On the Normalization of Haar Measures in the Representation Theory of Compact Semi-Simple Lie Groups

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1. Introduction. Let $\mathfrak{g}$ be a compact real semi-simple Lie algebra, $\mathfrak{j}$ a Cartan subalgebra of $\mathfrak{g}$; let $\mathfrak{g}_c$ (respectively $\mathfrak{j}_c$) denote the complexification of $\mathfrak{g}$ (respectively $\mathfrak{j}$); let $\Phi = \{\alpha\}$ denote the set of roots of the pair $(\mathfrak{g}_c, \mathfrak{j}_c)$, Φ^+ the positive roots (relative to some ordering of Φ); let W denote the Weyl group of the pair $(\mathfrak{g}_c, \mathfrak{j}_c)$. Because $\mathfrak{g}$ is compact, the Killing form B of $\mathfrak{g}$ is negative definite; agreeing to use $-B$, we see, therefore, that $\mathfrak{g}$ may be equipped with the structure of a Euclidean space in a natural way. It is well-known that the restriction of B to $\mathfrak{j}$ is non-degenerate; therefore $\mathfrak{j}$ may also be regarded as a Euclidean space in the obvious manner.

Now suppose that G is a connected Lie group with Lie algebra $\mathfrak{g}$; let J denote the analytic subgroup of G with Lie algebra $\mathfrak{j}$—then J is a maximal torus in G. The group G operates on $\mathfrak{g}$ in the usual way (namely through the adjoint representation); we shall agree to write xX in place of $Ad(x)X (x \in G, X \in \mathfrak{g})$. Let $d_{\mathfrak{g}}(X)$ (respectively $d_{\mathfrak{j}}(H)$) denote the canonical element of volume on $\mathfrak{g}$ (respectively $\mathfrak{j}$) as measured by $-B$; let $d_G(x)$ denote normalized Haar measure on G (hence $\int_G d_G(x) = 1$); let π denote the polynomial function on $\mathfrak{j}_c$ which is defined by the requirement $\pi(H) = \prod_{\alpha>0} \alpha(H) (H \in \mathfrak{j}_c)$—then, according to Harish-Chandra [1, p. 105], there exists a positive constant vol (G) such that, for all continuous functions f on $\mathfrak{g}$ with compact support, we have

$$\int_{\mathfrak{g}} f(X)\, d_{\mathfrak{g}}(X) = \text{vol}\,(G) \int_{\mathfrak{j}} \left|\pi(H)\right|^2 d_{\mathfrak{j}}(H) \int_G f(xX)\, d_G(x).$$

The objective of the present note is to compute vol (G).

Let r denote the cardinality of the set Φ^+, $[W]$ the order of the Weyl group W; let $(\cdot\,,\,\cdot)$ denote the usual scalar product on the real span (in the dual of $\mathfrak{j}_c$) of the elements α in Φ; let ρ be one-half the sum of the positive roots (i.e. let $\rho = 2^{-1} \sum_{\alpha>0} \alpha$)—then it turns out that

$$\text{vol}\,(G) = (2\pi)^r / ([W] \prod_{\alpha>0} (\rho, \alpha)) \qquad (\pi = 3.14 \cdots).$$

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Besides being of interest in its own right, this result has the following significance relative to some work of Kostant (cf. Pukanszky [1]). Thus let us suppose for the moment that G is, in addition, simply connected. As was pointed out by Kostant (and described in Pukanszky's paper), there exists a one-to-one correspondence between the set of all equivalence classes of finite dimensional irreducible representations of G and certain G-orbits in $\mathfrak{g}$ (assumed to be identified with its dual through the Killing form); the G-orbits in question here are those of the form $\Lambda + \rho$, Λ a dominant integral linear function on $\mathfrak{j}_c$; each such orbit admits a G-invariant measure—the K-measure of the orbit—of finite total volume V_Λ (say) (cf. Pukanszky [1, p. 256]); this being so, it can then be shown (cf. Pukanszky [1, p. 262]), for any dominant integral Λ, that

$$V_\Lambda = \mathrm{vol}\,(G) 2^r (r!) \prod_{\alpha>0} (\Lambda + \rho, \alpha).$$

