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On the motions of the Sun, the Galaxy and the Andromeda nebula

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Summary. The sum of the velocity of the Galaxy, **G**, and the velocity of the Sun may be determined from their reflection in the radial velocities of the members of the Local Group of galaxies excluding Andromeda. Since Andromeda and the Galaxy are much more massive than all other members combined their momenta must be equal and opposite. Thus the observed velocity of Andromeda is a reflection of the already determined sum, plus the mass ratio times a reflection of the Galaxy's velocity. The knowledge of the observed radial velocity of Andromeda and the mass ratio allows us to determine both the velocity of the Galaxy and the circular velocity of its rotation at the Sun, V_c.

The method gives

$$V_c = 294 \pm 42 \text{ km/s}$$

$$\mathbf{G} = (-34, +7, -16) \pm 23 \text{ km/s}$$

where **G** is with respect to the centre of mass of the Local Group. Uncertainty in the mass ratio gives little change and may be removed by using Tully & Fishers' observed law $M \propto (V_{\max})^{5/2}$.

The uncertainties in the above determination of V_c may be considerably lowered if we make the further assumptions:

(1) The Galaxy does not have an exceptionally large maximum circular velocity for a spiral. This gives us the limit $V_c < 300$ km/s.

(2) The Galaxy and Andromeda obey the Tully & Fisher relationship and move in a bound ellipse about one another.

(3) The mass of Andromeda is $< 4 \times 10^{11} M_\odot$.

We then obtain $V_c = 290^{+10}_{-15}$ km/s.

The new values are discussed in terms of known Galactic parameters and other determinations.

Finally we draw conclusions concerning the binding of the Local Group, and the epoch of galaxy formation.

1 Introduction

The Local Group of galaxies consists of a number of independent members together with two major subgroups centred on the Galaxy and the Andromeda nebula. Those subgroups contain more than 95 per cent of the known mass so we shall assume that their momenta with respect to the centre of mass of the Local Group must be almost equal and opposite. Likewise we shall assume that there is no net mean motion of the independent members of the Local Group with respect to that mass centre.

From looking at the radial velocities of these independent members which are well scattered about the sky we determine the velocity vector of the Sun with respect to the centre of mass of the Local Group. By applying a small correction we find the velocity, **U**, of an ideal local observer who travels under gravity in a circle about the Galaxy rather than with the slightly non-circular motion of the Sun. In Fig. 1 this velocity **U** is broken into the unknown circular velocity V_c in the known direction $\hat{\mathbf{c}}$ and the fully unknown velocity of the Galaxy **G**.

Now we may deduce from observations the radial velocity of Andromeda as seen by our ideal observer, v'_{Ar}. The velocity *vector* with respect to him, $\mathbf{v}'_A = \mathbf{AO}$ has the point A lying on the plane that allows for arbitrary transverse components. This plane we know, so we can deduce the line, L, in which it cuts the plane of the known vectors $\hat{\mathbf{c}}$ and **U**. The fixed elements of Fig. 1 are thus v'_{Ar}, the line L, the vector **U** and the direction $\hat{\mathbf{c}}$. The variable elements are V_c, **G**, $\mathbf{v}_A$, $\mathbf{v}'_A$. Now since Andromeda and the Galaxy have equal and opposite momenta $\mathbf{v}_A = -\mu\mathbf{G}$ where μ is the mass ratio $\mu = M_G/M_A$. Thus the point A lies in the plane of **U** and $\hat{\mathbf{c}}$ and hence in the line L; furthermore $AB = \mu BC$. Now let us choose a possible value of the mass ratio μ. We shall now have to swing the line ABC about B keeping it in the $\hat{\mathbf{c}}$, **U** plane until we find a place with $AB = \mu BC$. Once that is done we have the value of V_c consistent with our chosen μ and with V_c known, **G** and $\mathbf{v}_A$ follow. All therefore hinges on a choice of μ. In practice this is not of great importance because V_c is not strongly dependent on μ, see the $f = 0$ curve of Fig. 4. However, even that uncertainty can be eliminated by appealing to a relationship between the mass of a spiral galaxy and the maximum circular velocity achieved in its 21-cm rotation curve. Tully & Fisher discovered that the luminosity L is proportional to $(V_{max})^{5/2}$. Since V_{max} is dynamically determined they postulate that the mass M should be proportional to $(V_{max})^{5/2}$ with as great a precision. Now from $V_c(\mu)$ and 21-cm work on our Galaxy one can deduce that $V_{max}(\mu)$ for our Galaxy is a few km/s higher than $V_c(\mu)$. The Tully–Fisher relations for ourselves and Andromeda can be divided to give

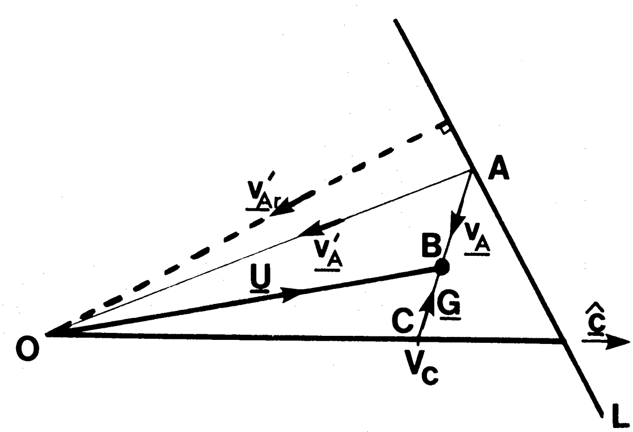

Figure 1. A plot of velocity vectors. O is our ideal observer, $\hat{\mathbf{c}}$ the direction of the circular motion V_c. The motion of O with respect to the Local Group is **U**. The known vector $v'_{Ar}\hat{\mathbf{r}}_A$ lies out of the plane of the paper but the plane normal to it and distant v'_{Ar} from O cuts the paper in the line L. Heavy lines are known; $\mathbf{V}_A = -\mu\mathbf{G}$ and the light lines slew around as μ is varied.

$\mu = M_G/M_A = [(V_{max}(\mu))/(265\ \text{km/s})]^{5/2}$ where 265 km/s is the V_{max} observed in Andromeda. This equation is solved graphically for μ in Fig. 4.

Let **v** be the velocity of a member of the Local Group of galaxies with respect to the centre of mass of the group. Let **v**′ be the velocity of that galaxy measured from the Sun but corrected to the local circular velocity. Further let **G** be the velocity of our Galaxy and $V_c\hat{\mathbf{c}}$ be the circular velocity of rotation at the solar distance from the centre of the Galaxy. Then the observed velocity is the true velocity less the motion of the observer so

$$\mathbf{v}' = \mathbf{v} - (V_c\hat{\mathbf{c}} + \mathbf{G}). \tag{1}$$

However, Andromeda has equal and opposite momentum to the Galaxy since the masses of other members of the Local Group are negligible in comparison to theirs. Hence for Andromeda

$$\mathbf{v}'_A = -(M_G/M_A)\,\mathbf{G} - (V_c\hat{\mathbf{c}} + \mathbf{G}). \tag{2}$$

Of course only radial components can be observed so denoting the unit vector in the direction of the chosen galaxy by $\hat{\mathbf{r}}$ we have

$$v'_r = \hat{\mathbf{r}}\cdot\mathbf{v}' = \hat{\mathbf{r}}\cdot[\mathbf{v} - (V_c\hat{\mathbf{c}} + \mathbf{G})] \tag{3}$$

and

$$v'_{Ar} = \hat{\mathbf{r}}_A\cdot\mathbf{v}'_A = -V_c\hat{\mathbf{r}}_A\cdot\hat{\mathbf{c}} - (1+\mu)\,\mathbf{G}\cdot\hat{\mathbf{r}}_A.$$

Solving this for V_c in terms of **G** we have

$$V_c = -[(1+\mu)\,\mathbf{G}\cdot\hat{\mathbf{r}}_A + v'_{Ar}]/(\hat{\mathbf{r}}_A\cdot\hat{\mathbf{c}}) \tag{4}$$

where

$$\mu = M_G/M_A.$$

Substituting this value in equation (3) we have on solving for $\hat{\mathbf{r}}\cdot\mathbf{v} = v_r$

$$\hat{\mathbf{r}}\cdot\mathbf{v} = v'_r - v'_{Ar}\frac{\hat{\mathbf{r}}\cdot\hat{\mathbf{c}}}{\hat{\mathbf{r}}_A\cdot\hat{\mathbf{c}}} + \mathbf{G}\cdot\left[\hat{\mathbf{r}} - (1+\mu)\,\hat{\mathbf{r}}_A\frac{\hat{\mathbf{r}}\cdot\hat{\mathbf{c}}}{\hat{\mathbf{r}}_A\cdot\hat{\mathbf{c}}}\right]. \tag{5}$$

Now some part of the observed radial velocities of Local Group members will be intrinsic, but some other part which varies systematically across the sky from member to member will be due to reflections of the observer's velocity. We may imagine ourselves trying different values of **G** working out V_c from (4) and the intrinsic part of the radial velocity from (5). If after one trial some of these systematic velocities are left in, we may expect that a different **G** would be able to lower the deduced radial velocities still further. In fact we expect that our best estimate of **G** will be obtained by minimizing the sum of the squares of the intrinsic velocities of the galaxies. Any alteration of **G** away from that one would cause some systematic appearance in the residual radial velocities $\hat{\mathbf{r}}\cdot\mathbf{v}$.

