, 1983.

_____, "The Misuse of Accounting Rates of Return: Comment," _American Economic Review_, June 1984, _74_, 501-06.

Ravenscraft, David J., "Structure-Profit Relationships at the Line of Business and Industry Level," _Review of Economics and Statistics_, February 1983, _65_, 22-31.

Salamon, Gerald L., "Accounting Rate of Return, Measurement Error, and Tests of Economic Hypotheses: The Case of Firm Size," mimeo., June 1983.

Salinger, Michael A., "Tobin's _q_, Unionization, and the Concentration-Profits Relationship," _Rand Journal of Economics_, Summer 1984, _15_, 159-70.

Scherer, F. M., Review of F. M. Fisher, J. J. McGowan, and J. E. Greenwood, _Folded, Spindled, and Mutilated: Economic Analysis and U. S. v. IBM, Journal of Economic Literature,_ June 1984, _22_, 620-21.

Smirlock, Michael, Gilligan, Thomas, and Marshall, William, "Tobin's _q_ and the Structure-Performance Relationship," _American Economic Review_, December 1984, _74_, 1051-60.

* I am indebted to Paul L. Joskow for helpful discussions but retain responsibility for error.

1. The problem is not limited to constant returns and the profits-sales ratio. Any attempt to calculate the Lerner measure must involve at least an implicit measurement of marginal cost. Such a measure, if direct, must take capital costs properly into account to produce a valid result. (This point is well made in S. J. Leibowitz.) For simplicity, I assume constant returns throughout this paper, as does most of the empirical literature, even though that seems a questionable assumption when studying the effect of firm size.

2. Some recent papers have used Tobin's q as a simple measure of monopoly power. While that measure seems definitely superior to either the profits-sales ratio or the accounting rate of return, its use does not avoid the capital-theoretic difficulties here discussed. I comment briefly on this in the Appendix.

3. There are other problems with Scherer's statement. My normal strategy for many years has been to weigh 150 pounds.

4. For simplicity, I assume that the price of a new unit of capital is always constant at unity so that there are no gains or losses stemming from capital price changes.

5. This can also be shown more laboriously by considering the optimal assignment of labor in the Cobb-Douglas case given (19).

6. This proviso essentially rules out the case in which valuations are discontinuous and the valuation used by the firm

is below Hotelling valuation only at isolated points.

7. For simplicity, the entire analysis and the examples have been worked out in before-tax terms. It is not hard to show that the same analysis applies if after-tax magnitudes are used, interpreting f(.) as the *after*-tax benefit stream. (See Fisher and McGowan, pp. 95-97.) Of course, in that case the examples below would involve different pre-tax f(.) for different depreciation methods.

8. Since Ravenscraft (like others) does not find a regression coefficient for his capital/sales variable that is easily interpretable as r, it is not simple further to adjust the 3.6 figure to take account of imputed interest. It is clear, however, that such an adjustment for any reasonable value of r would make the appropriate multiplier very much higher.

At the maximum value for profits/sales given by Ravenscraft (.5371), the lower bound for the multiplier is approximately .22; the minimum value for profits/sales is negative, so that the multiplier corresponding to the low end of the range is unbounded. For comparative purposes, using Census of Manufactures data (Ravenscraft, p. 30), the lower bound for the multiplier at the mean of L (.2188) is be roughly .89; at the maximum (.4232), it is about .34; at the minimum (.0096), it is about 25.8. Even these lower multipliers make the errors in the tables below loom very large.

9. See footnote 7. It should be noted, however, that a value of r of .15 may be a bit high for an after-tax example.

10. It may still be objected that a decay rate of unity is

very high, although how one is to know that without any empirical evidence on f(.) is not clear. If lower decay rates are used, Hotelling depreciation becomes less accelerated relative to the three depreciation methods examined. In the limit, a zero decay rate corresponds to a one-hoss shay example as in Tables 1 and 2. As we have seen, such an example generates positive, rather than negative errors at low rates of growth. Correspondingly, as the decay rate goes from unity down to zero, the errors at low rates of growth move upwards from negative values to positive ones. Of course there is _some_ decay rate at which such errors are small (although that rate will not necessarily be the same for each of the three depreciation methods used). It would be foolhardy, however, to suppose that such a decay rate must be the "realistic" one.

11. There is also the probability-zero case that u is correlated with the regressors in just such a way as to cancel out the effects of (1 - L*). There is no need to discuss this.

12. It could also result from a negative correlation of (B - A)/B and capital intensity stemming from a systematic relation between capital intensity and the shape of f(.).