The problem of explicitly computing vol (G) was, however, left open there. Using the above stated value for vol (G), we see that actually

$$V_\Lambda = (4\pi)^r r! [W]^{-1} \Bigl(\prod_{\alpha>0} (\Lambda + \rho, \alpha) / \prod_{\alpha>0} (\rho, \alpha)\Bigr) = (4\pi)^r r! [W]^{-1} d_\Lambda \,,$$

d_Λ the dimension of the irreducible representation determined by Λ (we have, of course, recalled Weyl's well-known formula for d_Λ). It is instructive to compare the above formula for V_Λ / d_Λ with the observations in Pukanszky [1, p. 264] on the nilpotent case.

The present method of calculating vol (G) depends upon some results of de Rham [1] and Schwartz [1] which deal with the fundamental solutions to certain partial differential equations; this technique has also been utilized by Harish–Chandra [2] in his study of the invariant integral on a non-compact semi-simple Lie algebra.

2. Preliminary Results.

(A) *Results of de Rahm.* Let E be an n-dimensional real vector space, Q a positive definite bilinear form on E; fix an orthonormal basis $\{X_i\}$ in E relative to the Euclidean norm on E determined by Q; let $x_1, \cdots, x_n$ denote the corresponding set of coordinates; put $dX = dx_1 \cdots dx_n$—thus dX is the usual canonical element of volume on E. Let n_1, n_2 be two non-negative integers such that $n_1 + n_2 = n \geqq 2$. Put

$$u = \sum_1^{n_1} x_i^2 - \sum_{n_1+1}^{n} x_i^2 \,, \qquad \Box = \sum_1^{n_1} (\partial/\partial x_i)^2 - \sum_{n_1+1}^{n} (\partial/\partial x_i)^2 \,.$$

De Rham [1] has shown that there exists a function ξ on $\mathbf{R}$ (the real numbers) with the following properties. Let $\Xi(X) = \xi(u(X)) (X \in E; X = \sum_{i=1}^n x_i X_i)$—then Ξ is a locally summable function on E and

$$\Box^{[n/2]} \cdot \Xi = \delta$$

in the sense of distributions. [Here δ denotes the Dirac measure concentrated at the origin while $[n/2]$ is the greatest integer in $n/2$.] It is actually possible to give explicit expressions for ξ in terms of the Heaviside function $t \mapsto Y(t)(Y(t)$ being 0 or 1 according as $t \leqq 0$ or $t > 0$, $t \in \mathbf{R})$. There are four cases:

(1) $\xi(t) = (-1)^{(n_1-1)/2} a_n^{-1} Y(t) \, |t|^{-1/2}$ if $n \equiv n_1 \equiv 1 \mod 2$;

(2) $\xi(t) = (-1)^{n_1/2} a_n^{-1} Y(-t) \, |t|^{-1/2}$ if $n \equiv n_2 \equiv 1 \mod 2$;

(3) $\xi(t) = -(-1)^{n_1/2} b_n^{-1} \log |t|$ if $n \equiv n_1 \equiv 0 \mod 2$;

(4) $\xi(t) = (-1)^{(n_1-1)/2} c_n^{-1} Y(t)$ if $n_1 \equiv n_2 \equiv 1 \mod 2$.

Here

$$a_n = 2^{n-1}\pi^{(n-1)/2}\Gamma((n-1)/2), \; b_n = \pi c_n = 2^n \pi^{n/2}\Gamma(n/2),$$

Γ the classical gamma function.

(B) *Results of Schwartz.* Retain the notations introduced in (A). Given $X \in E$, write $X = \sum_{i=1}^n x_i X_i$ and set $d(X) = (x_1^2 + \cdots + x_n^2)^{1/2}$; put $\Delta = \sum_{i=1}^n (\partial/\partial x_i)^2$—then Schwartz [1, p. 47] has established the following points. If n is odd