Now for each galaxy define an $\alpha = (\hat{\mathbf{r}}\cdot\hat{\mathbf{c}})/(\hat{\mathbf{r}}_A\cdot\hat{\mathbf{c}})$ and a vector $\boldsymbol{\beta}$,

$$\boldsymbol{\beta} = \hat{\mathbf{r}} - \alpha(1+\mu)\,\hat{\mathbf{r}}_A. \tag{6}$$

Notice that these are known for each galaxy. Equation (5) can be rewritten

$$v_r = v'_r - \alpha v'_{Ar} + \mathbf{G}\cdot\boldsymbol{\beta}. \tag{7}$$

Summing the squares over all galaxies except ourselves and Andromeda

$$\Sigma v_r^2 = \Sigma(v'_r - \alpha v'_{Ar})^2 + 2\,\Sigma[v'_r - \alpha v'_{Ar})\,\boldsymbol{\beta}\cdot\mathbf{G}] + \Sigma(\mathbf{G}\cdot\boldsymbol{\beta})^2 \tag{8}$$

the **G** that minimizes is given by $\mathbf{G}_0$ where;

$$\mathbf{B}\cdot\mathbf{G}_0 = -\Sigma[(\mathbf{v}'_r - \alpha v'_{Ar})\,\boldsymbol{\beta}] \tag{9}$$

and where **B** is the tensor $\Sigma\boldsymbol{\beta\beta}$. Thus

$$\mathbf{G}_0 = -\mathbf{B}^{-1}\cdot\Sigma(v'_r - \alpha v'_{Ar})\,\boldsymbol{\beta}. \tag{10}$$

This is our best estimate of **G**. Its error estimate is given by the dispersion tensor

$$\Delta\mathbf{G}\,\Delta\mathbf{G} = \sigma^2\mathbf{B}^{-1} \tag{11}$$

as we shall show in the statistics Section 2. σ^2 is the variance of the distribution of true radial velocities; in terms of our best estimates of them

$$\sigma^2 = (\Sigma v_r^2)/(N-3) \tag{12}$$

when the minimizing value of **G** is used in calculating Σv_r^2. Notice that **G** is determined in terms of known quantities by (10), with **G** known Σv_r^2 is determined by (8), σ^2 by (12) and thus the errors $\boldsymbol{\Delta}G\,\boldsymbol{\Delta}\mathbf{G}$ by (11). With **G** and its errors known the circular velocity follows from (4) and its error is

$$\Delta V_c = (1+\mu)[(\hat{r}_A\cdot\boldsymbol{\Delta}\mathbf{G}\,\boldsymbol{\Delta}G\cdot\hat{\mathbf{r}}_A)/(\mathbf{r}_A\cdot\hat{\mathbf{c}})^2]^{1/2}. \tag{13}$$

2 Proof of the statistical formulae

Suppose the v_r are a sample taken from a population of zero mean and variance σ^2. The observed $v'_r = v_r + \alpha v'_{Ar} - \mathbf{G}\cdot\boldsymbol{\beta}$ where **G** is the true velocity of our Galaxy and v'_{Ar} is the observed radial velocity of Andromeda. Only the velocities of the remaining galaxies are assumed to be taken from the aforesaid population. In general our estimate $\mathbf{G}_0$ of the velocity of the Galaxy will differ from its true value **G**. Our process for determining G_0 is to invent a trial v_r^*

$$v_r^* = v'_r - \alpha v'_{Ar} + (\mathbf{G}+\Delta\mathbf{G})\cdot\boldsymbol{\beta} = v_r + \Delta\mathbf{G}\cdot\boldsymbol{\beta} \tag{14}$$

and to minimize the sum of the squares of the v_r^* over all trials of $\Delta\mathbf{G}$. This is done by taking

$$\Delta\mathbf{G} = -\mathbf{B}^{-1}\cdot(\Sigma v_r\boldsymbol{\beta}) \tag{15}$$

$$\Sigma v_r^{*2} = \Sigma v_r^2 - 2\,\Sigma v_r\boldsymbol{\beta}\cdot\mathbf{B}^{-1}\cdot\Sigma v_r\boldsymbol{\beta} + \Delta\mathbf{G}\cdot\mathbf{B}\cdot\Delta\mathbf{G}. \tag{16}$$

Notice that although $\boldsymbol{\Delta}\mathbf{G}$ is not in general zero, nevertheless since v_r are random variables of mean zero, the expectation value of ΔG is zero. Thus the minimizing value $\mathbf{G}_0$ is an unbiased estimate of **G**. The dispersion tensor corresponding to this estimate is the expectation of the tensor $\boldsymbol{\Delta}\,\mathbf{G}\,\boldsymbol{\Delta}\mathbf{G}$

$$\langle\boldsymbol{\Delta}\mathbf{G}\,\boldsymbol{\Delta}\,\mathbf{G}\rangle = \langle[\mathbf{B}^{-1}\cdot(\Sigma v_r\boldsymbol{\beta})][\mathbf{B}^{-1}\cdot(\Sigma v_r\boldsymbol{\beta})]\rangle.$$

Now the v_rs of the different galaxies are independent random variables of mean zero, thus the only terms whose expectation values are non-zero are those in which the v_r of the same galaxy occurs squared. The expectation value of v_r^2 is σ^2 and thus

$$\langle\boldsymbol{\Delta}\mathbf{G}\,\boldsymbol{\Delta}\,\mathbf{G}\rangle = \langle\mathbf{B}^{-1}(\Sigma v_r^2\boldsymbol{\beta\beta})\cdot\mathbf{B}^{-1}\rangle = \sigma^2\mathbf{B}^{-1}. \tag{17}$$

Thus

$$\langle\Delta\mathbf{G}\cdot\Delta\mathbf{G}\rangle = \sigma^2\,\mathrm{tr}\,(\mathbf{B}^{-1}) = \sigma^2/(\Sigma\boldsymbol{\beta}\cdot\boldsymbol{\beta}). \tag{18}$$

To get an estimate of the value of σ^2 we consider the expectation value of Σv_r^{*2} when $\boldsymbol{\Delta}\mathbf{G}$ is taken as its minimizing value (15) in terms of v_r. The expectation of the middle term is $-2\langle \mathbf{\Delta G} \cdot \mathbf{B} \cdot \mathbf{\Delta G} \rangle$ and so

$$\langle \Sigma v_r^{*2} \rangle = \langle \Sigma v_r^2 \rangle - \langle \mathbf{\Delta G} \cdot \mathbf{B} \cdot \mathbf{\Delta G} \rangle = (N-3)\,\sigma^2 \tag{19}$$

where N is the number of galaxies in the sum, i.e. the number where Andromeda and the Galaxy are omitted.

3 Application to the Local Group

3.1 MEMBERSHIP

To get a good value for **G** and hence a good value for V_c we need our galaxies to move independently of one another about the Local Group and to be well spread about the sky. Further we can only use objects whose radial velocities have been determined. The Local Group contains a number of subgroups that are obviously bound together such as Andromeda and its satellites. M32, NGC 205, 185, 147. M33 may or may not be found to this subgroup but since it lies 14° apart in the sky and has quite a different velocity, −180 km/s in place of −300 km/s, we shall treat it as independent of Andromeda. Other Local Group members that move independently are IC 1613, NGC 6822, Fornax, Leo A = DD 69, Wolf–Lundmark–Melotte and IC 10. To these may be added the David Dunlap dwarfs [1] DD 210 and DD 216. It is not yet clear that the Large and Small Magellanic Clouds are bound together, but as it would be rash to consider them as statistically independent we shall replace them by their combined mass at their centre of mass with the same total momentum. However, we consider that Fig. 1(a) and 2(a) together with our recent work on the Magellanic Stream [2] makes it almost certain that the Magellanic Clouds are bound to the Galaxy so, in order to use their observed velocity, we shall assume that their velocity is not a random variable with respect to the centre of mass of the Local Group but rather with respect to the centre of mass of the Galaxy. To incorporate objects bound to the Galaxy we make a minor modification to the treatment given in the first section. There are numerous other objects in this category and if velocities of more of them were measured the method we are discussing would give more accurate results. Some picture of the totality of objects available is obtained by projecting the members of the Local Group on to the Galactic plane, Fig. 2(a), and on to the Galactic meridional plane through the Sun and the Galactic axis, Fig. 2(b). Table 1 lists members of the Local Group and in a subsidiary section doubtful members. In our selection we have been guided by de Vaucouleurs [3] but with the addition of DDO 216, since its velocity measured by Fisher & Tully [4] puts it clearly among the other members that are probably bound. Maffei I and II originally considered as possible members are now thought to be a few megaparsecs away [5] so we do not consider them as members.