Table 1
Values of (B-A)/B
10-Year One-Hoss Shay

r = .2

g	Straight line	Sum of Years Digits	Exponential
.00	0.1353	0.2794	0.1453
.02	0.1209	0.2505	0.1301
.04	0.1066	0.2216	0.1149
.06	0.0924	0.1926	0.0998
.08	0.0783	0.1639	0.0848
.10	0.0645	0.1353	0.0700
.12	0.0509	0.1072	0.0554
.14	0.0376	0.0795	0.0410
.16	0.0247	0.0523	0.0270
.18	0.0121	0.0258	0.0133
.20	-0.0000	0.0000	0.0000
.22	-0.0117	-0.0251	-0.0129
.24	-0.0230	-0.0494	-0.0254
.26	-0.0338	-0.0728	-0.0375
.28	-0.0442	-0.0955	-0.0491
.30	-0.0541	-0.1172	-0.0603

r = .15

g	Straight line	Sum of Years Digits	Exponential
.00	0.0937	0.2231	0.1026
.02	0.0807	0.1928	0.0887
.04	0.0678	0.1625	0.0747
.06	0.0550	0.1323	0.0608
.08	0.0424	0.1022	0.0470
.10	0.0300	0.0724	0.0333
.12	0.0178	0.0431	0.0198
.14	0.0058	0.0142	0.0065
.16	-0.0058	-0.0140	-0.0065
.18	-0.0170	-0.0416	-0.0191
.20	-0.0279	-0.0685	-0.0315
.22	-0.0385	-0.0945	-0.0435
.24	-0.0486	-0.1197	-0.0551
.26	-0.0583	-0.1440	-0.0664
.28	-0.0676	-0.1675	-0.0772
.30	-0.0765	-0.1900	-0.0877

Table 2

Values of (B-A)/B

2 Years Delay Followed By 10-Year One-Hoss Shay

	r = .2			r = .15		
g	Straight line	Sum of Years Digits	Exponential	Straight line	Sum of Years Digits	Exponential
.00	0.3624	0.4784	0.3871	0.2710	0.3861	0.2955
.02	0.3349	0.4413	0.3581	0.2416	0.3434	0.2638
.04	0.3055	0.4020	0.3271	0.2103	0.2981	0.2299
.06	0.2743	0.3603	0.2940	0.1769	0.2501	0.1937
.08	0.2412	0.3163	0.2588	0.1415	0.1994	0.1550
.10	0.2062	0.2699	0.2214	0.1038	0.1460	0.1139
.12	0.1691	0.2210	0.1818	0.0640	0.0897	0.0703
.14	0.1300	0.1696	0.1399	0.0219	0.0306	0.0241
.16	0.0888	0.1156	0.0956	-0.0225	-0.0314	-0.0248
.18	0.0455	0.0591	0.0490	-0.0692	-0.0963	-0.0763
.20	0.0000	0.0000	0.0000	-0.1184	-0.1643	-0.1306
.22	-0.0477	-0.0618	-0.0515	-0.1699	-0.2353	-0.1876
.24	-0.0977	-0.1263	-0.1056	-0.2240	-0.3094	-0.2476
.26	-0.1499	-0.1936	-0.1622	-0.2806	-0.3867	-0.3105
.28	-0.2045	-0.2637	-0.2215	-0.3399	-0.4672	-0.3763
.30	-0.2616	-0.3368	-0.2835	-0.4018	-0.5511	-0.4453

Table 3

Values of $(B-A)/B$

10-Year Exponential Decline $(C\bar{e}^{-\theta})$

	r = .2				r = .15		
g	Straight line	Sum of Years Digits	Exponential	g	Straight line	Sum of Years Digits	Exponential
.00	-0.6667	-0.3889	-0.6475	.00	-0.5218	-0.3044	-0.5068
.02	-0.5665	-0.3355	-0.5501	.02	-0.4269	-0.2529	-0.4146
.04	-0.4761	-0.2862	-0.4623	.04	-0.3416	-0.2053	-0.3317
.06	-0.3946	-0.2405	-0.3831	.06	-0.2647	-0.1613	-0.2570
.08	-0.3208	-0.1982	-0.3114	.08	-0.1952	-0.1206	-0.1896
.10	-0.2539	-0.1589	-0.2465	.10	-0.1325	-0.0829	-0.1286
.12	-0.1932	-0.1224	-0.1876	.12	-0.0756	-0.0479	-0.0734
.14	-0.1380	-0.0885	-0.1340	.14	-0.0240	-0.0154	-0.0233
.16	-0.0878	-0.0569	-0.0852	.16	0.0229	0.0149	0.0222
.18	-0.0419	-0.0275	-0.0407	.18	0.0656	0.0430	0.0637
.20	-0.0000	0.0000	0.0000	.20	0.1046	0.0693	0.1015
.22	0.0384	0.0257	0.0373	.22	0.1403	0.0938	0.1361
.24	0.0736	0.0497	0.0714	.24	0.1729	0.1168	0.1677
.26	0.1060	0.0723	0.1029	.26	0.2029	0.1383	0.1968
.28	0.1359	0.0934	0.1318	.28	0.2304	0.1584	0.2235
.30	0.1634	0.1133	0.1585	.30	0.2558	0.1773	0.2481