$$\Delta^{k+(n-1)/2} \cdot d^{2k-1} = c_{k,n}\, \delta \qquad (k \geqq 1)$$

and if n is even

$$\Delta^{k+n/2} \cdot (d^{2k} \log d) = c'_{k,n}\, \delta \qquad (k \geqq 0).$$

Here $c_{k,n}$ and $c'_{k,n}$ are non-zero numbers and the above relations are taken in the sense of the theory of distributions. The constants $c_{k,n}$, $c'_{k,n}$ are:

$$c_{k,n} = (2(k + (n-1)/2) - n)(2(k + (n-1)/2) - 2 - n) \cdots (4-n)(2-n) \cdot 2^{k+(n-1)/2-1}(k - 1 + (n-1)/2)! \, \frac{2(\sqrt{\pi})^n}{\Gamma(n/2)};$$

$$c'_{k,n} = [[(2(k + n/2) - n)(2(k + n/2) - 2 - n) \cdots (4-n)(2-n)]] \cdot 2^{k-1+n/2}(k - 1 + n/2)! \, \frac{2(\sqrt{\pi})^n}{\Gamma(n/2)}$$

where in the formula $[[\cdots]]$ the factor zero is omitted.

(C) *Results of Harish-Chandra.* (1) Let $S(\mathfrak{g}_c)$ denote the symmetric algebra over $\mathfrak{g}_c$, $I(\mathfrak{g}_c)$ the corresponding set of G-invariants in $S(\mathfrak{g}_c)$; we shall identify $S(\mathfrak{g}_c)$ with the algebra of polynomial functions on $\mathfrak{g}_c$ (via the identification of $\mathfrak{g}_c$ with its dual through the Killing form). Let us also bear in mind that the elements of $S(\mathfrak{g}_c)$ can be regarded, whenever it is convenient to do so, as differential operators on $\mathfrak{g}$; if $p \in S(\mathfrak{g}_c)$, then the corresponding differential operator on $\mathfrak{g}$ will be denoted by $\partial(p)$—the subalgebra $\mathfrak{P}(\mathfrak{g}_c)$ of the algebra of all differential operators on $\mathfrak{g}$ which is generated by $S(\mathfrak{g}_c) \cup \partial(S(\mathfrak{g}_c))$ is called the algebra of polynomial differential operators (on $\mathfrak{g}$). If f is a C^∞ function on $\mathfrak{g}$, D a differential operator on $\mathfrak{g}$, then, in what follows, we shall usually write $f(X; D)$ instead of $Df(X)(X \in \mathfrak{g})$. Similar definitions and notations apply to $\mathfrak{j}$.

(2) Given $f \in C_c^\infty(G)$, set

$$\phi_f(H) = \pi(H) \int_G f(xH)\, d_G(x) \qquad (H \in \mathfrak{j}).$$

It is easy to prove that $\phi_f \in C_c^\infty(\mathfrak{j})$; moreover it can be shown (cf. Harish–Chandra [1, p. 103]) that

$$\phi_{\partial(p)f} = \partial(\bar{p})\phi_f \qquad (\text{all} \quad p \in I(\mathfrak{g}_c)).$$

Here $\bar{p}$ denotes the restriction of the polynomial function p to $\mathfrak{j}_c$. Harish–Chandra [1, p. 104] has also established that

$$\phi_f(0; \partial(\pi)) = [W] \prod_{\alpha>0} (\rho, \alpha) f(0) \qquad (\rho = 2^{-1} \sum_{\alpha>0} \alpha).$$

(3) Let ω denote the Casimir polynomial of $\mathfrak{g}_c$, $\tilde{\omega}$ its restriction to $\mathfrak{j}_c$ (thus $\omega(X) = \operatorname{tr}(ad(X))^2$, $X \in \mathfrak{g}_c$); let R and L, respectively, denote the mappings $D \mapsto D \circ \partial(\tilde{\omega})$, $D \mapsto \partial(\tilde{\omega}) \circ D$ $(D \in \mathfrak{P}(\mathfrak{j}_c))$ of $\mathfrak{P}(\mathfrak{j}_c)$ into itself (the circle denoting composition of differential operators); put $\nabla = L - R$. Let r denote the number of positive roots of the pair $(\mathfrak{g}_c, \mathfrak{j}_c)$—then clearly the greatest integer in $n/2$, $[n/2]$, is $[l/2] + r$ (n the dimension of $\mathfrak{g}_c$, l the dimension of $\mathfrak{j}_c$). Harish–Chandra [2, p. 572] has shown that