To get some preliminary acquaintance with the data and to get some guidance on the membership problem, let us first see what happens if we try the assumption that the circular velocity is by far the most dominant motion. If then all the objects were at rest their radial velocities v_r' would be $-V_c \sin l \cos b$. In Fig. 3 we plot $v_r' \sec b$ against l for all objects with $|b| < 60°$. The $\sec b$ correction factors are large at larger b so that small intrinsic velocities are exaggerated and give large scatter. It is clear from the figure that the curve $-300 \sin l$ gives a surprisingly good fit, provided we omit the doubtful members which would by and large force us towards an even larger amplitude V_c. Of course if the motion of the whole Galaxy lies in the galactic plane, then this curve should still be a sinusoidal curve of amplitude $|V_c \hat{\mathbf{c}} + \mathbf{G}|$ but then we should expect to fit a curve with a phase shift l_0, $\sin(l - l_0)$. In fact some small phase shift seems to be present in that a ~10° shift would fit better for each certain member. The SMC is an exception, but since it is bound to the Galaxy it should not

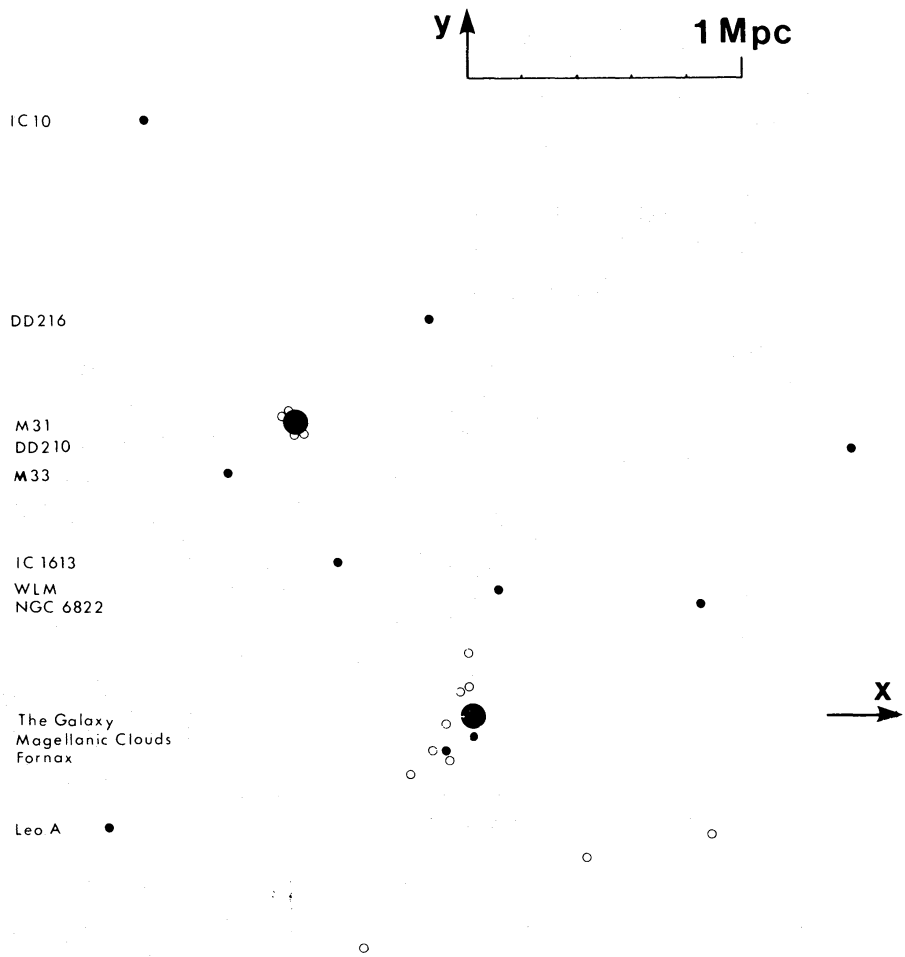

Figure 2(a).

Figure 2. (a) Plot of the Local Group projected on to the Galactic plane x, y. Black dots denote independently moving members used in solution. (b) Plot of the Local Group projected on to the meridional x, z. Plane through the Galactic axis and the Sun.

reflect the **G** motion. It is basically the good fit of $-300 \sin l$ that leads to de Vaucouleurs's velocity of the Sun with respect to the Local Group of 300 km/s in the $l = 90°$, $b = 0$ direction [7]. The reader may well think it odd that if the Sun's velocity is 250 in that direction the extra 50 which must then be attributed to the motion of the Galactic Centre happens to be in about that *same* direction. We believe that the origin of this apparent coincidence is that the true circular velocity is nearer 300 than 250 and that the **G** motion is really quite small and in a quite different direction. The second curve drawn is $V_c = -303 \sin(l - 7°.5)$ which is our final best value for the components of $V_c \hat{\mathbf{c}} + \mathbf{G}$ in the Galactic plane.

3.2 APPLICATION AND RESULTS

We now give the minor modification of the method for the situation in which some of the objects are bound to the Galaxy. For this subset the intrinsic velocities are assumed random, not with respect to the centre of mass of the Local Group but rather with respect to the Galactic Centre. Thus for them we have

$$v_r' = v_r - V_c \hat{\mathbf{c}} \cdot \hat{\mathbf{r}}$$

with v_r now with respect to the Galactic Centre so there is no correction for the motion of our Galaxy.

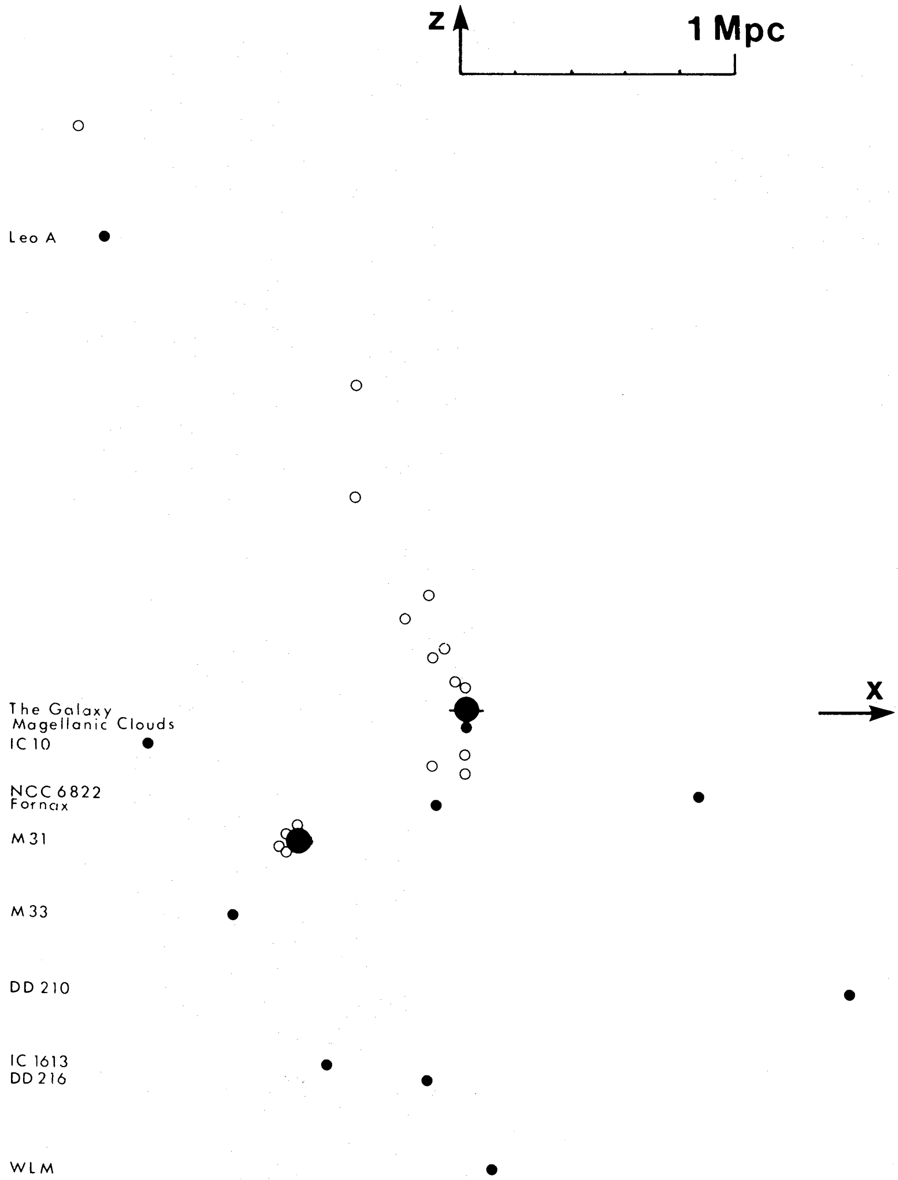

Figure 2 (b)

For this subset of galaxies we have

$$v_r = v_r' - v_{Ar}'[(\hat{\mathbf{c}}\cdot\hat{\mathbf{r}})/(\hat{\mathbf{c}}\cdot\hat{\mathbf{r}}_A)] - (1+\mu)[(\hat{\mathbf{c}}\cdot\mathbf{r})/(\hat{\mathbf{c}}\cdot\hat{\mathbf{r}}_A)]\,\hat{\mathbf{r}}_A\cdot\mathbf{G}$$

that is

$$v_r = v_r' - \alpha v_{Ar}' + \boldsymbol{\beta}^*\cdot\mathbf{G}$$

where

$$\boldsymbol{\beta}^* = -(1+\mu)\,\alpha\hat{\mathbf{r}}_A$$

thus the only change is that for these galaxies $\boldsymbol{\beta}^*$ replaces $\boldsymbol{\beta}$ wherever $\boldsymbol{\beta}$ occurred previously.