Table 4

Values of (B-A)/B

2-Year Delay Followed by 10-Year Exponential Decline $(Ce^{-\theta})$

	r = .2			r = .15		
g	Straight line	Sum of Years Digits	Exponential	Straight line	Sum of Years Digits	Exponential
.00	-0.2290	-0.0055	-0.1814	-0.2240	-0.0307	-0.1828
.02	-0.1852	0.0044	-0.1439	-0.1771	-0.0192	-0.1427
.04	-0.1474	0.0119	-0.1118	-0.1365	-0.0102	-0.1083
.06	-0.1149	0.0172	-0.0848	-0.1015	-0.0036	-0.0792
.08	-0.0873	0.0203	-0.0621	-0.0716	0.0008	-0.0547
.10	-0.0640	0.0214	-0.0436	-0.0462	0.0030	-0.0345
.12	-0.0446	0.0206	-0.0287	-0.0250	0.0032	-0.0181
.14	-0.0288	0.0180	-0.0171	-0.0075	0.0015	-0.0052
.16	-0.0163	0.0137	-0.0086	0.0067	-0.0020	0.0045
.18	-0.0068	0.0076	-0.0030	0.0177	-0.0072	0.0112
.20	-0.0000	0.0000	0.0000	0.0259	-0.0141	0.0152
.22	0.0042	-0.0092	0.0006	0.0314	-0.0227	0.0167
.24	0.0060	-0.0199	-0.0011	0.0345	-0.0328	0.0159
.26	0.0056	-0.0321	-0.0050	0.0354	-0.0445	0.0129
.28	0.0032	-0.0458	-0.0109	0.0341	-0.0576	0.0079
.30	-0.0013	-0.0609	-0.0187	0.0309	-0.0723	0.0008

Table 5

Values of (B-A)/B

5-Year Exponential Decline $(Ce^{-\theta})$

r = .2

g	Straight line	Sum of Years Digits	Exponential
.00	-0.2554	-0.1159	-0.2101
.02	-0.2245	-0.1028	-0.1847
.04	-0.1950	-0.0900	-0.1603
.06	-0.1667	-0.0776	-0.1371
.08	-0.1397	-0.0655	-0.1148
.10	-0.1138	-0.0538	-0.0935
.12	-0.0890	-0.0424	-0.0731
.14	-0.0653	-0.0314	-0.0536
.16	-0.0426	-0.0206	-0.0349
.18	-0.0208	-0.0102	-0.0171
.20	0.0000	0.0000	-0.0000
.22	0.0199	0.0099	0.0164
.24	0.0390	0.0194	0.0320
.26	0.0573	0.0288	0.0470
.28	0.0748	0.0378	0.0613
.30	0.0916	0.0466	0.0751

r = .15

g	Straight line	Sum of Years Digits	Exponential
.00	-0.1999	-0.0908	-0.1646
.02	-0.1693	-0.0776	-0.1393
.04	-0.1399	-0.0647	-0.1151
.06	-0.1119	-0.0521	-0.0920
.08	-0.0850	-0.0400	-0.0699
.10	-0.0594	-0.0281	-0.0488
.12	-0.0348	-0.0166	-0.0286
.14	-0.0114	-0.0055	-0.0093
.16	0.0111	0.0054	0.0091
.18	0.0326	0.0159	0.0268
.20	0.0532	0.0261	0.0436
.22	0.0728	0.0361	0.0598
.24	0.0917	0.0457	0.0752
.26	0.1097	0.0551	0.0899
.28	0.1270	0.0642	0.1041
.30	0.1435	0.0730	0.1176

bar code is on back cover

bar code is on back cover