$$\begin{aligned} \pi \circ \partial(\tilde{\omega}^{[n/2]}) &= (L - \nabla)^{[n/2]}\pi \\ &= \partial(\tilde{\omega}^{[l/2]}) \circ \eta \end{aligned}$$

where

$$\eta = \sum_{i=0}^{r} \binom{[l/2]+r}{i} (-1)^i L^{r-i} \nabla^i \pi,$$

the first term in the expression for η being the usual binomial coefficient. The local expression for η at the origin can be computed (cf. Harish–Chandra [2, p. 572])—the result is

$$\eta_0 = (-1)^r [l/2]([l/2] + 1) \cdots ([l/2] + r - 1) 2^r\, \partial(\pi).$$

[If D is any differential operator on $\mathfrak{j}$ (say), then the local expression of D at a point $H \in \mathfrak{j}$ is that unique element $p \in S(\mathfrak{j}_c)$ such that $f(H; D) = f(H; \partial(p))$ for all $f \in C^\infty(\mathfrak{j})\cdot$]

3. Computation of vol (G). We shall agree to retain the notations introduced in n^0's 1 and 2.

The computation of vol (G) will be divided into two separate cases, according to whether $l \geqq 2$ or $l = 1$.

The Case $l \geqq 2$. We shall employ the results of de Rham outlined in 2(A). Let ω denote the Casimir polynomial of $\mathfrak{g}_0$, $\tilde{\omega}$ the restriction of ω to $\mathfrak{j}_0$. Fix an orthonormal basis for $(\mathfrak{g}, \mathfrak{j})$ relative to $(-B)$ and let $x_1, \cdots, x_n$ denote the corresponding set of coordinates—then, in the notations of $2(A)$,

$$u = -\sum_1^n x_i^2 = \omega, \qquad \square = -\sum_1^n (\partial/\partial x_i)^2 = \partial(\omega), \qquad \Xi(X) = \xi(\omega(X))(X \in \mathfrak{g}).$$

[Hence we are taking $n_1 = 0$, $n_2 = n$—plainly $n \geqq 2$, $\mathfrak{g}$ being semi-simple.] Since $n \equiv l \bmod 2$, cases (2) and (3) of 2(A) are applicable simultaneously to both $\partial(\omega)$ and $\partial(\tilde{\omega})$ according to whether l is odd or even, respectively. Let $\tilde{\Xi}$ denote the function corresponding to the pair $(\mathfrak{j}, \partial(\tilde{\omega}))$—then we have

$$g(0) = \int_{\mathfrak{j}} \tilde{\Xi}(H) g(H; \partial(\tilde{\omega}^{[l/2]}))\, d_{\mathfrak{j}}(H)$$

for all $g \in C_0^\infty(\mathfrak{j})$. On the other hand, for $g \in C_0^\infty(\mathfrak{j})$,

$$\int_{\mathfrak{j}} \Xi(H) g(H; \partial(\tilde{\omega}^{[l/2]}))\, d\,(H) = \begin{cases} 2^{l-n}\pi^{(l-n)/2} \dfrac{\Gamma((l-1)/2)}{\Gamma(n-1)/2}\, g(0) & \text{if } l \text{ is odd;} \\[2ex] 2^{l-n}\pi^{(l-n)/2} \dfrac{\Gamma(l/2)}{\Gamma(n/2)}\, g(0) & \text{if } l \text{ is even.} \end{cases}$$

Thus, for example, consider the case when l is odd—then

$$\begin{aligned} g(0) &= \int_{\mathfrak{j}} a_l^{-1} Y(-\tilde{\omega}(H))\, |\tilde{\omega}(H)|^{-1/2}\, g(H; \partial(\tilde{\omega}^{[l/2]}))\, d_{\mathfrak{j}}(H) \\ &= a_l^{-1}/a_n^{-1} \int_{\mathfrak{j}} a_n^{-1} Y(-\tilde{\omega}(H))\, |\tilde{\omega}(H)|^{-1/2}\, g(H; \partial(\tilde{\omega}^{[l/2]}))\, d_{\mathfrak{j}}(H) \\ &= a_l^{-1}/a_n^{-1} \int_{\mathfrak{j}} \Xi(H) g(H; \partial(\tilde{\omega}^{[l/2]}))\, d\,(H). \end{aligned}$$