Taking axes x towards the Galactic Centre, y towards $l = 90$ and $b = 0$, and z towards $b = 90$, we obtain for $\mu = 1$ the following results from the 10 independent members besides

Table 1. The Local Group.

Name	Other names	r/kpc	l	b	v_r'	Type
The Galaxy	Milky Way					
LMC		53	280.5	−32.9	260	SBm
SMC		62	302.8	−44.3	159	Im
Draco		67	86.4	34.7		dE
U Mi	DD 199	84	105.1	44.8		dE
Sculptor		67	287.8	−83.2		dE
Ursa Maj	A 1127	120	202.3	71.8		dE
Sex C	A 1003	140	240.1	41.9		dE
Pegasus	A 2304	170	87.1	−42.7		dE
Fornax		188	237.3	−65.7	28	dE
Leo I	DD 74	230	226.0	49.1		E4
Leo II	DD 93	230	220.1	67.2		dE
M31		670	121.2	−21.6	−297	Sb
M32		660	121.2	−22.0		E3
NGC 205		640	120.7	−21.1		E5
NGC 147		660	119.8	−14.3		E5
NGC 185		640	120.8	−14.5		E3
And I						
And II						
And III						
M33	NGC 598	720	133.6	−31.3	−182	Scd
IC 10		1260	119.0	−3.3	−334	SBm
NGC 6822		500	25.4	−18.4	−56	Im
IC 1613		740	129.9	−60.6	−240	Im
Wolf Lundmark Melotte	DD 221, A 2359	870	75.7	−73.6	−126	Im
DD 210		1000	34.1	−31.3	−123	
Peg Irreg	DD 216, A 2326	1000	94.8	−43.6	−176	I
Leo A	DD 69, A 0956	1100	196.9	52.4	24	Im
Possible members (deduced to be non-members in text)						
Sex B	DD 70, A 0957	1500	233.2	43.8	289	Im
Sex A	DD 75, A 1009	600	246.2	39.9	314	IBm
GR 8	DD 155	1500	269.2	73.9	222	
DD 187		1500	25.6	70.5	164	
NGC 3109	DD 236	1500	262.1	23.1	394	

ourselves and Andromeda

$V_c = 291 \pm 51$ km/s

$G = (-74, -28, -51)$ km/s.

The natural size of the rms error is given by

$\langle \tfrac{1}{3} \mathbf{\Delta G} \cdot \mathbf{\Delta G} \rangle^{1/2} = 42$ km/s.

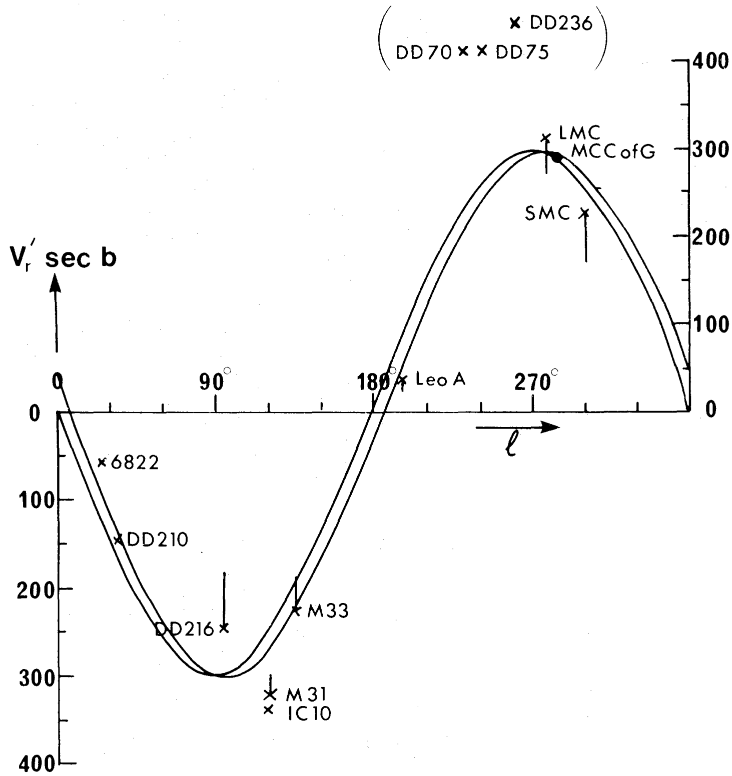

Figure 3. v_r' sec b for members of the Local Group with $b < 60°$ plotted against l. The curves are $-300 \sin l$ and $-303 \sin (l + 7°.5)$.

The velocity dispersion in the radial direction of the members is

$\sigma = 55$ km/s.

The error ellipsoid of **G** has eigenvalues and eigenvectors given by

$\sigma_1 = 17$ km/s along $(0.73, -0.66, +0.17)$

$\sigma_2 = 62$ km/s along $(0.50, -68, 0.53)$

$\sigma_3 = 33$ km/s along $(-0.47, -0.29, +0.83)$

G will have less than a one sigma variation if Δ**G** lies within that ellipsoid. Notice how elongated it is.

3.3 RESIDUALS AND GROUP MEMBERSHIP

With the above solution we may look at the residual radial velocities of the different galaxies. These are given in Table 2, column 1. It is then seen that Fornax, which we have treated as bound to our Galaxy on the basis of Figs 1 and 2, is left with one of the largest residuals. If instead we treat it as unbound to our Galaxy we get a new solution with $V_c = 316 \pm 48$ km/s and a very small residual for Fornax. Although this perhaps suggests that Fornax is not bound to our Galaxy, we thought that our more conservative readers might prefer $V_c = 291$ to $V_c = 316$ as our basic result. Notice that the difference is well within the error of either determination. Table 2 suggests Fornax is bound to our Galaxy when the v_e column is considered.

Seeking to reduce the rather large errors by adding more objects we added the two very distant globular clusters NGC 2419 and NGC 7006 to the group of objects bound to our

Table 2. Least-square solutions and residuals.

μ, f	1, 0	1, 1/3	1, 1	1.25, 1				
$V_c \pm \Delta V_c$	291 ± 51	287 ± 50	292 ± 43	294 ± 42				
$\mathbf{G}$	(−74, −28, −51)	(−54, −3, −31)	(−39, +9, −19)	(−34, +7, −16)				
$\sigma_1, \sigma_2, \sigma_3$	17, 62, 33	17, 49, 31	15, 32, 24	13, 29, 26				
$G \pm \langle(\Delta \mathbf{G})^2\rangle^{1/2}$	95 ± 42	62 ± 35	44 ± 24	38 ± 23				
σ, σ_r	55, 55	59, 57	54, 60	54, 61				
					$M_A/M_\odot = 3 \times 10^{11}$			$M_A/M_\odot = 3.5 \times 10^{11}$
	v_r	v_r	v_r	v_r	v_e	$v_e/3^{1/2}$	$v_e/6^{1/2}$	v_e
Fornax	−71	−70	−72	−72	145	84	59	156
Cof M SMC–LMC	+15	+19	+15	+13	246	142	100	266
IC 1613	−74	−91	−100	−103	99	57	40	107
NGC 6822	+1	+21	+37	+41	99	57	40	107
M33	+51	+41	+36	+33	134	76	55	145
DD 210	−23	−9	+3	+5	76	44	31	82
DD 216	+53	+52	+54	+53	90	52	37	97
WLM	−11	−24	−30	−32	85	49	35	91
Leo A	−19	−20	−22	−23	69	40	28	75
IC 10	−64	−57	−50	−51	80	46	33	87
Sextans A	+120	+112	+104	+104	69	50	36	94
GR 8	+116	+135	+146	+150	80	34	24	64
DD 187	+131	+160	+178	+182	87	34	24	64
DD 236	+143	+130	+118	+117	59	34	24	63
Sextans B	+137	+130	+123	+122	59	34	24	64
NGC 2419	+3	+3	+3	+3	264	152	108	285
NGC 7006	−89	−92	−89	+86	216	125	88	233

Galaxy and the five doubtful members of the Local Group listed in Table 1. The result gave $V_c = 334$ km/s. However, the newly-added doubtful members still had positive residuals in radial velocity larger than escape velocity suggesting that they were taking part in the expansion of the Universe; indeed at a probable distance of the order of 1.5 Mpc the expected Hubble flow should give velocities of 90 km/s or so which is typical of their residuals from the first solution. We regard this as rather satisfactory evidence that the group of distant dwarfs in Sextans are not *bona fide* members of the Local Group. Notice that no such explanation can work for IC10, which lies beyond Andromeda, because it is approaching, so its residual would be enhanced still further if we treated it as a non-member which ought to be taking part in the Universal expansion.

3.4 EFFECTS OF VARYING THE MASS RATIO, μ, AND ITS PROBABLE VALUE

In Fig. 4 we plot the value of our solutions for V_c against the value of the mass ratio $\mu = M_G/M_A$ which is used to obtain that solution. This gives the curve labelled $f = 0$. Since our errors on V_c are ± 45 km/s the precise location of this 'least-squares' curve of $V_c(\mu)$ is somewhat academic, however, the value of μ can not be chosen at will. Tully & Fisher recently found that the maximum rotational velocities of flat galaxies showed a very tight correlation with their absolute magnitudes. If we assume a constant mass-to-light ratio then their relationship becomes

$$M \propto (V_{max})^{5/2}$$

Thus we have

$$(V_{max})_G = \mu^{2/5}(V_{max})_A$$

where G stands for the Galaxy and A for Andromeda. This relationship is drawn as the steep heavy line in Fig. 4 and we must expect the Galaxy's V_{max} to lie close to that line within the

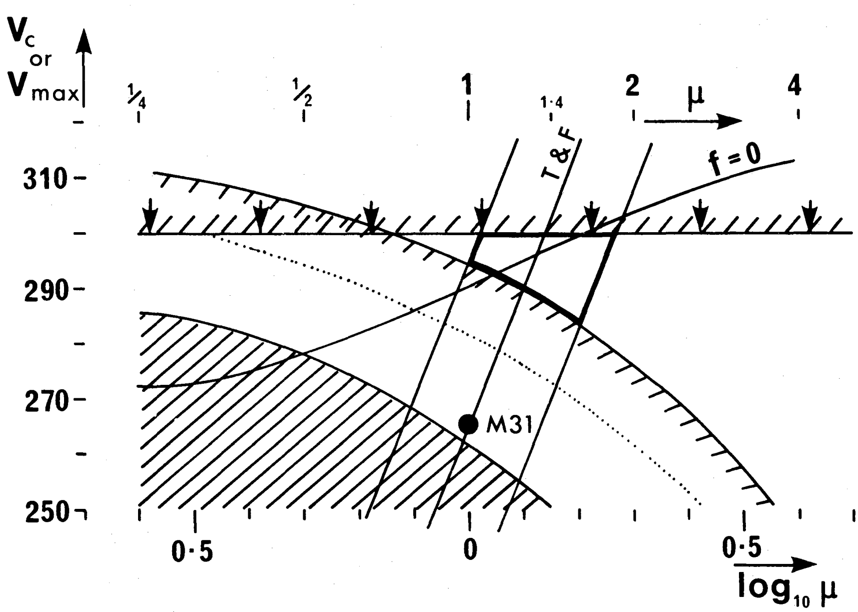

Figure 4. Effect of mass ratio μ on least-squares solution for $V_c(\mu)$ here labelled $f = 0$. In the shaded region bound motion is impossible if $M_A \leqslant 3 \times 10^{11} M_\odot$. In the region between that and the shaded line bound motion is improbable with M_A in that range. This improbable line moves to the dotted position for $M_A \leqslant 4 \times 10^{11} M_\odot$. External galaxies suggest a maximum rotational velocity of 300 km/s shown as an upper limit.

slot indicated. However, V_{max} must lie above, even if only a little above, the circular velocity near the Sun whose most probable values lie on the $f = 0$ curve. These considerations suggest that μ probably lies in the range $1.0 < \mu < 1.4$ and that the low values of μ near ¼ which are associated with a most probable $V_c \sim 270$ km/s would make the Galaxy an exception to Tully & Fishers' correlation.

3.5 DEMANDS OF A BOUND ORBIT

Emerson, modelling the Andromeda Nebula with no mass further than 28 kpc from the centre, finds [10] a total mass within that distance of $2.2 \times 10^{11} M_\odot$. However, considerably larger masses can be obtained by demanding that the circular velocity remains high to much greater distances where nothing has yet been observed [11]. Unless there is a new halo component that contributes at large radii it seems unlikely that the total mass of that galaxy can exceed $3 \times 10^{11} M_\odot$ even if we include the masses of its satellites. Now the condition that the two-body system composed of the Galaxy and Andromeda should be bound is

$$\tfrac{1}{2}[(1+\mu)\,\mathbf{G}]^2 - G(M_A + M_G)/r_A < 0$$

where the scalar G is Newton's constant and should be distinguished from the vector velocity of the Galaxy $\mathbf{G}$. Thus $|\mathbf{G}|^2 < 2GM_A r^{-1}(1+\mu)^{-1}$ and so $|\mathbf{G}| < [2(1+\mu)]^{1/2} \cdot 44$ km/s. If the Galaxy and Andromeda were moving straight at one another with such a velocity the remaining radial velocity would be $-297 + (1+\mu)[2(1+\mu)]^{1/2}44$ and this would have to be accounted for by circular velocity $V_c \sin 121^\circ \cos 22^\circ$. Thus

$$V_c > 1.26 \left[297 - 88\left(\frac{1+\mu}{2}\right)^{1/2}\right] = 262 - 110\left[\left(\frac{1+\mu}{2}\right)^{1/2} - 1\right] \text{ km/s.}$$

We have shaded the impossible region of Fig. 4.

If we say that it is unlikely that the system is just on the edge of being bound and it is also unlikely that the velocity will be directed exactly along the line to Andromeda, then it would seem wise to reduce the component of $\mathbf{G}$ towards Andromeda by a factor $\sqrt{2}$; we then have

$$V_c \geqslant 1.26 \left[297 - 62\left(\frac{1+\mu}{2}\right)^{1/2}\right] = 295 - 78\left[\left(\frac{1+\mu}{2}\right)^{1/2} - 1\right].$$

This curve is also drawn on Fig. 4 and gives an intersection with the Tully & Fisher relationship at $V_c = 290$ and $\mu = 1.25$. Drawn lightly in the same diagram is this last likely limit when Andromeda's mass is assumed to be less than $4 \times 10^{11} M_\odot$ instead of $3 \times 10^{11} M_\odot$.

The collection of data on rotation curves of external galaxies discussed in Section 4 shows that maximum circular velocities in excess of 300 km/s are rare. It seems reasonable to assume that the Galaxy is not such a collector's piece. We therefore deduce that the parameters of the Galaxy probably lie within the heavy-edged quadrilateral of Figs 4 and 5. Although this ties down the value of V_c very satisfactorily we still have to determine $\mathbf{G}$. To do this we would like to use our least-squares method, but with some constraint on the Galactic velocity. At present a small lowering of the sum of squares that can be achieved only by making $\mathbf{G}$ large, will nevertheless emerge as the formal solution to the mathematical problem. We believe that there ought to be a penalty for making the Galactic velocity large just as there is for the radial velocities of other galaxies of the Local Group. To see what this involves in principle, let us consider the method of least squares as being derived from the method of maximum likelihood with an assumed Gaussian distribution of the velocities of

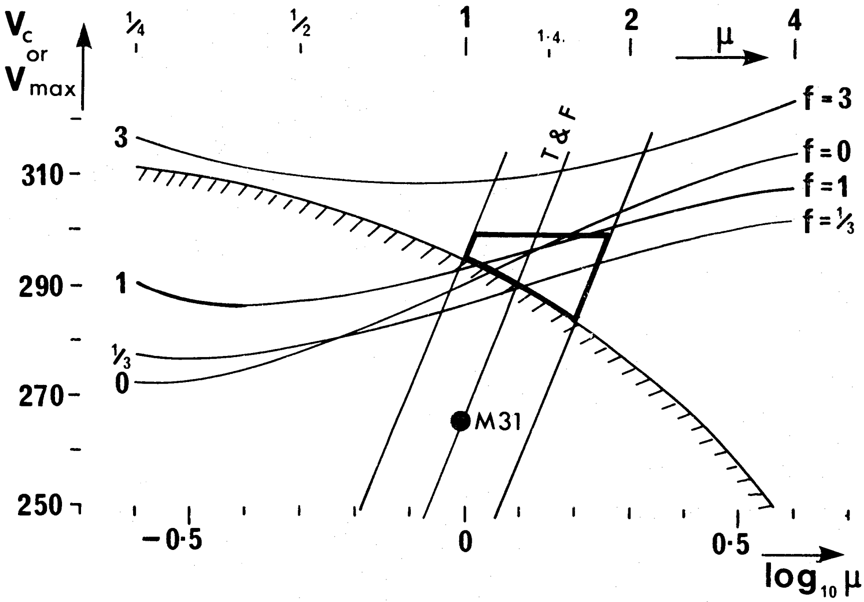

Figure 5. Effect of putting the velocities of the Galaxy and Andromeda into the least-square solution with various weights *f*.