Consequently

$$\int_{\mathfrak{j}} \Xi(H) g(H; \partial(\tilde{\omega}^{[l/2]}))\, d_{\mathfrak{j}}(H) = \frac{a_l}{a_n}\, g(0) = 2^{l-n}\pi^{(l-n)/2} \frac{\Gamma((l-1)/2)}{\Gamma((n-1)/2)}\, g(0).$$

The case of even l is treated in a similar way. Now choose an $f \in C_0^\infty(\mathfrak{g})$ such that $f(0) = 1$—then, as has been noted above, $\phi_f \in C_0^\infty(\mathfrak{j})$. Therefore

$$\begin{aligned} 1 = f(0) &= \int_{\mathfrak{g}} \Xi(X) \partial(\omega^{[n/2]}) f(X)\, d_{\mathfrak{g}}(X) \\ &= \operatorname{vol}(G)\, (-1)^r \int_{\mathfrak{j}} \Xi(H) \pi(H) \phi_{\partial(\omega^{[n/2]})f}(H)\, d_{\mathfrak{j}}(H) \\ &= \operatorname{vol}(G)\, (-1)^r \int_{\mathfrak{j}} \Xi(H) \pi(H) \phi_f(H; \partial(\tilde{\omega}^{[n/2]}))\, d_{\mathfrak{j}}(H) \qquad (\text{cf. } 2(C) - (2)) \end{aligned}$$

$$= \mathrm{vol}\,(G)\,(-1)^r \int_{\mathfrak{j}} \Xi(H)\phi_f(H; \pi \circ \partial(\tilde{\omega}^{[n/2]}))\, d_{\mathfrak{j}}(H)$$

$$= \mathrm{vol}\,(G)\,(-1)^r \int_{\mathfrak{j}} \Xi(H)\phi_f(H; \partial(\tilde{\omega}^{[l/2]}) \circ \eta)\, d_{\mathfrak{j}}(H)$$

where the η in the last expression is as in 2(C)–(3). We recall that the local expression for η at zero, η_0, is:

$$\eta_0 = (-1)^r \prod_{i=0}^{r-1} ([l/2] + i) 2^r\, \partial(\pi).$$

Hence

$$1 = \mathrm{vol}\,(G)\,(-1)^r \int_{\mathfrak{j}} \Xi(H)\phi_f(H; \partial(\tilde{\omega}^{[l/2]}) \circ \eta)\, d_{\mathfrak{j}}(H)$$

$$= \mathrm{vol}\,(G) \begin{cases} (-1)^r 2^{l-n} \pi^{(l-n)/2} \dfrac{\Gamma((l-1)/2)}{\Gamma((n-1)/2)} \phi_f(0; \eta) & \text{if } l \text{ is odd;} \\ (-1)^r 2^{l-n} \pi^{(l-n)/2} \dfrac{\Gamma(l/2)}{\Gamma(n/2)} \phi_f(0; \eta) & \text{if } l \text{ is even,} \end{cases}$$

and so, keeping in mind that $\phi_f(0; \partial(\pi)) = [W] \prod_{\alpha>0} (\rho, \alpha)$ (cf. 2(C)–(2)), we find that

$$1 = \begin{cases} \left\{[W] \prod_{\alpha>0} (\rho, \alpha) \prod_{i=0}^{r-1} ([l/2] + i) 2^{r+l-n} \pi^{(l-n)/2} \dfrac{\Gamma((l-1)/2)}{\Gamma((n-1)/2)}\right\} \mathrm{vol}\,(G) \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } l \text{ is odd;} \\ \left\{[W] \prod_{\alpha>0} (\rho, \alpha) \prod_{i=0}^{r-1} ([l/2] + i) 2^{r+l-n} \pi^{(l-n)/2} \dfrac{\Gamma(l/2)}{\Gamma(n/2)}\right\} \mathrm{vol}\,(G) \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } l \text{ is even.} \end{cases}$$

Let $z \in \mathbf{C}$; if z is neither zero nor a negative integer, then, as is well known,

$$\prod_{i=0}^{r-1} (z + i) = \frac{\Gamma(z + r)}{\Gamma(z)} \qquad (r \geqq 1).$$

Using this relation in the last expression for 1 above, we find that

$$\mathrm{vol}\,(G) = (2\pi)^r / ([W] \prod_{\alpha>0} (\rho, \alpha))$$

whether l be odd or even. Therefore the claimed value for vol (G) has been established in the case $l \geqq 2$.

(ii) The Case $l = 1$. In this situation $\mathfrak{g}$ is isomorphic to the compact real form for $sl(2, \mathbf{C})$—hence $n = 3$. Let us use 2(B) with $n = 3$, $k = 1$—thus

$$c_{1,3} f(0) = \int_{\mathfrak{g}} d(X) \Delta^2 f(X)\, d_{\mathfrak{g}}(X) \qquad (f \in C_c^\infty(\mathfrak{g})).$$

Here $\Delta = -\partial(\omega)$, $d(X) = (-\omega(X))^{1/2}(X \in \mathfrak{g})$. Let $\tilde{d}$ denote the distance function on $\mathfrak{j}$. Choose $f \in C_c^\infty(\mathfrak{g})$ such that $f(0) = 1$—then

$$\begin{aligned} c_{1,3} &= \int_{\mathfrak{g}} d(X)\Delta^2 f(X)\, d_{\mathfrak{g}}(X) \\ &= \operatorname{vol}(G)(-1) \int_{\mathfrak{j}} \tilde{d}(H)\pi(H)\phi_{\partial(\omega^2)f}(H)\, d_{\mathfrak{j}}(H) \\ &= \operatorname{vol}(G)(-1) \int_{\mathfrak{j}} \tilde{d}(H)\phi_f(H; \pi \circ \partial(\tilde{\omega}^2))\, d_{\mathfrak{j}}(H). \end{aligned}$$

Employing the notations introduced in 2(C)–(3), note that $\pi \circ \partial(\tilde{\omega}^2) = R^2\pi = (L - \nabla)^2\pi = (L^2 - 2L\nabla + \nabla^2)\pi = (L^2 - 2L\nabla)\pi = L(L - 2\nabla)\pi = \partial(\tilde{\omega}) \circ \mu$ where $\mu = (L - 2\nabla)\pi$. Now $L = R + \nabla$—thus $\mu = (R - \nabla)\pi$ and so $\mu_0 = -(\nabla\pi)_0 = -2\,\partial(\pi)$. Therefore

$$\begin{aligned} c_{1,3} &= \operatorname{vol}(G)(-1) \int_{\mathfrak{j}} \tilde{d}(H)\phi_f(H; \partial(\tilde{\omega}) \circ \mu)\, d_{\mathfrak{j}}(H) \\ &= \operatorname{vol}(G)(-1)^2 c_{1,1}\phi_f(0; \mu) \qquad (\text{cf. } 2(\text{B})) \\ &= \operatorname{vol}(G)(-1)^2 c_{1,1}(-2)\phi_f(0; \partial(\pi)) \\ &= (-2)[W](\rho, \alpha)c_{1,1} \operatorname{vol}(G). \end{aligned}$$

Consequently

$$\operatorname{vol}(G) = c_{1,3}/(-2)[W](\rho, \alpha)c_{1,1}\,.$$

The values of $c_{1,3}$ and $c_{1,1}$ are known—see 2(B) above. Moreover in the case at hand, $[W] = 2$, $(\rho, \alpha) = 2^{-1}(\alpha, \alpha) = 1/4$. Bearing in mind that $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(3/2) = \sqrt{\pi}/2$, we get

$$\operatorname{vol}(G) = \frac{(-4)2(\sqrt{\pi})^3\Gamma(1/2)}{(-2)2(1/4)4\sqrt{\pi}\Gamma(3/2)} = 4\pi,$$

as desired.

The computation of vol (G) is now complete.

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