the independent small galaxies about the Local Group's mass centre. The problem solved above gives the most probable V_c and $\mathbf{G}$, $\mathbf{v}_A$ *if* all values are equally likely *a priori*. But they are not, for $\mathbf{G}$ and v_A are themselves galaxy velocities and should therefore be Gaussianly distributed *a priori*. However, there is a problem here for $\mathbf{G}$ and v_A are correlated, in fact $\mathbf{v}_A = -\mu\mathbf{G}$, so they are not independent random variables. Furthermore, if one were distributed with the same Gaussian as the independent members then the other certainly would not be. There is a further reason for a difference between the dispersions of $\mathbf{G}$ and $\mathbf{v}_A$ and of the typical galaxy, in that any true random motion of the galaxy and Andromeda will have been reduced by our choosing their centre of mass as our zero of velocity. Thus, a kick given to Andromeda imparting a change of speed $\Delta\mathbf{v}$ will change by $\mathbf{v}_A$ only $\mu\Delta\mathbf{v}/(1+\mu)$ and will also change $\mathbf{G}$ by $-[\Delta\mathbf{v}/(1+\mu)]$; a kick $\Delta\mathbf{W}$ on the galaxy would change $\mathbf{G}$ by $[\Delta\mathbf{W}/(1+\mu)]$. These strongly suggest that $\mathbf{G}$ must be distributed with a smaller-than-typical dispersion which would indeed be zero if μ were so large that the Galaxy alone defined the mass centre of the Local Group. If the mean-square values of each component of $\Delta\mathbf{v}$ and $\Delta\mathbf{W}$ were σ^2 then the dispersion generated in each component of $\mathbf{G}$ would be $[(2\sigma^2)/(1+\mu)^2]$ which should then be compared with the dispersion σ^2 of the other galaxies. However, we note that if μ were very small the galaxy should be treated comparably with the others whereas the above formula would give twice the dispersion. We shall therefore take the *a priori* distribution of $\mathbf{G}$ to be given by

$$\exp -\left[\frac{1}{2\sigma^2}(\mathbf{G}^2+\mathbf{v}_A^2)\right] = \exp -[(1+\mu^2)\,\mathbf{G}^2/2\sigma^2]$$

which gives $\mathbf{G}$ a dispersion of $\sigma^2/(1+\mu^2)$. For $\mu = 1$, the important case for this paper, this gives the same as $2\sigma^2(1+\mu)^{-2}$ while for $\mu \to 0$ or ∞ it treats the lighter galaxy on a similar footing to the other light ones. Any more detailed justification for such an *a priori* probability distribution can only come from the physics of some theory of galaxy formation. Our aim is not to make extra assumptions but to make deductions from the observations.

To see the effect of other possible *a priori* distributions we insert a factor *f* in front of

$\mathbf{G}^2 + \mathbf{v}_A^2$. The likelihood of a given set of observations is then given by

$$L = \exp - \left\{ \frac{1}{2\sigma^2} \left[f(\mathbf{G}^2 + \mathbf{v}_A^2) + \Sigma v_r^2 \right] \right\} = \exp - \left\{ \frac{1}{2\sigma^2} \left[f(1 + \mu^2)\, \mathbf{G}^2 + \Sigma v_r^2 \right] \right\}$$

and the maximum likelihood leads to the minimizing of

$$f(\mathbf{G}^2 + v_A^2) + \Sigma v_r^2 = f(1 + \mu^2)\, \mathbf{G}^2 + \Sigma v_r^2.$$

Our old problem corresponded to minimizing this with $f = 0$ while our new problem has $f = 1$. To estimate σ^2 we must now divide this sum by $(N + 3) - 3 = N$ since the three components in $f(1 + \mu^2)\, \mathbf{G}^2$ each have dispersion σ^2. **B** is also suitably modified by the addition of a term $(1 + \mu^2)\, \mathbf{I}$ where **I** is the unit tensor. In practice we ran solutions with $f = 0$, 1/3, 1, 2, 3 and 400. The last was a check which of course reduces **G** to almost zero and gives a circular velocity of 374 km/s sufficient to reduce the radial velocity of Andromeda to zero. $f = 2$ and 3 are artificial ways of restricting the random motion of the Galaxy by more than the sensible $f = 1$ solution. For mass ratio $\mu = 1$, V_c and the magnitude and components of **G** are plotted in Fig. 6 as a function of the f used in making the least-squares solution. Evidently the $f = 0$ solutions are quite unstable as there is a rapid change as f is increased to 1/3 in all the components of **G**. However, near $f = 1$ this sensitivity becomes much less

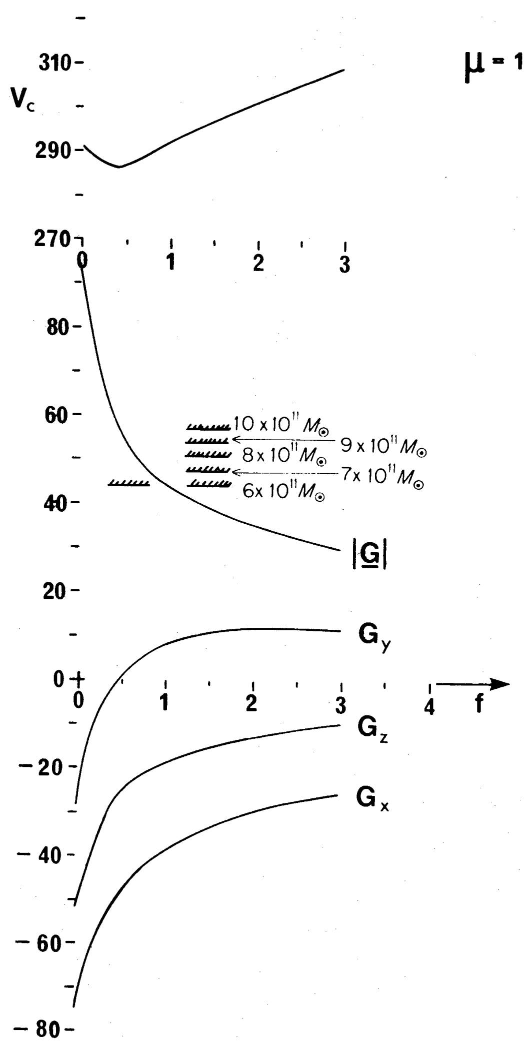

Figure 6. The Galactic motions and the binding condition.

marked. The limits on G which would bind us to Andromeda for various values of our combined total mass are also shown and in this respect the $f=1$ solution is acceptable. On detailed inspection of the residuals of the other Local Group members we find they are bound and that they are close to obeying the Virial theorem statistically. However, we considered it somewhat objectionable that two galaxies whose circular velocities are in the ratio of 292/265 should be treated as though they had the same mass whereas Tully and Fisher would expect a mass ratio of $\mu = (292/265)^{5/2} = 1.25$. We therefore ran the whole solution yet again with this mass ratio. The results are those quoted in the Summary which correspond to least squares with $f = 1$.

$V_c = 294 \pm 42$ km/s

$\mathbf{G} = (-34, +7, -16)$ km/s

$\mathbf{G} = 38$ km/s towards $l = 168$, $b = -25$.

The error ellipsoid of $\mathbf{G}$ is

$\sigma_1 = 13$ km/s along $(0.68, -0.70, 0.21)$

$\sigma_2 = 29$ km/s along $(0.51, 0.66, 0.55)$

$\sigma_3 = 23$ km/s along $(-0.52, -0.26, 0.81)$.

Notice that although this ellipsoid is still rather flat it is no longer very elongated as it was for our $f = 0$ solution.

From these results we deduce the motion of the Sun with respect to the Local Group to be

$\mathbf{S} = (-34, 301, -16) + (9, 12, 7) = (-25, 313, -9)$

with errors essentially the same as those of $\mathbf{G}$ given above. The motion of Andromeda is just $-\mu\mathbf{G}$, i.e.

$\mathbf{v}_A = (+42, -9, +20)$.

The rms velocity of one component of motion of a small independent member of the Local Group is $\sigma = 54$ km/s from the overall analysis but for the radial velocities of the other members excluding $\mathbf{v}_A$ and $\mathbf{G}$ we get $\sigma_r = 61$ km/s.

3.6 THE BINARY ORBIT OF ANDROMEDA AND THE GALAXY

The poor accuracy of both the Galaxy's velocity and the masses makes it somewhat premature to try to determine any definitive orbit. However, the results of investigating possible orbits are both interesting and rewarding [12]. In this section we investigate two cases:

(1) The Galaxy's velocity is $G = (-34, +7, -16)$ and the mass ratio $\mu = 1.25$. We take a variety of total masses for ourselves + M31 ranging from 6 to $13 \times 10^{11} M_\odot$.

(2) The circular velocity is $V_c = 290$ km/s, the ıelative velocity of Andromeda and the Galaxy is the least then possible to account for Andromeda's observed velocity of -297. This implies that the Galaxy's velocity is directed at Andromeda so that there is no orbital angular momentum. This procedure gives the most bound orbits possible and thus the shortest possible periods. However, it is not a ridiculous case since the simplest cosmological ideas concerning the formation of galaxies suggest a radial orbit when a group contains just two massive members. This gives $G = [2/(1+\mu)](-16, 27, -12)$.

Taking the first case, the constant of areas for the relative orbit of the Galaxy and Andromeda is $\mathbf{h} = \mathbf{r}_A \times (\mathbf{v}_A - \mathbf{G})$ and so $|\mathbf{h}| = 1.2 \times 10^{31}\,\mathrm{cm^2/s}$ and the direction of $\mathbf{h}$ is (0.38, −0.20, −0.90) which is towards $l = 333°$, $b = -65°$. This of course defines the direction of the normal to the plane of the orbit. The specific internal energy of binary motion is $\epsilon = \tfrac{1}{2}(\dot{r}_A^2 + h^2 r_A^{-2}) - G(M_G + M_A)\, r_A^{-1}$ and the period is $P = 2\pi G(M_G + M_A)(-2\epsilon)^{-3/2}$. The eccentricity is given by $1 - e^2 = h^2(-2\epsilon)[G(M_G + M_A)]^{-2}$. Table 3 gives the values of these quantities for various assumed values of $M_G + M_A$ from 6 to 13 times $10^{11} M_\odot$. Only for $M_G + M_A > 8 \times 10^{11} M_\odot$ is the maximum separation less than 2 Mpc which is extreme for distances within the Local Group today, thus it is likely that the $M_G + M_A$ must be $9 \times 10^{11} M_\odot$ or more. With all the resultant cases the eccentricity of the orbit is close to 3/4. However, even with these large masses the period of the binary orbit is very long even compared with the age of the Universe. In no case is there sufficient time for the material of the Galaxy and Andromeda to move apart with the expansion of the Universe and for the two bodies to return after condensation to their present positions and velocities. The age of the Universe is $\leqslant 2 \times 10^{10}$ yr and even the outward movement of our orbits normally lasts longer than that. With the aim of circumventing this difficulty we consider our second case when the period is the minimum possible consistent with the circular velocity of $V_c = 290$ km/s. This procedure gives acceptably small maximum separations for all the masses considered but again the period for separation and reapproach to the present configuration is unacceptably long except for masses of $13 \times 10^{11} M_\odot$ or above. Thus either we have to accept these even larger masses, or we abandon the idea that the material that now makes up the Galaxy and Andromeda has been moving freely under gravity since the decoupling era $z \sim 1000$. Both alternatives have far-reaching consequences. The first would imply the truth of the heavy halo hypothesis while the second would imply that the Galaxy and Andromeda did not fall into a natural union but rather that they were pushed into it. Indeed the entries for t, the period since apocentre, show that $2t$ is typically 2×10^{10} yr or more. Thus running back the gravitational motion of ourselves and Andromeda for the full age of the Universe would lead to a separation which is typically no less than their present separation. This implies that non-gravitational acceleration must have taken place and at a separation that is unlikely to be less than the present one. It is unlikely that non-gravitational acceleration can occur when the material of a Galaxy is already condensed with stars. Thus we deduce that galaxy formation took place at a separation as large as the present one, at a mean density typical of the Local Group's or less. That implies galaxy formation at an epoch we see now with $z < 5$. Further it implies that non-gravitational forces were probably involved in the formation of the Local Group. It also suggests that cluster formation and galaxy formation

Table 3. Relative orbits of the Galaxy and Andromeda.

	$h = 1.2 \times 10^{31}$, G = (−34, +7, −16), $\mu = 1.25$				$V_c = 290$ km/s, $h = 0$, $e = 1$, $\mu = 1.25$			
$M_G + M_A$/ $10^{11} M_\odot$	$P/10^{10}$ yr	e	r_{max}/Mpc	r_{min}/Mpc	$P/10^{10}$ yr	t/P	$\tfrac{1}{2}P + t$/ 10^{10} yr	r_{max}/Mpc
6	214	0.956	14	0.32	8.4	0.43	7.8	1.55
7	26.9	0.85	3.4	0.28	6.2	0.41	5.6	1.34
8	13.0	0.82	2.5	0.25	4.7	0.39	4.2	1.18
9	8.5	0.76	1.7	0.23	4.0	0.37	3.5	1.09
10	6.4	0.7415	1.4	0.21	3.4	0.35	2.9	1.04
11	5.1	0.736	1.2	0.18	3.1	0.34	2.6	0.98
12	4.3	0.736	1.1	0.17	2.7	0.32	2.2	0.94
13	3.8	0.739	1.0	0.15	2.5	0.31	2.0	0.91

occur at the same epoch rather than galaxy formation occurring first. The density of the Universe when all this occurred was probably the mean density of the Local Group (or some factor of up to 8 less to allow for the contraction involved in coming towards equilibrium).

To provide, the extra forces that appear to be needed it is interesting to look for possible pressures that could cause galaxy formation. The most likely known possibilities are cosmic ray pressures due to nearby quasars or radio galaxies and NGC 5128 (Centaurus A) might have been much more active in its youth and have provided energetic winds which helped to push the Local Group together. Alternatively either the Galaxy or Andromeda may be an escaped member of another group of galaxies and their present union may be due to their chance meeting in the dark when going in nearly the same direction.

3.7 STATISTICS OF THE RESIDUALS

Here we consider the residuals of the genuine members of the Local Group with respect to our fourth and final solution listed in Table 2. There is no obvious expansion – five radial velocities are positive and four negative. The mean $|v_r|$ is 42.6 km/s as compared with a mean v_r of -13.6. This is well within expectation for 10 members, so there is no significant net contraction. IC 1613 is unbound for our last solution unless the mass of Andromeda is greater than about $3.3 \times 10^{11} M_\odot$. It is interesting to compare the residual radial velocities with the velocity of escape v_e, the radial velocity of escape assuming the other two components are equal $3^{-1/2} v_e$ and with the radial velocity corresponding to the virial theorem $6^{-1/2} v_e$. Except for IC 1613 there are no cases of discrepancy with the observed radial velocity of much too great a magnitude. It seems likely that IC 1613 can be included at a chance of one in ten by mildly increasing our assumed mass for Andromeda from say 3 to $3.5 \times 10^{11} M_\odot$. Our best estimate of the sum of the squares of the true residual velocities is $3N\sigma_r^2 = 30(54)^2 = 8750\ (\text{km/s})^2$. For an assumed mass of Andromeda of $3 \times 10^{11} M_\odot$ our best estimate of the sum of the squares of the escape velocities is obtained from the appropriate column of Table 2. This gives $2 \times 7750\ (\text{km/s})^2$. We have put in no mass weighting, but from the Virial Theorem we might expect this second sum to be twice the first. If we multiplied our assumed masses by $8750/7750 = 1.16$ to give $M_A = 3.5 \times 10^{11} M_\odot$ equality would be achieved. However, there is no rigorous justification for using the Virial Theorem in this way, the virial theorem used properly is totally dominated by the Galaxy and Andromeda which are much more massive than the others.

4 Discussion of Galactic parameters

The conventional IAU values of the vital statistics of the Galaxy are the Oort constants and $R_\odot$,

$$A = 15\ \text{km/s/kpc} \qquad A-B = \Omega = 25\ \text{km/s/kpc}$$

$$B = -10\ \text{km/s/kpc} \qquad V_c = (A-B)\,R_0 = 250\ \text{km/s}$$

$$R_0 = 10\ \text{kpc}.$$

Over the last 20 years A has been measured to be various values between 12 and 19, but appears to have settled down satisfactorily although 16 ± 2 might be preferable to 15. Determination of B is in a bad state. Proper motions of stars carefully selected give values consistent with the IAU ones, but as Clube has pointed out [13, 14] different selections could give totally different values. The Lick proper motions with respect to extragalactic nebulae are independent of precession corrections and ought to give a more direct determination; from them Clube derives $A = 16 \pm 5$ but $B = -26 \pm 5$ km/s/kpc, much more negative than previous values. However, together with this he finds obvious evidence of disturbance in the differential rotation of the Galaxy which is no doubt present and probably invalidates determinations of B that depend on local stars within 1.5 kpc. Determinations of A depend on somewhat more distant stars though they should be viewed with caution since the Galaxy is not accurately obeying the theoretical model of circular motions which serves as the background to the definition and use of A. The determination of the ratio of the radial to the tangential velocity dispersions of stars moving close to the Galactic plane should theoretically give $\sigma_u^2/\sigma_v^2 = (B - A)/B$ a relation due to Lindblad. At present this is probably the best way of determining B since the value of A is not a subject of disagreement. However, the locally observed velocity dispersion tensor is skew to the direction toward the Galactic Centre and the theorists' smooth anisotropic Gaussian distribution of residual velocities is a severe idealization of the bumpy and clumpy distribution of observed stellar velocities with its vertex deviation. Work by Mayor [15] explains the vertex deviation in terms of density wave theory, but this implies a significant disturbance from the symmetrical galaxy which in turn may lead to a systematic change in σ_u^2/σ_v^2 away from the symmetrical theories values of $(B - A)/B$.

Allen gives a local velocity ellipsoid for all stars of axes $(\sigma_I, \sigma_{II}, \sigma_{III}) = (38, 24, 18)$ km/s with the largest two axes in the Galactic plane but offset by 13° from the Galactic Centre. Thus

$$\sigma_u^2 = 38^2 \cos^2 13 + 24^2 \sin^2 13 = 1389 \,(\text{km/s})^2$$

and

$$\sigma_v^2 = 24^2 \cos^2 13 + 38^2 \sin^2 13 = 581 \,(\text{km/s})^2$$

giving $\sigma_u^2/\sigma_v^2 = 2.4 \pm 0.2$ where the error has been estimated from the spread among stars of different types.

Woolley *et al.* [16] have circumvented the difficulty of measuring only local stars, by working on distant K stars in the $l \sim 90°$ and $l \sim 180°$ regions near the Galactic plane. They find a ratio of σ_u^2/σ_v^2 of about 2.0 from several hundred stars whose spectra were taken with the Isaac Newton telescope. It thus appears that local velocity ellipsoids may have been slightly lengthened by the local disturbance of the Galaxy. We shall take as our possible range of $(B - A)/B$

$$(B - A)/B = 2.3 \pm 0.2$$

thus

$$-B = A/(1.3 \pm 0.2).$$

Possible values of A, B, Ω are thus

A	18	16	14
B	-14 ± 2	-12 ± 2	-11 ± 2
Ω	32 ± 2	28 ± 2	25 ± 2

With $R_0 = 10$ kpc this would allow V_c between 230 and 340 km/s, but the highest values would be inconsistent with the value of $2AR_0$ from the 21-cm observations of differential rotation which gives $2AR_0 = 300 \pm 40$ km/s. Similarly for $R_0 = 8$ pc the lowest values of V_c close to 200 would be in contradiction to $2AR_0$ and so the most likely range for V_c lies in the range $250 \leqslant V_c \leqslant 300$.

From assuming that the highest velocity stars define a sphere in velocity space Fricke [17] deduced a V_c close to 275 km/s and Isobe $V_c = 275 \pm 20$ from more modern data [18].

Toomre, who was earlier an advocate of low circular velocities [19] has under Eggen's stimulus [19] looked at the details of high-velocity stars. Going back to original data on every star (which turns out to be very important!) and taking more measurements where necessary, Greenstein & Toomre (private communication) find a few stars that must be moving very fast on the basis of radial velocity alone. It is likely that the lower bound to the escape velocity from the solar neighbourhood lies between 400 and 450 km/s. The ratio of the escape velocity to the circular velocity is $\sqrt{2}$ for motion about a point mass, can be very large deep inside a body of material and is as low as 1.18 at the edge of a V_c = constant disk – Mestel [20]. Thus overall $\sqrt{2}$ is probably a reasonable compromise. Thus escape velocities between 400 and 450 km/s imply circular velocities between 280 and 320 unless we accept some escaping stars among the *known* high-velocity stars.

Evidence on maximum circular velocities in other galaxies is rapidly increasing due to the assiduousness of the radio observers [9]. The radio 21-cm profile width once corrected for the instrument gives $W = 2\,V_{max} \sin i$.

There is some tendency for the maximum velocities to be high at small i presumably due to underestimation of i. Table 4 gives values of V_{max} and i.

Table 4. Maximum circular velocities of galaxies with $V_{max} > 200$ km/s.

NGC	i	V_{max}	Type
224, M31	75	265	Sb I–II
1068	45	286	Sb
1808	60	307	S0 a
2146	70	210	SBa
2903	70	230	Sbc
3031, M81	59	256	Sab I–II
3521	66	260	Sbc
3623	75	227	Sab
3953	60	252	Sbc
3992	59	280	Sbc
4013	80	204	Sb
4088	65	203	Sbc
4100	67	206	Sb
4111	85	230	S0
4157	84	214	Sb
4192	77	234	Sab I–II
4217	73	214	Sb
4254	28	301	Sc I
4258	64	250	Sbc
4321	23	342	Sbc I
4394	25	229	Sbc II
4501	60	308	Sb I
4535	38	265	Sc I:
4571	20	277	Scd
4639	40	248	Sbc II–III
4651	48	259	Sc II:
4698	58	256	Sab II:
4826	57	305	Sab
5005	23	293	Sb
5055	59	279	Sbc
5457, M101	18	291	Scd I
7331	74	284	Sbc
7469	41	204	Sa
7479	46	221	SBc

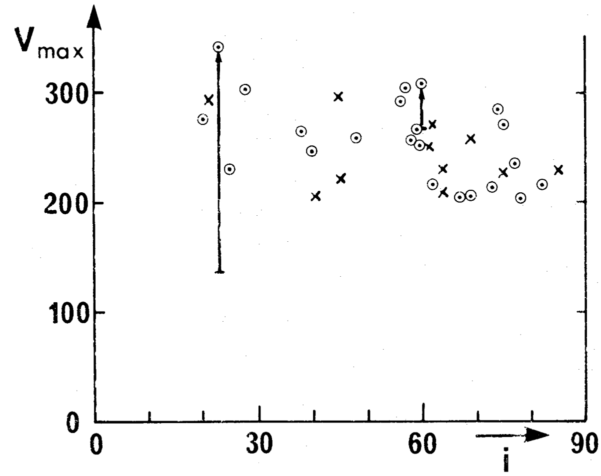

Figure 7. V_{max} for spirals with $V_{max} > 200$ km/s. × denotes an optical determination, arrows show two inclination corrections.

In Fig. 7 we plot V_{max} against i for galaxies with $V_{max} > 200$ km/s from reliable data found in the recent literature [9, 21, 22]. Galaxies in the Sb–Sbc–Sc classes can have maximum circular velocities up to 300 km/s but velocities in excess of that are rare if they exist. Only four points like above the 300 level and two of those have got there through very large inclination corrections. Thus it is unlikely that V_{max} for the Galaxy can be greater than 300 km/s. The circular velocity at the Sun must be less than V_{max} as $|B| < A$ as we have seen and

$$-\frac{R}{V}\frac{dV}{dR} = \frac{A+B}{A-B} = \frac{0.3 \pm 0.2}{2.3 \pm 0.2}$$

which lies between 0.2 and 0.05 since the ± signs are coupled together. Thus $V(R)$ is certainly falling at the Sun, although it may be falling very slowly. Thus the Sun is past the peak of the curve but possibly not very far past it. Indeed these values of $(R/V)(dV/dR)$ can readily be obtained at a velocity of only 4 km/s below V_{max} as one may show by taking a smooth velocity law such as $V = (AR)/(b^3 + R^3)^{1/2}$. Since such values are very small compared with 250–300 we deduce that the circular velocity at the Sun may be insignificantly smaller than V_{max}.

5 Conclusions

We recommend the following values of the Galactic parameters:

$V_c = 290$ km/s at $R_0 = 10$ kpc

$V_{max} \simeq 295$ km/s

$A = 16$ km/s/kpc

$B = -13$ km/s/kpc

$\Omega = 29$ km/s/kpc

$M_G = 4.4 \times 10^{11} M_\odot$

$M_A = 3.5 \times 10^{11} M_\odot$

$\mathbf{G} = (-30, 10, -15)$ km/s.

The Galaxy moves mainly towards us and slightly downwards towards the South Galactic Pole.

We also consider that the galaxies DD 187, DD 236, GR 8 and Sextans A and B take part in the universal expansion and are not members of the Local Group, whereas DD 210, DD 216 and Leo A are.

The Local Group of galaxies appears to be bound and its members obey the Virial Theorem in a statistical sense. Although some galaxies are quite massive, the masses required here are smaller than those advocated by Halo enthusiasts. Unless the mass of the Local Group is at least $1.3 \times 10^{12} M_{\odot}$ for the formation of the Local Group must have been due to non-gravitational forces occurring at a time corresponding to $z \leqslant 5$ and galaxies formed then or later.

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Note added in proof

Attention is drawn to a recent preprint by Yahil Tammann & Sandage which discusses the same problems especially the large probable errors.